On Milgrom's Law

F. Winterberg

Department of Physics, University of Nevada, Reno, NV, USA

E-mail:
winterbe@unr.edu

Abstract

To explain Milgrom's law (MOND) and the accelerated expansion of the universe, in both instances involving an acceleration of the same order of magnitude, it is assumed that the vacuum of space in a kind of plasma made up of an equal number of positive and negative mass Planck mass particles, gravitationally interacting with each other. This hypothesis leads to positive mass Dirac spinors, as composed particles, made up of positive and negative quasiparticles of this plasma. The formation of the positive mass Dirac spinors is accompanied by the emission of negative masses. The gravitationally self interacting positive mass spinors lead to the formation of a filamentary vortex tangle connecting neighboring galaxies with the galaxies formed by the Jeans instability of a cylindrical mass cylinder. This hypothesis not only explains the filamentary structure of the metagalaxy, but also Milgrom's law by Newton's gravitational force of a mass cylinder. The negative masses released during the formation of the positive mass Dirac spinors fill the space in between the positive mass filaments, explaining the observed voids.

PACS numbers: 04.50, 12.10G, 04.60, 95.85S

1. Introduction

Milgrom's law (Modified Newtonian Dynamics, or MOND) [1], states that Newton's $1/r^2$ force law for a star in circular motion around the center of its galaxy, fails outside a critical radius where the acceleration would become less than $\approx 10^{-8}$ cm/s^2, from where on the force decreases only in proportion to 1/r. For a spherical mass as is the core of a galaxy this result is in gross violation of Gauss's law. It would, however, not be a violation of this law if the spherical mass is replaced by a cylindrical mass. In the context of Milgrom's law this can only mean that the galaxy us part if an invisible, much larger cylindrical mass. It would be visible if it would be composed of a light emitting plasma. For this reason it is most likely made up of the non-baryonic cold dark matter. In such a configuration there would be bridges connecting the galaxies should be composed of vortices held together by their own gravitational force.

2. Negative Masses in Einstein's Gravitational Field Equation

As shown by Hund [2], Einstein's gravitational field equations lead to the existence of negative masses. For the proof it is sufficient to consider the gravitational field outside a spherical symmetric mass distribution, described by Schwarzschild's s solution. With the line element in spherical coordinates

$$ds^2 = f^2c^2dt^2 - h^2dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2) \tag{1}$$

expressing the components of the metric tensor g_{ik} in space-time by two functions h(r) and f(r), and inserting these components of the metric tensor into Einstein's vacuum field equation

$$R_{ik} = 0 \tag{2}$$

one obtains

$$\left.\begin{aligned} (hf)' &= 0 \\ h\left(f'' + \frac{2}{r}f'\right) - h'f' &= 0 \end{aligned}\right\} \tag{3}$$

For the gravitational field F, if measured in eigen-time fdt, and eigen-length hdr, one obtains for the acceleration, and hence the radial force F:

$$F = \frac{d}{fdt}\left(\frac{h}{f}\frac{dr}{dt}\right) = -\frac{c^2f'}{hf} \tag{4}$$

With F given by (4) one can write for the second equation (3):

$$\frac{1}{hr^2}(r^2F)' - \frac{F^2}{c^2} = 0 \tag{5}$$

The first term is identical to the definition of the divergence of a radial vector $\mathbf{F} = F\mathbf{r}/r$, which is the increase $d(4\pi r^2F)$ of the flux of $\mathbf{F}$ through a spherical surface of radius r, divided by the increase in the volume of this sphere $4\pi r^2hdr$. One therefore can write for (5)

$$div\mathbf{F} - \frac{1}{c^2}\mathbf{F}^2 = 0 \tag{6}$$

Comparing this result with Newton's law of gravity where (G is Newton's constant, ρ density)

$$div\mathbf{F} = -4\pi G\rho \tag{7}$$

one can see that the gravitational field **F** has a negative mass density

$$\rho_g = -\frac{F^2}{4\pi Gc^2} \tag{8}$$

One can test this result for

$$F = -\frac{GM}{r^2} \tag{9}$$

the gravitational field of a spherical mass of radius R, for r > R. One there finds

$$\rho_g = -\frac{GM}{4\pi c^2 r^4} \tag{10}$$

To obtain the total amount of negative mass M_g outside the mass M, we integrate and obtain

$$M_g = \int_R^\infty \rho_g 4\pi r^2 dr = -\frac{GM^2}{c^2 R} \tag{11}$$

or

$$M_g c^2 = -\frac{GM^2}{R} = E_{pot} \tag{12}$$

where E_{pot} is the negative gravitational potential energy of a spherical shell of radius R and mass M. This example shows, that to obtain the gravitational field mass M_g, one simply has to equate the gravitational potential energy with $M_g c^2$.

Next we analyze the two body problem treated by Bondi [3]. The Newtonian gravitational potential energy for the two bodies of mass m_1 and m_2, and separated by the distance r, is

$$E_{pot} = -\frac{Gm_1 m_2}{r} \tag{13}$$

It would become positive if one of the masses is negative.

Lengthy calculations with Einstein's nonlinear gravitational field theory, including quantum field theoretical corrections, give for the potential energy of two masses [4]

$$E_{pot} = -\frac{Gm_1 m_2}{r}\left(1 + 3\frac{G(m_1 + m_2)}{2rc^2} + \frac{41}{10\pi}\frac{r_p{}^2}{r^2}\right) \tag{14}$$

where $r_p \approx 10^{-33}$cm is the Planck length. The last term in the bracket on the r.h.s. of (14) involves quantum mechanical corrections and is small for r >> r_p.

It is remarkable that for $m_2 = -m_1$ the second term vanishes, and one obtains there for the gravitational field energy ($m_1=|m_2|=m$)

$$E_{pot} = m_g c^2 = \frac{G\,|m|^2}{r}\left(1+\frac{41}{10\pi}\frac{r_p^{\;2}}{r^2}\right) \tag{15}$$

$$\approx \frac{Gm^2}{r}, \quad r >> r_p \tag{16}$$

Therefore, to complete Bondi's calculation, one may simply add m_g to the positive mass of the positive mass of his positive-negative two body problem.

3. Planck's Doctrine

In a farsighted paper published in 1899, Planck expresses the view that the most fundamental equations in physics should contain only three parameters: Newton's constant G, the velocity of light c, and Planck's constant h [5]. According to this doctrine, Einstein's gravitational field equation cannot be fundamental because it is derived from Einstein-Hilbert Lagrangian.

$$\mathscr{L} = \frac{c^3}{16\pi G}\sqrt{-g}\,R \tag{17}$$

It appears the most obvious way to introduce *h*, is by a modification of the Einstein-Hilbert Lagrangian, which does not contain Planck's constant. A way how this can be done is suggested by drawing an analogy drawn between the ideal gas equation and the Van der Waals equation. In the ideal gas equation the pressure diverges if the density becomes infinite. In the Van der Waals equation the pressure diverges when the density reaches a maximum $\rho = \rho_0$. This is done by making in the ideal gas equation the substitution:

$$\frac{p}{\rho} \to \frac{p}{\rho\left(1-\rho/\rho_0\right)} \tag{18}$$

Making the analogy with the Van der Waals equation therefore suggests to replace (17) by [6]

$$\mathscr{L} = \frac{c^3}{16\pi G}\frac{\sqrt{-g}\,R}{1-r_p^{\;2}R} \tag{19}$$

There then, the curvature invariant is limited by $R \leq 1/r_p^2$. The inclusion of negative masses would rather suggest that

$$\mathscr{L} = \frac{c^3}{16\pi G}\frac{\sqrt{-g}\,R}{\left(1-r_p^{\;2}R\right)\left(1+r_p^{\;2}R\right)} \tag{20}$$

such that $R \leq \pm 1/r_p^2$, with $\mathscr{L} \to \infty$ for $R = \pm 1/r_p^2$. In the weak field limit $R \approx -(1/c^2)\nabla^2\Phi$, where Φ is the gravitational potential, one therefore has

$$\nabla^2\Phi = \mp c^2 / r_p^2 \tag{21}$$

Comparing this with

$$\nabla^2\Phi = \pm 4\pi G\rho \tag{22}$$

and setting $\rho = \pm m_p/r_p^3$, one obtains (21) and

$$\Phi = \mp\frac{Gm_p}{r_p}, \quad r = r_p \tag{23}$$

This means the vacuum for the Lagrangian (20) can be a medium made up of positive and negative Planck masses. One may call this medium a Planck mass plasma [7, 8, 9], or alternatively a Planck aether. As it was shown by Redington [10], its metric can be described by a "Literal Rippling of Spacetime."

3. Negative Masses in Dirac's Equation

While Einstein's gravitational field equation cannot be final, because it does not contain Planck's constant h, the Dirac's equation can likewise not be final since it does not contain G. Following Schrödinger's Zitterbewegung analysis [11, 12] and the work by Hönl and Papapetrou [13, 14, 15], one can consider a Dirac particle to be made up of a positive and a negative mass, with the positive mass larger than the absolute value of the negative mass. It is therefore possible to introduce the gravitational constant into Dirac's equation, by assuming that the surplus in positive mass over the absolute value of the negative mass comes from the positive mass of the gravitational field energy of a positive mass gravitationally interacting with a negative mass [7, 8, 9]. If the positive gravitational field mass is added to the positive mass of the mass dipole, one obtains a pole-dipole mass configuration from which one can derive the Dirac equation. It is the small residual mass of the gravitational field which is the mass of a Dirac particle.

While without the mass of the gravitational field a mass dipole would lead to the self-acceleration, a pole-dipole configuration leads to a helical motion, along the helix reaching the velocity of light. It is from this configuration that one can derive the Dirac equation [13, 14, 15]. We therefore call this configuration a spinor roton, and suggest that the non-baryonic cold dark matter is made up of spinor rotons.

The much lighter elementary particles of the standard model are in a likewise way made up from much smaller pole-dipole configurations of quantized lower energy vortex configurations of the Planck mass plasma [7, 8, 9].

4. Planck Mass Plasma Hypothesis

With the Planck length $r_p = \sqrt{\hbar G/c^3}$ and the Planck force $F_p = c^4/G$, the Planck mass plasma hypothesis makes the following assumptions [7, 8, 9]:

1. The ultimate building blocks of matter are Planck mass particles which obey the laws of classical Newtonian mechanics, but there are also negative Planck mass particles.
2. A positive Planck mass particle exerts a short range repulsive and a negative Planck mass particle a likewise attractive force, with the magnitude and range of the force equal to the Planck force F_p and the Planck length r_p.

3. Space is filled with an equal number of positive and negative Planck mass particles, with each Planck length volume in the average occupied by one Planck mass particle.

This hypothesis makes the following predictions:

1. Nonrelativistic quantum mechanics as an approximation with departures from the approximation suppressed by the Planck length.
2. Lorentz invariance as a dynamic symmetry for energies small compared to the Planck energy.
3. A spectrum of quasiparticles very much like the particles of the standard model, and beyond the standard model also phonons and rotons.

5. Phonons and Rotons of the Planck Mass Plasma

In the ground state of the Planck mass plasma, made up of positive and negative mass Planck masses, each mass component possesses a phonon-roton spectrum [6]. Assuming that the shape of the phonon-roton spectrum obtained by Henshaw and Woods [16] is universal, the phonon-roton spectrum of the Planck mass plasma is obtained by replacing the Debye-length with the Planck length, or the Debye energy with the Planck energy (see figure 1).

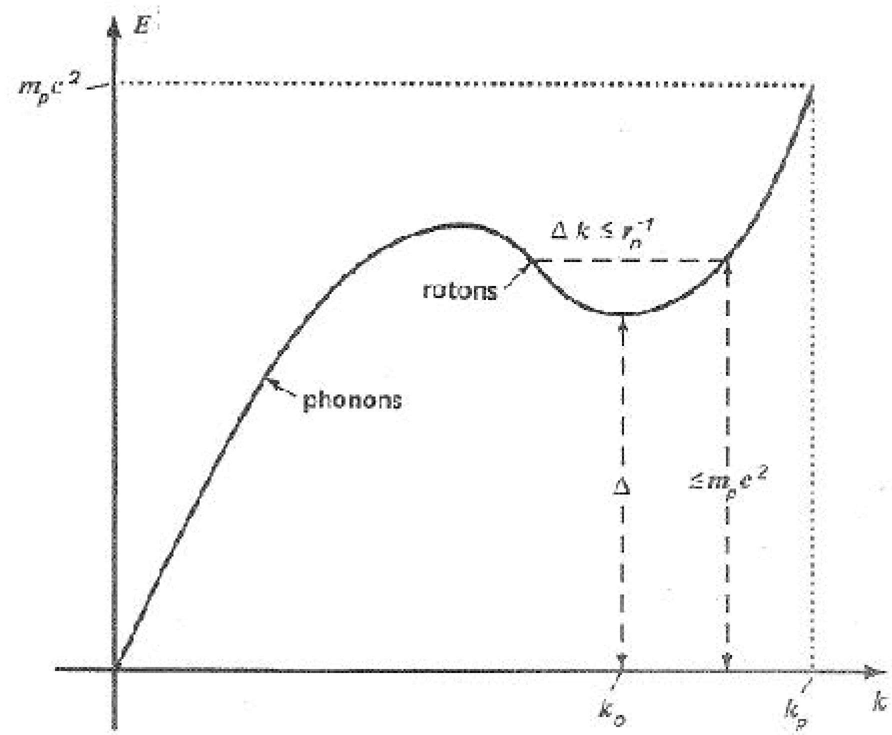

Figure 1: Phonon-roton distribution within superfluid helium

6. Spinor Rotons

From the experimentally established phonon-roton spectrum in superfluid helium one obtains for the roton energy a value about 0.16 times the Debye energy. Replacing the Debye energy with the Planck energy $m_p c^2$, where m_p is the Planck mass, the mass of the rotons in the Planck mass plasma should be $m_r \approx 0.16 m_p$.

We now consider the gravitational interaction of a positive mass roton with a negative mass roton, separated by the distance r. For this (positive-negative) mass dipole the energy of the gravitational field is positive and with $m_r^- = -m_r^+$ given by

$$E = -\frac{Gm_r^- m_r^+}{r} = \frac{G\left|m_r^\pm\right|}{r} \tag{24}$$

According to the mass-energy equivalence, this field has the mass

$$m = \frac{E}{c^2} = \frac{G\left|m_r^\pm\right|}{c^2 r} \tag{25}$$

A second equation is given by the uncertainty relation[1]

$$\left|m_r^\pm\right| rc \approx h \tag{26}$$

Eliminating r from (25) and (26) one obtains

$$m = \frac{G\left|m_r^\pm\right|^3}{\hbar c} \tag{27}$$

or because of $\mathrm{Gm_p^2} = \hbar \mathrm{c}$,

$$\frac{m}{m_p} = \left(\frac{\left|m_r^\pm\right|}{m_p}\right)^3 \tag{28}$$

With $m_r^\pm \approx 0.16 m_p$, one finds that $m / m_p \approx 4\times 10^{-3}$, or $mc^2 \approx 4\times 10^{16} GeV$. For r one finds that $r / r_p = (m_p / m)^{1/3} \approx 2.5$. According to (15) this result is correct within 21%.

For the other, much smaller mass quasiparticles of the Planck mass plasma, arising from the quantized resonance energy of the small ring vortices, the quantum correction to the gravitational interaction potential (15) can be neglected. This situation somehow resembles the situation in Bohr's model of the hydrogen atom, where it is sufficient to take the classical Coulomb potential to compute the electron orbits, without waiting for the quantum electrodynamic corrections of the Coulomb potential.

4. The Structure of the Metagalaxy and Milgrom's Law

The metagalaxy shows a filamentary spider-net structure in which the galaxies are arranged along invisible lines, for which the Planck (positive-negative) mass plasma can give a plausible explanation. It leads to the gravitationally interacting bound positive mass spinor rotons, forming by their mutual gravitational attraction large linear vortex filaments. A multitude of such vortex filaments is called a vortex sponge. If these vortex filaments are filling all of space, they can explain the filamentary structure of the metagalaxy, with the light emitting visible matter coming from the much smaller quasiparticles of the Planck mass plasma [7, 8, 9].

In shedding thermal energy by the emission of light, it is then plausible that by the Jeans instability, visible matter forms galaxies, situated inside these large intergalactic vortices, with the total mass of the galaxies smaller than the total mass of the invisible mass made up by the spinor rotons (see figure 2). It is conjectured that the mass of the spinor rotons makes up the non-baryonic cold dark matter, with the energy gap in the phonon-roton spectrum explaining the negative pressure of the dark energy [9].

---

[1] A more correct quantum mechanical treatment leads to $2\left|m_r^\pm\right| rc = \hbar$ [7, 8, 9].

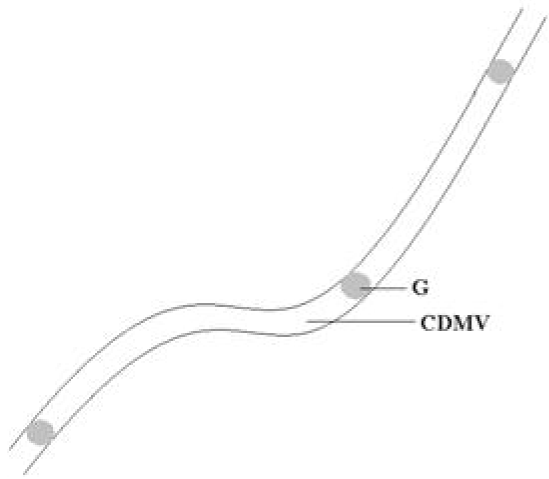

Figure 2: G galaxies (visible matter), CDMV non-baryonic cold dark matter

Milgrom's law simply states that from a certain radius, r_c, measured from the center of a galaxy of mass M, Newton's inverse square law for a spherical mass M

$$F = -\frac{GM}{r^2}\,, \quad r < r_c \tag{29a}$$

is replaced by

$$F = -\frac{const}{r}\,, \; r < r_c \tag{29b}$$

From the conjectured vortex structure of the metagalaxy, with the spherical clumping of the galaxies made up from light-emitting ordinary matter, the force law expressed by (29a-b) becomes plausible. It simply means that for $r < r_c$ it is determined by Newton's force of a spherical mass, while for $r > r_c$ it is determined by Newton's force law for a cylindrical mass of length l,

$$F = -\frac{Gm}{lr} \tag{30}$$

where m is the mass of the cylinder per length l. If l is the distance in between two galaxies within the same vortex, then the mass ratio of the vortex filaments making up the invisible cold dark matter, to the mass of the galaxies must be of the order l/R, where R is the radius of the galaxy. For $l/R \approx 10$, as it is between our and the neighboring Andromeda galaxy, this mass ratio is by the order of magnitude the same as between the invisible cold dark matter and the matter inside the galaxies.

5. Conclusion

According the Weinberg [17], the dark energy – small positive cosmological constant – problem is the most important unsolved problem facing elementary particle physics. String theories can accommodate a small cosmological constant only with great difficulty [18]. With a "string landscape" of about 10^{500} possible universes, it means that our universe would have to be one of them where all the parameters of minimally supersymmetric standard model have the right value to make our existence possible. This seems very implausible.

The Planck mass plasma model, where supersymmetry is replaced by a plasma of positive and negative masses, and where physical reality is in the three space and one time dimension as for all physics laboratories, has no difficulty to explain the dark energy and the small positive cosmological constant. The density of the phonons and rotons of this plasma decreases during the cosmic expansion in the same way as the density of ordinary matter. However, for the total amount of all energies to add up to zero, requires that in the formation of the positive mass Dirac spinor rotons, and to a lesser degree the Dirac spinor rotons of ordinary matter, negative energy and hence negative mass must be carried away. This leads to the question, where did this negative mass go? It is for this reason that the negative mass hypothesis makes it more plausible why the initial entropy of the universe is so small. In the standard cosmological model the initial entropy of the expanding hot fireball should be rather high. The generation of positive mass Dirac spinors, resulting from the interaction of positive with likewise negative masses, is accompanied by the repulsive gravitational force of the released negative masses accumulating in the voids between the positive masses of the galaxies, conceivably explaining the observed expansion and filamentary structure of the meta-galaxy.

The by order of magnitude about equal acceleration in Milgrom's law and the acceleration for the cosmic expansion, would then be simply a consequence of the virial theorem for the assumed Planck mass plasma [19].

References

1. M. Milgrom, Astrophysical Journal **270**, 365 (1983).
2. F. Hund, Z.f. Physik **124**, 742 (1948).
3. H. Bondi, Reviews of Modern Physics **29**, 423 (1957).
4. N.E.J. Bjerrum-Bohr et al. Phys. Rev. D, **67**, 084033.
5. M. Planck, Sitzungsberichte der Königlich Preußichen Akademie der Wissenschaften zu Berlin **5**: 440-480, 1899.
6. F. Winterberg, Z. f. Naturforsch. **47a**, 1217 (1992).
7. F. Winterberg, Z. Naturforsch.-Physical Sciences **43a**, 1 (1988).
8. F. Winterberg, The Planck Aether Hypothesis, C.F. Gauss Press, Reno, Nevada, 2nd Edition 2002.
9. F. Winterberg, Z. Naturforsch.-Physical Sciences **58a**, 231 (2003).
10. N. Redington, arXiv:gr-qc/9701063v1.
11. E. Schrödinger, Berl. Ber. **1930**, 416.
12. E. Schrödinger, Berl. Ber. **1931**, 418.
13. H. Hönl and A. Papapetrou, **112**, 512 (1939).
14. H. Hönl and A. Papapetrou, **114**, 478 (1939).
15. H. Hönl and A. Papapetrou, **116**, 153 (1940).
16. D. G. Henshaw and A.D. Woods, Phys. Rev. **121**, 1266 (1961).
17. S. Weinberg, Lake Views, The Belknap Press of Harvard University Press, Cambridge, Massachusetts, London, England, 2009, p. 46ff.
18. R. Kallosh, A. Linde, S. Kachru, S. Trivedi Phys. Rev. D **68** (2003), 046005.
19. F. Winterberg, Physica Scripta **84** (2011) 065902.