A DISSERTATION ON THE ORIGINS OF QUANTIZATION IN THE UNIVERSE

(WITH **UNIFICATION GRAVITY ELECTROMAGNETISM** AS APPENDIX**)**

Leonardo ***Rubino***
leonrubino@yahoo.it
For www.vixra.org
June 2011 – Rev. 00

Abstract: The Universe is quantized simply because its age is very long, so its cycle frequency is very small, but not zero! From this, the quantizations of all physical quantities can be derived.

Contents:

-Contents. Page.1

-Chapter 1: Quantization and Indetermination from the Universe. Page.2
Par. 1.1: Introductory concepts. Page.2
Par. 1.2: Quantization, Indetermination and Universe. Page.3
Par. 1.3: The Planck/Einstein Equation and the quantization. Page.3

-Chapter 2: The birth of Quantum Physics. Page.5
Par. 2.1: The Photoelectric Effect and the walk to quantization. Page.5
Par. 2.2: Planck's Black Body Spectrum. Page.6
Par. 2.3: The Stefan-Boltzmann's Law. Page.14
Par. 2.4: The Wien's Law. Page.16
Par. 2.5: The Compton Effect. Page.17

-Chapter 3: A more formal treatise on Quantum Mechanics. Page.19
Par. 3.1: The Schrödinger's Equation (formal deduction). Page.19
Par. 3.2: The Heisenberg's Indetermination Relations (formal deduction). Page.23

-Chapter 4: Physical constants as an effect of the Universe (the origins of physical constants). Page.26
Par. 4.1: The speed of light. Page.26
Par. 4.2: Mass and radius of the electron. Page.26
Par. 4.3: Planck's Constant. Page.28
Par. 4.4: Stephan-Boltzmann's Constant Page.29
Par. 4.5: The Fine Structure Constant. Page.29
Par. 4.6: The Boltzmann's Constants. Page.31
Par. 4.7: The Universal Gravitational Constant. Page.31

-APPENDIXES. Page.32
Appendix 1: As I see the Universe (Unification Gravity Electromagnetism). Page.32

-Bibliography. Page.49

Chapter 1: Quantization and Indetermination from the Universe.

Par. 1.1: Introductory concepts.

If the world had ever existed, then what is happening now should have already happened.
A. SCHOPENHAUER.

If an event, after having had at its disposal an infinite time, hasn't happened yet, then it's because it can never happen.
In physics an infinite time is meaningless. The infinite is something you can just say and you can assign a symbol, but it can be neither imagined nor really handled.
In mathematics they talk about a tendency to infinite; just a tendency. The Universe cannot be born an infinite time ago; and so, what was before it? Well, we cannot say there isn't any answer, but rather we can say this question is wrong. Time was born together with the Universe and in the Universe, so the expression "before the Universe" is a contradiction. It exists since the moment when it started to exist and that's it. Or better, it exists and that's it. Rather, there is something more interesting: to understand how the Universe can "appear" without violating the conservation laws and laws of physics in general (see my explaination in App. 1).
Well, we have to admit that if matter shows mutual attraction as gravitation, then we are in a harmonic and oscillating Universe in contraction towards a common point, that is the center of mass of all the Universe. As a matter of fact, the acceleration towards the center of mass of the Universe and the gravitational attractive properties are two faces of the same medal. Moreover, all the matter around us shows it want to collapse: if I have a pen in my hand and I leave it, it drops, so showing me it wants to collapse; then, the Moon wants to collapse into the Earth, the Earth wants to collapse into the Sun, the Sun into the centre of the Milky Way, the Milky Way into the centre of the cluster and so on; therefore, all the Universe is collapsing. Isn't it?
So why do we see far matter around us getting farther and not closer? Easy. If three parachutists jump in succession from a certain altitude, all of them are falling towards the center of the Earth, where they would ideally meet, but if parachutist n. 2, that is the middle one, looks ahead, he sees n. 1 getting farther, as he jumped earlier and so he has a higher speed, and if he looks back at n. 3, he still sees him getting farther as n. 2, who is making observations, jumped before n. 3 and so he has a higher speed. Therefore, although all the three are accelerating towards a common point, they see each other getting farther. Hubble was somehow like parachutist n. 2 who is making observations here, but he didn't realize of the background acceleration g (a_{Univ}).
At last, I remind you of the fact that recent measurements on far galaxies type Ia supernovae, used as standard candles, have shown an accelerating Universe; this fact is against the theory of our supposed current post Big Bang expansion, as, after that an explosion has ceased its effect, chips spread out in expansion, ok, but they must obviously do that without accelerating.
Anyway, as the world wasn't born an infinite time ago, collapsing matter cannot come from an infinite distance; therefore, hundreds of billions years ago there was an expansion (post Big Bang), in the opposite direction with respect to the collapse we have now, and so all that with a repulsive gravity. On the basis of all that, the Universe is cyclic and so it has a cyclic frequency **and this is the right key to understand why it is quantized!** All the frequencies which are in the Universe must so be, directly or indirectly, a multiple of the Universe one and this one is the smallest existing frequency.

In App. 1 I prove that the period T_{Univ} of the Universe is: $T_{Univ} = \frac{2pR_{Univ}}{c} = 2{,}47118 \cdot 10^{20} s$ (7.840 billion years) ($n_{Univ} = 4{,}05 \cdot 10^{-21} Hz$), as we know from physics that: v=ωR and $w = 2p / T$, and, for the whole Universe: c=ωR_{Univ} and $w = 2p / T_{Univ}$. And about the value of the angular frequency, we have: $w_{Univ} \cong c / R_{Universo-New} = 2{,}54 \cdot 10^{-20} rad / s$, and it is the right parameter for a reinterpretation of the global Hubble's constant H_{global}, which is H_{local} only for the Universe visible to us ($w_{Univ} = H_{Global}$).
Moreover, still in App. 1, by starting from data on the Coma galaxy cluster, we prove the Universe, while collapsing with speed c, accelerates with acceleration $a_{Univ} = 7{,}62 \cdot 10^{-12}\ m/s^2$.

Par. 1.2: Quantization, Indetermination and Universe.

As per App. 1, we derive The Heisenberg Uncertainty Principle as a consequence of the essence of the macroscopic, collapsing and a_{Univ} accelerating Universe.
According to this principle, the product Δx Δp must keep above $\mathbf{h}/2$, and with the equal sign, when Δx is at a maximum, Δp must be at a minimum, and vice versa:
$\Delta p \cdot \Delta x \geq \mathbf{h}/2$ and $\Delta p_{\max} \cdot \Delta x_{\min} = \mathbf{h}/2$ $(\mathbf{h} = h/2p)$

Now, as $\Delta p_{\max}$ we take, for the electron ("stable" and base particle in our Universe!), $\Delta p_{\max} = (m_e \cdot c)$ and as $\Delta x_{\min}$ for the electron, as it is a harmonic of the Universe in which it is (just like a sound can be considered as made of its harmonics), we have: $\Delta x_{\min} = a_{Univ} / (2p)^2$, as a direct consequence of the characteristics of the Universe in which it is; in fact, $R_{Univ} = a_{Univ} / w_{Univ}^2$, as we know from physics that $a = w^2 R$, and then $w_{Univ} = 2p / T_{Univ} = 2pn_{Univ}$, and as w_e of the electron (which is a harmonic of the Universe) we therefore take the "n_{Univ}–th" part of w_{Univ} , that is:
$|w_e| = |w_{Univ} / n_{Univ}|$ like if the electron of the electron-positron pairs can make oscillations similar to those of the Universe, but through a speed-amplitude ratio which is not the (global) Hubble Constant, but through H_{Global} divided by n_{Univ}, and so, if for the whole Universe: $R_{Univ} = a_{Univ} / w_{Univ}^2$,

then, for the electron: $\Delta x_{\min} = \frac{a_{Univ}}{(w_e)^2} = \frac{a_{Univ}}{(|w_{Univ}/n_{Univ}|)^2} = \frac{a_{Univ}}{(|H_{Global}/n_{Univ}|)^2} = \frac{a_{Univ}}{(2p)^2}$, from which:

$$\Delta p_{\max} \cdot \Delta x_{\min} = m_e c \frac{a_{Univ}}{(2p)^2} = 0{,}527 \cdot 10^{-34} \text{ [Js]} \qquad (1.1)$$

and such a number ($0{,}527 \cdot 10^{-34}$ Js), as chance would have it, is really $\mathbf{h}/2$!!

Par. 1.3: The Planck/Einstein Equation and the quantization.

As we said, there is a Universe with its frequency, the smallest and most basic one. Then, through (1.1) we got $\hbar/2 = h/4\pi$ and here Planck's constant $h = 6{,}625 \cdot 10^{-34} J \cdot s$ starts showing, as a function of macroscopic quantities, as a_{Univ} and c. Moreover, still in App. 1, we show that if we imagine an electron ("stable" and base particle in our Universe!) irradiating all energy it's made of in time T_{Univ} , we get a power which is exactly ½ of Planck's constants, expressed in watt! In fact:

$$L_e = \frac{m_e c^2}{T_{Univ}} = \frac{1}{2} h_W = 3{,}316 \cdot 10^{-34} W$$

And Planck/Einstein Equation $E = h\nu$ (and $E_T = nh\nu$, in case of many photons), which tells us the energy of a photon is ν (frequency) times the energy box h (in joule), is held somehow as the father of quantum physics, of energy boxes etc.
Before, we got such a special constant h from visual reasonings about the Universe and particles, but in the last century it appeared through the Planck/Einstein Equation, mainly through two separate phenomena: one, the Photoelectronic Effect, was studied mainly by Einstein, while the other, that is the studies on the Black Body Radiation Spectrum, was mainly treated by Planck. On the opinion of who is writing, here, both Einstein and Planck didn't intuite in advance their equation and the quantization, but rather were forced by circumstances to such suppositions in order to just make the theoretical interpretation match the results from the experiments!
Moreover, as a quantum is not as small as zero, but it has its own size somehow, then, in the opinion of who is writing, here, in the evaluation of physical quantities, uncertainties cannot be zero (The Heisenberg Uncertainty Principle, Schrödinger's Equation etc). If you see a particle, in order to figure out its position, you must interfere with it somehow, although through the smallest quantum of energy, and so you "touch" it, so you move it, so you change what you are going to measure.
In thermodynamics, too, where quantum physics acts deeply, if, for instance, I try to make a liquid in a calorimeter reach the absolute zero, I'll put a thermometer inside and start cooling as well as I can, through a refrigerator, but whenever I decide to check the temperature reached, in order to see if the absolute zero has been reached, then, in the opinion of the writer, I have to see the thermometer, so I have to illuminate it, although through just the smallest quantum of luminous energy, and so I heat it and it transmits some heat to to the liquid and therefore I'll never get the absolute zero.
Now, let's analyse both the above mentioned phenomena: the Photoelectronic Effect and the Black Body Radiation Spectrum.

Chapter 2: The birth of Quantum Physics.

Par. 2.1: The Photoelectric Effect and the walk to quantization.

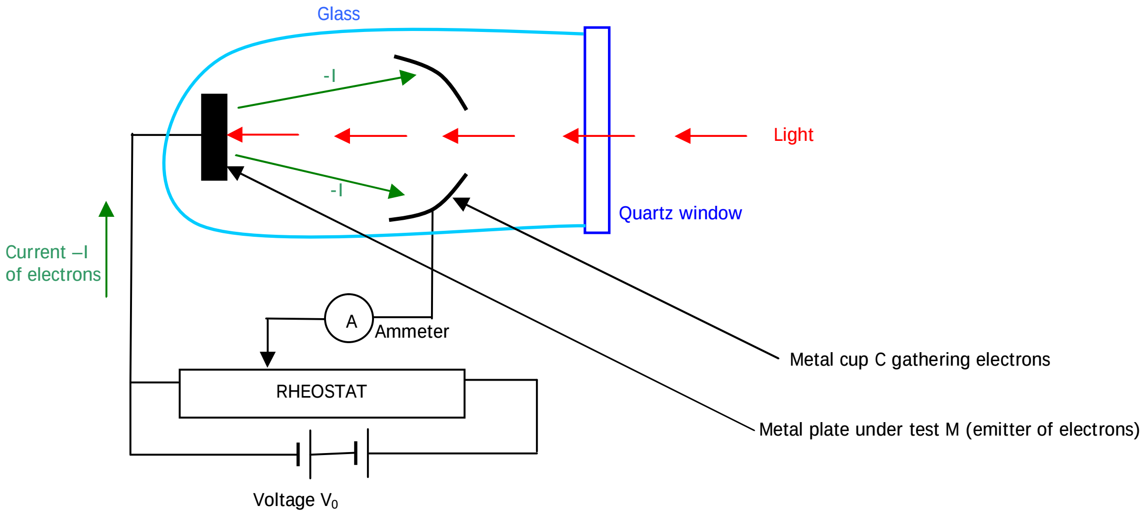


Fig. 2.1: Device for the Photoelectronic Effect.

Let the voltage between the cup C and plate M be: $\Delta V = V_C - V_M$ and let I be the current measured by the ammeter. Then, let I_∞ be the saturation current, that is the maximum current you can have with a certain light flux Φ.
From the experiments, we have:

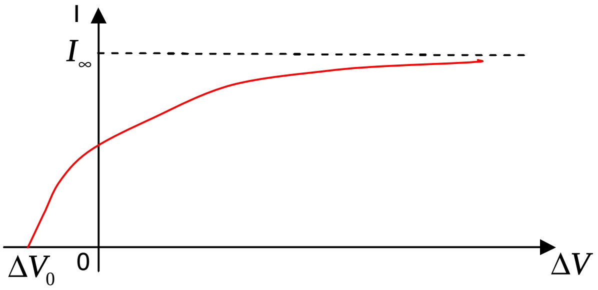


Fig. 2.2: Voltage-current graph.

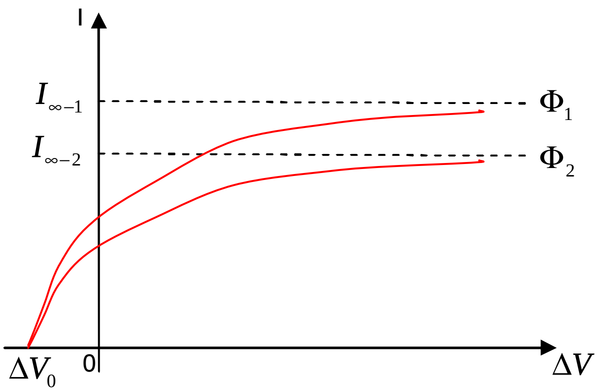


Fig. 2.3: Voltage-current graph for different light fluxes Φ.

Incident light makes electrons jump out of the plate M, and they are then gathered by the cup C, and accelerated, too, by a voltage.

We have that the electrons are emitted with a kinetic energy E_K which can be measured by supplying an inverse $\Delta V = \Delta V_0$ (stop voltage) so that the current of electrons emitted also with $\Delta V = 0$ is reduced to zero; when this happens, we have: $-e\Delta V_0 = E_K$.
From the experiments, we see that $\Delta V_0 \neq f(\Phi)$, that is: ΔV_0 does not depend on Φ, but, on the contrary, it depends on the frequency n of the incident light.
All this is in a complete disagreement with classic physics.
The experiments show what is in Fig. 2.4:

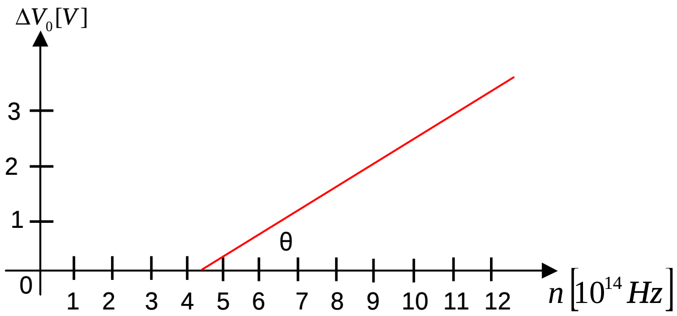


Fig. 2.4: Stop voltage-frequency of the incident radiation.

tgq is fixed and is always: $tgq = h/e$. The equation of this line, known as Einstein Relation, is, of course:

$E_K = -e\Delta V_0 = hn - L_e = \frac{1}{2} m_e V^2$, where L_e is the extraction energy needed for the electron, hn is the energy brought from the photon to the electron and $E_K = \frac{1}{2} m_e V^2$ is the kinetic energy with which the electron comes out.
The big news, here, is the relation $E = hn$ (Planck/Einstein relation) through which light brings energy: it depends on the frequency through a constant $h = 6{,}625 \cdot 10^{-34}\, Js$ (Planck's constant).

Par. 2.2: Planck's Black Body Spectrum.

<u>preamble on Boltzmann's Distribution Law:</u>
now we try to understand how changes, in a material, the number of molecules per unit of volume, when the energy changes.
Suppose to have a column of gas at a constant temperature, in a container and under the effect of the gravitational field.

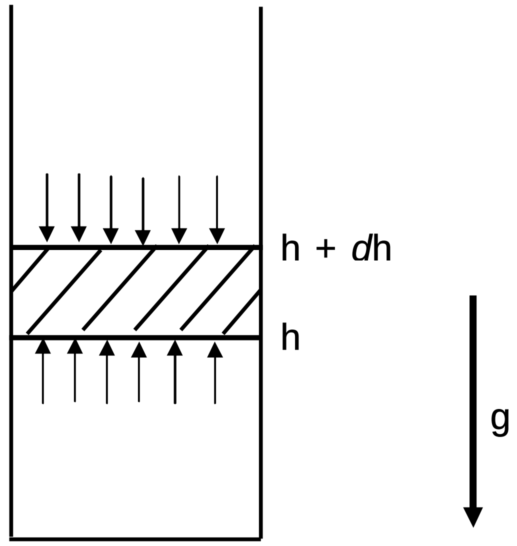


Fig. 2.5: Column of gas.

If this container has a volume V in which we have N gas particles, we define **n** as the number of particles per unit of volume.
With reference to the above figure, we examine a section S of the column of gas at the height h. The pressure P_h at the height h is obviously higher than that at the height h+dh, as at h the mass of gas pushing downwards is higher.
Being pressure P defined as dF/dS =(weight of the disc dh high and section S) / S, we have:

$$P_{h+dh} - P_h = dP = \frac{-m \cdot n \cdot S \cdot dh \cdot g}{S} = -mgndh \,, \qquad (2.1)$$

where m is the mass of every single particle of gas, n is the number of particles per unit of volume, S dh is the volume of the disc, g is the gravitational acceleration end the negative sign tells us that dP is negative (P goes down while we go up).
We also know from thermodynamics that:

$$PV = n_{kmoles}RT = n_{kmoles}N_A \cdot \frac{R}{N_A}T = N \cdot k \cdot T \,, \qquad (2.2)$$

where the first equality is the law of ideal gases (R=const), N_A is the number of particles in a kilomole, i.e. the Number of Avogadro, N= n_{kmoles} N_A= is the total number of gas particles (made of n_{kmoles}) and k=R/N_A is the Boltzmann's constant.
For a proof of the equation of state of ideal gases, see any of the books on general Physics.
From the previous equation, we have:

$$P = \frac{N}{V}kT = nkT \,.$$

By differentiating this equation, we get:

$$dP = dnkT \qquad (2.3)$$

By eq. (2.1) and (2.3) , we have:

$$\frac{dn}{n} = -\frac{mg}{kT}dh = -\frac{dE_p}{kT} \,,$$

where dE_p=mgdh is the differential of the potential energy of every particle.
The integration of this differential equation easily yields the following result:

$$n = n_0 e^{-E_p/kT} \,, \qquad (2.4)$$

where n_0 is constant.

In case the particles are subject not to the gravitational field, but to any other conservative force, F_i (for instance, the intermolecular forces themselves), which we suppose is oriented along x, in (2.4), instead of the potential energy E_p, we'll have the corresponding potential energy E_i coming from the force F_i, that is:

$$E_i = -\int F_i \cdot dx .$$

Finally:

$$n = n_0 e^{-Ei/kT} \tag{2.5}$$

The situation with non conservative forces is here not taken into account, as in this case it wouldn't be even possible to claim the thermal equilibrium.

In our opinion, the Boltzmann's equation (2.5) can be considered as proved and we want to remind you of what it means:

the probability to find molecules in a certain spatial disposition changes exponentially with the opposite of the potential energy of that disposition, divided by kT.

<u>Preamble on the linear harmonic oscillator:</u>

We consider a mass fixed to one end of a spring; the other end is fixed to a wall.

When the mass starts oscillating, as F=ma and, by Hooke, F=-kx, we can write the following differential equation:

$$ma + kx = m\frac{d^2x}{dt^2} + kx = 0$$, whose solution is:

$$x = x_0 \sin(wt + q) , \tag{2.6}$$

where $w = \sqrt{k/m}$.

Now, we write the expression for the total energy E (which is the sum of the kinetic energy with the elastic potential one) of such an oscillating mass:

$$\frac{m}{2}\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}kx^2 = E = E_k + E_p ; \tag{2.7}$$

This is true because:

$$E_p = -\int F \cdot dx = \int kx \cdot dx = \frac{1}{2}kx^2 .$$

Using (2.6) in (2.7) and taking into account the expression for w , we get:

$$E = \frac{m}{2}w^2 x_0^{\,2} \cos^2(wt+q) + \frac{1}{2}kx_0^{\,2} \sin^2(wt+q) = \tag{2.8}$$

$$= \frac{1}{2}kx_0^{\,2}[\cos^2(wt+q) + \sin^2(wt+q)] = \frac{1}{2}kx_0^{\,2}$$

As, from the previous expression, kinetic and potential components are the same, we have justified the reason why we assigned two identical values (½)kT for the total energy of the oscillators in the cavity of a black body.

<u>preamble on standing waves:</u>

If a wave S1 propagates in a limited mean, the superposition of it with its reflected one S2 generates a standing wave S:

$S_1 = A\sin(kx - wt)$, $\qquad S_2 = A\sin(kx + wt)$.

The difference in sign in the arguments is due to the fact that those waves propagate in opposite directions; moreover, the term $wt = 2pnt$ tells us that if we fix a point x, we have an oscillation in time, while the term kx tells us that, if we fix a time t, we see an oscillation by moving along x.
Therefore, a propagating wave oscillates in time and also along the space through which it's propagating indeed.

$$S = S_1 + S_2 = 2A \cdot \sin kx \cdot \cos wt = 2A \cdot \sin \frac{2p}{l} x \cdot \cos 2pnt \ ; \qquad (2.9)$$

after that we take into account the following trigonometric equality:

$$\sin a + \sin b = 2\cos\frac{(a-b)}{2} \cdot \sin\frac{(a+b)}{2} \ .$$

<u>Planck's Black Body Spectrum:</u>
Let's consider a cavity whose sides are at temperature T, uniform and constant.
Microscopic charges which makes the sides move because of the thermal agitation and, so doing, they radiate electromagnetic waves which fill the cavity; there is an energy transfer from the cavity sides to the electromagnetic field. Simultaneously, electromagnetic waves move into the cavity and hit the sides; so doing, they transfer energy from the field to the cavity sides. An equilibrium is so settled.
The **black body radiation spectrum** is the function $f(n)$ so that $f(n)dn$ is the energy had by the electromagnetic field in the unity of volume of the cavity, and with frequency between n and $n + dn$, that is:

$$f(n)dn = du \qquad [J/m^3]$$

Cavity sides emit and absorb radiation and can be held as made by small oscillating dipole. Moreover, we can assign the radiation in the cavity two degrees of freedom corresponding to two polarization planes which are perpendicular and independent each other and on which every electromagnetic wave can oscillate; in simpler words, an electromagnetic wave which propagates along z can oscillate transversally on both planes zx and zy.
We know from the kinetic theory of gases that for every particle, and so for every em wave emitted by the particles, and for every degree of freedom we can assign an energy equal to twice $\frac{1}{2}kT$, that is kT, as the total energy is made of a kinetic part and a potential part and their mean values are the same (see (2.8)).
For a proof of the fact that the total energy to be conferred is really kT/degree of freedom see any of the available general physics books.
Now, suppose we have, out of simplicity, a cubic cavity whose electromagnetic radiation propagates along the three axis, so generating standing waves; moreover, we consider just one polarization plane per propagation axis (y), and we'll later take into account the

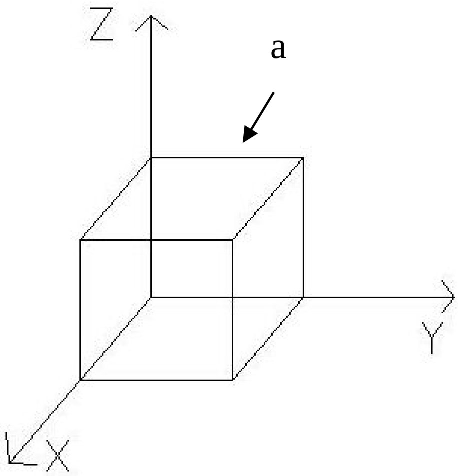


Fig. 2.6.

real existence of two degrees of freedom.
As the cavity is place of standing waves, and considering the x axis as the propagation one, we will write the following equation for a standing wave (see (2.9)):
$E_y(x,t) = E_{oy}\sin(kx)\cdot\sin(2pnt)$, k is the wave number = $2p/l$ and l is the wavelength.
We remind that : $c = ln$, and: $w = 2p/T = 2pn$.
As the standing wave must be zero in x = 0 and in x = a , we have:

$$ka = np \rightarrow n = 2a/l \rightarrow n = \frac{c}{l} = \frac{c\cdot n}{2a}\ .$$

<u>n is positive and not zero, otherwise we don’t have any wave.</u>
In general, for a wave propagating along a random direction, we have, component by component:

$$E_y(x,t) = E_{oy}\sin(k_x x)\cdot\sin(2pnt) \qquad k_x = (2p/l)\cdot\cos a$$
$$E_z(y,t) = E_{oz}\sin(k_y y)\cdot\sin(2pnt) \qquad k_y = (2p/l)\cdot\cos b$$
$$E_x(z,t) = E_{ox}\sin(k_z z)\cdot\sin(2pnt) \qquad k_z = (2p/l)\cdot\cos g$$

where the three direction cosines are the components of the versor $\hat{k}$ which indicates the direction of propagation of the wave.
Still by analogy with the single dimension case, we have:

$$k_x a = n_x p \qquad \rightarrow (2a/l)\cos a = n_x$$
$$k_y a = n_y p \qquad \rightarrow (2a/l)\cos b = n_y$$
$$k_z a = n_z p \qquad \rightarrow (2a/l)\cos g = n_z$$

$$n_x^{\ 2} + n_y^{\ 2} + n_z^{\ 2} = (2a/l)^2(\cos^2 a + \cos^2 b + \cos^2 g) = 4a^2/l^2,$$

from which:

$$n = \frac{c}{l} = \frac{c}{2a}\sqrt{n_x^{\ 2} + n_y^{\ 2} + n_z^{\ 2}} \qquad (2.10)$$

With all values of n, we have all possible ways of vibration. If we put such values n_x , n_y , n_z on three axes and considering the example n_x , n_y , n_z = (1 , 2 , 2) , we see that the number of possible vibrations corresponding to terns n_x , n_y , n_z (n_x , n_y , n_z $\neq 0$, or we have a singularity case) are the vertexes of the following graph, where the n values are different from zero, so they are all the red spots.

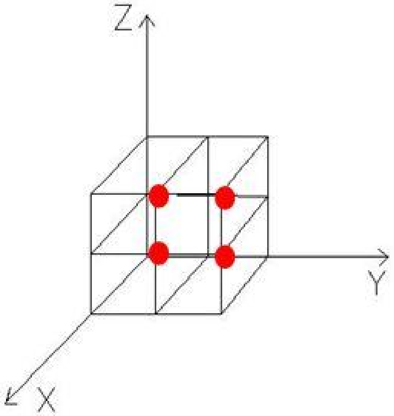


Fig. 2.7.

The fundamental thing we must take into account now (and this has a general validity) is that such *possible ways of vibration (●) correspond, in number, to the small unit side cubes (which are four, too).*

So: ***n . of possible ways of vibration= total volume V located by the tern*** n_x , n_y , n_z .

The radical in the expression (2.10) is just the radius of an octant of sphere located by the three components n_x , n_y , n_z (of course, we consider just the octant where n_x , n_y , n_z are positive, as those must be positive and not zero).

The last remark makes us use the more suitable polar coordinates:

as the volume of an octant of a sphere is equal to $\frac{1}{8}\frac{4}{3}p \cdot r^3$, the number N of modes of possible vibrations for a value of r between 0 and r is:

$$N = \frac{1}{8}\frac{4}{3}p \cdot r^3 \cdot$$

As a consequence, the number N(r) dr of possible modes of vibration for a value of r between r and r + dr can be obtained by differentiating the previous equation:

$$N(r)dr = \frac{p}{2} r^2 dr \cdot$$

Now, let's define an N(n) so that N(r)dr = N(n)dn =number of possible modes of vibration for frequencies between n and n +dn ; we see that, according to (2.10), n = r c / (2 a) , and by differentiating the last equation, we have:

$$dn = \frac{c}{2a} dr$$; and then we get:

$$N(n)dn = \frac{p}{2}\left(\frac{2a}{c}\right)^3 n^2 dn = \frac{4p}{c^3} V n^2 dn$$, where $V = a^3 =$ volume of the cavity.

Now, in order to pass from the previous equation to f(n), and remembering that, according to the definition of f(n) itself we gave before, we have to:

-divide by V to refer to the unity of volume

-multiply by two to take into account the two possible states of polarization of the radiation (as well as we will do when we'll consider the black body)

-multiply by kT, that is, by the mean energy corresponding to each degree of freedom.

Therefore:

$$f(n)dn = \frac{8p}{c^3} kTn^2 dn \quad , \qquad (2.11)$$

and this equation is known to be the Rayleigh-Jeans equation**.**

Of course:

$$f(n) = \frac{8p}{c^3} kTn^2$$

The graph of this equation is here below:

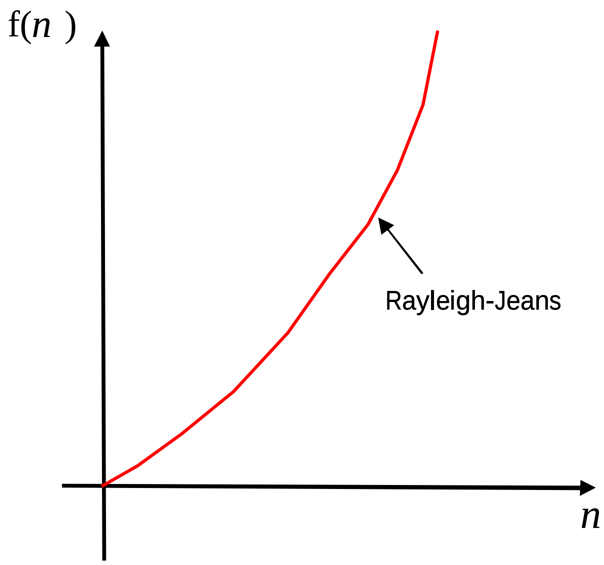


Fig. 2.8: Rayleigh-Jeans' graph.

The experiments, on the contrary, show a different behaviour:

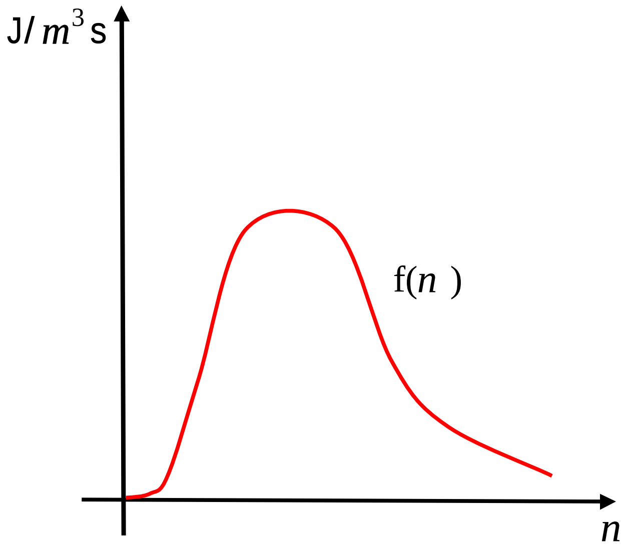


Fig. 2.9: Real emission.

In the real situations, there is a peak, that is a value of frequency around which the emission of the black body concentrates.
Of course, the above curve is for a fixed temperature T and we'll see the more the temperature increases, the higher the frequency values are.
That's why, for instance, a piece of iron at ambient temperature emits an electromagnetic radiation in the range of the infrared waves, or around it, while if you heat it, it will emit visible radiation, at temperatures around some hundreds of centigrade degrees (white heat, red heat).
Similarly, you can find many characteristics of the surface of a star by just studying the frequency spectrum of the light the star irradiates.

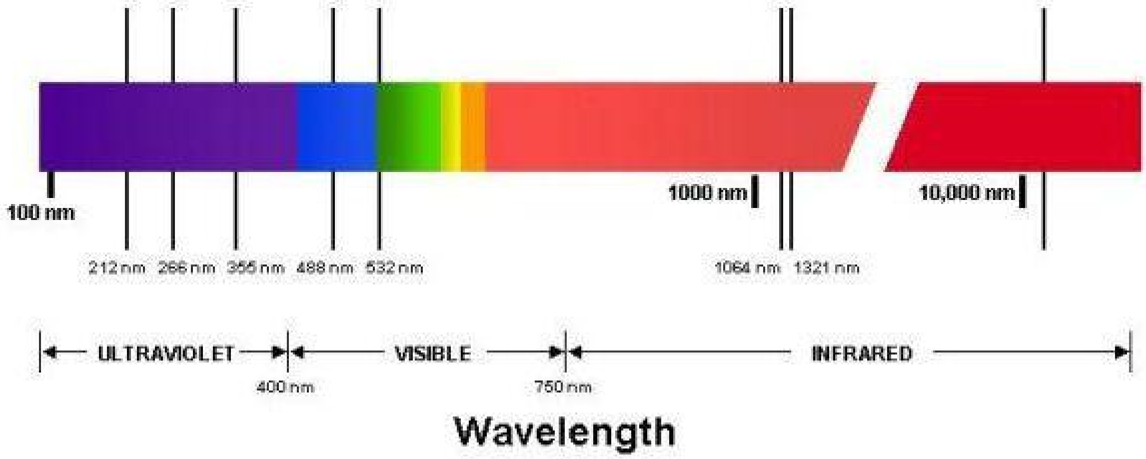


Fig. 2.10: Spectrum of the electromagnetic radiation.

Nothing similar is shown by the Rayleigh-Jeans graph, which leads to an ultraviolet catastrophe. All this was the beginning of the chrisis of classic physics, and there was the need to bring new ideas and quantum hypotheses to make the theoretical deductions match the reality; as an example, we bring the Max Planck's supposition:
first of all, we see that if we want to figure out the mean energy $\overline{E}$ among all energies of the elements of a system, we can carry out a weighed average of all energies, which are distributed according to the already proved Boltzmann's formula (2.5) for n(E); therefore:

$$\overline{E} = \frac{\sum E_i \cdot n(E_i)}{\sum n(E_i)} = \frac{\sum E_i \cdot n_0 e^{-E_i/kT}}{\sum n_0 e^{-E_i/kT}} = \frac{\sum E_i \cdot e^{-E_i/kT}}{\sum e^{-E_i/kT}}; \qquad (2.12)$$

the numerator is the sum of all energies and each of them is weighed according to the number of components which have it, while the denominator is the total number of particles. For the moment, such an average value should be kT, and this is exactly the energy value we conferred to every constituent.
In order to jump from the Rayleigh-Jeans equation to one whose graph is that of the Planck's black body above reported, Planck supposed that for every value of frequency n , the energy of the system could have just discrete (quantized!) values:

$E = hn, 2hn, \ldots\ldots, nhn$, (n integer). [Planck/Einstein equation]

By such an assumption, (2.12) becomes (summation over n):

$$\overline{E} = \frac{\sum_0^{\infty} nhn \cdot e^{-nhn/kT}}{\sum_0^{\infty} e^{-nhn/kT}} \quad .$$

The result is:

In fact, by assuming that $\frac{hn}{kT} = z$, we have: $\overline{E} = kT \frac{\sum_0^\infty nz \cdot e^{-nz}}{\sum_0^\infty e^{-nz}}$; by defining:

$f(z) = \sum_0^\infty e^{-nz}$, we have: $-z \cdot df/dz = z\sum_0^\infty n \cdot e^{-nz} = \sum_0^\infty nz \cdot e^{-nz}$, so:

$$\overline{E} = -kTz \frac{df/dz}{f} = -kTz \frac{d}{dz}\ln z = -kTz\frac{d}{dz}\ln\sum_0^\infty e^{-nz}.$$

Now, for Taylor's series, or for the study on geometrical series:

$\sum_0^\infty x^n = \frac{1}{1-x}$, and if we say: $e^{-z} = x$, we have:

$$\overline{E} = -kTz\frac{d}{dz}\ln(1-e^{-z})^{-1} = -kTz\frac{1}{(1-e^{-z})^{-1}}(1-e^{-z})^{-2}e^{-z} = kTz\frac{e^{-z}}{1-e^{-z}} = \frac{kTz}{e^z-1} = \frac{hn}{e^{hn/kT}-1}$$

that is, the assumption , after that we have taken into account the expression for z.
Therefore, Planck's news was to put in Rayleigh-Jeans' equation (2.11), the value of $\overline{E}$, just found, instead of the mean energy per component, that is, kT:

$$\boxed{f(n)dn = \frac{8pn^2}{c^3}\frac{hn}{e^{hn/kT}-1}dn} \tag{2.13}$$

and this is really the Planck's equation.
By dividing both sides by dn, we get an expression for f(n) which excellently describes the experimental graph above reported on the black body emission!

Par. 2.3: The Stefan-Boltzmann's Law.

We defined the black body as a cavity. Now, let's make a hole to make some radiation (u [J/m^3]) come out from the cavity, as in the figure below:

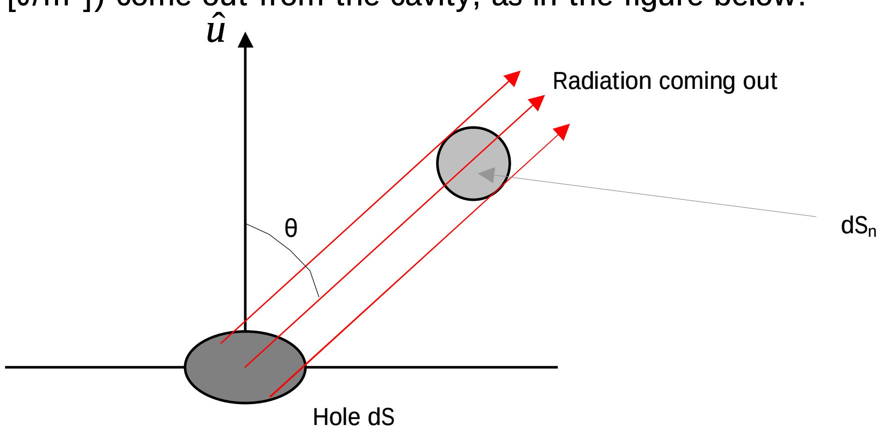


Fig. 2.11: Hole and radiation coming out.

Then, we had, through (2.11) that:

$$f(n)dn = \frac{8p}{c^3}kTn^2dn \qquad [J/m^3] \tag{2.14}$$

If now we introduce the power W [J/s=W] and the solid angle Ω [sr], we easily have, about dS_n:

$$(\frac{d^2W}{d\Omega dv})d\Omega dv = du \cdot c \cdot dS_n \frac{d\Omega}{4p} \quad [W] \text{ (power in the interval } dv \text{ and } d\Omega\text{.)} \qquad (2.15)$$

as such watt on dS_n are due to du [J/m^3] which comes out of the hole by speed c, which is the speed of the radiation, and get [Jm/(m^{3}s)]= [W/m^2], then again by square meters of dS_n (and we get watt), but all by the fraction of solid angle (dimensionless fraction) $\frac{d\Omega}{4p}$ under which dS_n is seen.

We remind, now, that $dS_n = dS\cos q$ and $du = f(v)dv$, and (2.15) becomes:

$$(\frac{d^2W}{d\Omega dv})d\Omega dv = f(v)dv \cdot c \cdot dS\cos q \frac{d\Omega}{4p} \quad [W]$$

If now we introduce the intensity of radiation, that is I [W/m^2], we have, of course:

$$(\frac{d^2I}{d\Omega dv})d\Omega dv = \frac{1}{dS}(\frac{d^2W}{d\Omega dv})d\Omega dv = \frac{cf(v)}{4p}\cos q \cdot d\Omega dv \quad [W/m^2]$$

(the cosine law just seen is the Lambert's Cosine Law).

If now we remind a solid angle can be expressed as a function of polar coordinates angles in the following way: $d\Omega = \sin q dq dj$, we also have:

$$(\frac{d^2I}{d\Omega dv})d\Omega dv = \frac{1}{dS}(\frac{d^2W}{d\Omega dv})d\Omega dv = \frac{cf(v)}{4p}\cos q \cdot d\Omega dv = \frac{cf(v)}{4p}\cos q \cdot \sin q \cdot dq dj\, dv \quad [W/m^2]$$

By integrating this equation over $d\Omega$, that is, over $dq dj$ (θ between 0 and p) (φ between 0 and p), and considering that:

$\int_0^p \cos q \sin q dq = 2\int_0^{p/2} \cos q \sin q dq = 2(1/2) = 1$, while the integral over φ is obviously p , we have, in the end:

$$\iint_{q-j}(\frac{d^2I}{d\Omega dv})d\Omega dv = (\frac{dI}{dv})dv = dI = \frac{cf(v)}{4p}\cdot 1 \cdot p \cdot dv = \frac{cf(v)}{4}dv$$

but as $\frac{dI}{dv} = e(n) \quad [W/(Hz \cdot m^2)] = [J/m^2]$, we have:

$$e(n)dv = (\frac{c}{4})f(v)dv \quad [W/m^2] \qquad (2.16)$$

Now, through (2.14) and the following one: $n = c/l$, we have:

$$e(n)dv = (\frac{c}{4})f(v)dv = (\frac{c}{4})\frac{8p}{c^3}kTn^2 \cdot dv = \frac{2p}{c^2}n^2kTdv$$

Now, by differentiating $n = c/l$, we easily have: $dn = c \cdot dl / l^2$ and defining $f(l)$ and $e(l)$ as follows (of course):

$$f(l)dl = f(v)dv$$
$$e(l)dl = e(v)dv$$

we'll have:

$$f(l)dl = f(v)\frac{dv}{dl}dl = \frac{8p}{c^3}kTn^2 \cdot \frac{dv}{dl}dl = \frac{8p}{c^3}kTn^2 \cdot \frac{c}{l^2}dl = \frac{8p}{l^4}kT \cdot dl \qquad (2.17)$$

$$e(l)dl = e(v)\frac{dv}{dl}dl = \frac{2p}{c^2}n^2kT\frac{dv}{dl}dl = \frac{2p}{c^2}n^2kT\frac{c}{l^2}dl = \frac{2pc}{l^4}kT \cdot dl \qquad (2.18)$$

If now, as well as we did with (2.11) to get (2.13), in (2.17) and (2.18) we put, in place of kT, the expression: $\frac{hv}{e^{hv/kT}-1}$, we'll have the following versions of the Planck's Equation:

$$f(v)dv = \frac{8pv^2}{c^3}\frac{hv}{e^{hv/kT}-1}dv \qquad [J/m^3] \tag{2.19}$$

$$f(l)dl = \frac{8phc}{l^5}\frac{1}{e^{hc/kTl}-1}dl \qquad [J/m^3] \tag{2.20}$$

$$e(v)dv = \frac{2pv^2}{c^2}\frac{hv}{e^{hv/kT}-1}dv \qquad [W/m^2] \tag{2.21}$$

$$e(l)dl = \frac{2phc^2}{l^5}\frac{1}{e^{hc/kTl}-1}dl \qquad [W/m^2] \tag{2.22}$$

Then, by integrating (2.21), we have:

$$e = \frac{2ph}{c^2}\int_0^\infty \frac{v^3}{e^{hv/kT}-1}dv = \frac{2ph}{c^2}\int_0^\infty \frac{e^{-hv/kT}v^3}{1-e^{-hv/kT}}dv = \frac{2ph}{c^2}\int_0^\infty [v^3e^{-hv/kT}\sum_0^\infty (e^{-hv/kT})^n]dv = \frac{2ph}{c^2}\sum_1^\infty\int_0^\infty v^3e^{-n(hv/kT)}dv$$

If now we put: $b = \frac{h}{kT}$ and $a = \frac{2ph}{c^2}$, we have again:

$$e = a\sum_1^\infty\left(-\frac{d^3}{d(bn)^3}\right)\int_0^\infty e^{-bnv}dv = a\sum_1^\infty\left(-\frac{d^3}{d(bn)^3}\right)\left(\frac{1}{bn}\right) = \frac{6a}{b^4}\sum_1^\infty\frac{1}{n^4} = \frac{6a}{b^4}\frac{p^4}{90} = \frac{2p^5k^4}{15c^2h^3}T^4 = \boxed{sT^4 = e}$$

$[W/m^2]$ (Stefan-Boltzmann's Law)

where $s = \frac{2p^5k^4}{15c^2h^3} = 5{,}670\cdot 10^{-8}\frac{W}{m^2K^4}$ (Stefan-Boltzmann's constant)

In order to prove that $\sum_1^\infty \frac{1}{n^4}$ yields a number equal to $\frac{p^4}{90}$ you can just sum the first terms of that series.

Par. 2.4: The Wien's Law.

From (2.22) we have: $e(l) = \frac{2phc^2}{l^5}\frac{1}{e^{hc/kTl}-1}$; with reference to Fig. 9, here shown:

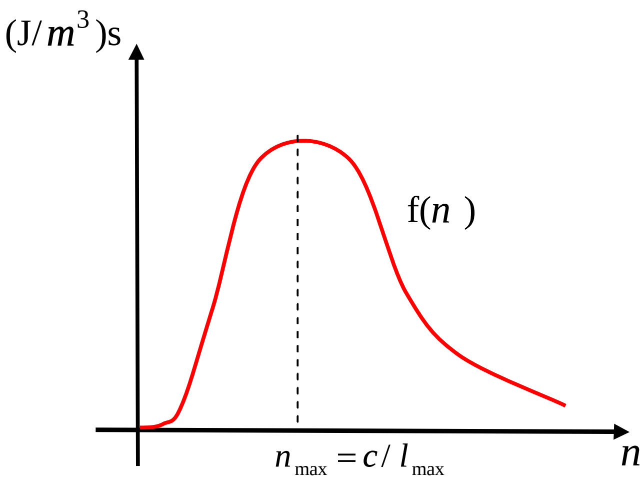


Fig. 2.12: The maximum frequency.

if we want to understand through what λ_{max} the emission takes place, by mathematical analysis we put $\frac{de(l)}{dl} = 0$, that is, we put to zero the first derivative, so:

$5l^4(e^{hc/kTl}-1)+l^5e^{hc/kTl}(-\frac{hc}{kTl^2})=0$, so: $5l\,e^{hc/kTl}-5l-\frac{hc}{kT}e^{hc/kTl}=0$, so, again:

$\frac{(e^{hc/kTl}-1)}{e^{hc/kTl}}=1-e^{-hc/kTl}=\frac{hc}{5kTl}$; this trascendental equation, if numerically solved, but also graphically solved, if you like, yields: $\frac{hc}{kTl}=4{,}965$, from which:

$$l_{\max}=\frac{C}{T}=\frac{hc}{k\cdot 4{,}965}\frac{1}{T}=\boxed{\frac{0{,}2897\cdot 10^{-2}}{T}=l_{\max}}\quad [m] \quad \text{(Wien's Law)} \qquad (2.23)$$

and $C=0{,}2897\cdot 10^{-2}\ [K\cdot m]$ is the Wien's Constant.

Par. 2.5: The Compton Effect.

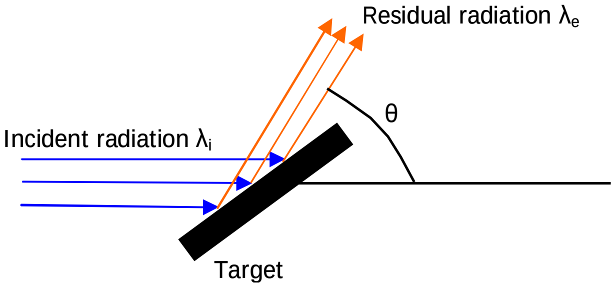


Fig. 2.13: The Compton Effect.

We are here in a situation similar to that of the Photoelectronic Effect; but here, on the contrary, the incident radiation on the target has a very small wavelength λ_i , equal to some tenths of an Å. Therefore, we are talking about very energetic photons.
The electrons will have a certain angle θ', but we'll see also a residual radiation at λ_e. Being this a very energetic collision, as much as the kinetic energy of the electron can be compared with its rest one m_ec^2, it will be held as well as a collision of a photon against a free electron, as if it weren't linked to its nucleus. And we'll have to use the relativistic formulas anyway.
Such an effect, of course, cannot be understood on a classic physics basis.
Now, we show that: $l_e=l_i+l_c(1-\cos q)$ (2.24)

$l_c=\frac{h}{m_ec}=0{,}025$ Å is the Compton's wavelength.

Now, we show the vectorial composition of the linear momenta involved:

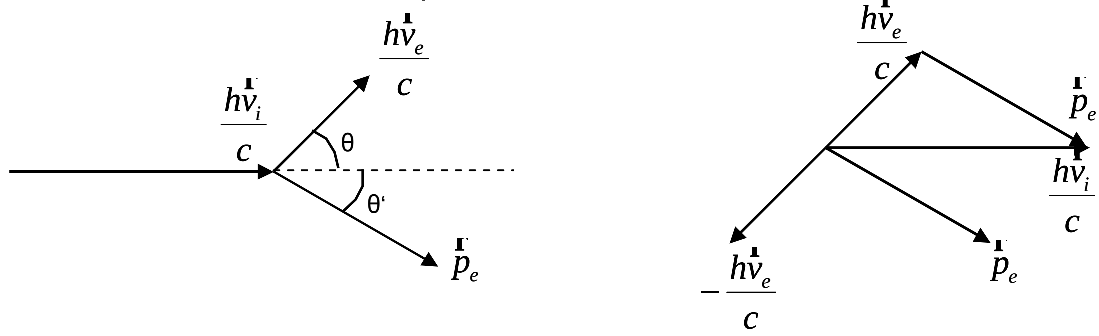


Fig. 2.14: Vectorial composition in the Compton Effect.

We have $\vec{p}_e = \frac{h\vec{\nu}_i}{c} - \frac{h\vec{\nu}_e}{c}$, so, by scalarly multiplying side to side with itself: $\vec{p}_e \cdot \vec{p}_e = \frac{h\vec{\nu}_i}{c} \cdot \frac{h\vec{\nu}_i}{c} + \frac{h\vec{\nu}_e}{c} \cdot \frac{h\vec{\nu}_e}{c} - 2\frac{h\vec{\nu}_i}{c} \cdot \frac{h\vec{\nu}_e}{c}$, that is:

$$p_e^2 = (\frac{h\nu_i}{c})^2 + (\frac{h\nu_e}{c})^2 - 2\frac{h\nu_i}{c}\frac{h\nu_e}{c}\cos\theta \qquad (2.25)$$

Moreover, because of the energy conservation:

$$E_0 + h\nu_i = E + h\nu_e \qquad (2.26)$$

Now, about the rest quantities, we have: $E_0 = m_e c^2$, (2.27)

$\vec{p}_0 = 0$, while, about the dynamic ones:

$$E = \frac{m_e c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_e c^2 \text{ and} \qquad (2.28)$$

$$\vec{p}_e = \frac{m_e \vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_e \vec{v} \qquad (2.29)$$

and moreover, from relativity and from the two previous equations, we have:

$$c^2 \vec{p}_e^{\,2} - E^2 = -m_e c^4 \qquad (2.30)$$

Now, multiply (2.25) by c^2 and in (2.26) isolate E and then square, so getting:

$$c^2 p_e^2 = (h\nu_i)^2 + (h\nu_e)^2 - 2h^2\nu_i\nu_e\cos\theta$$

$$E^2 = (E_0 + h\nu_i - h\nu_e)^2 = m_e c^4 + (h\nu_i)^2 + (h\nu_e)^2 + 2m_e c^2(h\nu_i - h\nu_e) - 2h\nu_i h\nu_e$$

and by subtracting side to side those two equations and taking into account (2.30),

$$h\nu_e - h\nu_i = \frac{h^2\nu_i\nu_e}{m_e c^2}(1 - \cos\theta) \qquad (2.31)$$

and now, by multiplying by $\frac{c}{\nu_i\nu_e}(= \frac{\lambda_e}{\nu_i} = \frac{\lambda_i}{\nu_e})$, we get: $\lambda_i - \lambda_e = \frac{h}{m_e c}(1 - \cos\theta)$ and (2.24) has been proved.

Now, we calculate θ' by projecting the already introduced equation $\frac{h\vec{\nu}_i}{c} = \frac{h\vec{\nu}_e}{c} + \vec{p}_e$ on axes; we have:

$0 = \frac{h\nu_e}{c}\sin\theta + p_e\sin\theta'$ and $\frac{h\nu_i}{c} = \frac{h\nu_e}{c}\cos\theta + p_e\cos\theta'$, that is:

$-\frac{h\nu_e}{c}\sin\theta = p_e\sin\theta'$ and $\frac{h\nu_i}{c} - \frac{h\nu_e}{c}\cos\theta = p_e\cos\theta'$

and by dividing side to side, we have: $tg\theta' = \frac{\nu_e\sin\theta}{\nu_i - \nu_e\cos\theta} = \frac{\sin\theta}{\frac{\nu_i}{\nu_e} - \cos\theta}$, but for the (2.31):

$\frac{\nu_i}{\nu_e} = 1 + \frac{h\nu_i}{m_e c^2}(1 - \cos\theta)$, so, finally: $tg\theta' = \frac{\sin\theta}{(1 + \frac{h\nu_i}{m_e c^2})(1 - \cos\theta)} = \frac{\cot(\theta/2)}{(1 + \frac{h\nu_i}{m_e c^2})}$.

Chapter 3: A more formal treatise on Quantum Mechanics.

Par. 3.1: The Schrödinger's Equation (formal deduction).

We know the Planck/Einstein's Equation:

$$E = hn \tag{3.1}$$

And we also know the relation between pulsation (angular velocity) w and frequency n :

$$w = 2pn \tag{3.2}$$

Then, for the energy of a particle:

$$E = m_0 c^2 = \vec{p} \cdot \vec{c} \tag{3.3}$$

and then the linear momentum:

$$\vec{p} = m_0 \vec{c} \tag{3.4}$$

and, moreover, the general relations $c = ln$ (velocity is wavelength by frequency) $\left|\vec{k}\right| = \frac{2p}{l}$ (modulus of the wave vector $\vec{k} = \frac{2p}{l}\hat{k}$) and $\mathbf{h} = \frac{h}{2p}$ (Dirac's constant – barred $\mathbf{h}$).

Now, from (3.1) and (3.3), we have: $p = h\frac{n}{c} = \frac{h}{l} = \frac{h}{2p}\frac{2p}{l} = \mathbf{h}k$ (3.5)

Moreover: $E = hn = \frac{h}{2p}2pn = \mathbf{h}w$. (3.6)

And for a particle, $E = \frac{1}{2}mv^2 = \frac{1}{2m}m^2v^2 = \frac{p^2}{2m}$ (3.7)

and $E = \mathbf{h}w = \frac{p^2}{2m} = \frac{\mathbf{h}^2k^2}{2m}$. (3.8)

Now, as in order to locate a particle I have to interfere with it, by illuminating it, or perturbing it somehow, and as, simply speaking, the smaller a particle is, the more that perturbation disturbs it, diverts it, slows sit down, accelerates it etc, one is led not to imagine anymore it as a single point, but rather through a wave.

With De Broglie, we can associate a wavelength to a particle, through (3.5):

$l = \frac{h}{p} = \frac{h}{mV}$, where, now, V is the velocity of the particle and p is the modulus of $\vec{p} = m_0\vec{V}$.

For what has been just said, we are also led to introduce a wave function $\Psi = \Psi(\vec{r},t) = \Psi(\vec{x},t)$ which describes the particle when moving along $\vec{r}(x,y,z)$ (or $\vec{x}(x,y,z)$).

<u>wave function</u>:

for all what previously said, the particle isn't anymore a dimensionless point, but rather something like a cloud which is the space in which the probability to find the particle is higher; if we put $r(\vec{x},t)d^3x$ the probability to find the particle in the volume between $\vec{x}$ and $\vec{x} + d^3x$ (d^3x as we are thinking in three dimensions), it must be proportional, through a proportionality constant, to the square modulus $\left|\Psi(\vec{x},t)\right|^2$ of the wave function $\Psi = \Psi(\vec{x},t)$. We are talking here about a square modulus, as, in general, we can express a wave through trigonometric functions, and so also in a complex form, that is, with complex numbers and we have quantifiable quantities in the real field, as long as we take their moduli:

$|\Psi(\vec{x},t)|^2 d^3x = |N|^2 \rho(\vec{x},t) d^3x$ $\quad$ $(|\Psi(\vec{x},t)|^2 = \Psi(\vec{x},t)\Psi^*(\vec{x},t))$, where $\Psi^*(\vec{x},t)$ is the complex conjugated of $\Psi(\vec{x},t)$, (i swapped with –i).

Ψ is typical of every single electron. Now, by the definition of probability, the integration over all the space must yield the maximum probability:

$\int \rho(\vec{x},t) d^3x = 1$, so: $\int |\Psi(\vec{x},t)|^2 d^3x = |N|^2$

Let's normalize the function Ψ so that $\int |\Psi(\vec{x},t)|^2 d^3x = 1$, and we have:

$$\Psi_N(\vec{x},t) = \frac{1}{N}\Psi(\vec{x},t)$$

Let's write down a list of some of the properties Ψ must have:

-it must be continuous, as the probability to find the particle, for instance, in x_0, must be the same, whatever you tend to x_0 , whether from left or from right.

-it must be limited everywhere, as well as the probability to find the particle in a certain place is.

-for a particle which is localized in a region Ω, we must have $\Psi = 0$ for $x \notin \Omega$.

-it must be a monodrome function (just one value)

-wave functions which differs just by the normalization describe the same physical system (and $\Psi = 0 \rightarrow$Vacuum)

-if a system can stay in a state Ψ_1 and also in a state Ψ_2 , then it can stay also in a generic state $\Psi = a\Psi_1 + b\Psi_2$.

<u>wave function of a free particle</u>:

we know from wave physics that, of course, a wave propagating through time and through x, must have, as an argument, a function like:

$\frac{2\pi}{\lambda}\hat{k}\cdot\vec{x} - \frac{2\pi}{\lambda}vt = \vec{k}\cdot\vec{x} - \omega t$, as if we fix a point in time (as: t=0) we have a variability with x and fixing x we have a variability in time, that is a real wave.

Now, according to (3.5) and (3.6) we have: $\vec{k}\cdot\vec{x} - \omega t = \frac{\vec{p}}{\hbar}\vec{x} - \frac{E}{\hbar}t$ and so the wave function must be like:

$$f(\vec{k}\cdot\vec{x} - \omega t) = f(\frac{\vec{p}}{\hbar}\vec{x} - \frac{E}{\hbar}t) \qquad (3.9)$$

We notice that deriving (3.9) over t means to factor ω, while deriving it over x means to factor k.

Now, as according to (3.8): $\omega = \frac{\hbar k^2}{2m}$, we understand, for all what has been just said, that we have to take a t-first order wave equation which is also an x-second order:

$$\frac{\partial\Psi}{\partial t} = \gamma\frac{\partial^2\Psi}{\partial x^2}. \qquad (3.10)$$

Now, Fourier should suggest to propose base functions as candidates to be solutions of (3.10), the following four:

$$A\sin(\vec{k}\cdot\vec{x}-wt) \tag{3.11}$$

$$B\cos(\vec{k}\cdot\vec{x}-wt) \tag{3.12}$$

$$Ce^{i(\vec{k}\cdot\vec{x}-wt)} \tag{3.13}$$

$$De^{-i(\vec{k}\cdot\vec{x}-wt)} \tag{3.14}$$

So, we notice that (3.11) and (3.12), in their monodimensional form, (x in place of $\vec{x}$ etc), cannot satisfy (3.10), while (3.13) and (3.14) can, provided that we consider:

$-iw=-gk^2$, from which: $g=i\frac{w}{k^2}=i\hbar\frac{\hbar w}{\hbar^2k^2}=i\hbar\frac{E}{p^2}=\frac{i\hbar}{2m}$ and we notice that g is here independent from dynamic quantities as p, therefore it works for us.

If, on the contrary, if we chose the d'Alembert wave equation $\frac{\partial^2\Psi}{\partial^2 t}=g\frac{\partial^2\Psi}{\partial x^2}$ (not ok), all four candidates should have satisfied it, but for g we would have had:

$g=\frac{w^2}{k^2}=(\frac{\hbar w}{\hbar k})^2=\frac{E^2}{p^2}=\frac{p^2}{4m^2}$, not ok, as such a g should be a dynamic parameter, as it has p inside, so such an equation would have changed its characteristics with p.

So, we put (3.13) in our good candidate (3.10), so getting:

$\frac{\partial\Psi}{\partial t}=\frac{i\hbar}{2m}\frac{\partial^2\Psi}{\partial x^2}$, and, after multiplying both sides by $i\hbar$:

$$i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} \tag{3.15}$$

(Schrödinger's Equation for a free particle and on a monodimensional motion)

If now we put the expression for $\Psi(x,t)$ ((3.13) monodimensional) in (3.15), we get:

$\hbar w\Psi=\frac{\hbar^2k^2}{2m}\Psi$, that is:

$$E\Psi=\frac{p^2}{2m}\Psi \quad ; \tag{3.16}$$

in fact, we already had: $E=\frac{p^2}{2m}$.

Now, we rewrite, one over another, (3.15) and (3.16):

$$\boxed{i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}}$$

$$\boxed{E\Psi=\frac{p^2}{2m}\Psi}$$

By a comparison side to side, we see that it is possible to make the following associations of operators:

$E\to i\hbar\frac{\partial}{\partial t}$ and $p^2\to-\hbar^2\frac{\partial^2}{\partial x^2}$ >>> $p\to-i\hbar\frac{\partial}{\partial x}$

In three dimensions, (3.15) becomes:

$$i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\Delta\Psi \quad , \tag{3.17}$$

which is the three-dimension Schrödinger's equation for a free particle, where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplacian, then $\Psi(\vec{x},t) = Ce^{i(\vec{k}\cdot\vec{x}-wt)}$, $|\Psi(\vec{x},t)| = C$, $E \to i\hbar\frac{\partial}{\partial t}$, $p^2 \to -\hbar^2\Delta$, $p \to -i\hbar\nabla$, $\vec{p} = \hbar\vec{k}$, $w = \frac{\hbar k^2}{2m}$, con $k = |\vec{k}|$.

We notice that the velocity of the wave is $v_f = \frac{w}{k} = \frac{E}{p} = \frac{p}{2m}$, that is, a phase velocity, while the particle velocity is $v_g = \frac{p}{m} = \frac{dw}{dk} = \frac{d}{dk}\frac{\hbar k^2}{2m} = 2v_f$, and so it is a group velocity.

Now, as in (3.17) the quantity $-\frac{\hbar^2}{2m}\Delta$ has got the dimension of an energy E, a kinetic one, in this case, and this quantity corresponded to:

$$-\frac{\hbar^2}{2m}\Delta \to \frac{p^2}{2m} = \frac{1}{2m}m^2v^2 = E_k, \tag{3.18}$$

if the particle is also in a potential V, we'll have, in place of the mere kinetic energy, the total energy $H=T+V=E_k+V$ (H is the Hamiltonian) and (3.17) will become: ($\Psi(\vec{x},t) = Ce^{i(\vec{k}\cdot\vec{x}-wt)}$, wave function and $\Psi^*(\vec{x},t) = Ce^{-i(\vec{k}\cdot\vec{x}-wt)}$ is its complex conjugated)

$$\boxed{i\hbar\frac{\partial\Psi}{\partial t} = (-\frac{\hbar^2}{2m}\Delta + V)\Psi} \tag{3.19}$$

Complete Schrödinger's Equation! (3.19)

As an alternative, according to (3.18) we can write:

$$E_k = \frac{p^2}{2m} = \boxed{H - V = \frac{p^2}{2m}} \tag{3.20}$$

and also:

$$\boxed{-\frac{\hbar^2}{2m}\Delta\Psi = (H-V)\Psi} \tag{3.21}$$

that is : $\boxed{\Delta\Psi + \frac{2m}{\hbar^2}(H-V)\Psi = 0}$ **An alternative for the complete Schrödinger's Equation!** (3.22)

Regarding phase and group velocities, for a photon, which is monocromatic and follows the d'Alembert equation, those two velocities are the same ($v_f = v_g = c$), and all this shows us once again that Schrödinger's Equation is not the same as the d'Alembert wave equation and for it we have: $v_f \neq v_g$.

The Schrödinger's Equation sounds like a tied wave, standing like. As chance would have it. **Wanna see the Schrödinger's Equation, in the formulation of the (3.22), is a standing wave equation???**

Let's try and see:

first of all, we notice that (3.22) really looks like the equation of standing waves:

$$\frac{\partial^2\Psi}{\partial x^2} + k^2\Psi = 0; \quad \text{(standing waves equation)} \tag{3.23}$$

Out of simplicity, we consider (3.22) in a monodimensional form:

$\frac{\partial^2\Psi}{\partial x^2}+\frac{2m}{\hbar^2}(H-V)\Psi=0$; well, it's exactly the same.

(3.23) is the standing wave equation, indeed; as a matter of fact, if a <u>generic</u> Ψ_1 propagates in a limited mean, the superposition of it with its reflection Ψ_2 makes a standing wave $\Psi=\Psi_1+\Psi_2$: $\Psi_1=A\sin(kx-wt)$, $\Psi_2=A\sin(kx+wt)$.

The difference in sign in the arguments shows that those two waves propagate in opposite directions; moreover, the term $wt=2pnt$ tells us that, if you fix a point x, you have an oscillation in time, while the term kx tells us that if you fix a time t, you'll see an oscillation when you move along x.

Ψ , therefore, oscillates in time and along the direction of propagation.

$$\Psi=\Psi_1+\Psi_2=2A\sin kx\cdot\cos wt=2A\sin\frac{2p}{l}x\cdot\cos 2pnt\ ; \tag{3.24}$$

after that we have used the following trigonometric identity:

$\sin a+\sin b=2\cos\frac{(a-b)}{2}\cdot\sin\frac{(a+b)}{2}$.

Now, if you fix t in (3.24), you'll have: $\Psi=const\cdot\sin kx$, from which:

$\frac{\partial^2\Psi}{\partial x^2}=-const\cdot k^2\sin kx=-k^2\Psi$, from which, again: $\frac{\partial^2\Psi}{\partial x^2}+k^2\Psi=0$, so the (3.23), that is, the standing wave equation!

Therefore, as a <u>further intuitive proof of the Schrödinger's Equation</u>, we give the following: let Ψ be the wave function; it must withstand the following wave equation: $\frac{\partial^2\Psi}{\partial x^2}+k^2\Psi=0$;

then we know from the previous pages that $p=\hbar k$, from which: $k^2=\frac{p^2}{\hbar^2}$ and so:

$$\frac{\partial^2\Psi}{\partial x^2}+\frac{p^2}{\hbar^2}\Psi=0. \tag{3.25}$$

Then, we know through (3.20) that: $H-V=\frac{p^2}{2m}$, and so: $\frac{2m}{\hbar^2}(H-V)=\frac{p^2}{\hbar^2}$ and (3.25) yields: $\frac{\partial^2\Psi}{\partial x^2}+\frac{p^2}{\hbar^2}\Psi=\frac{\partial^2\Psi}{\partial x^2}+\frac{2m}{\hbar^2}(H-V)\Psi=0$ so really the (3.22) monodimensional!

Par. 3.2: The Heisenberg's Indetermination Relations (formal deduction).

<u>preamble on the mean value of an operator</u>:

we know that by (Ψ,Ψ) we mean the following: $\int\Psi^*(\vec{x},t)\Psi(\vec{x},t)d^3x$, which is 1 for normalized Ψ.

Before, we talked about probability P as a function of the space (x or $\vec{x}$) and proportional ro the square modulus of the wave function:

$P\propto|\Psi(\vec{x},t)|^2=\Psi(\vec{x},t)\Psi^*(\vec{x},t)$, where $\Psi^*(\vec{x},t)$ is the complex conjugated of $\Psi(\vec{x},t)$ (i swapped with –i). If then you want to calculate the mean value (over the space) for an operator F, we can use the weighed mean value calculation, where the weight evaluated for every point where you want to calculate the mean value, is $\Psi(\vec{x},t)\Psi^*(\vec{x},t)$:

$$\langle F\rangle=(\Psi,F\Psi)=\int\Psi^*(\vec{x},t)F\Psi(\vec{x},t)d^3x \tag{3.26}$$

preamble on fundamental commutators:

we define the commutator of the operator A with the operator B: $[A,B]=AB-BA$. Now, in case A and B are just numbers, their commutator will be zero, but if they are operators, then things can be different.

For fundamental commutators, we have:

$[x_i,x_j]=x_ix_j-x_jx_i=0$ (x=position)

$[p_i,p_j]=(-i\hbar\frac{\partial}{\partial x_i})(-i\hbar\frac{\partial}{\partial x_j})-(-i\hbar\frac{\partial}{\partial x_j})(-i\hbar\frac{\partial}{\partial x_i})=0$, (we saw that $p\to -i\hbar\frac{\partial}{\partial x}$).

$[x_i,p_j]=i\hbar\delta_{ij}$;

in fact, if you apply the commutator to an auxiliary and generic operator φ:

$$[x_i,p_j]\varphi=x_i(-i\hbar\frac{\partial\varphi}{\partial x_j})-(-i\hbar\frac{\partial}{\partial x_j})(x_i\varphi)=-i\hbar x_i\frac{\partial\varphi}{\partial x_j}+i\hbar\frac{\partial x_i}{\partial x_j}\varphi+i\hbar x_i\frac{\partial\varphi}{\partial x_j}=i\hbar\delta_{ij}\varphi$$

where δ_{ij} is the Kronecker's Delta, and is 0 if $i\neq j$ and 1 if $i=j$. In fact, as x_i and x_j are ortogonal and linearly independent (as x, y and z are), we really have $\frac{\partial x_i}{\partial x_j}=\delta_{ij}$.

About the commutator $[t,E]$: (as $E\to i\hbar\frac{\partial}{\partial t}$)

$[t,E]\varphi=it\hbar\frac{\partial\varphi}{\partial t}-i\hbar\frac{\partial}{\partial t}(t\varphi)=it\hbar\frac{\partial\varphi}{\partial t}-i\hbar\frac{\partial t}{\partial t}\varphi-it\hbar\frac{\partial\varphi}{\partial t}=-i\hbar\frac{\partial t}{\partial t}\varphi=-i\hbar\varphi$ and so:

$[t,E]=-i\hbar$

preamble on the eigenvalue equation and on deviations:

as x_i is a certain position on a certain axis (for instance, x_1=x, x_2=y, x_3=z), then also Ψ_i is a certain state i, considered as a component i of a wave functio Ψ in a maybe infinite-dimension space i=infinite).

If states "i" exist, where an operator F (which can be simply a real number f) has a well defined value, then we have: $\langle F\rangle_i=f_i$.

F should be an "observable", likely. Then, we know the definition of mean square deviation ΔF for F and we want it becomes zero:

$\Delta F=\sqrt{\langle F^2\rangle_i-\langle F\rangle_i^2}=0$. We also define the "simple deviation" Δ_F:

$\Delta_F=F-\langle F\rangle_i$. Then, we have:

$\langle\Delta_F^2\rangle_i=\langle(F-\langle F\rangle_i)^2\rangle_i=\langle F^2\rangle_i+\langle F\rangle_i^2-2\langle F\rangle_i\langle F\rangle_i=\langle F^2\rangle_i-\langle F\rangle_i^2=(\Delta F)^2$. Now, the request according to which: $\Delta F=0$, becomes as follows: $\langle\Delta_F^2\rangle_i=0=(\Psi_i,\Delta_F^2\Psi_i)=0$. And as F is an observable, then hermitian (F*=F), also Δ_F will be hermitian, and so we can write:

$\langle\Delta_F^2\rangle_i=(\Psi_i,\Delta_F^2\Psi_i)=(\Delta_F\Psi_i,\Delta_F\Psi_i)=\int|\Delta_F\Psi_i|^2dx=0$, from which: $\Delta_F\Psi_i=0$, that is:

$F\Psi_i=f_i\Psi_i$, which is the eigenvalue equation for F.

preamble on the Schwarz's Inequality:

if we consider the scalar product between two vectors as the projection of one over the other, we have: $\vec{u}\cdot\vec{w}=|\vec{u}||\vec{w}|\cos\theta\le|\vec{u}||\vec{w}|=\sqrt{\vec{u}\cdot\vec{u}}\sqrt{\vec{w}\cdot\vec{w}}=\sqrt{\vec{u}^2}\sqrt{\vec{w}^2}$, as $\cos\theta\le 1$.

$\vec{u}\cdot\vec{w}\le\sqrt{\vec{u}\cdot\vec{u}}\sqrt{\vec{w}\cdot\vec{w}}$ is a general form for the Schwarz's Inequality.

If now we go back to our quantum operatorial mean values formalism, we have, from analogy: $|(\Psi, FG\Psi)| \leq \sqrt{(\Psi, F^2\Psi)}\sqrt{(\Psi, G^2\Psi)}$, that is, also (by squaring both sides, if we like):

$|(\Psi, FG\Psi)|^2 \leq (\Psi, F^2\Psi)(\Psi, G^2\Psi) = (\Psi, FF\Psi)(\Psi, GG\Psi)$ and as F and G are hermitian, we'll also have: $|(F\Psi, G\Psi)|^2 \leq (F^*\Psi, F\Psi)(G^*\Psi, G\Psi) = (F\Psi, F\Psi)(G\Psi, G\Psi)$, (3.27)

as, from the definition of (3.26), it's very easy to see that an operator between round brackets can be moved from left to right, with respect to the comma, provided that you turn it into its complex conjugated and if it is hermitian, its complex conjugated is equal to itself.

(3.27) is the Schwarz's Inequality we're interested in.

at last, the Heisenberg's Indetermination Relations:

as now we can well manage with all quantum terminology and formalism, as per all what has been said so far, let's try and evaluate the following expression: $(i\langle[F,G]_\Psi\rangle)^2$, where F and G are hermitian:

$(\langle i[F,G]_\Psi\rangle)^2 = |(\Psi, FG\Psi) - (\Psi, GF\Psi)|^2$, but we can also say that:

$|(\Psi, FG\Psi) - (\Psi, GF\Psi)|^2 \leq (|(\Psi, FG\Psi)| + |(\Psi, GF\Psi)|)^2$, as the sum of moduli is for sure not less than the simple difference.

As F and G are hermitian, we can say:

$(\Psi, GF\Psi) = (G\Psi, F\Psi) = (FG\Psi, \Psi) = (\Psi, FG\Psi)^*$ and $(\Psi, FG\Psi) = (F\Psi, G\Psi)$ and so, about the previous equations:

$(\langle i[F,G]_\Psi\rangle)^2 \leq 4|(F\Psi, G\Psi)|^2$; then, according to Schwarz:

$|(F\Psi, G\Psi)|^2 \leq (F\Psi, F\Psi)(G\Psi, G\Psi)$ and so:

$$(\langle i[F,G]_\Psi\rangle)^2 \leq 4(\Psi, F^2\Psi)(\Psi, G^2\Psi) = 4\langle F^2\rangle_\Psi \langle G^2\rangle_\Psi \tag{3.28}$$

Before we said: $\Delta_F = F - \langle F\rangle_\Psi$, and, from analogy: $\Delta_G = G - \langle G\rangle_\Psi$, that is:

$$\begin{cases} \Delta_F = F - \langle F\rangle_\Psi \\ \Delta_G = G - \langle G\rangle_\Psi \end{cases} \tag{3.29}$$

and we also got:: $\langle \Delta_F^2\rangle_\Psi = \langle F^2\rangle_\Psi - \langle F\rangle_\Psi^2 = (\Delta F)^2$ and, still from analogy, then also: $\langle \Delta_G^2\rangle_\Psi = \langle G^2\rangle_\Psi - \langle G\rangle_\Psi^2 = (\Delta G)^2$, that is:

$$\begin{cases} \langle \Delta_F^2\rangle_\Psi = \langle F^2\rangle_\Psi - \langle F\rangle_\Psi^2 = (\Delta F)^2 \\ \langle \Delta_G^2\rangle_\Psi = \langle G^2\rangle_\Psi - \langle G\rangle_\Psi^2 = (\Delta G)^2 \end{cases} \tag{3.30}$$

From (3.29) we have: $[\Delta_F, \Delta_G] = [F, G]$, (3.31)

as, in making $[\Delta_F, \Delta_G]$ explicit, products of F and G with the m.v. cancel each other (while FG and GF don't). Now, in (3.28) let's make a replacement: $F \to \Delta_F$ and $G \to \Delta_G$; we have:

$$(\langle i[\Delta_F, \Delta_G]_\Psi\rangle)^2 \leq 4\langle {\Delta_F}^2\rangle_\Psi \langle {\Delta_G}^2\rangle_\Psi \tag{3.32}$$

and also taking into account (3.30) and (3.31), (3.32) changes again:

$(\langle i[F,G]_\Psi\rangle)^2 \leq 4(\Delta F)^2(\Delta G)^2$, from which:

$$\boxed{\Delta F \cdot \Delta G \geq \frac{1}{2}\left|\langle i[F,G]_{\Psi}\rangle\right|} \tag{3.33}$$

which is the Heisenberg's Indetermination Relation.

If we now put F=x and G=p and remembering the preambles on fundamental commutators, from (3.33) we have the famous: $\boxed{\Delta x \cdot \Delta p \geq \frac{\hbar}{2}}$. (if I want to know well the position of an electron, then I have to give up some accuracy on the evaluation of its speed $\propto$ p, and vice versa)

On the contrary, if we put F=t and G=E and still remembering preambles on fundamental commutators, still according to (3.33) we'll have the famous (as well): $\boxed{\Delta E \cdot \Delta t \geq \frac{\hbar}{2}}$.

Chapter 4: Physical constants as an effect of the Universe (the origins of physical constants).

Par. 4.1: The speed of light.

We know from physics that for a gravitating body at a distance R from the center of mass of the system in which it's gravitating, the centrifugal acceleration is:

$$a_c = \frac{v^2}{R} \tag{4.1}$$

Now, talking about our Universe ([1]), it's contracting with an acceleration $a_{Univ} \cong 7{,}62 \cdot 10^{-12}\, m/s^2$ towards its center of mass and we are, of course, at a certain distance from it, and we call it $R_{Univ} \cong 1{,}17908 \cdot 10^{28}\, m$.

We also know that in the place of the Universe where we are, the speed of light is c, of course, so (4.1) becomes:

$$a_{Univ} = \frac{c^2}{R_{Univ}} \quad \text{, from which:} \tag{4.2}$$

$$\boxed{c = \sqrt{a_{Univ} \cdot R_{Univ}} \cong 3 \cdot 10^8\, m/s\,.} \tag{4.3}$$

We also have: $c = \sqrt{\frac{GM_{Univ}}{R_{Univ}}} \cong 3 \cdot 10^8\, m/s$.

([1]): for an analytic treatment of our Universe, see Appendix 1.

Par. 4.2: Mass and radius of the electron.

Our Universe is Harmonic and oscillating, contracting towards its center of mass. Such a contraction makes the getting closer of all matter, and towards the center of mass of the Universe, indeed. This mutual getting closer physical effect is what we commonly call, from centuries, force of gravity. Then, as we are talking about a "harmonic" motion, we are led to think that harmonics of the Universe can exist, as well as a sound can be held made of its harmonics (of Fourier) and there are strong reasons to think that the electron is a harmonic of the Universe, or better a real Universe in small size or, if we like, a smal particle of "God", or a small Higgs boson of nowadays, better known as God's particle; the

only difference is that the God's boson hasn't been found yet, provided that it exists and that it serves to what they say, while we know the electron somewhat well.
Well then, we think the electron is so similar to the Universe, in the microscopic range, that the "classic" gravitational acceleration on its surface, as if it were a small planet, is the same as the contraction cosmic acceleration a_{Univ} of the Universe; so we have to write that:

$m_x \cdot g_e = G\frac{m_x \cdot m_e}{r_e^2}$, from which:

$g_e = G\frac{m_e}{r_e^2} = a_{Univ} = 7{,}62 \cdot 10^{-12}\, m/s^2$ and so:

$$\frac{m_e}{r_e^2} = \frac{a_{Univ}}{G} \quad . \tag{4.4}$$

Now, as the Coulomb's electric force between an electron e^- and a positron e^+, or between an electron e^- and a proton p^+ in an atom of hydrogen, is very higher than the gravitational one, I suspect such an enormous force is due to the huge gravitational force all the surrounding Universe transmits to the electron itself; and vice versa, too, that is, the composition of all electrical forces from all particles in the Universe shows, on a macroscopic range, like a gravitational force.
Now, as we do not have other different reasons to explain such an intense electric force, and as the gravitational force, for the moment, is the only one I clearly know, if I don't want to invent new unknown forces, and I do not intend to do that, I can suppose that the electrostatic energy of a charge in a pair e^- – e^+ at distance r_e (classic radius of the electron) is due to the gravitational influence of the Universe around, that is:

$$\frac{1}{4pe_0} \cdot \frac{e^2}{r_e} = \frac{GM_{Univ} m_e}{R_{Univ}} \tag{4.5}$$

from which: $m_e r_e = \frac{1}{4pe_0} \cdot \frac{R_{Univ} e^2}{GM_{Univ}}$ (4.6)

($M_{Univ} = 1{,}59486 \cdot 10^{55}\, kg$)

If now we combine (4.4) and (4.6), we get:

$$\boxed{r_e = \left(\frac{1}{4pe_0} \cdot \frac{R_{Univ} e^2}{a_{Univ} M_{Univ}}\right)^{\frac{1}{3}} \cong 2{,}8179 \cdot 10^{-15}\, m}$$

$$\boxed{m_e = \frac{a_{Univ}}{G} r_e^2 = 9{,}1 \cdot 10^{-31}\, kg}$$

which are exactly the values physics has always taught us!

Par. 4.3: Planck's Constant.

1) We know from physics that: v=ωR and $w = 2p/T$, and, for the whole Universe: $c=\omega R_{Univ}$ and $w = 2p/T_{Univ}$, from which:

$$T_{Univ} = \frac{2pR_{Univ}}{c} = 2{,}47118 \cdot 10^{20}\, s \qquad \text{(7.840 billion years)} \tag{4.7}$$

About the angular frequency, we have: $w_{Univ} \cong c / R_{Universo-New} = 2{,}54 \cdot 10^{-20} rad/s$, and it is a right parameter for a reinterpretation of the global Hubble's constant H_{global}, whose value is H_{local} only in the portion of Universe visible by us ($w_{Univ} = H_{Global}$).

Now, if we imagine an electron ("stable" and base particle in our Universe!) irradiating all energy it's made of in time T_{Univ}, we get a power which is exactly ½ of Planck's constants, expressed in watt!
In fact:

$$L_e = \frac{m_e c^2}{T_{Univ}} = \frac{1}{2} h_W = 3{,}316 \cdot 10^{-34} W \qquad (4.8)$$

(One must not be surprised by the coefficient ½; in fact, at fundamental energy levels, it's always present, such as, for instance, on the first orbit of the hydrogen atom, where the circumference of the orbit of the electron (2πr) really is $\frac{1}{2} l_{DeBroglie}$ of the electron. The photon, too, can be represented as if it were contained in a small cube whose side is $\frac{1}{2} l_{photon}$).

Therefore, $\boxed{h = |2L_e| = \left|2\frac{1}{2} h_W\right| = 6{,}625 \cdot 10^{-34} Js\,.}$

2) As an alternative, according to the Principle of Indetermination of Heisenberg, as the product Δx Δp must keep above $\hbar/2$, and with the equal sign, when Δx is at a maximum, Δp must be at a minimum, and vice versa:

$$\Delta p \cdot \Delta x \geq \hbar/2 \quad \text{and} \quad \Delta p_{max} \cdot \Delta x_{min} = \hbar/2 \quad (\hbar = h/2\pi)$$

Now, as Δp_{max} we take, for the electron ("stable" and base particle in our Universe!), $\Delta p_{max} = (m_e \cdot c)$ and as Δx_{min} for the electron, as it is a harmonic of the Universe in which it is (just like a sound can be considered as made of its harmonics), we have: $\Delta x_{min} = a_{Univ} / (2\pi)^2$, as a direct consequence of the characteristics of the Universe in which it is; in fact, from (1.15), $R_{Univ} = a_{Univ} / w_{Univ}^2$, as we know from physics that $a = w^2 R$, and then $w_{Univ} = 2\pi / T_{Univ} = 2\pi n_{Univ}$, and as w_e of the electron (which is a harmonic of the Universe) we therefore take the "n_{Univ}–th" part of w_{Univ}, that is: $|w_e| = |w_{Univ} / n_{Univ}|$ like if the electron of the electron-positron pairs can make oscillations similar to those of the Universe, but through a speed-amplitude ratio which is not the (global) Hubble Constant, but through H_{Global} divided by n_{Univ}, and so, if for the whole Universe: $R_{Univ} = a_{Univ} / w_{Univ}^2$, then, for the electron: $\Delta x_{min} = \frac{a_{Univ}}{(w_e)^2} = \frac{a_{Univ}}{(|w_{Univ}/n_{Univ}|)^2} = \frac{a_{Univ}}{(|H_{Global}/n_{Univ}|)^2} = \frac{a_{Univ}}{(2\pi)^2}$, from which: $\Delta p_{max} \cdot \Delta x_{min} = m_e c \frac{a_{Univ}}{(2\pi)^2} = 0{,}527 \cdot 10^{-34}$ [Js] and such a number ($0{,}527 \cdot 10^{-34}$ Js), as chance would have it, is really $\hbar/2$!!

Then, as $\hbar = h/2\pi$, we have: $\boxed{h = 2\pi\hbar = m_e c \frac{a_{Univ}}{\pi} = 6{,}625 \cdot 10^{-34} Js\,.}$

Par. 4.4: Stephan-Boltzmann's Constant.

Let's go on considering the electron as a harmonic of the Universe, so a small Universe, and consider the proportion electron-Universe between mass and irradiated power, also using (4.8):

$\frac{L_e}{m_e} = \frac{L_{Univ}}{M_{Univ}}$, so: $L_{Univ} = \frac{1}{2} h_W \frac{M_{Univ}}{m_e} = \frac{M_{Univ} c^2}{T_{Univ}} = 5{,}80 \cdot 10^{51} W$

If now we remember the Stephan-Boltzmann's Law: $\frac{L_{[W]}}{4pR^2} = sT^4$, and if we use it for the Universe, after having given the Universe the same temperature of the Cosmic Microwave Background Radiation T(CMBR)= $Temp_{Univ} \cong 2{,}73K$ (isn't it?!...), we get:

$$\boxed{s = \frac{L_{Univ}}{4pR_{Univ}{}^2 (Temp_{Univ})^4} = 5{,}67 \cdot 10^{-8} W / m^2 K^4 \ !}$$

which is the very value all general physics books show.
Remark: the mean temperature you can "give" to an electron is:

$$T_e = \left(\frac{L_e}{4pr_e{}^2 s}\right)^{1/4} = \left(\frac{\frac{1}{2}h}{4pr_e{}^2 s}\right)^{1/4} \cong 2{,}73K \,!$$

Par. 4.5: The Fine Structure Constant.

We know that $a = \frac{1}{137} = \frac{\frac{1}{4pe_0} e^2}{\frac{h}{2p} c}$ (Alonso-Finn) is the Fine Structure Constant; let's try and understand the physical meaning of such a constant, by multiplying numerator and denominator by $\frac{1}{r_e}$, or also by $\frac{1}{a_0}$, where r_e is the classic radius of the electron and a_0 is the Bohr's radius, that is the radius of the orbit of the electron in a hydrogen atom:

$$a = \frac{1}{137} = \frac{\frac{1}{4pe_0} \frac{e^2}{r_e}}{\frac{h}{2pr_e} c} = \frac{\frac{1}{4pe_0} \frac{e^2}{a_0}}{\frac{h}{2pa_0} c} ; \qquad (4.9)$$

The numerator is the electrostatic energy of the electron, while the denominator is the energy that can be irradiated by the electron itself, through a photon whose frequency is $\frac{c}{2pa_0}$ ($E = hn = h\frac{c}{2pa_0}$) as, if the electron had the speed of light, it ran the circumference of the orbit $2pa_0$ in the period $T = \frac{2pa_0}{c}$, and so we have a frequency $n_c = \frac{1}{T} = \frac{c}{2pa_0}$.

Let's write down (4.9) again:

$$a = \frac{\frac{1}{4pe_0}\frac{e^2}{a_0}}{hn_c} = \frac{\frac{1}{4pe_0}\frac{e^2}{a_0}}{\frac{hc}{2pa_0}}$$, so: $\frac{1}{4pe_0}\frac{e^2}{a_0} = \frac{hac}{2pa_0} = \frac{hV}{2pa_0} = hn_V = ahn_c$. From this equation, we see that the real speed V of the electron is ac, that is $\frac{1}{137}$ of the speed of light, and so also the energy of the photon which is emitted from such an electron in the H atom, that is hn_V, is $\frac{1}{137}$ of the energy hn_c which would be emitted if the electron were not in H, but in a pair electron-positron at distance r_e and so at speed c, that is: ahn_c. As a matter of fact, we know from physics that speed V of the electron in H is $\frac{1}{137}c$.

Besides, a is also given by the speed of an electron in a hydrogen atom and the speed of light ratio: $a = v_{e\_in\_H} / c = e^2 / 2e_0 hc$, or also as the ratio between Compton wavelenght of the electron (which is the minimum λ of e^- when it's free and has the speed of light c) and the wavelength of e^- indeed, on the first orbit of H: $a = l_{Compton} / l_{1-H} = (h/m_e c)/(h/m_e v_{e\_in\_H})$. Moreover, $a = \sqrt{r_e/a_0}$, where $a_0 = 0{,}529$ Å is the Bohr's radius.

But we also see that the Fine Structure Constant can be expressed by the following equation:

$$a = \frac{1}{137} = \frac{Gm_e^2}{r_e} \Big/ hn_{Univ} \quad , \tag{4.10}$$

where, of course, $n_{Univ} = \frac{1}{T_{Univ}}$. Before, we've also seen that the other expression for it, is:

$$a = \frac{1}{137} = \frac{\frac{1}{4pe_0}e^2}{\frac{h}{2p}c} \tag{4.11}$$

We could so set the following equality and deduce the relevant consequences: $(a = \frac{1}{137}) = \frac{\frac{1}{4pe_0}e^2}{\frac{h}{2p}c} = \frac{\frac{Gm_e^2}{r_e}}{hn_{Univ}}$, from which: $\frac{1}{4pe_0}e^2 = \frac{c}{2pn_{Univ}}\frac{Gm_e^2}{r_e} = \frac{c}{H_{global}}\frac{Gm_e^2}{r_e} = R_{Univ}\frac{Gm_e^2}{r_e}$

Therefore, we can write that: $\frac{1}{4pe_0}\frac{e^2}{R_{Univ}} = \frac{Gm_e^2}{r_e}$.

Now, if we temporarily imagine, out of simplicity, that the mass of the Universe is made of N electrons e^- and positrons e^+, we could write:

$M_{Univ} = N \cdot m_e$, from which: $\frac{1}{4pe_0}\frac{e^2}{R_{Univ}} = \frac{GM_{Univ}m_e}{\sqrt{N}\sqrt{N}r_e}$, or also:

$$\frac{1}{4pe_0} \cdot \frac{e^2}{(R_{Univ}/\sqrt{N})} = \frac{GM_{Univ}m_e}{\sqrt{N}r_e} \quad . \tag{4.12}$$

If now we suppose that $R_{Univ} = \sqrt{N}r_e$, or, by the same token, $r_e = R_{Univ}/\sqrt{N}$, then (4.12) becomes:

$$\boxed{\frac{1}{4pe_0}\cdot\frac{e^2}{r_e}=\frac{GM_{Univ}m_e}{R_{Univ}}} \quad !! \qquad (4.13)$$

Now, first of all we see that the supposition $R_{Univ}=\sqrt{N}r_e$ is very right, as from the definition of N above given and from the value of the mass of the Universe, we have:

$N=\frac{M_{Univ}}{m_e}\cong 1{,}75\cdot 10^{85}$ (~Eddington), from which: $\sqrt{N}\cong 4{,}13\cdot 10^{42}$ (~Weyl) and

$R_{Univ}=\sqrt{N}r_e\cong 1{,}18\cdot 10^{28}m$, that is the very R_{Univ} value we know.

Summing up: (4.10), (4.11) and (4.13) tell us that the Fine Structure Constant comes out not only from the characteristics of atoms and particles, but also from those of the Universe and there is a more important particular: such characteristics, microscopic and macroscopic ones, are deeply linked each other!

Par. 4.6: The Boltzmann's Constants.

The integration of the Planck's Black Body Radiation Equation gives the Stephan-Boltzmann's Law:

$e=\frac{2ph}{c^2}\int_0^{\infty}\frac{n^3dn}{(e^{hn/kT}-1)}=\frac{2p^5k^4}{15c^2h^3}T^4=sT^4$, that is: $s=\frac{2p^5k^4}{15c^2h^3}$, from which:

$k=(\frac{15c^2h^3s}{2p^5})^{\frac{1}{4}}=1{,}38\cdot 10^{-23}J/K$, coming out from constants we've already found, so far.

Par. 4.7: The Universal Gravitational Constant.

In Par. 4.1 we saw that:

$c=\sqrt{\frac{GM_{Univ}}{R_{Univ}}}\cong 3\cdot 10^8 m/s$, from which, if we like:

$G=\frac{c^2R_{Univ}}{M_{Univ}}\cong 6{,}67\cdot 10^{-11}N\cdot m^2/kg^2$.

Appendixes:

App. 1: As I see the Universe (Unification Gravity Electromagnetism).

Contents of App. 1:

-Contents of App. 1. Page 32

-App. 1-Chapter 1: A new Universe, 100 times bigger, more massive and older. Page 32
App. 1-Par. 1.1: No dark matter! Page 32
App. 1-Par. 1.2: Cosmic acceleration a_{Univ}. Page 33
App. 1-Par. 1.3: The new density of the Universe. Page 34
App. 1-Par. 1.4: Further considerations on the meaning of a_{Univ}. Page 35
App. 1-Par. 1.5: Further confirmations and encouragements from other branches of physics. Page 35
App. 1-Par. 1.6: On discrepancies between calculated and observed rotation speeds of galaxies. Page 37

-App. 1-Chapter 2: The unification of electromagnetic and gravitational forces (Rubino). Page 38
App. 1-Par. 2.1: The effects of M_{Univ} on particles. Page 38
App. 1-Par. 2.2: The discovery of the common essence of gravity and electromagnetism. Page 39
App. 1-Par. 2.3: The oscillatory essence of the whole Universe and of its particles. Page 40

-App. 1-Chapter 3: The unification of magnetic and electric forces. Page 41
App. 1-Par. 3.1: Magnetic force is simply a Coulomb's electric force(!). Page 41

-App. 1-Chapter 4: Justification of the equation $R_{Univ} = \sqrt{N} r_e$ previously used for the unification of electric and gravitational forces (Rubino). Page 43
App. 1-Par. 4.1: The equation $R_{Univ} = \sqrt{N} r_e$ (!). Page 43

-App. 1-Chapter 5: "a_{Univ}" as absolute responsible of all forces. Page 44
App. 1-Par. 5.1: Everything from "a_{Univ}". Page 44
App. 1-Par. 5.2: Summarizing table of forces. Page 45
App. 1-Par. 5.3: Further considerations on composition of the Universe in pairs +/-. Page 45
App. 1-Par. 5.4: The Theory of Relativity is just an interpretation of the oscillating Universe just described contracting with speed c and acceleration a_{univ}. Page 45
App. 1-Par. 5.5: On "Relativity" of lost energies. Page 47

-App. 1-SUBAPPENDIXES. Page 48
App. 1-Subappendix 1: Physical constants. Page 48

App. 1-Chapter 1: A new Universe, 100 times bigger, more massive and older.

App. 1-Par. 1.1: No dark matter!

ON DISCREPANCIES BETWEEN CALCULATED AND OBSERVED DENSITIES **ρ**Univ :
The search for 99% of matter in the Universe, after that it has been held invisible sounds somewhat strange. And it's a lot of matter, as dark matter should be much more than the visible one (from 10 to 100 times more).

Astrophysicists measure a ρ value of visible Universe which is around: $r \cong 2 \cdot 10^{-30} kg/m^3$.
Prevailing cosmology nowadays gives the following value of ρ: (see also (A1.6)):

$$r_{Wrong} = H_{local}^2 / (\frac{4}{3} pG) \cong 2 \cdot 10^{-26} kg/m^3 \text{ (too high!)} \quad . \qquad \text{(A1.1)}$$

Let's use the following plausible value for Hlocal (local Hubble's constant – see (A1.7) below):

$$H_{local} \cong 75 km/(s \cdot Mpc) \cong 2{,}338 \cdot 10^{-18} [(\frac{m}{s}) \Big/ m] \qquad \text{(A1.2)}$$

confirmed by many measurements on Coma cluster, for instance, (see (A1.7) below) and this also confirms that the farthest objects ever observed are travelling away with a speed close to that of light:

$$H_{local} \approx c / R_{Universe-Old} \text{ , from which: } R_{Univ-Old} \approx c / H_{local} \approx 4000 Mpc \approx 13{,}5 \cdot 10^9 light\_year \qquad \text{(A1.3)}$$

Moreover, one can easily calculate the speed of a "gravitating" mass m at the edge of the visible Universe, by the following equality between centrifugal and gravitational forces:

$$m \cdot a = m \cdot \frac{c^2}{R_{Univ-Old}} = G \cdot m \cdot M_{Univ-Old} / R^2_{Univ-Old} \quad , \qquad \text{(A1.4)}$$

from which, also considering (A1.3), we have:

$$M_{Univ-Old} = c^3 /(G \cdot H_{local}) \cong 1{,}67 \cdot 10^{53} kg \qquad \text{(A1.5)}$$

and so:

$$r_{Wrong} = M_{Univ-Old} /(\frac{4}{3} p R^3_{Univ-Old}) = (c^3 / GH_{local}) \Big/ [\frac{4}{3} p (\frac{c}{H_{local}})^3] = H^2_{local} /(\frac{4}{3} pG) \cong 2 \cdot 10^{-26} kg / m^3 \qquad \text{(A1.6)}$$

i.e. (A1.1) indeed (too high value!)

Good..., sorry, bad; this value is ten thousand times higher than the observed density value, which has been measured by astrophysicists. Moreover, galaxies are too "light" to spin so fast (see further on). As a consequence, they decided to take up searching for dark matter, and a lot of, as it should be much more than the visible one (from 10 to 100 times more).

On the contrary, astrophysicists detect a value for ρ around: $r \cong 2 \cdot 10^{-30} kg / m^3$.

Let's try to understand which arbitrary choices, through decades, led to this discrepancy. From Hubble's observations on, we understood far galaxies and clusters got farther with speeds determined by measurements of the red shift. Not only; the farthest ones have got higher speeds and it quite rightly seems there's a law between the distance from us of such objects and the speeds by which they get farther from us.

Fig. A1.1 below is a picture of the Coma cluster, about which hundreds of measurements are available; well, we know the following data about it:

distance Δx=100 Mpc = 3,26 10^8 l.y. = 3,09 10^{24} m

speed Δv=6870 km/s=6,87 10^6 m/s.

Fig. A1.1: Coma cluster.

If we use data on Coma cluster to figure out the Hubble's constant Hlocal, we get:

$$H_{local} = \Delta v \Big/ \Delta x \cong 2{,}22 \cdot 10^{-18} [(\frac{m}{s}) \Big/ m] , \qquad \text{(A1.7)}$$

That is a good value for "local" Hubble's constant.

App. 1-Par. 1.2: The cosmic acceleration a_{Univ}.

As a confirmation of all we just said, we also got the same Hlocal value from (A1.3) when we used data on the visible Universe of 13,5 10^9 l.y. radius and ~c speed, instead of data on Coma cluster. By the same reasonings which led us so far to get the Hlocal constant definition, we can also state that if galaxies increase their own speeds with going farther, then they are accelerating with an acceleration we call a_{Univ} , and, from physics, we know that:

$\Delta x = \frac{1}{2} a \cdot \Delta t^2 = \frac{1}{2}(a \cdot \Delta t) \cdot \Delta t = \frac{1}{2} \Delta v \cdot \Delta t$, from which: $\Delta t = \frac{2 \cdot \Delta x}{\Delta v}$, which, if used in the definition of acceleration a_{Univ} , yields:

$$a_{Univ} = \frac{\Delta v}{\Delta t} = \frac{\Delta v}{\frac{2 \cdot \Delta x}{\Delta v}} = \frac{(\Delta v)^2}{2 \cdot \Delta x} = a_{Univ} \cong 7{,}62 \cdot 10^{-12} m/s^2 , \quad \text{cosmic acceleration} \qquad \text{(A1.8)}$$

after that we used data on Coma cluster.
This is the acceleration by which all our visible Universe is accelerating towards the center of mass of the whole Universe.

App. 1-Par. 1.3: The new density of the Universe.
Now, we say the Universe is 100 times bigger and heavier:

$$R_{Univ-New} \cong 100 R_{Univ} \cong 1{,}17908 \cdot 10^{28} m \qquad \text{(A1.9)}$$

$$M_{Univ-New} \cong 100 M_{Univ} \cong 1{,}59486 \cdot 10^{55} kg \qquad \text{(A1.10)}$$

This value of radius is 100 times the one previously calculated in (A1.3) and it should represent the radius between the center of mass of the Universe and the place where we are now, place in which the speed of light is c.
((as we are not exactly on the edge of such a Universe, we can demonstrate the whole radius is larger by a factor $\sqrt{2}$, that is R_{Univ}=1,667 10^{28}m.))

Anyway, we are dealing with linear dimensions 100 times those supported in the prevailing cosmology nowadays. We can say that there is invisible matter, but it is beyond the range of our largest telescopes and not inside galaxies or among them; the dark matter should upset laws of gravitations, but they hold very well.

By these new bigger values, we also realize that:

$$c^2 = \frac{GM_{Univ}}{R_{Univ}} \; ! \qquad \text{(A1.11)}$$

By the assumptions in the (A1.9) and (A1.10), we get:

$$\boxed{r = M_{Univ-New} / (\frac{4}{3} p \cdot R^3_{Univ-New}) = 2.32273 \cdot 10^{-30} kg/m^3 \quad !} \qquad \text{(A1.12)}$$

which is the right measured density!

And we also see that:

$a_{Univ} = \frac{c^2}{R_{Univ-New}} = 7{,}62 \cdot 10^{-12} \; m/s^2$, (as we know, from physics, that $a = \frac{v^2}{r}$)

as well as:

$a_{Univ} = G \cdot M_{Univ-New} / R^2_{Univ-New} = 7{,}62 \cdot 10^{-12} \; m/s^2$ (from the Newton's Universal Law of Gravitation)

The new density in the (A1.12) is very very close to that observed and measured by astrophysicists and already reported at page 32.
Nature fortunately sends encouraging and convincing signs on the pursuit of a way, when confirmations on what one has understood are coming from branches of physics very far from that in which one is investigating.
On the basis of that, let's remind ourselves of the classic radius of an electron ("stable" and base particle in our Universe!), which is defined by the equality of its energy E=$m_e c^2$ ant its electrostatic one, imagined on its surface (in a classic sense):

$m_e \cdot c^2 = \frac{1}{4pe_0} \frac{e^2}{r_e}$, so:

$$r_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{m_e \cdot c^2} \cong 2{,}8179 \cdot 10^{-15} m \qquad \text{(A1.13)}$$

Now, still in a classic sense, if we imagine, for instance, to figure out the gravitational acceleration on an electron, as if it were a small planet, we must easily conclude that: $m_x \cdot g_e = G\frac{m_x \cdot m_e}{r_e^2}$, so:

$$g_e = G\frac{m_e}{r_e^2} = 8\pi^2\varepsilon_0^2\frac{Gm_e^3c^4}{e^4} = a_{Univ} = 7{,}62 \cdot 10^{-12}\ m/s^2 \text{ !!!} \qquad \text{(A1.14)}$$

that is the very value obtained in (A1.8) through different reasonings, macroscopic, and not microscopic, as it was for (A1.14). All in all, why should gravitational behaviours of the Universe and of electrons (making it) be different?

App. 1-Par. 1.4: Further considerations on the meaning of a_{Univ}.

Well, we have to admit that if matter shows mutual attraction as gravitation, then we are in a harmonic and oscillating Universe in contraction towards a common point, that is the center of mass of all the Universe. As a matter of fact, the acceleration towards the center of mass of the Universe and the gravitational attractive properties are two faces of the same medal. Moreover, all the matter around us shows it want to collapse: if I have a pen in my hand and I leave it, it drops, so showing me it wants to collapse; then, the Moon wants to collapse into the Earth, the Earth wants to collapse into the Sun, the Sun into the centre of the Milky Way, the Milky Way into the centre of the cluster and so on; therefore, all the Universe is collapsing. Isn't it?

So why do we see far matter around us getting farther and not closer? Easy. If three parachutists jump in succession from a certain altitude, all of them are falling towards the center of the Earth, where they would ideally meet, but if parachutist n. 2, that is the middle one, looks ahead, he sees n. 1 getting farther, as he jumped earlier and so he has a higher speed, and if he looks back at n. 3, he still sees him getting farther as n. 2, who is making observations, jumped before n. 3 and so he has a higher speed. Therefore, although all the three are accelerating towards a common point, they see each other getting farther. Hubble was somehow like parachutist n. 2 who is making observations here, but he didn't realize of the background acceleration g (a_{Univ}).

At last, I remind you of the fact that recent measurements on Ia type supernovae in far galaxies, used as standard candles, have shown an accelerating Universe; this fact is against the theory of our supposed current post Big Bang expansion, as, after that an explosion has ceased its effect, chips spread out in expansion, ok, but they must obviously do that without accelerating.

Moreover, on abundances of U^{235} and U^{238} we see now (trans-CNO elements created during the explosion of the primary supernova, we see that (maybe) the Earth and the solar system are just (approximately) five or six billion years old, but all this is not against all what just said on the real age of the Universe, as there could have been sub-cycles from which galaxies and solar systems originated, whose duration is likely less than the age of the whole Universe.

About T_{Univ} of the Universe, we know from physics that: v=ωR and $\omega = 2\pi / T$, and, for the whole Universe: $c=\omega R_{Univ}$ and $\omega = 2\pi / T_{Univ}$, from which:

$$T_{Univ} = \frac{2\pi R_{Univ}}{c} = 2{,}47118 \cdot 10^{20} s \qquad \text{(7.840 billion years)} \qquad \text{(A1.15)}$$

About the angular frequency: $\omega_{Univ} \cong c / R_{Universo-New} = 2{,}54 \cdot 10^{-20} rad / s$, and it is a right parameter for a reinterpretation of the global Hubble's constant H_{global} , whose value is H_{local} only in the portion of Universe visible by us ($\omega_{Univ} = H_{Global}$).

App. 1-Par. 1.5: Further confirmations and encouragements from other branches of physics.

1) Stephan-Boltzmann's law:

$e = \sigma T^4$ [W/m²], where $\sigma = 5{,}67 \cdot 10^{-8}\, W/(m^2K^4)$

It's very interesting to notice that if we imagine an electron ("stable" and base particle in our Universe!) irradiating all energy it's made of in time T_{Univ} , we get a power which is exactly ½ of Planck's constants, expressed in watt!

In fact:

$$L_e = \frac{m_e c^2}{T_{Univ}} = \frac{1}{2}h_W = 3{,}316 \cdot 10^{-34} W$$

(One must not be surprised by the coefficient ½; in fact, at fundamental energy levels, it's always present, such as, for instance, on the first orbit of the hydrogen atom, where the circumference of the orbit of the electron (2πr) really is $\frac{1}{2}\lambda_{DeBroglie}$ of the electron. The photon, too, can be represented as if it were contained in a small cube whose side is $\frac{1}{2}\lambda_{photon}$).

2) Moreover, we notice that an electron and the Universe have got the same luminosity-mass ratio:

in fact, $L_{Univ} = \frac{M_{Univ}c^2}{T_{Univ}} = 5{,}80 \cdot 10^{51} W$ (by definition) and it's so true that:

$$\frac{L_{Univ}}{M_{Univ}} = \frac{\frac{M_{Univ}c^2}{T_{Univ}}}{M_{Univ}} = \frac{c^2}{T_{Univ}} = \frac{L_e}{m_e} = \frac{\frac{m_e c^2}{T_{Univ}}}{m_e} = \frac{c^2}{T_{Univ}} = \frac{\frac{1}{2}h_W}{m_e}$$

and, according to Stephan-Boltzmann's law, we can consider that both an "electron" and the Universe have got the same temperature, the cosmic microwave background one:

$$\frac{L}{4pR^2} = sT^4 \text{, so: } T = \left(\frac{L}{4pR^2 s}\right)^{1/4} = \left(\frac{L_{Univ}}{4pR_{Univ}{}^2 s}\right)^{1/4} = \left(\frac{L_e}{4pr_e{}^2 s}\right)^{1/4} = \left(\frac{\frac{1}{2}h}{4pr_e{}^2 s}\right)^{1/4} = 2{,}73K \quad !!!$$

3) The Heisenberg Uncertainty Principle as a consequence of the essence of the macroscopic and a_{Univ} accelerating Universe:

according to this principle, the product Δx Δp must keep above $\mathbf{h}/2$, and with the equal sign, when Δx is at a maximum, Δp must be at a minimum, and vice versa:

$$\Delta p \cdot \Delta x \geq \mathbf{h}/2 \quad \text{and} \quad \Delta p_{\max} \cdot \Delta x_{\min} = \mathbf{h}/2 \quad (\mathbf{h} = h/2p)$$

Now, as $\Delta p_{\max}$ we take, for the electron ("<u>stable</u>" and base particle in our Universe!), $\Delta p_{\max} = (m_e \cdot c)$ and as $\Delta x_{\min}$ for the electron, as it is a harmonic of the Universe in which it is (just like a sound can be considered as made of its harmonics), we have: $\Delta x_{\min} = a_{Univ}/(2p)^2$, as a direct consequence of the characteristics of the Universe in which it is; in fact, $R_{Univ} = a_{Univ}/w_{Univ}^2$, as we know from physics that $a = w^2 R$, and then $w_{Univ} = 2p/T_{Univ} = 2pn_{Univ}$, and as w_e of the electron (which is a harmonic of the Universe) we therefore take the "n_{Univ}–th" part of w_{Univ}, that is:

$|w_e| = |w_{Univ}/n_{Univ}|$ like if the electron of the electron-positron pairs can make oscillations similar to those of the Universe, but through a speed-amplitude ratio which is not the (global) Hubble Constant, but through H_{Global} divided by n_{Univ}, and so, if for the whole Universe: $R_{Univ} = a_{Univ}/w_{Univ}^2$, then, for the electron:

$$\Delta x_{\min} = \frac{a_{Univ}}{(w_e)^2} = \frac{a_{Univ}}{(|w_{Univ}/n_{Univ}|)^2} = \frac{a_{Univ}}{(|H_{Global}/n_{Univ}|)^2} = \frac{a_{Univ}}{(2p)^2}$$, from which:

$\Delta p_{\max} \cdot \Delta x_{\min} = m_e c \frac{a_{Univ}}{(2p)^2} = 0{,}527 \cdot 10^{-34}$ [Js] and such a number ($0{,}527 \cdot 10^{-34}$ Js), as chance would have it, is really $\mathbf{h}/2$!!

4) As we previously did, let's remind ourselves of the classic radius of an electron ("<u>stable</u>" and base particle in our Universe!), which is defined by the equality of its energy $E=m_e c^2$ ant its electrostatic one, imagined on its surface (in a classic sense):

$$m_e \cdot c^2 = \frac{1}{4pe_0}\frac{e^2}{r_e} \text{, so:}$$

$$r_e = \frac{1}{4pe_0}\frac{e^2}{m_e \cdot c^2} \cong 2{,}8179 \cdot 10^{-15} m$$

Now, still in a classic sense, if we imagine, for instance, to figure out the gravitational acceleration on an electron, as if it were a small planet, we must easily conclude that: $m_x \cdot g_e = G\frac{m_x \cdot m_e}{r_e^2}$, so:

$$g_e = G\frac{m_e}{r_e^2} = 8p^2 e_0^2 \frac{Gm_e^3 c^4}{e^4} = a_{Univ} = 7{,}62 \cdot 10^{-12}\, m/s^2 \text{ !!!}$$

5) We know that $a = \dfrac{1}{137}$ is the value of the Fine structure Constant and the following formula $\dfrac{Gm_e^2}{r_e} \Big/ hn$ yields the same value only if n is the one of the Universe we just described, that is: $a = \dfrac{1}{137} = \dfrac{Gm_e^2}{r_e} \Big/ hn_{Univ}$, where, clearly: $n_{Univ} = \dfrac{1}{T_{Univ}}$ (see (A1.15)) !!

6) If I suppose, out of simplicity, that the Universe is made of just harmonics, as electrons e^- (and/or positrons e^+), their number will be: $N = \dfrac{M_{Univ}}{m_e} \cong 1{,}75 \cdot 10^{85}$ (~Eddington); the square root of such a number is: $\sqrt{N} \cong 4{,}13 \cdot 10^{42}$ (~Weyl).

Now, we are surprised to notice that $\sqrt{N} r_e \cong 1{,}18 \cdot 10^{28} m$ (!), that is, the very R_{Univ} value we had in (A1.9) ($R_{Univ} = \sqrt{N} r_e \cong 1{,}18 \cdot 10^{28} m$) !!!

App. 1-Par. 1.6: On discrepancies between calculated and observed rotation speeds of galaxies.

Fig. A1.2: Andromeda galaxy (M31).

Andromeda galaxy (M31):

Distance: 740 kpc; R_{Gal}=30 kpc;
Visible Mass M_{Gal} = 3 $10^{11} M_{Sun}$;
Suspect Mass (+Dark) M_{+Dark} = 1,23 $10^{12} M_{Sun}$;
M_{Sun}=2 10^{30} kg; 1 pc= 3,086 10^{16} m;

By balancing centrifugal and gravitational forces for a star at the edge of a galaxy:

$m_{star} \dfrac{v^2}{R_{Gal}} = G \dfrac{m_{star} M_{Gal}}{R_{Gal}^2}$, from which: $v = \sqrt{\dfrac{GM_{Gal}}{R_{Gal}}}$

On the contrary, if we also consider the tidal contribution due to a_{Univ}, i.e. the one due to all the Universe around, we get:

$v = \sqrt{\dfrac{GM_{Gal}}{R_{Gal}} + a_{Univ} R_{Gal}}$; let's figure out, for instance, in M31, how many R_{Gal} (how many k times) far away from the center of the galaxy the contribution from a_{Univ} can save us from supposing the existence of dark matter:

$\sqrt{\dfrac{GM_{+Dark}}{kR_{Gal}}} = \sqrt{\dfrac{GM_{Gal}}{kR_{Gal}} + a_{Univ} kR_{Gal}}$, so: $k = \sqrt{\dfrac{G(M_{+Dark} - M_{Gal})}{a_{Univ} R_{Gal}^2}} \cong 4$, therefore, at 4$R_{Gal}$ far away, the existence of a_{Univ} makes us obtain the same high speeds observed, without any dark matter. Moreover, at 4R_{Gal} far away, the contribution due to a_{Univ} is dominant.

At last, we notice that a_{Univ} has no significant effect on objects as small as the solar system; in fact:

$G \dfrac{M_{Sun}}{R_{Earth-Sun}} \cong 8{,}92 \cdot 10^8 >> a_{Univ} R_{Earth-Sun} \cong 1{,}14$.

All these considerations on the link between a_{Univ} and the rotation speed of galaxies are widely open to further speculations and the equation through which one can take into account the tidal effects of a_{Univ} in the galaxies can have a somewhat different and more difficult look, with respect to the above one, but the fact that practically all galaxies have dimensions in a somewhat narrow range (3 – 4 $R_{Milky\ Way}$ or not so much more) doesn't seem to be like that just by chance, and, in any case, none of them have radii as big as tents or hundreds of $R_{Milky\ Way}$, but rather by just some times. In fact, the part due to the cosmic acceleration, by zeroing the centripetal acceleration in some phases of the revolution of galaxies, would fringe the galaxies themselves, and, for instance, in M31, it equals the gravitational part at a radius equal to:

$\frac{GM_{M31}}{R_{Gal-Max}} = a_{Univ} R_{Gal-Max}$, from which: $R_{Gal-Max} = \sqrt{\frac{GM_{M31}}{a_{Univ}}} \cong 2{,}5 R_{M31}$; in fact, maximum radii ever observed in galaxies are roughly this size.

App. 1-Chapter 2: The unification of electromagnetic and gravitational forces (Rubino).

App. 1-Par. 2.1: The effects of M_{Univ} on particles.

We remind you that from the definition of r_e in (A1.13): $\frac{1}{4pe_0} \cdot \frac{e^2}{r_e} = m_e c^2$ and from the (A1.11): $c^2 = \frac{GM_{Univ}}{R_{Univ}}$ (~Eddington), we get:

$$\boxed{\frac{1}{4pe_0} \cdot \frac{e^2}{r_e} = \frac{GM_{Univ} m_e}{R_{Univ}}} \ !! \qquad \text{(A2.1)}$$

As an alternative, we know that the Fine structure Constant is 1 divided by 137 and it's given by the following equation:

$a = \frac{1}{137} = \frac{\frac{1}{4pe_0} e^2}{\frac{h}{2p} c}$ (Alonso-Finn), but we also see that $\frac{1}{137}$ is given by the following equation, which can be considered suitable, as well, as the Fine structure Constant:

$a = \frac{1}{137} = \frac{\frac{Gm_e^2}{r_e}}{hn_{Univ}} = \frac{E_{Box\_Min}}{E_{Emanable}}$, where $n_{Univ} = \frac{1}{T_{Univ}}$. E_{Box_Min} is the smallest box of energy in the Universe (the electron), while $E_{Emanable}$ is the smallest emanable energy, as n_{Univ} is the smallest frequency.

Besides, a is also given by the speed of an electron in a hydrogen atom and the speed of light ratio: $a = v_{e\_in\_H} / c = e^2 / 2e_0 hc$, or also as the ratio between Compton wavelenght of the electron (which is the minimum λ of e^- when it's free and has the speed of light c) and the wavelength of e^- indeed, on the first orbit of H: $a = l_{Compton} / l_{1-H} = (h/m_e c)/(h/m_e v_{e\_in\_H})$. Moreover, $a = \sqrt{r_e / a_0}$, where $a_0 = 0{,}529$ Å is the Bohr's radius.

So, we could set the following equation and deduce the relevant consequences (Rubino):

$(a = \frac{1}{137}) = \frac{\frac{1}{4pe_0} e^2}{\frac{h}{2p} c} = \frac{\frac{Gm_e^2}{r_e}}{hn_{Univ}}$, from which: $\frac{1}{4pe_0} e^2 = \frac{c}{2pn_{Univ}} \frac{Gm_e^2}{r_e} = \frac{c}{H_{global}} \frac{Gm_e^2}{r_e} = R_{Univ} \frac{Gm_e^2}{r_e}$

after that (A1.15) has been used.

Therefore, we can write: $\frac{1}{4pe_0} \frac{e^2}{R_{Univ}} = \frac{Gm_e^2}{r_e}$ (and this intermediate equation, too, shows a deep relationship between electromagnetism and gravitation, but let's go on...)

Now, if we temporarily imagine, out of simplicity, that the mass of the Universe is made of N electrons e^- and positrons e^+, we could write:

$M_{Univ} = N \cdot m_e$, from which: $\frac{1}{4pe_0}\frac{e^2}{R_{Univ}} = \frac{GM_{Univ}m_e}{\sqrt{N}\sqrt{N}r_e}$,

or also: $$\frac{1}{4pe_0} \cdot \frac{e^2}{(R_{Univ}/\sqrt{N})} = \frac{GM_{Univ}m_e}{\sqrt{N}r_e} \quad . \qquad \text{(A2.2)}$$

If now we suppose that $R_{Univ} = \sqrt{N}r_e$ (see also (A4.2)), or, by the same token, $r_e = R_{Univ}/\sqrt{N}$, then (A2.2) becomes:

$\boxed{\frac{1}{4pe_0} \cdot \frac{e^2}{r_e} = \frac{GM_{Univ}m_e}{R_{Univ}}}$!! (Rubino) that is (A2.1) again.

Now, first of all we see that the supposition $R_{Univ} = \sqrt{N}r_e$ is very right, as from the definition of N above given (A1.10), we have:

$N = \frac{M_{Univ}}{m_e} \cong 1{,}75 \cdot 10^{85}$ (~Eddington), from which: $\sqrt{N} \cong 4{,}13 \cdot 10^{42}$ (~Weyl) and

$R_{Univ} = \sqrt{N}r_e \cong 1{,}18 \cdot 10^{28}\, m$, that is the very R_{Univ} value obtained in (A1.9).

App. 1-Par. 2.2: The discovery of the common essence of gravity and electromagnetism.

Now, (A2.1) is of a paramount importance and has got a very clear meaning (Rubino) as it tells us that the electrostatic energy of an electron in an electron-positron pair (e^+e^- adjacent) is exactly the gravitational energy given to this pair by the whole Universe M_{Univ} at an R_{Univ} distance! (and vice versa)

Therefore, an electron gravitationally cast by an enormous mass M_{Univ} for a very long time T_{Univ} and through a long travel R_{Univ}, gains a gravitationally originated kinetic energy so that, if later it has to release it all together, in a short time, through a collision, for instance, and so through an oscillation of the e^+e^- pair - spring, it must transfer a so huge gravitational energy indeed, stored in billion of years that if this energy were to be due just to the gravitational potential energy of the so small mass of the electron itself, it should fall short by many orders of size. Therefore, the effect due to the immediate release of a big stored energy, by e^-, which is known to be $\frac{GM_{Univ}m_e}{R_{Univ}}$, makes the electron "appear", in the very moment, and in a narrow range (r_e), to be able to release energies coming from forces stronger than the gravitational one, or like if it were able to exert a special gravitational force, through a special Gravitational Universal Constant G', much bigger than G:

$(\frac{1}{4pe_0} \cdot \frac{e}{m_e} \cdot \frac{e}{m_e}) \cdot \frac{m_e m_e}{r_e} = G' \cdot \frac{m_e m_e}{r_e}$; it's only that during the sudden release of energy by the electron, there is a run taking effect due to its eternal free (gravitational) falling in the Universe. And, at the same time, gravitation is an effect coming from the composition of many small electric forces.

I also remark here, that the energy represented by (A2.1), as chance would have it, is really $m_e c^2$!!!, that is a sort of run taking kinetic energy, had by the free falling electron-positron pair , and that Einstein assigned to the rest matter, unfortunately without telling us that such a matter is never at rest with respect to the center of mass of the Universe, as we all are inexorably free falling, even though we see one another at rest; from which is its essence of gravitationally originated kinetic energy $m_e c^2$:

$$m_e c^2 = \frac{1}{4pe_0} \cdot \frac{e^2}{r_e} = \frac{GM_{Univ}m_e}{R_{Univ}} .$$

App. 1-Par. 2.3: The oscillatory essence of the whole Universe and of its particles.

We're talking about oscillations as this is the way the energy is transferred, and also in collisions, such as those among billiards balls, where there do are oscillations in the contact point, and how, even though we cannot directly see them (those of peripheral electrons, of molecules, of atoms etc, in the contact point).

So, we're properly talking about oscillations also because, for instance, a single hydrogen atom, or a e^+e^- pair, which are ruled by laws of electromagnetism, behave as real springs: in fact, in polar coordinates, for an electron orbiting around a proton, there is a balancing between the electrostatic attraction and the centrifugal force:

$$F_r = -\frac{1}{4pe_0}\frac{e^2}{r^2} + m_e(\frac{dj}{dt})^2 r = -\frac{1}{4pe_0}\frac{e^2}{r^2} + \frac{p^2}{m_e r^3}$$, where $\frac{dj}{dt} = w$ and $p = m_e v \cdot r = m_e w r r = m_e w r^2$

Let's figure out the corresponding energy by integrating such a force over the space:

$$U = -\int F_r dr = -\frac{1}{4pe_0}\frac{e^2}{r} + \frac{p^2}{2m_e r^2}. \qquad \text{(A2.3)}$$

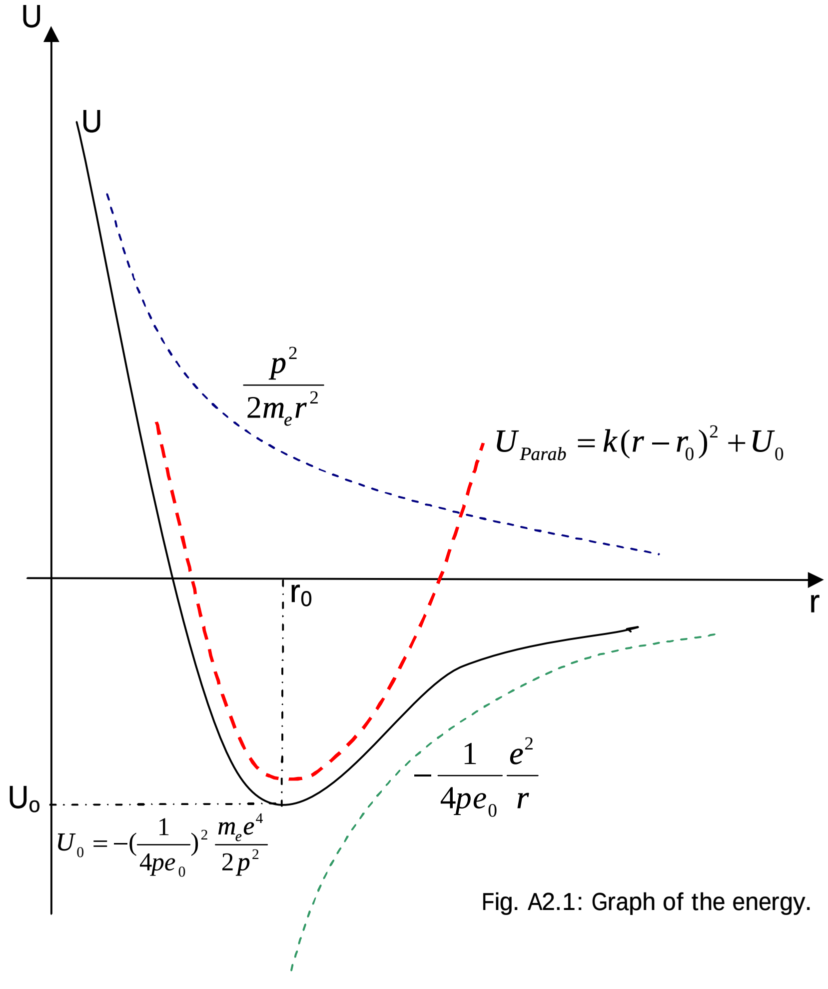
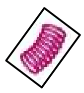


Fig. A2.1: Graph of the energy.

The point of minimum in (r_0,U_0) is a balance and stability point (F_r=0) and can be calculated by zeroing the first derivative of (A2.3) (i.e. setting F_r=0 indeed).

Moreover, around r_0, the curve for U is visibly replaceable by a parabola U_{Parab}, so, in that neighbourhood, we can write:

$U_{Parab} = k(r - r_0)^2 + U_0$, and the relevant force is: $F_r = -\partial U_{Parab}/\partial r = -2k(r - r_0)$

Which is, as chance would have it, an elastic force ($F = -kx$ - Hooke's Law). 

Moreover, the gravitational law which is followed by the Universe is a force which changes with the square value of the distance, just like the electric one, so the gravitational force, too, leads to the Hooke's law for the Universe.

By means of (A2.1) and of its interpretation, we have turned the essence of the electric force into that of the gravitational one; now we do the same between the electric and magnetic force, so accomplishing the unification of electromagnetic and gravitational fields. At last, all these fields are traced back to a_{Univ} , as gravitation does.

App. 1-Chapter 3: The unification of magnetic and electric forces.

App. 6-Par. 3.1: Magnetic force is simply a Coulomb's electric force(!).

Concerning this, let's examine the following situation, where we have a wire, of course made of positive nuclei and electrons, and also a cathode ray (of electrons) flowing parallel to the wire:

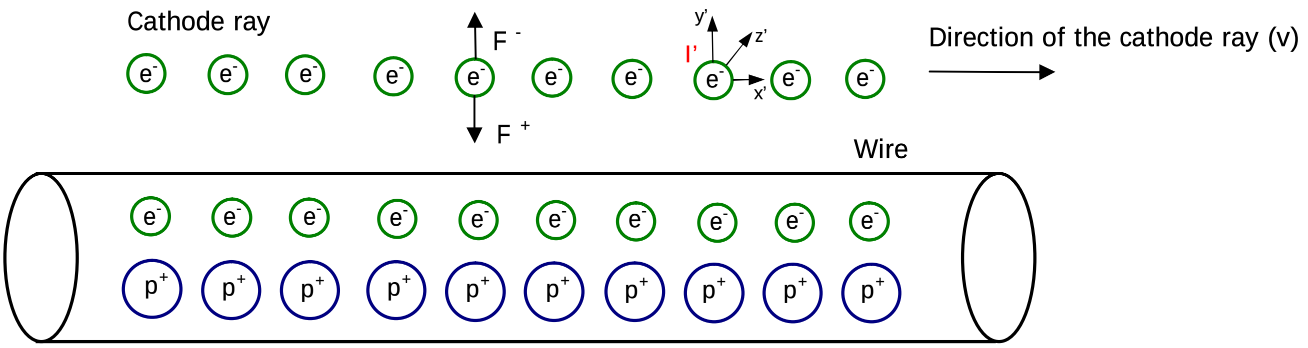


Fig. A3.1: Wire not flown by any current, seen from the cathode ray steady ref. system I' (x', y', z').

We know from magnetism that the cathode ray will not be bent towards the wire, as there isn't any current in it. This is the interpretation of the phenomenon on a magnetic basis; on an electric basis, we can say that every single electron in the ray is rejected away from the electrons in the wire, through a force F^- identical to that F^+ through which it's attracted from positive nuclei in the wire.

Now, let's examine the situation in which we have a current in the wire (e^- with speed u)

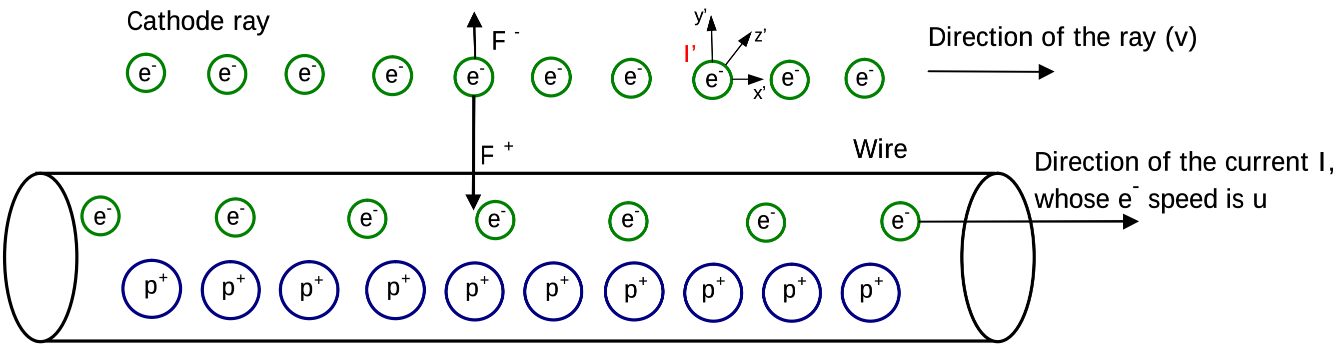


Fig. A3.2: Wire flown by a current (with e^- speed=u), seen from the cathode ray steady ref. system I' (x', y', z').

In this case we know from magnetism that the cathode ray must bend towards the wire, as we are in the well known case of parallel currents in the same direction, which must attract each other.

This is the interpretation of this phenomenon on a magnetic basis; on an electric basis, we can say that as the electrons in the wire follow those in the ray, they will have a speed lower than that of the positive nuclei, in the system I', as such nuclei are still in the wire. As a consequence of that, spaces among the electrons in the wire will undergo a lighter relativistic Lorentz contraction, if compared to that of the nuclei's, so there will be a lower negative charge density, if compared to the positive one, so electrons in the ray will be electrically attracted by the wire.

This is the interpretation of the magnetic field on an electric basis. Now, although the speed of electrons in an electric current is very low (centimeters per second), if compared to the relativistic speed of light, we must also acknowledge that the electrons are billions and billions...., so a small Lorentz contraction on so many spaces among charges, makes a substantial magnetic force to appear.

But now let's see if mathematics can prove we're quantitatively right on what asserted so far, by showing that the magnetic force is an electric one itself, but seen on a relativistic basis.

On the basis of that, let's consider a simplified situation in which an electron e^- , whose charge is q, moves with speed v and parallel to a nuclei current whose charge is Q^+ each (and speed u):

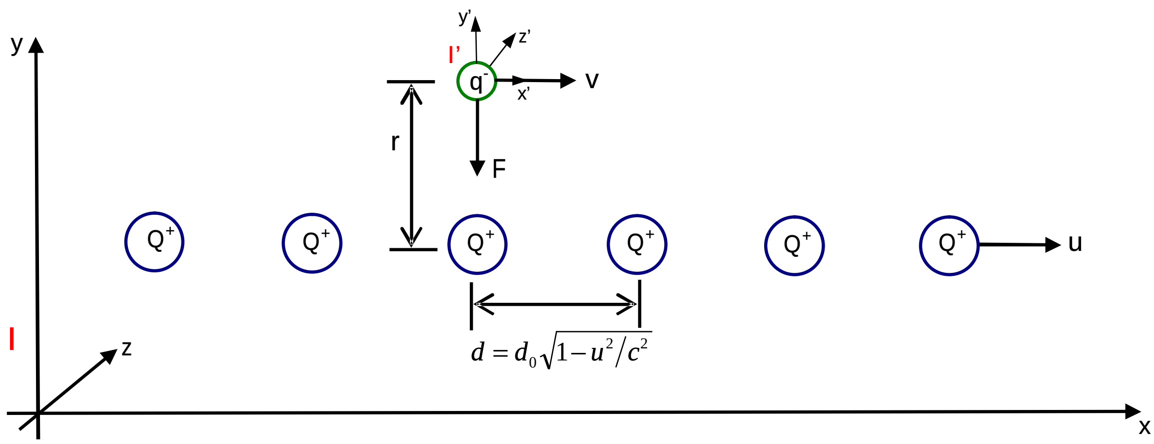


Fig. A3.3: Current of positive charge (speed u) and an electron whose speed is v, in the reader's steady system I.

a) Evaluation of F on an electromagnetic basis, in the system I :
First of all, we remind ourselves of the fact that if we have N charges Q in line and d spaced (as per Fig. A3.3), then the linear charge density λ will be:

$$l = N \cdot Q / N \cdot d = Q/d \quad .$$

Now, still with reference to Fig. A3.3, in the system I, for the electromagnetics the electron will undergo the Lorentz force $F_l = q(E + v \times B)$ which is made of an originally electrical component and of a magnetic one:

$$F_{el} = E \cdot q = (\frac{1}{e_0}\frac{l}{2pr})q = (\frac{1}{e_0}\frac{Q/d}{2pr})q$$ due to the electric attraction from a linear distribution of charges Q, and:

$$F_{magn} = m_0 \frac{I}{2pr} = m_0 \frac{Q/t}{2pr} = m_0 \frac{Q/(d/u)}{2pr} = m_0 \frac{uQ/d}{2pr}$$ (Biot and Savart).

So: $$F_l = q(\frac{1}{e_0}\frac{Q/d}{2pr} - vm_0 \frac{uQ/d}{2pr}) = q\frac{Q/d_0}{2pr}(\frac{1}{e_0} - m_0 uv)\frac{1}{\sqrt{1-u^2/c^2}} \quad , \qquad \text{(A3.1)}$$

where the negative sign tells us the magnetic force is repulsive, in that case, because of the real directions of those currents, and where the steady distance d_0 is contracted to d, according to Lorentz, in the system I where charges Q have got speed u ($d = d_0\sqrt{1-u^2/c^2}$).

b) Evaluation of F on an electric base, in the steady system I' of q:
in the system I' the charge q is still and so it doesn't represent any electric current, and so there will be only a Coulomb electric force towards charges Q:

$$F'_{el} = E' q = (\frac{1}{e_0}\frac{l'}{2pr})q = (\frac{1}{e_0}\frac{Q/d'}{2pr})q = q(\frac{1}{e_0}\frac{Q/d_0}{2pr})\frac{1}{\sqrt{1-u'^2/c^2}} \quad , \qquad \text{(A3.2)}$$

where u' is the speed of the charge distribution Q in the system I', which is due to u and v by means of the well known relativistic theorem of composition of speeds:

$$u' = (u-v)/(1-uv/c^2) \quad , \qquad \text{(A3.3)}$$

and d_0, this time, is contracted indeed, according to u': $d' = d_0\sqrt{1-u'^2/c^2}$.

We now note that, through some algebraic calculations, the following equality holds (see (A3.3)):

$$1-u'^2/c^2 = \frac{(1-u^2/c^2)(1-v^2/c^2)}{(1-uv/c^2)^2}$$, which, if replacing the radicand in (A3.2), yields:

$$F'_{el} = E' q = (\frac{1}{e_0}\frac{l'}{2pr})q = (\frac{1}{e_0}\frac{Q/d'}{2pr})q = q(\frac{1}{e_0}\frac{Q/d_0}{2pr})\frac{(1-uv/c^2)}{\sqrt{1-u^2/c^2}\sqrt{1-v^2/c^2}} \qquad \text{(A3.4)}$$

We now want to compare (A3.1) with (A3.4), but we still cannot, as one is about I and the other is about I'; so, let's scale F'_{el} in (A3.4), to I, too, and in order to do that, we see that, by definition of the force itself, in I':

$$F'_{el}(in\_I') = \frac{\Delta p_{I'}}{\Delta t_{I'}} = \frac{\Delta p_I}{\Delta t_I \sqrt{1-v^2/c^2}} = \frac{F_{el}(in\_I)}{\sqrt{1-v^2/c^2}}$$, where $\Delta p_{I'} = \Delta p_I$, as Δp extends along y, and not along the direction of the relative motion, so, according to the Lorentz transformations, it doesn't change, while Δt, of course, does. So:

$$F_{el}(in\_I) = F'_{el}(in\_I')\sqrt{1-v^2/c^2} = q(\frac{1}{e_0}\frac{Q/d_0}{2pr})\frac{(1-uv/c^2)}{\sqrt{1-u^2/c^2}\sqrt{1-v^2/c^2}}\sqrt{1-v^2/c^2} =$$

$$= q(\frac{1}{e_0}\frac{Q/d_0}{2pr})\frac{(1-uv/c^2)}{\sqrt{1-u^2/c^2}} = F_{el}(in\_I) \qquad \text{(A3.5)}$$

Now we can compare (A3.1) with (A3.5), as now both are related to the I system.
Let's write them one over another:

$$F_l(in\_I) = q(\frac{1}{e_0}\frac{Q/d}{2pr} - vm_0\frac{uQ/d}{2pr}) = q\frac{Q/d_0}{2pr}(\frac{1}{e_0} - m_0uv)\frac{1}{\sqrt{1-u^2/c^2}}$$

$$F_{el}(in\_I) = q(\frac{1}{e_0}\frac{Q/d_0}{2pr})\frac{(1-uv/c^2)}{\sqrt{1-u^2/c^2}} = q\frac{Q/d_0}{2pr}(\frac{1}{e_0} - \frac{uv}{e_0c^2})\frac{1}{\sqrt{1-u^2/c^2}}$$

Therefore we can state that these two equations are identical if the following identity holds: $c = 1/\sqrt{e_0m_0}$, and this identity is known since 1856. As these two equations are identical, the magnetic force has been traced back to the Coulomb's electric force, so the unification of electric and magnetic fields has been accomplished!!

App. 1-Chapter 4: Justification of the equation $R_{Univ} = \sqrt{N}r_e$ previously used for the unification of electric and gravitational forces (Rubino).

App. 1-Par. 4.1: The equation $R_{Univ} = \sqrt{N}r_e$ (!).

First of all, we have already checked the validity of the equation $R_{Univ} = \sqrt{N}r_e$, used in (A2.2), as it has proved to be numerically correct.
And it's also justified on an oscillatory basis and now we see how; such an equation tells us the radius of the Universe is equal to the classic radius of the electron multiplied by the square root of the number of electrons (and positrons) N in which the Universe can be thought as made of. (We know that in reality almost all the matter in the Universe is not made of e^+e^- pairs, but rather of p^+e^- pairs of hydrogen atoms H, but we are now interested in considering the Universe as made of basic bricks, or in fundamental harmonics, if you like, and we know that electrons and positrons are basic bricks, as they are stable, while the proton doesn't seem so, and then it's neither a fundamental harmonic, and so nor a basic brick).
Suppose that every pair e^+e^- (or, for the moment, also p^+e^- (H), if you like) is a small spring (this fact has been already supported by reasonings made around (A2.3)), and that the Universe is a big oscillating spring (now contracting towards its center of mass) with an oscillation amplitude obviously equal to R_{Univ} , which is made of all microoscillations of e^+e^- pairs.
And, at last, we confirm that those micro springs are all randomly spread out in the Universe, as it must be; therefore, one is oscillating to the right, another to the left, another one upwards and another downwards, and so on.
Moreover e^+ and e^- components of each pair are not fixed, so we will not consider N/2 pairs oscillating with an amplitude $2r_e$, but N electrons/positrons oscillating with an amplitude r_e.

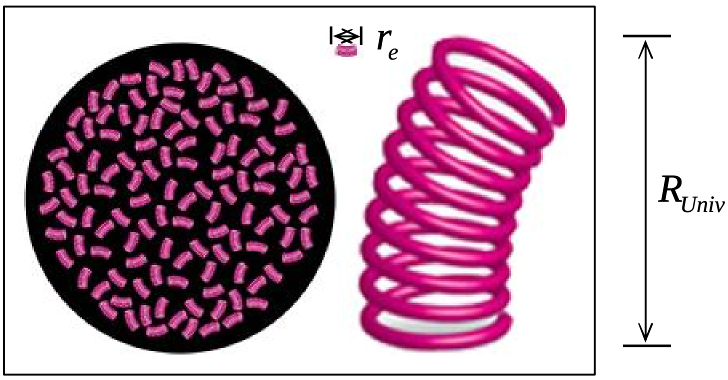


Fig. A4.1: The Universe represented as a set of many (N) small springs, oscillating on <u>random</u> directions, or as a single big oscillating spring.

Now, as those micro oscillations are randomly oriented, their random composition can be shown as in Fig. A4.2.

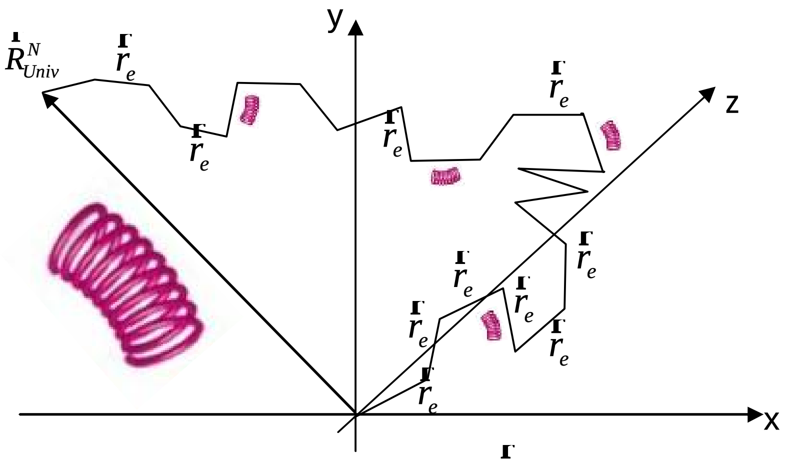


Fig. A4.2: Composition of N micro oscillations $\vec{r}_e$ randomly spread out, so forming the global oscillation R_{Univ}.

We can obviously write that: $\vec{R}_{Univ}^{N} = \vec{R}_{Univ}^{N-1} + \vec{r}_e$ and the scalar product $\vec{R}_{Univ}^{N}$ with itself yields: $\vec{R}_{Univ}^{N} \cdot \vec{R}_{Univ}^{N} = (R_{Univ}^{N})^2 = (R_{Univ}^{N-1})^2 + 2\vec{R}_{Univ}^{N-1} \cdot \vec{r}_e + r_e^2$; we now take the mean value:

$$\left\langle (R_{Univ}^{N})^2 \right\rangle = \left\langle (R_{Univ}^{N-1})^2 \right\rangle + \left\langle 2\vec{R}_{Univ}^{N-1} \cdot \vec{r}_e \right\rangle + \left\langle r_e^2 \right\rangle = \left\langle (R_{Univ}^{N-1})^2 \right\rangle + \left\langle r_e^2 \right\rangle, \qquad \text{(A4.1)}$$

as $\left\langle 2\vec{R}_{Univ}^{N-1} \cdot \vec{r}_e \right\rangle = 0$, because $\vec{r}_e$ can be oriented randomly over 360° (or over $4p$ sr, if you like), so a vector averaging with it, as in the previous equation, yields zero.

We so rewrite (A4.1): $\left\langle (R_{Univ}^{N})^2 \right\rangle = \left\langle (R_{Univ}^{N-1})^2 \right\rangle + \left\langle r_e^2 \right\rangle$ and proceeding, on it, by induction:

(by replacing N with N-1 and so on):

$\left\langle (R_{Univ}^{N-1})^2 \right\rangle = \left\langle (R_{Univ}^{N-2})^2 \right\rangle + \left\langle r_e^2 \right\rangle$, and then: $\left\langle (R_{Univ}^{N-2})^2 \right\rangle = \left\langle (R_{Univ}^{N-3})^2 \right\rangle + \left\langle r_e^2 \right\rangle$ etc, we get:

$\left\langle (R_{Univ}^{N})^2 \right\rangle = \left\langle (R_{Univ}^{N-1})^2 \right\rangle + \left\langle r_e^2 \right\rangle = \left\langle (R_{Univ}^{N-2})^2 \right\rangle + 2\left\langle r_e^2 \right\rangle = ... = 0 + N\left\langle r_e^2 \right\rangle = N\left\langle r_e^2 \right\rangle$, that is:

$\left\langle (R_{Univ}^{N})^2 \right\rangle = N\left\langle r_e^2 \right\rangle$, from which, by taking the square roots of both sides:

$\sqrt{\left\langle (R_{Univ}^{N})^2 \right\rangle} = R_{Univ} = \sqrt{N}\sqrt{\left\langle r_e^2 \right\rangle} = \sqrt{N} \cdot r_e$, that is:

$$R_{Univ} = \sqrt{N} \cdot r_e \quad \text{!!!} \quad \text{(Rubino)} \qquad \text{(A4.2)}$$

Anyway, it's well known that, in physics, for instance, the walk R made over N successive steps r, and taken in random directions, is really the square root of N by r (see, for instance, studies on Brownian movement).

App. 1-Chapter 5: "a_{Univ}"as absolute responsible of all forces.

App. 1-Par. 5.1: Everything from "a_{Univ}".

Still in agreement with what has been said so far, the cosmic acceleration itself a_{Univ} is responsible for gravity all, and so for the terrestrial one, too. In fact, just because the Earth is dense enough, it's got a gravitational acceleration on its surface g=9,81 m/s^2, while if today we could consider it as composed of electrons randomly spread, just like in Fig. A4.1 for the Universe, then it would have a radius $\sqrt{\frac{M_{Earth}}{m_e}} \cdot r_e = \sqrt{N_{Earth}} \cdot r_e$, and the gravitational acceleration on its surface would be:

$$g_{New} = G\frac{M_{Earth}}{(\sqrt{N_{Earth}} \cdot r_e)^2} = a_{Univ} = 7{,}62 \cdot 10^{-12} m/s^2 \quad \text{!!!}$$

Therefore, once again we can say that the gravitational force is due to the collapsing of the Universe by a_{Univ}, and all gravitational accelerations we meet, time after time, for every celestial object, are different from a_{Univ} according to how much such objects are compressed.

App. 1-Par. 5.2: Summarizing table of forces.

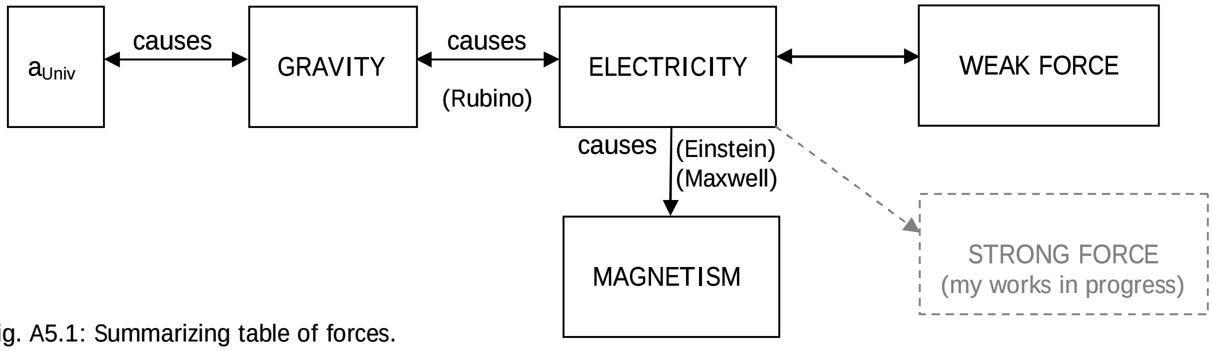


Fig. A5.1: Summarizing table of forces.

App. 1-Par. 5.3: Further considerations on composition of the Universe in pairs + / -.

The full releasing of every single small spring which stands for the electron-positron pair, is nothing but the annihilation, with turning into photons of those two particles. In such a way, that pair wouldn't be represented anymore by a pointed wave, pointed in certain place and time, (for instance $\sin(x-vt)/(x-vt)$, or the similar $d(x-vt)$ of Dirac), where the pointed part would stand for the charge of the spring, but it will be represented by a function like $\sin(x-ct)$, omogeneous along all its trajectory, and this is what a photon is. This will happen when the collapsing of the Universe in its center of mass will be accomplished.

Moreover, the essence of the pairs e^+e^-, or, in this era, of e^-p^+, is necessary in order not to violate Principle of Conservation of Energy. In fact, the Universe seems to vanish towards a singularity, after its collapsing, or taking place from nothing, during its inverse Big Bang-like process, and so doing, it would be a violation of such a conservation principle, if not supported by the Indetermination Principle, according to which an energy ΔE is legitimated to appear anyhow, unless it lasts less than Δt, in such a way that $\Delta E \cdot \Delta t \le \mathbf{h}/2$; in other words, it can appear provided that the observer doesn't have enough time, in comparison to his means of measure, to figure it out, so coming to the ascertainment of a violation. And, by the same token, the whole Universe, which is made of pairs +/-, has this property. And the appearing of a ΔE made of a pair of particles, shows the particles to reject each other first, so showing the same charge, while the successive annihilation after Δt shows a successive attraction, showing now opposite charges. So, the appearing and the annihilation correspond to the expansion and collapsing of the Universe. Therefore, if we were in an expanding Universe, we wouldn't have any gravitational force, or it were opposite to how it is now, and it's not true that just the electric force can be repulsive, but the gravitational force, too, can be so (in an expanding Universe); now it's not so, but it was!

The most immediate philosophical consideration which could be made, in such a scenario, is that, how to say, anything can be born (can appear), provided that it dies, and quick enough; so the violation is avoided, or better, it's not proved/provable, and the Principle of Conservation of Energy is so preserved, and the contradiction due to the appearing of energy from nothing is gone around, or better, it is contradicted it itself.

App. 1-Par. 5.4: The Theory of Relativity is just an interpretation of the oscillating Universe just described, contracting with speed c and acceleration a_{univ}.

On composition of speeds:

1) <u>Case of a body whose mass is m</u>. If in our reference system I, where we (the observers) are at rest, there is a body whose mass is m and it's at rest, we can say: $v_1 = 0$ and $E_1 = \frac{1}{2}mv_1^2 = 0$. If now I give kinetic energy to it, it will jump to speed v_2, so that, obviously: $E_2 = \frac{1}{2}mv_2^2$ and its delta energy of GAINED energy $\Delta_\uparrow E$ (delta up) is:

$$\Delta_\uparrow E = E_2 - E_1 = \frac{1}{2}mv_2^2 - 0 = \frac{1}{2}m(v_2 - 0)^2 = \frac{1}{2}m(\Delta v)^2$$, with $\Delta v = v_2 - v_1$.

Now, we've obtained a Δv which is simply $v_2 - v_1$, but this is a PARTICULAR situation and it's true only when it starts from rest, that is, when $v_1 = 0$.

On the contrary: $\Delta_{\uparrow}E = E_2 - E_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2}m(\Delta_V v)^2$, where Δ_V is a vectorial delta: $\Delta_V v = \sqrt{(v_2^2 - v_1^2)}$; therefore, we can say that, apart from the particular case when we start from rest (v_1 = 0), if we are still moving, we won't have a simple delta, but a vectorial one; this is simple base physics.

2) Case of the Earth. In our reference system I, in which we (the observers) are at rest, the Earth (E-Earth) rotates around the Sun with a total energy:

$E_{Tot} = \frac{1}{2}m_E v_E^2 - G\frac{M_{Sun}m_E}{R_{E-S}}$, and with a kinetic energy $E_K = \frac{1}{2}m_E v_E^2$. If now we give the Earth a delta up $\Delta_{\uparrow}E$ of kinetic energy in order to make it jump from its orbit to that of Mars (M-Mars), then, just like in the previous point 1, we have:

$\Delta_{\uparrow}E = \frac{1}{2}m_E v_E^2 - \frac{1}{2}m_E v_M^2 = \frac{1}{2}m_E(v_E^2 - v_M^2) = \frac{1}{2}m_E(\Delta_V v)^2$, with $\Delta_V v = \sqrt{(v_E^2 - v_M^2)}$, and so also here the speed deltas are vectorial-like (Δ_V).

3) Case of the Universe. In our reference system I, where we (the observers) are at rest, if we want to make a body, whose mass is m_0 and originally at rest, get speed V, we have to give it a delta v indeed, but for all what has been said so far, as we are already moving in the Universe, (and with speed c), as for above points 1 and 2, such a delta v must withstand the following (vectorial) equality:

$$V = \Delta_V v = \sqrt{(c^2 - v_{New-Abs-Univ-Speed}^2)} \,, \tag{A5.1}$$

where $v_{New-Abs-Univ-Speed}$ is the new absolute speed the body (m_0) looks to have, not with respect to us, but with respect to the Universe and its center of mass.
As a matter of fact, a body is inexorably linked to the Universe where it is, in which, as chance would have it, it already moves with speed c and therefore has got an intrinsic energy m_0c^2.
In more details, as we want to give the body (m_0) a kinetic energy E_k , in order to make it gain speed V (with respect to us), and considering that, for instance, in a spring which has a mass on one of its ends, for the harmonic motion law, the speed follows a harmonic law like:

$v = (wX_{Max})\sin a = V_{Max}\sin a \quad (v_{New-Abs-Univ-Speed} = c\sin a$, in our case),

and for the harmonic energy we have a harmonic law like:

$E = E_{Max}\sin a \quad (m_0c^2 = (m_0c^2 + E_K)\sin a$, in our case),

we get $\sin a$ from the two previous equations and equal them, so getting:

$$v_{New-Abs-Univ-Speed} = c\frac{m_0c^2}{m_0c^2 + E_K} \,,$$

now we put this expression for $v_{New-Abs-Univ-Speed}$ in (A5.1) and get:

$V = \Delta_V v = \sqrt{(c^2 - v_{New-Abs-Univ-Speed}^2)} = \sqrt{[c^2 - (c\frac{m_0c^2}{m_0c^2 + E_K})^2]} = V$, and we report it below:

$$V = \sqrt{[c^2 - (c\frac{m_0c^2}{m_0c^2 + E_K})^2]} \tag{A5.2}$$

If now we get E_K from (A5.2), we have:

$E_K = m_0c^2(\frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} - 1)$!!! which is exactly the Einstein's relativistic kinetic energy!

If now we add to E_K such an intrinsic kinetic energy of m_0 (which also stands "at rest" – rest with respect to us, not with respect to the center of mass of the Universe), we get the total energy:

$$E \ = E_K + m_0 c^2 = m_0 c^2 + m_0 c^2 \left(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}} - 1\right) = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}} m_0 c^2 = g \cdot m_0 c^2$$, that is the well known

$E = g \cdot m_0 c^2$ (of the Special Theory of Relativity). (A5.3)

All this after that we supposed to bring kinetic energy to a body at rest (with respect to us). Equation (A5.3) works wery well on particle accelerators, where particles gain energy, but there are cases (collapsing Universe and Atomic Physics) where masses lose energy and radiate, instead of gaining it, and in such cases (A5.3) is completely inapplicable, as it's in charge for added energies, not for lost ones.

App. 1-Par. 5.5: On "Relativity" of lost energies.

In case of lost energies (further phase of the harmonic motion), the following one must be used:

$$E = \frac{1}{g} \cdot m_0 c^2$$ (Rubino) (A5.4)

which is intuitive just for the simple reason that, with the increase of the speed, the coefficient $1/g$ lowers m_0 in favour of the radiation, that is of the lost of energy; unfortunately, this is not provided for by the Theory of Relativity, like in (A5.4).

For a convincing proof of (A5.4) and of some of its implications, I have further files about.

By using (A5.4) in Atomic Physics in order to figure out the ionization energies $\Delta_{\downarrow} E_Z$ of atoms with just one electron, but with a generic Z, we come to the following equation, for instance, which matches very well the experimental data:

$$\Delta_{\downarrow} E_Z = m_e c^2 [1 - \sqrt{1 - (\frac{Ze^2}{2e_0 hc})^2}\,]$$ (A5.5)

and for atoms with a generic quantum number n and generic orbits:

$$\Delta_{\downarrow} E_{Z-n} = m_e c^2 [1 - \sqrt{1 - (\frac{Ze^2}{4ne_0 hc})^2}\,]$$ (Wåhlin) (A5.6)

Orbit (n)	Energy (J)	Orbit (n)	Energy (J)
1	2,1787 10^{-18}	5	8,7147 10^{-20}
2	5,4467 10^{-19}	6	6,0518 10^{-20}
3	2,4207 10^{-19}	7	4,4462 10^{-20}
4	1,3616 10^{-19}	8	3,4041 10^{-20}

Tab. A5.1: Energy levels in the hydrogen atom H (Z=1), as per (A5.6).

On the contrary, the use of the here unsuitable (A5.3) doesn't match the experimental data, but brings to complex corrections and correction equations (Fock-Dirac etc), which tries to "correct", indeed, an unsuitable use.

Again, in order to have clear proofs of (A5.5) and (A5.6), I have further files about.

App. 1-SUBAPPENDIXES.

App. 1-Subppendix 1: Physical constants.

Boltzmann's Constant k: $1{,}38 \cdot 10^{-23} J / K$

Cosmic Acceleration a_{Univ}: $7{,}62 \cdot 10^{-12} m / s^2$

Distance Earth-Sun AU: $1{,}496 \cdot 10^{11} m$

Mass of the Earth M_{Earth}: $5{,}96 \cdot 10^{24} kg$

Radius of the Earth R_{Earth}: $6{,}371 \cdot 10^{6} m$

Charge of the electron e: $-1{,}6 \cdot 10^{-19} C$

Number of electrons equivalent of the Universe N: $1{,}75 \cdot 10^{85}$

Classic radius of the electron r_e: $2{,}818 \cdot 10^{-15} m$

Mass of the electron m_e: $9{,}1 \cdot 10^{-31} kg$

Fine structure Constant $a\,(\cong 1/137)$: $7{,}30 \cdot 10^{-3}$

Frequency of the Universe n_{Univ} : $4{,}05 \cdot 10^{-21} Hz$

Pulsation of the Universe $w_{Univ} (= H_{global})$: $2{,}54 \cdot 10^{-20}\, rad/s$

Universal Gravitational Constant G: $6{,}67 \cdot 10^{-11} Nm^2 / kg^2$

Period of the Universe T_{Univ} : $2{,}47 \cdot 10^{20} s$

Light Year l.y.: $9{,}46 \cdot 10^{15} m$

Parsec pc: $3{,}26\_a.l. = 3{,}08 \cdot 10^{16} m$

Density of the Universe ρ_{Univ}: $2{,}32 \cdot 10^{-30} kg / m^3$

Microwave Cosmic Radiation Background Temp. T: $2{,}73 K$

Magnetic Permeability of vacuum μ_0: $1{,}26 \cdot 10^{-6} H / m$

Electric Permittivity of vacuum ε_0: $8{,}85 \cdot 10^{-12} F / m$

Planck's Constant h: $6{,}625 \cdot 10^{-34} J \cdot s$

Mass of the proton m_p: $1{,}67 \cdot 10^{-27} kg$

Mass of the Sun M_{Sun}: $1{,}989 \cdot 10^{30} kg$

Radius of the Sun R_{Sun}: $6{,}96 \cdot 10^{8} m$

Speed of light in vacuum c: $2{,}99792458 \cdot 10^{8} m / s$

Stephan-Boltzmann's Constant σ: $5{,}67 \cdot 10^{-8} W / m^2 K^4$

Radius of the Universe (from the centre to us) R_{Univ}: $1{,}18 \cdot 10^{28} m$

Mass of the Universe (within R_{Univ}) M_{Univ}: $1{,}59 \cdot 10^{55} kg$

Thank you for your attention.
Leonardo RUBINO
leonrubino@yahoo.it

Bibliografia:

1) *(M. Alonso & E.J. Finn)* FUNDAMENTAL UNIVERSITY PHYSICS III, **Addison-Wesley**.

2) *(C. Rossetti)* ISTITUZIONI DI FISICA TEORICA (Intr. alla M.Q.), **Levrotto & Bella**.

3) *(R. Gautreau & W. Savin)* FISICA MODERNA – Schaum.

4) *(L. Wåhlin)* THE DEADBEAT UNIVERSE, 2nd Ed. Rev., Colutron Research.

5) *(R. Feynman)* LA FISICA DI FEYNMAN I-II e III – Zanichelli.

6) *(Lionel Lovitch-Sergio Rosati)* FISICA GENERALE, Elettricità, Magnetismo, Elettromagnetismo Relatività Ristretta, Ottica, Meccanica Quantistica , 3^ Edizione; Casa Editrice Ambrosiana-Milano.

7) *(C. Mencuccini e S. Silvestrini)* FISICA I – Meccanica-Termodinamica, Liguori.

8) *(C. Mencuccini e S. Silvestrini)* FISICA II – Elettromagnetismo-Ottica, Liguori.

9) *(R. Sexl & H.K. Schmidt)* SPAZIOTEMPO – Vol. 1, Boringhieri.

10) *(V.A. Ugarov)* TEORIA DELLA RELATIVITA' RISTRETTA, **Edizioni Mir.**

11) *(A. Liddle)* AN INTRODUCTION TO MODERN COSMOLOGY, 2nd Ed., Wiley.

12) *(A. S. Eddington)* THE EXPANDING UNIVERSE, Cambridge Science Classics.

13) ENCYCLOPEDIA OF ASTRONOMY AND ASTROPHYSICS, Nature Publishing Group & Institute of Physics Publishing.

14) *(Keplero)* THE HARMONY OF THE WORLD.

15) *(H. Bradt)* ASTROPHYSICS PROCESSES, Cambridge University Press.

16) *(L. Rubino)* Publications on physics in the Italian physics website fisicamente.net.
