Advancements over a Geometrodynamical Model

Sandro Antonelli
via Madonna del Piano 41 A 03017 Morolo (Fr) Italia
Email: antonelli41@live.it

Abstract This article is mainly conceived to gain more interest into a recent trustworthy development. The dynamics of the relativistic Space-Time structure, as discussed in the model, exhibits unforeseen analogies with the electromagnetic theory. As direct continuation of the analysis of the gravitational wave propagation in free space, one should realize (unlike Lorentz gauge in General Relativity) that the polarization state is in general a mixture of six independent modes as many as the independent components of the Riemann tensor determining the tidal forces, although one can always recover two polarizations for particular symmetry conditions on the direction of propagation and observation. Actually, in this gravitational framework, at least for one polarization state, transverse waves are expected to propagate causing equal in-phase deformation displacement for a symmetric source, not counterphase as in General Relativity. At this aim a new interferometry methodology is designed. Calculation of gravitational power losses for the keplerian system PSR 1913+16 in the solution by approximations of inhomogeneous problem is carried out to the first order, which allows the assessment of a second gravitational constant.

PACS. 04.30.
Mathematical subject codes: 35Q75; 83D05.

Keywords: General Relativity; Gravitational waves; Continuum mechanics; Tensorial Curl; Vorticity.

Introduction

Despite its impressive success [10], linear approximation in general relativity (GR) seems to contradict in that the exact theory does not on principle entail emission of gravitational radiation i.e. waves physically meaningful as pointed out by some authors [1][1]. Other ways have then been explored to search the mechanism responsible of gravitational wave genesis. Tailherer's model [8] (onwards Vortex Model) is one of these. Actually, the analysis of the motion of a continuum has shown a resemblance with Space-Time that might be more than superficial, for, as has been seen, it has been possible to establish valuable results by making use of a geometrodynamics theory whose basic equations are those of the vortex encountered in lagrangian description of continua [4,3] relating the angular velocity tensor to the deformation velocity $K_{\mu\nu}$. In particular, the ansatz has been tried, thereby the presence of vorticity does also mean the presence of curvature of space time through the constant *S*, which showed itself fundamental of all reasoning bringing to the gravitational wave propagation scenario[2] [8]:

$$C_{\mu\nu} = -kY_{\mu\nu} = S\omega_{\mu\nu} \tag{1}$$

where $k = 8\pi G/c^4$ with G Newton's constant and $C_{\mu\nu}$ the skew-symmetric contracted Riemann tensor (hereafter the *C* Riemann tensor), $Y_{\mu\nu}$ skew-symmetric energy-momentum tensor -see (24) further- and

$$\nabla_\sigma \omega_{\mu\nu} = \nabla_\mu K_{\nu\sigma} - \nabla_\nu K_{\mu\sigma} \quad (\mu,\nu,\sigma = 0,1,2,3)^3 \tag{2}$$

or by performing another curl (rotor) [cf. Tailherer (2007) equ. (4.18)]:

$$\begin{aligned} K_{\mu\nu/\alpha}{}^{\alpha} - K_{\alpha\nu/}{}^{\alpha}{}_{\mu} &= (|\nabla|^2 \boldsymbol{K})_{\mu\nu} - (\text{grad div } \boldsymbol{K})_{\mu\nu} \\ &= - K_\mu{}^\alpha R_{\alpha\nu} + K_\beta{}^\alpha R_{\nu\alpha\tau\mu} g^{\beta\tau} - [1/(2S)]\, C_{\mu\alpha}{}^{/}{}_{\nu}{}^{\alpha} \end{aligned} \tag{3}$$

with $R_{\alpha\sigma}$ the symmetric Riemann tensor and the slash standing for covariant or controvariant derivative. The previous ones have respectively a cinematic, dynamic and geometric content, in particular the last one states that a variation of curvature reveals itself in a propagation of metric deformation or, as it will be presently seen, in a "gravitational current" in the the Space-Time. For this purpose let us call total dynamic flux crossing a two dimensional oriented surface Σ of parametric equations $x^{\mu}(r,s)$ and complete contour l , the double covariant integral with respect to the skew indexes μ, ν (making use of the equality $\nabla_\mu K_{\nu\sigma} - \nabla_\nu K_{\mu\sigma} = \partial_\mu K_{\nu\sigma} - \partial_\nu K_{\mu\sigma}$ (see equ. 4.8 of [8])) :

---

[1] The simplest argument is that any moving body will travel a geodesic line and that will do without gravitational losses.

[2] See also equ (24), and (25) here . The reader should notice the change of notation with respect to [8] to banish any misleading of the skew symmetric Riemann tensor obtained as $C_{\mu\nu} = R_{\alpha\beta\mu\nu}\in^{\alpha\beta}$ (cf. [8] equ. 3.3; 4.12) with that of classical relativity as well as the skew energy – momentum tensor $Y_{\mu\nu}$ with that symmetric $T_{\mu\nu}$ in Einstein's equations .

[3] It apparently satisfies the Bianchi identity $\nabla_\sigma \omega_{\mu\nu} + \nabla_\mu \omega_{\nu\sigma} + \nabla_\nu \omega_{\sigma\mu} = 0$

$$\Phi_\Sigma\left(curl K\right)=\iint\sum_{\mu<\nu}^{0,3}\left(\nabla_\mu K_{\nu\sigma}-\nabla_\nu K_{\mu\sigma}\right)\left(\frac{\partial x^\mu}{\partial r}\frac{\partial x^\nu}{\partial s}-\frac{\partial x^\nu}{\partial r}\frac{\partial x^\mu}{\partial s}\right)drds$$
$$=\frac{1}{2}\iint\left(\partial_\mu K_{\nu\sigma}-\partial_\nu K_{\mu\sigma}\right)\left(\frac{\partial x^\mu}{\partial r}\frac{\partial x^\nu}{\partial s}-\frac{\partial x^\nu}{\partial r}\frac{\partial x^\mu}{\partial s}\right)drds \tag{4}$$

then the Green-Stokes theorem states that the previous expression equals the dynamic circulation of $K_{\mu\sigma}$ along the circuit l namely the integral:

$$J_\sigma=\oint_l K_{\mu\sigma}dx^\mu \tag{5}$$

We have therefore in our case that the "flux" of the derivative of the C Riemann tensor is:

$$\Phi_\Sigma\left(\nabla_\sigma C_{\mu\nu}\right)=S\,J_\sigma \tag{6}$$

If we let the surface be far enough from the masses distribution so that it allows us to consider the Space-Time nearly flat, we may take the derivative out of the sign of the integral to get:

$$\nabla_\sigma\Phi_\Sigma\left(C_{\mu\nu}\right)\approx S\,J_\sigma \tag{7}$$

We surprisingly recognize in the preceding one the striking formal resemblance with the Faraday Neumann law of electromagnetism in which the magnetic field and the current take respectively the place of the C Riemann tensor and the "gravitational current" J_σ. Thus we can state that a variation of "flux" of the energy momentum tensor through (24) must manifest itself in an induced gravitational current of gravitational radiation. In the following sections we shall ensue the vortex 4-dimensional formula going deeply into the resolution of a well studied gravitational binary system.

1. Overview: two Fundamental Cinematic Tensors

In inferring the main equations, all the points of Space-Time are considered parametrized with their co-ordinates which define the position vector ***OP*** in a 4-dimensional differentiable variety and a local frame relating to a local basis of vectors at each point of the cronotope $\boldsymbol{e}_\alpha=\partial \boldsymbol{OP}/\partial x^\alpha$ from which the metric tensor $g_{\alpha\beta}=\boldsymbol{e}_\alpha\cdot\boldsymbol{e}_\beta$.
Whereas in Special Relativity $g_{\alpha\beta}$ is a constant tensor, in General Theory we think of $g_{\alpha\beta}(x^\mu/\tau)$ as function of the variables x^μ and τ so as $\boldsymbol{e}_\alpha$. For simplicity's sake, let us restrict ourselves to a sub-space of $g_{\alpha\beta}$ spanned by any tern of vectors $\boldsymbol{e}_\alpha$. It follows that if the relations hold in each subspace, as they do, they hold in all the 4-dimensional space for any equation presenting three indexes or less. Therefore, let us choose without loss of generality the tern referring to the space indexes h = 1,2,3. Consider now the gradient of the space components of 4-velocity which will be of the type:

$$\partial_h\boldsymbol{v}=q_{hk}\,\boldsymbol{e}^k \qquad (\partial_h=\partial/\partial x^h \qquad h,k=1,2,3) \tag{8}$$

The matrix q_{hk} can always be split up in two parts

$$q_{hk} = (\partial_h \mathbf{v} \cdot \mathbf{e}_k)/c = K_{hk} + \omega_{hk} \tag{9}$$

with symmetrical part the deformation velocity of the metric

$$K_{hk} = 1/2c(\partial_\tau \mathbf{e}_h \cdot \mathbf{e}_k + \partial_\tau \mathbf{e}_k \cdot \mathbf{e}_h) = 1/2c\partial_\tau (\mathbf{e}_h \cdot \mathbf{e}_k) = 1/2c\, \partial_\tau g_{hk} \tag{10}$$

which can be referred to the second quadratic form as already outlined in [8] , and the skew-symmetric part

$$\omega_{hk} = (\partial_h \mathbf{v} \cdot \mathbf{e}_k - \partial_k \mathbf{v} \cdot \mathbf{e}_h)/2c = -\omega_{kh} \tag{11}$$

Following the classical approach presented in the Ferrarese's works [4] we thus get equ.(2) restricted to the three-dimensional space and readily generalized therefore to the whole Space-Time according to our reasoning. Let us then differentiate $\mathbf{e}_\mu (x^\nu/\tau)$ with respect to x^ν, we get for definition of Christoffel symbols [6]: $\partial_\nu \mathbf{e}_\mu = \Gamma^\alpha{}_{\nu\mu}\, \mathbf{e}_\alpha$. From that it turns out that $\partial_\nu \mathbf{v} = \partial_\nu (v_\mu \mathbf{e}^\mu) = (\partial_\nu v_\mu - \Gamma^\alpha{}_{\nu\mu}\, v_\alpha)\, \mathbf{e}^\mu = (\nabla_\nu v_\mu)\, \mathbf{e}^\mu$ leading (11) to the expression:

$$\omega_{\mu\nu} = \left(\nabla_\mu v_\nu - \nabla_\nu v_\mu\right)/2c = \left(\partial_\mu v_\nu - \partial_\nu v_\mu\right)/2c \tag{12}$$

taking advantage of the symmetry of Christoffel symbols with respect to inferior indexes.
We may put (cf.[7]) the previous expression in a form evidencing the curl of the 4-velocity:

$$\Omega^{\alpha\beta} = \varepsilon^{\alpha\beta\mu\nu} v_{\nu/\mu} \tag{13}$$

with $\varepsilon^{\alpha\beta\mu\nu}$ Levi-Civita tensor . Thus (12) is nothing but the relativistic version of the well-known relation of hydrodynamics $\boldsymbol{\omega} = ½\ curl\ \mathbf{v}_{EUL}$ with $\mathbf{v}_{EUL}$ eulerian velocity to which the known generalized Lagrange and Helmoltz theorems can be applied [2,5]. In particular, standing the identity (1) advanced at the beginning between the vortex and the skew Riemann tensor, the Lagrange theorem[4] maintains that if present, a gravitational field can never destroy itself and if absent can never originate.

2. Characteristics of deformation

The analysis of the deformations of metric is very interesting in that we may catch remarkable proprieties of the solution of the differential system (2) without passing through its resolution but by only qualitative analysis. Indeed by taking a free falling frame and assuming in a fixed point P^* initial geodesic co-ordinates we find the finite deformation of the metric at the τ proper time to be [5]:

$$2c\int_{\tau^*}^{\tau} K_{\mu\nu}(x^\rho/\tau')\,d\tau' = g_{\mu\nu}(x^\rho/\tau) - \eta_{\mu\nu} = h_{\mu\nu}(x^\rho/\tau) \tag{14}$$

---

[4] Although proved by reduction to absurdity from the case of flat space-time, a demonstration in general coordinates has not given yet.

[5] We should not be surprised at noting the fundamental tensor $g_{\mu\nu}$ to be inferred from a second quadratic form $K_{\mu\nu}$ not more than in kinematics the position elicited from the velocity through integration of the law of motion.

with $\eta_{\mu\nu}$ Minkowski tensor. Because of the Gauss gauge $g_{00}=1$, $g_{0k}=0$ $(k=1,2,3)$, it follows at once that $h_{00}=h_{0k}=0$, and hence $K_{00}=0$, $K_{0k}=0$. Moreover, we note $h_{\mu\nu}$ to depend only on the initial condition and on the displacement in terms of cronotope co-ordinates $\boldsymbol{s}=\boldsymbol{x}-\boldsymbol{x}_0$. In fact, let us try to put (14) in function of the displacement; since

$$\boldsymbol{OP}=\boldsymbol{OP}^*+\boldsymbol{s} \tag{15a}$$

let us see what happens to the local base at the point P :

$$\boldsymbol{e}_\mu=\frac{\partial \boldsymbol{OP}}{\partial x^\mu}=\boldsymbol{c}_\mu+\frac{\partial \boldsymbol{s}}{\partial x^\mu} \tag{15b}$$

with $\boldsymbol{c}_\mu$ rectangular base such that $\boldsymbol{c}_\mu\cdot\boldsymbol{c}^\nu=\delta_\mu{}^\nu$.

Then, remembering that $g_{\alpha\beta}=\boldsymbol{e}_\alpha\cdot\boldsymbol{e}_\beta$, equ.(14) becomes:

$$h_{\mu\nu}(x^\rho/\tau)=\left(\frac{\partial \boldsymbol{s}}{\partial x^\mu}\cdot\boldsymbol{c}_\nu+\frac{\partial \boldsymbol{s}}{\partial x^\nu}\cdot\boldsymbol{c}_\mu\right)+\frac{\partial \boldsymbol{s}}{\partial x^\mu}\cdot\frac{\partial \boldsymbol{s}}{\partial x^\nu} \tag{16a}$$

Since $\boldsymbol{s}=s_\alpha\boldsymbol{c}^\alpha=s^\alpha\boldsymbol{c}_\alpha$ equ. (16a) reads:

$$h_{\mu\nu}(x^\rho/\tau)=(\partial_\mu s_\nu+\partial_\nu s_\mu)+\partial_\mu s_\alpha\partial_\nu s^\alpha . \tag{16b}$$

Thus, the characteristics of deformation consist of two terms whose the first, linear and symmetric in the derivatives of the displacement, occurs in infinitesimal problems well studied in deformations of Elasticity [7] and that the reader is also addressed to for the analysis of the effects of the gravitational wave on the matter. For a wave propagating along the z axis, that is to say, perpendicularly to the direction of observation of a free falling frame (so expecting $h_{zz}(x^\rho/\tau)=0$ if we consider $s_3=0$ in (16b) at first order), we may take two wave polarizations, say

$$A_\oplus=\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix}$$ for deformations along the axes and $$A_\otimes=\begin{bmatrix}0&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&0\end{bmatrix}$$

for angular deformations of axes directions[6].

It may be shown [4,7] that after the deformation the angle between the axes r and s is given by:

$$\Theta_{rs}=\frac{\pi}{2}-arcsin\frac{h_{rs}}{\sqrt{1-h_{rr}}+\sqrt{1-h_{ss}}}$$

so the presence of off diagonal elements in the matrix h does mean angular deformations between axes directions.

[6] Other combinations of sign as the polarization $A_{11}=-A_{22}=1$ lead to area conservation in the transverse plane (x,y) according to eq. (17) referring to pure shear (so that Tr $(h_{ij})=0$), which evidently is not what we expect from symmetry reasons.

Moreover, because the lengthening of any vector $\boldsymbol{OA_0}$ whose versor $\boldsymbol{a_0}$ into OA is determined by the linear dilatation coefficient [4] :

$$\delta_a = \frac{\overline{OA} - \overline{OA_0}}{\overline{OA_0}} = \sqrt{1 - h_{hk}\, a_0^h\, a_0^k} - 1 \tag{17}$$

we may note that the polarization $A_\otimes$ would break a cylindrical symmetry since would involve opposite length variations in any perpendicular rotated directions having $\boldsymbol{a_0}$ as axis (e.g. $\boldsymbol{a_{01}}$: $[a_{0x}, a_{0y}]$ and $\boldsymbol{a_{02}}$: $[- a_{0y}, a_{0x}]$. In summary, there are in the general case six polarization modes as direct consequence of the independent components of the Riemann tensor [9] included in the tide forces expression:
$f_i = -m\, R^i_{0k0}\, x^k$ (m being the mass of a test particle in non-relativistic motion and x^k the displacement from the origin), that in our case are reduced to two.

3. Homogeneous solution

In order to work out the "gravitational billow", we must refer to the homogeneous solution of (4.21) of [8] $K_{\mu\nu}(x^\rho, \tau) = \Phi_{\mu\nu} + \tau\, \psi_{\nu/\mu}$ trying to find the solution with regard to symmetric cylindrical boundary conditions. Hence, let us consider a wave of $A_\oplus$ type. Thus, using the harmonic function

$$\Phi_{\mu\nu} = A_\oplus \frac{\Phi(\rho - ct)}{\rho} \tag{18}$$

with Φ any function, we get the following initial condition[7]:

$$K_{\mu\nu}(t=0) = \Phi_{\mu\nu}(t=0) = A_\oplus\, f(\rho) = f_{\mu\nu}(\rho) \tag{19}$$

Then, on expressing the harmonic function by a Fourier integral

$$\Phi_{\mu\nu}(\boldsymbol{x}, \tau) = \int F_{\mu\nu}\, e^{\delta\tau + i(\alpha x_1 + \beta x_2 + \gamma x_3)}\, d\alpha\, d\beta\, d\gamma \tag{20a}$$

and introducing spherical co-ordinates, such that $\alpha x^1 + \beta x^2 + \gamma x^3 = \rho\, r \cos\theta$, that is to say, taking the distance vector along the direction of propagation $z = x^3$ and $d\alpha\, d\beta\, d\gamma = \rho^2 \sin\theta\, d\rho\, d\theta\, d\varphi$, the following transforms have to calculated :

$$F_{\mu\nu}(\alpha, \beta, \gamma) = \frac{1}{(2\pi)^3} \int f_{\mu\nu}\, e^{i(\alpha\varsigma + \beta\eta + \gamma\chi)}\, d\varsigma\, d\eta\, d\chi = A_\oplus F \tag{20b}$$

and (remember that the harmonic condition yields the relation $\delta = -(\alpha^2 + \beta^2 + \gamma^2)^{1/2}$, i.e. the usual one between frequency and wave number components $\omega = c\,\delta$) :

$$K_{xx} = K_{yy} = \int \left\{ F(\alpha, \beta, \gamma) + i \left(\frac{\alpha\tau}{\delta} \right) \left[i\alpha F(\alpha, \beta, \gamma) \right] \right\} e^{\delta\tau + i(\alpha x_1 + \beta x_2 + \gamma x_3)}\, d\alpha\, d\beta\, d\gamma \tag{21}$$

[7] To simplify working out the Fourier expressions we do a Wick rotation $\tau = ict$ and so making euclidean the Minkowski tensor and imaginary the initial deformation velocity $\Phi_{\alpha\beta} = \frac{1}{2}\, \partial g_{\alpha\beta} / \partial (i\, ct)|_0$. At the end of calculation one has to take the imaginary part changed of sign.

Then, by considering the imaginary part, sign changed, after the integration, the relative deformations along the two axes will be:

$$h_{xx}(r)=h_{yy}(r)=2\int_0^{\tau} K_{11}\left(x^{\rho}/\tau'\right)d\tau'=2\int_0^{\tau} K_{22}\left(x^{\rho}/\tau'\right)d\tau' \qquad (22)$$

with r the distance between source and observer. Even examples with simple initial conditions entail cumbersome expressions involving delicate problems of convergence. So we skim over.

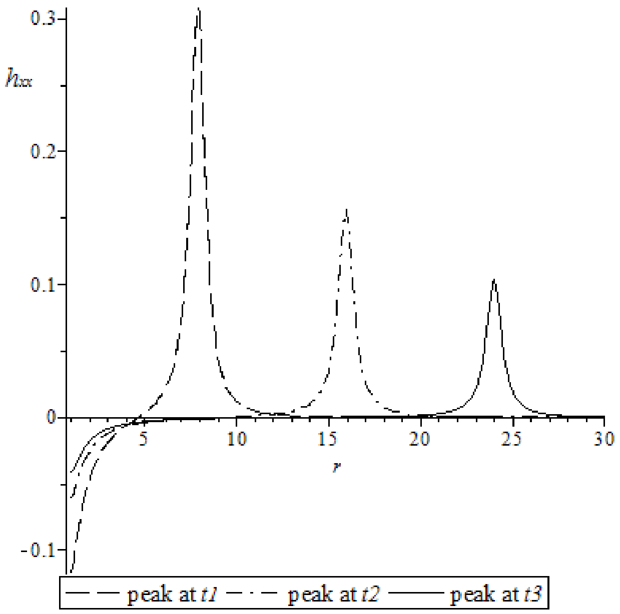

Fig1: Plot in function of r (arbitrary units) of the characteristic gravitational billow wave shape solution (numerical) h_{xx} with respect to a discontinuity source located about $r=0$ (as e.g. $K_{xx}(t=0)=1/\cosh r$) at fixed times $t1<t2<t3$ with its attenuating moving peak at $r = ct$.

4. Non-homogeneous solution in gravitational energy losses of a punctual keplerian system

By referring back to reference [8] we see the principal problem to be related to the following integral:

$$K_{\mu\sigma}=\int N_{\sigma\alpha\mu}{}^{\beta} y_{\beta}{}^{\alpha} d^4\xi + y_{\mu\sigma} \qquad (23a)$$

where the integral is over the coordinates ξ of the an astrophysical source and r the cronotope distance from the observation point $\boldsymbol{x}$ to the source point ξ where the energy-momentum tensor is not null. Hence $d^4\xi = d^3\boldsymbol{x}'d\tau$, with $\boldsymbol{x}'$ space co-ordinates of the keplerian system under discussion (binaries self-gravitating stars,…). The kernel and the other term result to be:

$$N_{\sigma\alpha\mu}{}^{\beta} = 3/(2\pi^2)\,[\,(\delta_\mu{}^\beta R_{\alpha\sigma} + R_{\sigma\alpha}{}^\beta{}_\mu)/r^2 + \nabla_\mu(1/r^2)\,(\xi^\rho - x^\rho) \times (\delta_\rho{}^\beta R_{\alpha\sigma} - R_{\sigma\alpha}{}^\beta{}_\rho)\,] \qquad (23b)$$

with $R_{\alpha\sigma}$ symmetrical Riemann tensor [8] and

$$y_{\mu\sigma} = -3\frac{1}{4Sr^2\pi^2}\int\left[C_{\mu\nu}{}^{/}{}_{\sigma}{}^{\nu} + \nabla_\mu\left(\frac{1}{r^2}\right)\cdot\left(x^\rho - \xi^\rho\right)C_{\rho\nu}{}^{/}{}_{\sigma}{}^{\nu}\right]d^4\xi \qquad (23c)$$

Moreover, by the attempting to estimate $R_{\mu\nu\rho\sigma}$ as ${}^{(1)}R_{\mu\nu\rho\sigma} = g_{\alpha\nu}R_{\beta\mu}\ /4$, since $R_{\beta\mu} = g^{\alpha\beta}\,R_{\alpha\beta\mu\nu}$, as starting position in a successive approximations process , we have for the "skew" C Riemann tensor:

$$C_{\mu\nu} = {}^{(1)}R_{\alpha\beta\mu\nu}\in^{\alpha\beta} = -kY_{\mu\nu} = -k(g_{\alpha\nu}/4)(T_{\beta\mu} - 1/2g_{\beta\mu}T)\in^{\alpha\beta} \qquad (24)$$

with $T_{\beta\mu}$ einstenian energy-momentum tensor of a punctual keplerian system.
As from ref. [8], we pick out the generic skew-symmetric cartesian tensor $\in^{\alpha\beta}$:

$$\in^{\alpha\beta} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad \text{and} \quad \in_{\alpha\beta} = \begin{bmatrix} 0 & -1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \qquad (25)$$

as the inverse matrix.

It is apparent in the previous ones (23)s we may neglect the derivative of the inverse squared of r in account of very large astronomical distances between source and observer and bring r out of the integral sign. Once computed $y_{\mu\sigma}$, developing the new metric the Riemann tensor in the first term of (23b) will be taken- however a rough estimate of it may seem - as ${}^{(2)}R_{\sigma\rho\nu\mu} \approx (\in_{\nu\mu}/4)C_{\sigma\rho}/k$. This shall profit to get a radiated power $\sim 1/S^2$.
Weird to note how in the last expression we may put $C_{\mu\nu/\sigma}{}^{\nu}$ in a form evidencing a curl by means of the Bianchi identity (footnote 3): $C_{\mu\nu/\sigma}{}^{\nu} = -\,C_{\nu\sigma/\mu}{}^{\nu} + C_{\mu\sigma/\nu}{}^{\nu} = -1/2\ \varepsilon_{\mu\alpha\delta\gamma}\,\varepsilon^{\delta\gamma\kappa\lambda}\,C_{\lambda\sigma/\kappa}{}^{\alpha} = -1/2\ \mathrm{curl}\ (\varepsilon^{\delta\gamma\kappa\lambda}\,C_{\lambda\sigma/\kappa})$.
The previous equ.(23*a*) consists of 16 equations. The C Riemann tensor is obtained by contraction through the tensor $\in^{\alpha\beta}$ whose skew-symmetry is preserved by a transformation of coordinates $U^{\mu\nu} = \partial x'^{\mu}/\partial x^{\nu}$ such that $U\in U^{+} = \in'$. Hence we may say that $K_{\mu\sigma}$ is known to within an antisymmetric matrix . But this does mean calibrate the constant S. In fact, I could choose whatever new $\in$ such that $\in' = \lambda\in$ into equation (24) for which S could be dimensioned as wanted in the fitting procedure. Nevertheless, once S has been determined in association with a given energy-momentum $T_{\mu\nu}$, I can arbitrarily use no longer any $\in$, but only that one being bond at that S through a similarity relation. This might amaze, that is, the fact that I associate a given cinematic quantity like the energy momentum with a determined constant tensor like $\in$, but as in geometry I need to adopt a coordinate system and a sample unit to represent a vector, so here to describe his

[8] We take occasion to remark the evident error of [8] in its notation, in equ. (4.20b) the tensor $R_{\alpha\sigma}$ stands for $\Re_{\alpha\sigma}$

way of being represented in terms of the contraction tensor according $Y_{\mu\nu} = (g_{\alpha\nu}/4)(T_{\beta\mu} - 1/2 g_{\beta\mu} T) \in^{\alpha\beta}$. On the other hand, we know in physics the case of Neumann-Faraday law in which the variation of magnetic flux is related to the electromotive force through the constant 1 in S.I. units. If we had chosen a different factor this would have meant to rescale the electromagnetic constants and the units, so in short $\in^{\alpha\beta}$ gauges S . So $\in'$ is fixed by means of six numbers. Moreover, the 4 conditions of free-divergence of the energy momentum tensor reduce the number of independent equations of equ.(23*a*) to 6. Thus the 10 unknown quantities $K_{\mu\nu}$ are defined through 10-6 = 4 accessory conditions that we chose as gaussian conditions for the metric tensor.

Since the keplerian motion occurs in a plane and the azimuthal angle is function of the time and the radial coordinate of the azimuthal angle, the quadruple integral in spherical co-ordinates will be taken on one independent angular variable of the reduced mass. The energy-momentum tensor of the single star is is $T^{\mu\nu}(1,2) = d(1,2)\, u^{\mu} u^{\nu}$, with u^{μ} the 4-velocity components of the (1,2) star mass and *d(*1,2*)* density mass which in the case of a punctual mass will be a three dimensional Dirac delta function. Given therefore the keplerian system defined by the equations in polar coordinates $(\rho(1,2), \theta, \varphi)$ in the center of mass system :

$$\rho(1,2) = [m(2,1)/(m1+m2)]\,\rho \qquad (26a)$$

with $\rho = \beta/(1 + e\cos\varphi)$, $\beta = a(1 - e^2)$, a major semi-axis of the ellipse, e the eccentricity[9] and φ the delayed phase and

$$\frac{\partial\varphi}{\partial\tau} = \frac{\alpha}{\rho^2} \quad , \qquad \alpha = \left[G(m1+m2)\,a\left(1-e^2\right)\right]^{1/2} \qquad (26b)$$

the velocity will be taken in the non-relativistic limit so that evaluating $u^{\rho}(1,2) = \partial\rho(1,2)/\partial\tau$ and $u^{\varphi}(1,2) = \partial\varphi/\partial\tau$ the energy-momentum tensor of the system wrt the center of mass comes out taking in account that it equals the sum of the components:

$$T^{\mu\nu}(1,2) = d(1,2)\begin{bmatrix} c^2 & c\dfrac{m(2,1)}{(m1+m2)}\dfrac{\alpha}{\beta}e\sin\varphi & 0 & c\dfrac{\alpha}{\rho^2} \\ c\dfrac{m(2,1)}{(m1+m2)}\dfrac{\alpha}{\beta}e\sin\varphi & \dfrac{m^2(2,1)}{(m1+m2)^2}\left(\dfrac{\alpha}{\beta}e\sin\varphi\right)^2 & 0 & \dfrac{m(2,1)}{(m1+m2)}\dfrac{\alpha^2}{\beta\rho^2}e\sin\varphi \\ 0 & 0 & 0 & 0 \\ c\dfrac{\alpha}{\rho^2} & \dfrac{m(2,1)}{(m1+m2)}\dfrac{\alpha^2}{\beta\rho^2}e\sin\varphi & 0 & \dfrac{\alpha^2}{\rho^4} \end{bmatrix}$$

d(1,2) being the point mass density of mass (1,2).

---

[9] We recall some data of the PSR1913+16 binary system: m1: 1.4 M_{sun}; m2:1.38 M_{sun}; *e*: 0.617; orbital period T : 7.72 hours; d major semi-axis a : 1.950E9 m; distance: 5kpc; *dT/dt*: -2.4E-12.

By using the method of successive approximations to work out (23*a*), we choose $K_{\mu\sigma}=0$ ($g_{\mu\sigma}=const=\eta_{\mu\sigma}$ Minkowski cartesian metric $\eta_{00}=-\eta_{11}=-\eta_{22}=-\eta_{33}=1$) as zero order approximation. So, as known, the metric tensor whereby indexes are lowered/raised and the matrix of transformation of tensors into spherical coordinates $x'^{\rho}\equiv(ct,r,\Theta,\Phi)$ read:

$$g'_{\mu\nu}=\begin{bmatrix}1&0&0&0\\0&-1&0&0\\0&0&-r^2&0\\0&0&0&-r^2\sin^2\Theta\end{bmatrix}\qquad \frac{\partial x'^{\mu}}{\partial x^{\nu}}=\begin{bmatrix}1&0&0&0\\0&\sin\Theta\cos\Phi&\sin\Theta\sin\Phi&\cos\Theta\\0&\dfrac{\cos\Theta\cos\Phi}{r}&\dfrac{\cos\Theta\sin\Phi}{r}&-\dfrac{\sin\Theta}{r}\\0&-\dfrac{\sin\Phi}{r\sin\Theta}&\dfrac{\cos\Phi}{r\sin\Theta}&0\end{bmatrix}$$

Moreover, taking in account that $d\tau=(\partial\tau/\partial\varphi)d\varphi$, the proper time in (26a)-(26b) is parametrized by the azimuthal variable . According to (24) the zeroth order C Riemann tensor $C_{\mu\nu}$ will, on using the previous contraction tensor, be of a form that is all but an antisymmetric tensor but this is the rudest approximation. So the skew-symmetry of the C Riemann as well as the symmetry of $K_{\mu\nu}$ will be considered a goodness index for the order of approximations. As shown below in fig.2, the C Riemann tensor antisymmetry check for the time space component $C_{21}+C_{12}$ at orbital distances of PSR1913+16 , seems quite adequate , although $K_{21}-K_{12}$ does not, working out the integral of equ.(23a) , and yet behaving well at sidereal distances. Anyway, better can be expected at further orders.

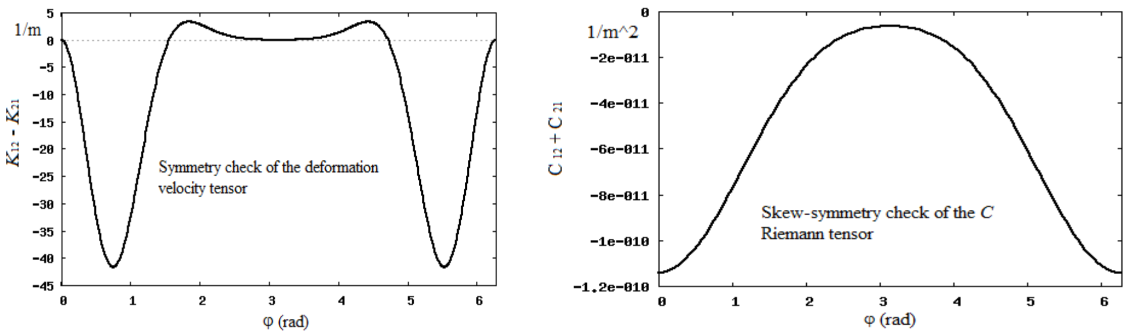

Fig 2: Plot in function of the azimuthal angle along the elliptic orbit of the approximation precision $K_{12}-K_{21}$ and $C_{12}+C_{21}$ respectively.

To first order we then get $K_{\mu\sigma}=y_{\mu\sigma}$. The next step will be if possible to determine the new metric $g_{\mu\nu}(x^{\rho}/\tau)d\tau=2c\int K_{\mu\nu}(x^{\rho}/\tau)d\tau+\eta_{\mu\nu}$ on the grounds of the definition of the deformation velocity, then the new trace Υ and in account of equ. (24) the new Riemann tensors; hence one determines the kernel simplified (23b) $N_{\sigma\alpha\mu}{}^{\beta}$, so that the next approximation will be:

$$K_{\mu\sigma}=\int N_{\sigma\alpha\mu}{}^{\beta}y_{\beta}{}^{\alpha}d^4\xi+y_{\mu\sigma}\tag{27}$$

and so on. Moreover, inserting the final worked out tensor metric into the geodesic equation $\partial_\tau^2 x^\rho + \Gamma^\rho{}_{\mu\nu}\partial_\tau x^\mu \partial_\tau x^\nu = f^\rho$ with f^ρ representing a field matter coupling term, would give us the modified acceleration law $\partial_\tau^2 \rho = a(\rho)$ which should fit in as experimental verification with local and solar observations (perihelion precession of planetary orbits, bending of light) as well as galaxies rotation curves. Also (27) is the obliged passage in inferring the cosmological behaviour of fundamental parameters of the evolution of the universe relatively to Friedmann solutions. The treatment of this matter will be object of next publications. The most great difficulty in expressing the energy loss against the major semi-axis of the elliptic orbit lies in working out the definite integrals for which numerical methods soon overflow in allocated memory (growing up to nearly 400 pages for single integrand) on behalf of the wxMaxima software preferred to Maple at this stage. Adopting numerical algorithms like Cavalieri-Simpson is therefore necessary (we took 10 steps for single variable of integration). Nonetheless it allows to give an estimate however rough may be of the constant S from known orbital data of PSR 1913+16 [10]. Actually, by differentiating with respect to the time the third Keplero law we have (mean values for orbit are concerned):

$$2T\frac{dT}{dt} = \frac{4\pi^2}{G\left(m1+m2\right)}3a^2\frac{da}{dt} \tag{28a}$$

On the other hand, differentiating the mean gravitational system energy $<E>=-G\, m1\, m2\, /\, (2r)$ one gets:

$$\frac{dr}{dt} = \frac{dE}{dt}\frac{2r^2}{G m1 m2} \tag{28b}$$

On substituting it into the previous one taking in account an orbital average value $< r> = a\cdot\gamma$ with $\gamma = (1-e^2)$ (read discussion further), from the free divergence of energy momentum tensor[10] for which $dE/d\tau = -c \int t_s{}^0 n^s r^2 d\Omega$, where for our purpose the energy-momentum tensor is $t_s{}^0 = -\Phi_s{}^{\sigma\alpha}\Phi^0{}_{\sigma\alpha}$ with the field $\Phi_{\nu\sigma\mu} = K_{\nu\sigma/\mu} - K_{\mu\sigma/\nu}$, one obtains S = 5.677E19 m^{-1} or S^{-1} = 1.761E-20 m. While the mean energy loss of Einstein's theory at a= 1.950E9 m amounts to 7.959E24 J/s - we remember that Einstein averaged emitted power is [11]:

$$< P > = \frac{32}{5}\frac{G^4}{c^5}\frac{m1^2 m2^2 (m1+m2)}{a^5(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$

-in the Vortex model it equals 1.387E25 J/s to the first approximation, thus 74% higher, as follows from the expression worked out (equ.29); however orbital phase average is taken here rather than time average, so to parallel them we should multiply the latter by $(1-e^2)$.

10 We make note in [8] the omission of a c factor in $dE/d\tau$ and a -¼ in front of the energy momentum in [8].

$$< P >_{\varphi} = \frac{1}{S^2}(7.463 \cdot 10^{-52} + \frac{2.831 \cdot 10^{-38}}{\sqrt{a}} - \frac{7.345 \cdot 10^{48}}{a} - \frac{2.752 \cdot 10^{-38}}{a^{3/2}} + \frac{6.572 \cdot 10^{56}}{a^2}$$
$$+ \frac{2.649 \cdot 10^{87}}{a^{5/2}} + \frac{1.884 \cdot 10^{92}}{a^3} + \frac{1.141 \cdot 10^{96}}{a^{7/2}} + \frac{2.962 \cdot 10^{98}}{a^4} - \frac{6.836 \cdot 10^{100}}{a^{9/2}} - \frac{1.725 \cdot 10^{103}}{a^5}$$
$$- \frac{1.511 \cdot 10^{105}}{a^{11/2}} - \frac{1.167 \cdot 10^{107}}{a^6} - \frac{3.86 \cdot 10^{108}}{a^{13/2}} + \frac{3.58 \cdot 10^{108}}{a^7} + \frac{7.454 \cdot 10^{94}}{a^{15/2}} + \frac{8.709 \cdot 10^{90}}{a^8}$$
$$+ \frac{5.492 \cdot 10^{89}}{a^{17/2}} + \frac{2.642 \cdot 10^{88}}{a^9} - \frac{6.224 \cdot 10^{90}}{a^{19/2}} - \frac{5.574 \cdot 10^{91}}{a^{10}} + \frac{8.372 \cdot 10^{93}}{a^{21/2}} + \frac{6.431 \cdot 10^{94}}{a^{11}}) \quad (29)$$

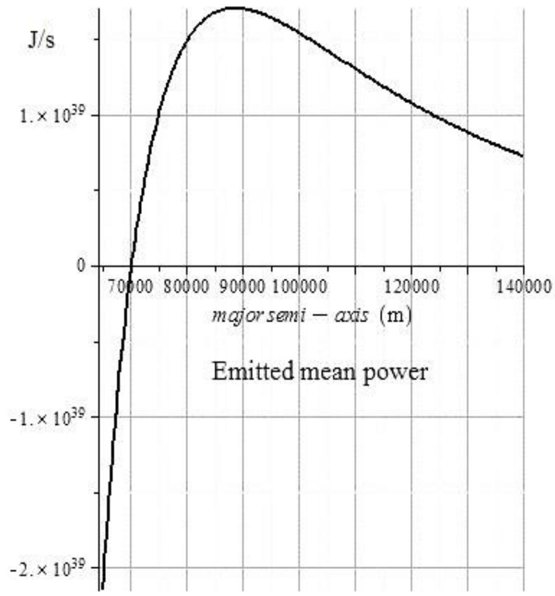

Fig. 3: Emitted mean power for Vortex model according to equ.(29), much lower than Einstein's amounting to 10E48.

Unfortunately, the previous expression is not monotonous as $1/a^5$ as in Einstein's behaviour and under 1E7*m* shows up sign variations preventing us from yielding an accurate comparison between experimental data and model previsions. We think it of as due to the early stage of approximation algorithm but successive approximations require very powerful calculus skill. We also show in fig. 4a/b the differential instantaneous energy loss $d^2E/dtd\Omega$ versus the orbital angle in both Vortex and Einstein cases - as well-known the latter proportional to the third derivative of quadrupole moment [9]:

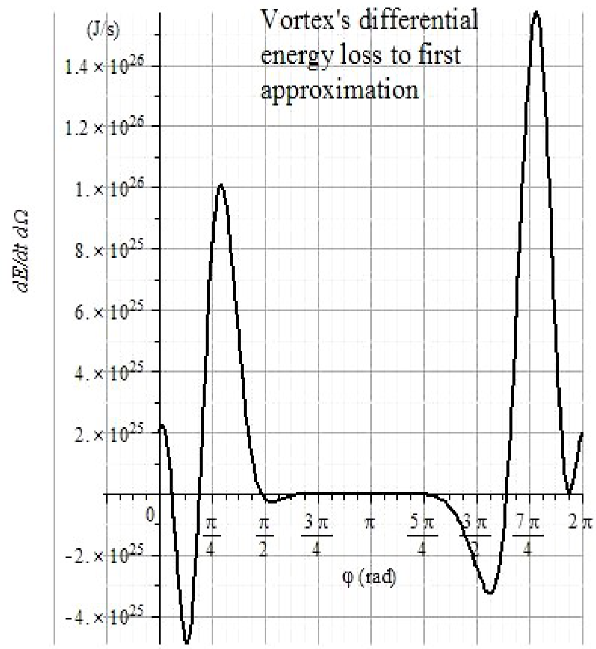

Fig. 4(a): 2π periodic emitted instantaneous power at given direction for Vortex model at polar angle $\Theta=\pi/2$ and azimuth $\Phi=\pi/2$.

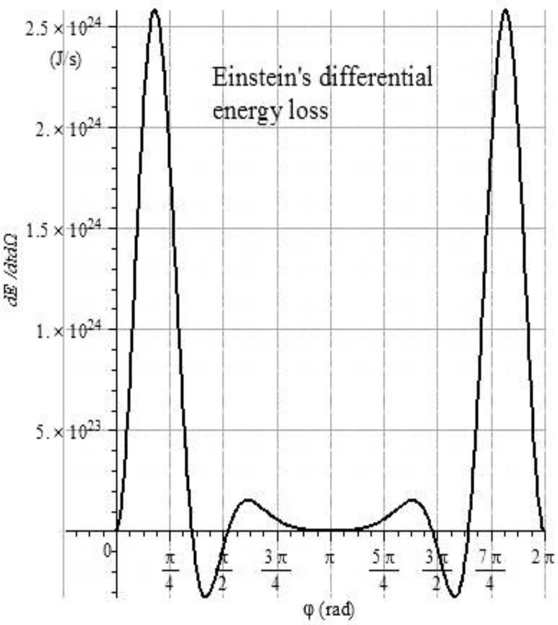

Fig. 4(b): 2π periodic emitted instantaneous power at given direction for Einstein's model at polar angle $\Theta=\pi/2$ and azimuth $\Phi=\pi/2$.

Finally, in order to detect a ⊕ polarized wave whose effect is in-phase deformations of linear distances in a plan normal to the direction of propagation, Michelson interferometer alike VIRGO is not suitable in that it works counter-phase. A possible advisable apparatus could be instead a large scale version of the interferometer set up by T.J.Herzog et al. [12,13] in the quantum entanglement equipment making use of a nonlinear crystal (e.g. Li B_3O_5) whose intensity at the output is proportional to $(1+cos(\Delta\varphi))$ where $\Delta\varphi$ is the overall arms phase-displacement (fig.5).

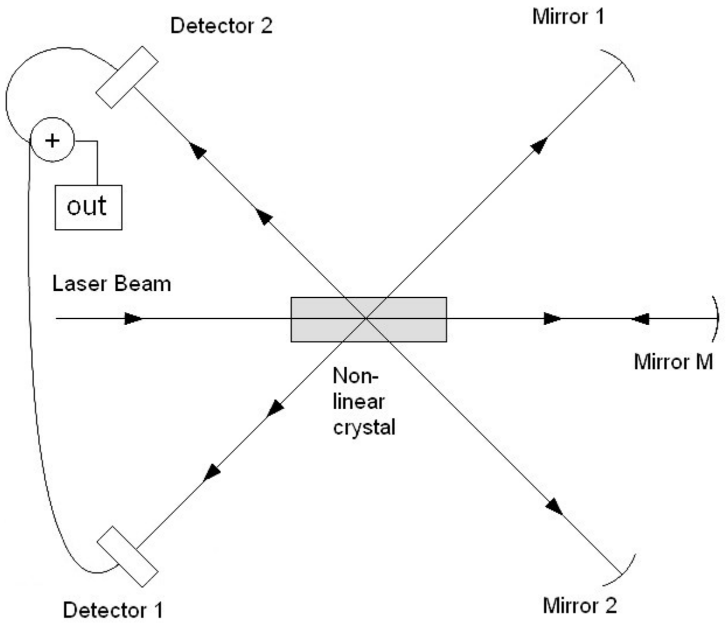

Fig. 5: Scheme of interferometric antenna for detection of gravitational wave in the ⊕ polarization in which a linear crystal splits the incident beam in two ones forth and back.

Conclusions

We have described a gravitational propagation framework that we expect to be appropriate in the theoretical interpretation of gravitational wave detection data, in which perturbations of metric produced by a source are detected over macroscopic scale like interferometry. We have shown that taking $K_{\mu\nu}$ as a symmetric potential of a massless particle (graviton) recovers the two independent polarizations of the field as in Einstein theory, but with the difference that counter-phase effects are checkable only in the ⊗ polarization. Great difficulty is encountered in resolving the field equations, which makes very hard verification of experimental data through ordinary numerical algorithms, so limiting the analysis to the leading approximation. Certainly, higher order approximations are mandatory to make a telltale check of the theory ; nevertheless for our purposes it is sufficient to be acquainted with the possibility of the evaluation of the coupling constant $S = 5.677E19\ m^{-1}$ as measure of the intrinsic inertia of the Space-Time to sweep out gravitational radiation and to assure causality in relativity as stressed in the reference paper [8]. Also, comparable agreement has been shown between the Vortex Model and the Einstein energy losses amount. As further consequence of this approach, a new acceleration law for test particles and a new cosmology are being developed and proved to be consistent with cosmic observational data.

Acknowledgements : I thank Dr.Mazzoli for introducing me to Tailherer's work and Maplesoft development team for suggestions.

References

[1] A.Loinger, *GW's towards fundamental principles of GR*, arXiv.org/abs/0709.0490, (2007), and references therein.

[2] C.Cattaneo, Introduction à la Théorie Macroscopique des Fluides Relativistes, Lectures held at the Collège de France, italian translation, Pitagora Eds. Bologna, (1980).

[3] G.Ferrarese: *Compatibility Conditions in Relativistic Continuum Dynamics*, Proc. of the Int. Conf. At Elba Is. in *Advances in modern continuum dynamics*, Pitagora Eds., Bologna, (1992).

[4] G.Ferrarese, Stazi: *Lezioni di Meccanica Razionale* vol 2 chap.VIII, §1.10, Pitagora Eds, Bologna, (1989).

[5] G.Ferrarrese-L.Stazi: *Il teorema di Lagrange nei fluidi perfetti relativistici*, Rend. Di Mat. **3,** pp. 409-421 (1989)

[6] G.Ferrarrese: *Lezioni di relatività Generale* , chap. 1, Pitagora Eds, Bologna, (2001).

[7] I.S.Sokolnikoff : *Tensor Analysis*, Chap. 6, §105, Wiley, New York, (1951).

[8] M.Tailherer : *A Critical Reading on the Theory of Gravitational Wave Propagation*, Journal of Natural Sciences, Vol.**1**, Issue 1, www.scientificjournals.org/journals2007/articles/1085.pdf (2007).

[9] H.C. Ohanian, R. Ruffini, *Gravitation and Spacetime*, sec.6.6; sec.5.2. W.W. Norton & Company, (1994).

[10] J.M.Weisberg and J.H. Taylor, *Relativistic Binary Pulsar B1913+16: Thirty Years of Observations and Analysis, arXiv:astro-ph/0407149 v1*, (2004).

[11] Peters, Mathews, *Gravitational radiation for point masses in a keplerian orbit*, Phys. Rev., **131,** (1963)

[12] T.J. Herzog, P.G.Kwiat, et al., Phys. Rev. Letters, 75**:** 3034, (1995).

[13] S. Gasiorowicz, *Quantum Physics*, 3th edition, Ch, 20.2 , Wiley, N.Y., (2003).