THE OBSCURE PRECESSION OF MERCURY'S PERIHELION

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Abstract.

The Sun's motion around the Solar System barycentre produces a small quadrupole moment in the gravitational energy of Mercury. This moment has until now gone undiscovered, but it actually generates 7arcsec/cy precession of Mercury's perihelion. Consequently, the residual 43arcsec/cy previously attributed to general relativity theory must account for this new component and only 36arcsec/cy for GR.

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1. Introduction

The orbit of planet Mercury has been calculated by several investigators; see Clemence (1947), Brouwer & Clemence (1961), and review in Pireaux & Rozelot (2003). In their calculations, the inverse square law has been applied to set up the differential equations of motion using the measured distances and velocities between Mercury, the Sun and planets. Then the observed precession of the perihelion of Mercury was explained as being due to general precession in longitude, perturbation by the planets, solar oblateness, and 43arcsec/cy for general relativity.

In this paper, an obscure contribution to Mercury's orbital precession has been identified due to the Sun moving around the barycentre, producing a small quadrupole moment in the energy of Mercury. This can be understood in theory by first imagining the Sun rapidly orbiting the barycentre so that it takes on the blurred appearance of a toroidal or oblate Sun. Then Mercury would orbit the average position of the Sun at the barycentre, and describe a *precessing* elliptical orbit. Now, it is proposed that for the existing Solar velocity there will be a residual component of this effect, such that Mercury is currently orbiting a centre of mass position, moving with the Sun, located a small distance towards the barycentre. This corresponds to a tiny variation in Newton's law, which causes Mercury to describe a precessing orbit.

As an aid to understanding, consider an analogous system of a pendulum bob suspended from a slowly *moving* pivot, instead of the usual fixed pivot. The pivot would drag the bob and it would not be able to describe the usual simple harmonic motion. Likewise, the Sun drags Mercury around with it, and Mercuty's binding energy is different *in form* from that for a stationary Sun. At present, received wisdom does not acknowledge that such an accelerating Sun affects the potential of Mercury.

An exaggerated example of this proposed phenomenon will now be derived, followed by a transformation to the real Solar System.

2. Theoretical precession due to a rapidly moving Sun

The absolute binding energy of Mercury, in the field of the Sun orbiting around the barycentre, may be calculated by using Newton's law. First, consider the *hypothetical system* shown in Figure 1. Let Mercury (mass M_1) be regarded as *stationary* at distance (r_{1C} = 57.9x10^{6}km) from the origin at barycentre C, while the Sun (mass M) travels *rapidly* around C at radius (r_{SC} = 7.43x10^{5}km). Then, for the Sun at instantaneous distance r_1 from Mercury we can write:

$$r_1^{\ 2} = r_{1C}^{\ 2} + r_{SC}^{\ 2} - 2 r_{1C} r_{SC} \cos\theta \qquad . \qquad (1a)$$

The *instantaneous* gravitational force exerted by the Sun on Mercury is given by the inverse square law, ($F_1 = -GMM_1 / r_1^2$), and the force directed towards barycentre C is $F_1\cos\alpha$, where $\cos\alpha$ is given by:

$$r_{SC}^{\ 2} = r_{1C}^{\ 2} + r_1^{\ 2} - 2 r_{1C} r_1 \cos\alpha \ . \qquad (1b)$$

Upon eliminating $\cos\alpha$, this force towards C is:

$$F = F_1 \cos\alpha = -\left(\frac{GMM_1}{r_1^{\ 2}}\right)\left(\frac{r_{1C} - r_{SC}\cos\theta}{r_1}\right) . \qquad (2a)$$

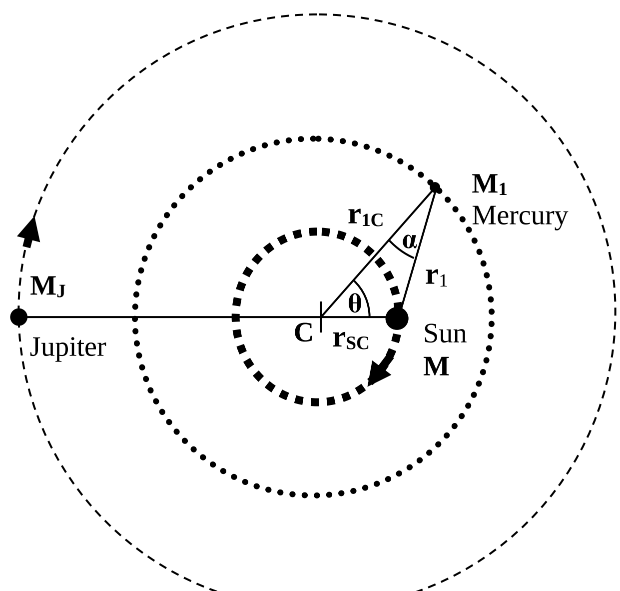

Fig. 1 Schematic diagram showing Jupiter and the Sun moving around their barycentre C. Mercury is here considered to be stationary during one orbit of the Sun.

Now eliminate variable r_1 and get all the $\cos\theta$ terms in the numerator:

$$F \approx -\left(\frac{GMM_1}{r_{1C}^{\ 2}}\right)\left(1-\frac{3}{2}\frac{r_{SC}^{\ 2}}{r_{1C}^{\ 2}}+\frac{2r_{SC}}{r_{1C}}\cos\theta+\frac{9}{2}\frac{r_{SC}^{\ 2}}{r_{1C}^{\ 2}}\cos^2\theta\right). \quad (2b)$$

After averaging θ over a complete orbit of the Sun, the *average* force towards C becomes:

$$\tilde{F} \approx -\left(\frac{GMM_1}{r_{1C}^{\ 2}}\right)\left[1+\frac{3}{4}\left(\frac{r_{SC}^{\ 2}}{r_{1C}^{\ 2}}\right)\right] . \quad (2c)$$

This is the force that would govern the orbit of inertial Mercury, rather than an instantaneous inverse square law. It is slightly stronger than an inverse square law for a stationary Sun located at C. By integrating from r_{1C} to infinity, the absolute potential energy of Mercury in this system would be:

$$PE \approx -\left(\frac{GMM_1}{r_{1C}}\right)\left[1+\frac{1}{4}\left(\frac{r_{SC}^{\ 2}}{r_{1C}^{\ 2}}\right)\right] . \quad (3)$$

These two expressions would apply to the equation of motion for Mercury *around C*, with angular momentum being greater than that around a fixed Sun at C. Inertial Mercury focuses on the centre of gravity at C, and the quadrupole moment would produce precession, similar to an oblate Sun.

3. Actual precession due to the real moving Sun

These expressions for a rapidly moving Sun will now be adjusted to cover a slowly moving Sun, in order to reveal the final residual quadrupole moment. Mercury is still chasing an elusive accelerating Sun, so the quadrupole moment cannot be eliminated. The Sun orbits the barycentre at $7.43\text{x}10^5$km radius over 11.86 years due to Jupiter. Its slow motion around C allows Mercury time to respond to the Sun's wobble such that the effective centre of gravity, where inertial Mercury focuses, lies between the Sun's centre and C. Radius r_{SC} in Eq.(2c) has to be replaced by a derived smaller value r'_{SC} in order to give the actual average acceleration of Mercury around this effective centre of gravity:

$$\tilde{a} \approx -\frac{GM}{r_{1C}{}^2}\left[1+\frac{3}{4}\left(\frac{r'_{SC}}{r_{1C}}\right)^2\right] \quad . \tag{4}$$

After substituting ($u = 1/r_{1C}$), plus Mercury's specific angular momentum [$h \approx (GMr_{1C})^{1/2}$], then orbit theory yields a differential equation for the trajectory:

$$\frac{d^2u}{d\phi^2}+u=\frac{-\tilde{a}}{h^2u^2}=\frac{GM}{h^2}+\left(\frac{GM}{h^2}\frac{3}{4}(r'_{SC})^2\right)u^2 \quad . \tag{5}$$

This type of equation has previously been solved because general relativity theory gives a similar expression for the trajectory of Mercury, (see Rindler, 2001):

$$\frac{d^2u}{d\phi^2}+u=\frac{GM}{h^2}+\left(\frac{3GM}{c^2}\right)u^2 \quad , \tag{6}$$

where the final term accounts for 43arcsec/cy precession of Mercury's orbit. Hence, by direct comparison, we can calculate the precession to expect from the quadrupole moment in Eq.(5):

$$\delta\omega \approx \left(\frac{c}{2h}r'_{SC}\right)^2 \times 43\text{arc sec/ cy} \quad . \tag{7}$$

Here, the effect of orbit eccentricity is included in the 43arcsec/cy term.

The value of r'_{SC} will be found by considering Figure 2, wherein Mercury is now depicted orbiting the Sun with period T_1 while the Sun is moving *slowly* around barycentre C with period ($T_{SC} = 49.2T_1$). Inertial Mercury will actually be focussed on a moving centre of gravity P (with period T_{PC}), at radius r_{PC} from C towards the Sun and distance r'_{SC} from the Sun. Logically, during a complete orbit of Mercury, the Sun moves arc distance ($2\pi r_{SC}/49.2$); and Mercury tries continuously to compensate for the Sun's movement. So we will expect r'_{SC} to be around $r_{SC}/(2 \times 49.2)$. An accurate radius will be derived using an action principle involving Mercury as it *orbits around* P with period T_1. By defining *action* as (Kinetic Energy x Time), let it be a conserved quantity if the centre of gravity is related to the real Solar mass M by:

$$\left(\tfrac{1}{2}Mv_{PC}{}^2\right)\times\left(T_{PC}+T_1\right)=\left(\tfrac{1}{2}Mv_{SC}{}^2\right)\times T_{SC} \quad . \tag{8}$$

On the left, the lower effective KE of the centre of gravity mass at P is compensated by an extension in time of one extra period of Mercury. Here, period ($T_{PC} = T_{SC}$), therefore ($r_{SC} / v_{SC} = r_{PC} / v_{PC}$), and simplification gives:

$$r_{PC} = r_{SC}\left(\frac{T_{SC}}{T_{SC} + T_1}\right)^{1/2} \quad . \qquad (9)$$

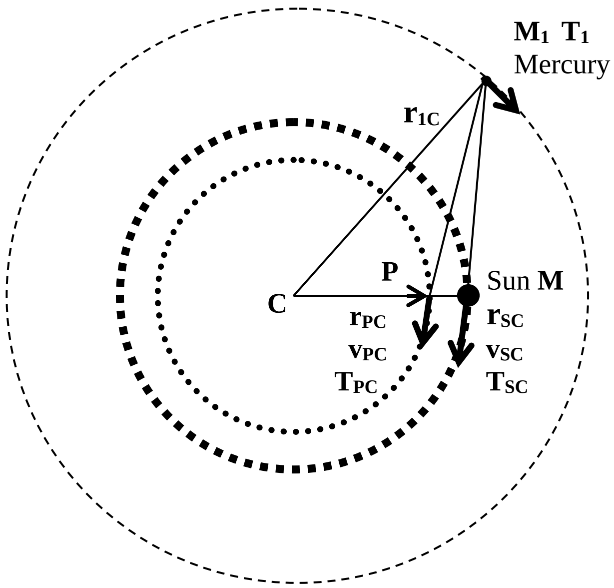

Fig. 2 Schematic diagram showing the Sun moving slowly around the barycentre at C. Mercury is considered to be orbiting the effective centre of gravity at P, between C and the Sun.

Consequently, the position of P is determined by Mercury's period being synchronised with a harmonic of the Sun's movement around the barycentre. From the viewpoint of an observer on Mercury, after 49.2 orbits, the Sun has moved in a circle around P of radius r'_{SC} given by:

$$r'_{SC} = r_{SC} - r_{PC} = r_{SC}\left[1 - \left(\frac{T_{SC}}{T_{SC} + T_1}\right)^{1/2}\right] \quad . \qquad (10)$$

It is this circular motion of the Sun *around* P which will now determine the quadrupole moment operating on Mercury, by introducing r'_{SC} into Eq.(7). Inspection of this expression confirms that for any arbitrary (T_1/T_{SC}), the position of P can vary as one would expect, from C to the Sun's centre. Thus, for (T_1 = 88.0days), (T_{SC} = 4331days) and (r_{SC} = 7.43x10^5km), it evaluates to (r'_{SC} = 7433km). Substitution in Eq.(7), with (h = 2.76x10^{15}m^{2}s^{-1}), yields the residual precession due to the quadrupole moment:

$$\delta\omega \approx 0.162 \times 43 \approx 7.0 \mathrm{arc\,sec/\,cy} \quad . \qquad (11)$$

Therefore, the well-known 43arcsec/cy residual precession, previously attributed to GR, must be reduced by 7arcsec/cy to 36arcsec/cy for GR.

The absolute potential energy of Mercury, in this realistic Solar System, is of the same form as Eq.(3):

$$PE_{av} \approx -\left(\frac{GMM_1}{r_{1C}}\right)\left[1 + \frac{1}{4}\left(\frac{r_{SC}'^{\,2}}{r_{1C}^{\,2}}\right)\right] . \qquad (12)$$

Precession due to other planets increasing the Sun's wobble is variable because together they cause great fluctuation in r_{SC} , with a long-term average at around 8.4x10^5km (Landscheidt, 2007).

Precessions currently attributed to general relativity in the orbits of Venus, Earth and Icarus, will also be affected by the Sun's movement, (Shapiro et al (1968), Lieske & Null (1969), Sitarski (1992)).

4. Conclusion

Motion of the Sun around the Solar System barycentre produces a small quadrupole moment in the average gravitational energy of Mercury. The effect of this is to generate 7arcsec/cy precession in Mercury's orbit, just as an oblate Sun would. This has not been included previously, so only 36arcsec/cy precession due to general relativity theory is now required for a fit to the observations. Unfortunately, received wisdom does not include the possibility of an accelerating Sun affecting the potential of Mercury. And complacency is currently impeding progress because investigators believe this theory does not apply when using heliocentric coordinates.

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