Commentary

The master formula for the U-matrix finally found?

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Abstract

In zero energy ontology U-matrix replaces S-matrix as the fundamental object characterizing the predictions of the theory. U-matrix is defined between zero energy states and its orthogonal rows define what I call M-matrices, which are analogous to thermal S-matrices of thermal QFTs. M-matrix defines the time-like entanglement coefficients between positive and negative energy parts of the zero energy state. M-matrices identifiable as hermitian square roots of density matrices. In this article it is shown that M-matrices form in a natural manner a generalization of Kac-Moody type algebra acting as symmetries of M-matrices and U-matrix and that the space of zero energy states has therefore Lie algebra structure so that quantum states act as their own symmetries. The generators of this algebra are multilocal with respect to partonic 2-surfaces just as Yangian algebras are multilocal with respect to points of Minkowski space and therefore define generalization of the Yangian algebra appearing in the Grassmannian twistor approach to $\mathcal{N} = 4$ SUSY.

1 Introduction

In zero energy ontology U-matrix replaces S-matrix as the fundamental object characterizing the predictions of the theory. U-matrix is defined between zero energy states and its orthogonal rows define what I call M-matrices, which are analogous to thermal S-matrices of thermal QFTs. M-matrix defines the time-like entanglement coefficients between positive and negative energy parts of the zero energy state.

A dramatic development of ideas related to the construction of U-matrix has taken place during the last year. In particular, twistorialization becomes possible in zero energy ontology and leads to the generalization of the Yangian symmetry of $\mathcal{N} = 4$ SUSY to TGD framework [22] with the replacement of finite-dimensional super-conformal group of M^4 with infinite-D super-conformal group assignable to partonic 2-surfaces. What is so beautiful is that the physical IR cutoff due to the formation of bound states of massless wormhole throats resolves the infrared divergence problem whereas UV divergences are solved by on mass shell propagation of wormhole throats for virtual particles. This also guarantees that

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Yangian invariance is not lost. There are excellent reasons to expect that the twistorial constructions generalize.

Recently quite dramatic further developments have taken place in the understanding of the notions of U-matrix, M-matrix and S-matrix- a trinity of matrices replacing in zero energy ontology the notion of S-matrix of positive energy ontology. Also twistorialization reduces to pure group theory-albeit infinite-dimensional: zero energy states define Yangian algebra. In the following I summarize these developments. It is however good to summarize first various loosely related ideas developed during years which converge to a tight pattern in in the resulting conceptual framework.

1. The realization that the hermitian square roots of density matrices form infinite-D unitary algebra and that their commutativity with universal S-matrix implies that zero energy states define the generalization of Kac-Moody algebra became only after I had realized the possibility to construct U-matrix. It is this observation which reduces the construction of U-matrix (or matrices if they form algebra) to that for S is expected to correspond directly to the ordinary S-matrix. A possible interpretation of the Kac-Moody type algebra of U-matrices is in terms of scales of CDs coming as positive integer powers of two. Another possibility more in line with the usual interpretation of S-matrix as time evolution operator is that scales of CDs come as integers and these integers correspond to powers of S.

   What is most fascinating is that zero energy states themselves define the symmetry algebra of the theory and that this algebra can be interpreted as a generalization of Yangian responsible for the successes of Grassmannian twistor approach by replacing finite-dimensional conformal group of Minkowski space with infinite-dimensional super-conformal algebras associated with partonic 2-surfaces in accordance with the replacement of point-like particles with surfaces. The basic characteristic of Yangian algebra is the multilocality of its generators and zero energy states are indeed multilocal since they involve partonic surfaces at both light-like boundaries of CD. Quantum TGD reduces to pure group theory! Note only states but also dynamics is coded completely by symmetries since M-matrices code for quantum dynamics! This aspect of zero energy ontology I have not realized before.

2. In ordinary QFT Feynman diagrams are purely algebraic objects. In TGD framework they reduce to space-time topology and geometry with Euclidian regions of space-time surfaces having interpretation as generalized Feynman diagrams. At the vertices of generalized Feynman diagrams in coming partonic 2-surfaces meet just like in ordinary Feynman diagrams which means deep difference from string theory. A more general assumption is that entire 4-D lines of generalized Feynman diagram meet at vertices. This could apply to the Euclidian regions only.

   There is also a second kind of branching involved with the hierarchy of Planck constants [17]. In Minkowskian regions similar meeting would take place for the branches of space-time sheets with same values of canonical momentum densities of Kähler action at the ends of CDs and have interpretation in terms of fractionization and hierarchy of Planck constants. The value of Planck constant for single branch would be effectively and integer multiple of the ordinary one. For the entire multi-sheeted structure describable naturally in terms of singular covering space of $M^4 \times CP_2$ it would be just the ordinary value.

3. Zero energy ontology with massless external wormhole wormholes implies as such twistorialization of the theory [10] although external wormhole momenta must be assumed to be massive bounds states of massless throats. This also guarantees exact Yangian symmetry and the absence of IR divergences. If also virtual wormhole throats are massless, twistorialization takes place in strong sense. This is possible only in zero energy ontology and accepting the identification of wormhole throats as basic building blocks of particles.

4. The notion of bosonic emergence [16] means that bosonic propagators emerge as radiative loops for wormhole contacts. The emergence generalizes to all states associated with wormhole contacts and also to flux tubes having wormhole contacts at their ends. What is nice that coupling constants emerge as normalization factors of propagators. Note that for single wormhole throat as opposed to wormhole contact having two throats bosonic propagator would result as a product of two collinear

fermionic propagators and have the standard form. For states with higher total number of fermions and ant-ifermions the propagator of wormhole throat behaves as p^n, $n > 2$. Here however p is replaced with what I call pseudo-momentum.

5. Number theoretical universality based on the extension of physics to p-adic number fields [13] suggest that at given level (CD) only finite sum of diagrams appears: otherwise there is danger that one obtains sum of rational functions which is not rational anymore. This gives strong constraints on generalized Feynman diagrams at the lowest level of the hierarchy. This follows naturally if twistor diagrams are identified as sums of Feynman diagrams which are irreducible in the sense that they do not represent two subsequence scatterings. Only these diagrams contribute to twistor diagram and the number of these diagrams is finite if all particles have small mass (even photon which would eat the remaining Higgs component).

6. Category theoretical approach [11] based on planar operad proposed for few years ago fits nicely with the twistorial construction of amplitudes interpreting radiative corrections in terms of CDs within CDs picture. The generalized Feynman diagrams with radiative corrections define the analog of planar operad with disk containing within itself disks containing.... replaced with causal diamond containing causal diamonds containing....

2 What is the master formula for the U-matrix?

The basic challenge is however still there and boils down to a simple question represented in the title. This master formula should be something extremely simple and should generalize the formula for S-matrix defined between positive energy states and identified formally as the exponential of Hamiltonian operator. In TGD framework the notion of unitary time development is given up so that something else is required and this something else should be manifestly Lorentz invariant and characterize the interactions.

Thinking the problem from this point of view allows only one answer: replace the time evolution operator defined by the Hamiltonian with the exponent for the action containing both bosonic and fermionic term. Bosonic term is the action for preferred extremal of Kähler action, which is indeed the unique Lorentz invariant defining interactions! Fermionic term would given by Chern-Simons Dirac action associated with light-like three surfaces and space-like 3-surfaces at the ends of CDs. The formula is as simple as it is obvious and still I had to use 32 years to discover it!

It took however one day to realize that the situation is not so simple as one might think first. The question is whether this action should be interpreted as the counterpart of action or effective action obtained by performing path integral in presence of external sources in QFT framework. Since one restricts space-time surfaces to preferred extremals so that there is no path integral, the only possible interpretation as the effective action. Also the condition that one obtains fermionic propagators correctly allows only this interpretation. For the Chern-Simons Dirac action the propagator would be the inverse of the correct propagator which obviously makes no sense. For the corresponding effective action the kinetic term is replaced with propagator and correct fermionic Feynman rules result when spinor basis selected to represent generalized eigenstates of the Chern-Simons Dirac operator.

The action interpreted as a counterpart of QFT effective action reduces to the sum of fermionic and bosonic terms. To make the representation more fluent I will mean with 3-surfaces in the following either the light-like orbits of wormhole throats at which the signature of the induced metric changes or the ends of space-time sheets at the boundaries of CDs. Note that it is possible to have CDs within CDs and these give rise to loop corrections having interpretation as zero energy states in shorter length scale. Finite measurement resolution means that one integrates over these degrees of freedom below the resolution scale. This gives rise to discrete variant of gauge coupling evolution based on scalings by factor two for CDs.

The next unpleasant question was whether this U-matrix is actually only the S-matrix appearing in the expression of a given M-matrix as a product of a hermitian square root of density matrix and unitary S-matrix having interpretation as the TGD counterpart of the ordinary S-matrix. The physical picture suggests this strongly. This observation led to a realization that the square roots of density matrices can be identified as generators of infinite-dimensional Lie-algebra of unitary matrices. Unit norm requires that hyper-finite factor of type II_1 is in question. The construction reduces to that for unitary S-matrix.

2.1 Universal formula for the hermitian square roots of density matrix

Zero energy ontology replaces S-matrix with M-matrix and groups M-matrices to rows of U-matrix. S-matrix appears as factor in the decomposition of M-matrix to a product of hermitian square root of density matrix and unitary S-matrix interpreted in standard sense.

$$M_i = \rho_i^{1/2} S \ \ .$$

Note that one cannot drop the S-matrix factor from M-matrix since M-matrix is neither unitary nor hermitian and the dropping of S would make it hermitian. The analog of the decomposition of M-matrix to the decomposition of Schrödinger amplitude to a product of its modulus and of phase is obvious.

The interpretation is in terms of square root of thermodynamics. This interpretation should give the analogs of the Feynman rules ordinary quantum theory producing unitary matrix when one has pure quantum states so that density matrix is projector in 1-D sub-space of state space (for hyper-finite factors of type II_1 something more complex is required).

This is the case. M-matrices are in this case just the projections of S-matrix to 1-D subspaces defined by the rows of S-matrix. The state basis is naturally such that the positive energy states at the lower boundary of CD have well-defined quantum numbers and superposition of zero energy states does not contain different quantum numbers for the positive energy states. The state at the upper boundary of CD is the state resulting in the interaction of the particles of the initial state. Unitary of the resulting U-matrix reduces to that for S-matrix.

A more general situation allows square roots of density matrices which are diagonalizable hermitian matrices satisfying the orthogonality condition that the traces

$$Tr(\rho_i^{1/2}\rho_j^{1/2}) = \delta_{ij} \ \ .$$

The matrices span the Lie algebra of infinite-dimensional unitary group. The hermitian square roots of M-matrices would reduce to the Lie algebra of infinite-D unitary group. This does not hold true for zero energy states.

If one however assumes that S commutes with the algebra spanned by the square roots of density matrices and allows powers of S one obtains a larger algebra complely analogous to Kac-Moody algebra in the sense that powers of S takes the role of powers of $exp(in\phi)$ in Kac-Moody algebra generators. The commutativity of S and density matrices means that the square roots of density matrices span symmetry algebra of S. The Hermitian sub-Lie-algebra commuting with S is large: for $SU(N)$ it would correspond to $SU(N-1)\times U(1)$ so that the symmetry algebra is huge in infinite-D case.

A possible interpretation for the sub-space spanned by M-matrices proportional to S^n is in terms of the hierarchy of CDs. If one assumes that the size scales of CDs come as octaves 2^m of a fundamental scale then one would have $m = n$. Second possibility is that scales of CDs come as integer multiples of the CP_2 scale: in this case the interpretation of n would be as this integer: this interpretation conforms with the intuitive picture about S as TGD counterpart of time evolution operator. This interpretation could also make sense for the M-matrice associated with the hierarchy of dark matter for which the scales of CDs indeed come as integers multiples of the basic scale.

If the square roots of density matrices are required to have only non-negative eigenvalues -as I have carelessly proposed in some contexts,- only projection operators are possible for Cartan algebra so that only pure states are possible. If one allows both signs one can have more interesting density matrices and this is the only manner to obtain square root of thermodynamics. Note that the standard representation for the Cartan algebra of finite-dimensional Lie group corresponds to non-pure state. For $\rho = Id$ one obtains $M = S$ defining the ordinary S-matrix. The orthogonality of this zero energy state with respect to other ones requires

$$Tr(\rho_i^{1/2}) = 0$$

stating that $SU(N=\infty)$ Lie algebra element is in question.

The reduction of the construction of U to that of S is an enormous simplification and reduces to the problem of finding the TGD counterpart of S-matrix. Note that the finiteness of the norm of $SS^\dagger = Id$

requires that hyper-finite factor of type II_1 is in question with the definining property that the infinite-dimensional unit matrix has unit norm. This means that state function reduction is always possible only into an infinite-dimensional subspace only [15].

The natural guess is that the Lie algebra generated by zero energy states is just the generalization of the Yangian symmetry algebra (see this) of$\mathcal{N} = 4$ SUSY postulated to be a symmetry algebra of TGD . The characteristic feature of the Yangian algebra is the multi-locality of its generators. The generators of the zero energy algebra are zero energy states and indeed form a hierarchy of multi-local objects defined by partonic 2-surfaces at upper and lower light-like boundaries of causal diamonds. Zero energy states themselves would define the symmetry algebra of the theory and the construction of quantum TGD also at the level of dynamics -not only quantum states in sense of positive energy ontology- would reduce to the construction of infinite-dimensional Lie-algebra! It is hard to imagine anything simpler!

2.2 Bosonic part of the action

Consider now the bosonic part of the action in detail.

1. The first term is the exponent of Kähler action which is purely classical quantity defining vacuum functional as the exponent of the modified Dirac action for the interior. Since there is no path integral over 4-surfaces, the only possible interpretation for Kähler action is as the counterpart of the effective action of quantum field theories to which one can indeed assign unique field pattern one the boundary values are fixed. For the preferred extremals with boundary conditions satisfying the weak form of electric-magnetic duality Kähler action reduces to Chern-Simons term with a constraint guaranteing the weak form of electric-magnetic duality. This constraint implies that the theory does not reduce to topological QFT. One must perform functional integral over 3-surfaces.

2. What is interesting that the Kähler action reduces to Chern-Simons action with constraint term. Could one replace exponent of real Kähler action with the imaginary one so that the situation would resemble very strongly ordinary QFT? Note however that one can also consider the replacement of imaginary unit with real unit in Chern-Simons action exponential and that in Abelian case the quantization argument for the coefficient of Chern-Simons action does not apply: the coefficient is however fixed by the weak form of electric-magnetic duality. In fact unitarity does not allow imaginary exponent: a simpler example is function space endowed with inner product defined by integration with weighting by exponent of some function. Unitarity requires real exponent.

3. Bosonic term involves also measurement interaction term which formally reduces to an addition of gauge part to Kähler gauge potential linear in momentum, color isospin and hyper charge, and possible other measured quantum numbers. This term couples space-time geometry to conserved quantum numbers and in this manner guarantees quantum classical correspondence. This term is added either to interior or with opposite sign to 3-surfaces but not both and therefore does not reduce to gauge transform. This term induces to Chern-Simons term at boundary an effective gauge term as addition to the induced Kähler gauge potential appearing in the Chern-Simons Dirac action. There it is not necessary add this term separately as done earlier.

2.3 Fermionic part of the action

It took some time to understand the identification of the fermionic term of the action.

1. By holography the fermionic term should reduce to modified Chern-Simons Dirac action with kinetic term replaced with its inverse. Otherwise kinetic term would replace propagator in the perturbative expansion. This replacement is new as compared to the earlier work.

2. The assumption familiar already from earlier work [9] is that spinors are generalized eigen modes of Chern-Simons Dirac operator with eigenvalues given by $\lambda^k\gamma_k$, where λ^k having only M^4 components is what I have called pseudo-momentum having region momentum as in Grassmannian approach to twistorialization. This gives the analog of massless propagator.

The natural assumption is that pseudo-momenta relate to the massless incoming and outgoing momenta propagating along wormhole lines via twistorial formula: in other words, the difference of pseudomomenta in the vertex of polygon to which external particle line is attached equals to the incoming real massless momentum. This allows to identify virtual particles as composites of massless wormhole throats. Incoming particles consists also of massless wormhole throats but are bound states so that their mass is quantized. The precise relationship between pseudo-momenta and real massless momenta in loops remains to be understood.

3. One could postulate the form of the fermionic effective action directly. It is also possible to obtain it by interpreting Chern-Simons Dirac action as being associated with primary spinor field and the spinor fields associated with the interior as the analog of external spinor source. These fields can be coupled to each other in standard manner by the term $\overline{\Psi}\Phi + \overline{\Phi}\Psi$, which couples quark and lepton chiralities but does not lead to the breaking of baryon and lepton number conservation in perturbation theory as terms of form $\overline{\Psi}\Psi$ and $\overline{\Phi}\Phi$ would lead. The Grassmannian path integral over Φ gives the fermionic effective action as the integral of $\overline{\Psi}D_{CSD}^{-1}\Psi$ over 3-surface with D^{-1} identified as the propagator for Chern-Simons Dirac action. The assumption that spinors are generalized eigenmodes of D at the 3-surface implies the reduction of propagator to $1/\lambda^k\gamma_k$ in the basis of generalized eigen modes.

4. In the spirit of holography the resulting fermionic effective action reduces to the terms assignable to 3-surfaces (as defined above) since in the interior Kähler Dirac equation is satisfied. Although Kähler Dirac action vanishes, its function of Kähler Dirac equation is highly non-trivial in holography since it correlates the modes of the induced spinor fields at different wormhole throats. One ask whether one should add to the fermionic effective action also measurement interaction term. Since this term correspond formally to a gauge term in Kähler gauge potential and is already induced by the corresponding bosonic term, the addition of this term seems un-necessary.

5. The explicit expression of the interaction term is obtained by expressing the second quantized induced spinor fields in terms of the fermionic oscillator operators. The quantization in these degrees of freedom has been discussed in [9]. Therefore the action of the exponential is completely well-defined and gives rise to a perturbation series in terms of massless pseudo-momentum propagators. The triviality of the perturbation series comes from the fact vertices are topological defined by partonic two-surfaces at which the lines of generalized Feynman diagrams meet.

2.4 Definition of U-matrix

The definition of U-matrix would be shockingly simple once the reduction to the construction of S-matrix is accepted. Just the exponential effective Chern-Simons Dirac action besides Kähler action reducing to Chern-Simons term and defining the weight for the functional integral over 3-surface. What is encouraging that the resulting U-matrix would be more or less the same as the one expected on basis of heuristic considerations.

1. The basis for bare zero energy states is obtained by using pairs of positive and negative energy states assigned to the boundaries of CD and having opposite quantum numbers. The action of the exponent of Kähler action and Chern-Simons Dirac effective action generates from these states "dressed" states and U-matrix is defined between these stressed states and bare states. M-matrix in turn is defined by the action of L on given bare zero energy states as entanglement coefficients.

2. U-matrix is automatically unitary in the fermionic degrees of freedom since the effective Chern-Simons Dirac action with the inverse of the usual kinetic term on the role of kinetic term is Hermitian operator. In bosonic degrees of freedom one expects unitarity by the analogy with finite dimensional function space endowed with inner product with vacuum functional defining the weighting. This would mean a beautiful solution to the long standing problem of how to achieve unitarity.

3. There are strong reasons to believe that a duality prevails in the sense that one can restrict the interior part of action to either the Euclidian regions of space-time surfaces defining 4-D Feynman

diagram or to their Minkowskian exterior. Number theoretic vision [12] suggests this duality and the recent considerations [9] support the same conclusion. Obviously this duality brings in mind Wick rotation of quantum field theories.

4. The fermionic action corresponds formally to free action so that there are no explicit interaction vertices: the situation in the geometric formulation of string theory is same. There is however no need for non-linear interaction terms which are also responsible for the divergences of quantum field theories. The interaction terms are replaced with topological interaction vertex at which the light-like 3-surfaces associated defining the orbits of partonic 2-surfaces (wormhole throats) meet like lines of the ordinary Feynman diagram.

   Note that this vertex distinguishes between TGD and string models where trouser vertex is a typical vertex: in TGD framework this kind of geometric decay does not correspond to particle decay but to the propagation of particle along different paths. The conservation of quantum numbers is required at the vertices. Also massless-ness property for the particles propagating along the lines is natural in zero energy ontology and makes possible twistorialization with the constraint that physical particles are massive bound states of massless wormhole throats.

5. The non-trivial propagation of state with total number n of fermions and antifermions is possible only if n contractions of the propagator appears along the line (otherwise one would obtain only quark lepton contractions forbidden by conservation laws). This implies the proportionality $1/p^n$ of the propagator so that only total fermion number $n = 1, 2$ is possible for non-vacuum wormhole throat. This proportionality was earlier deduced from the SUSY limit of TGD based on a generalization of SUSY algebra [14]. As a consequence, wormhole contact having two throats can carry at most spin 2 and the large SUSY defined by the fermionic oscillator operators is badly broken and effectively reduced to that generated by the right-handed neutrino which is also broken.

6. The assumption that all particles have non-vanishing mass means that given state can decay only to a virtual state with finite number of particles. This together with massless propagation along virtual lines simplifies enormously the perturbation series and is expected to imply finiteness.

7. The integration over WCW could spoil the unitarity. Although the exponent of Kähler action is positive it could give rise to divergent integral if the Kähler action has definite sign. The reduction to Chern-Simons term does not make obvious the positivity. If one believes on Minkowskian-Euclidian duality in the sense that one can define vacuum functional either as the exponent of Kähler action for the Minkowskian or Euclidian regions, one obtains definite sign for the Kähler function since for the Euclidian signature Kähler action indeed has definite sign.

   What is remarkable that in Chern-Simons term the non-analytic $1/g_K^2$ dependence on Kähler coupling strength disappears by the the weak form of electric-magnetic duality so that perturbation series with respect to the small parameter g_K^2 should make sense. One expects that this expansion gives small contributions to coupling constants determined in lowest order by bosonic emergence and involving fermionic loops.

8. The resulting generalized Feynman diagrammatics differs from the standard one in many respects. The lines of Feynman diagrams are replaced with 3-surfaces in the sense specified above. Only a very restricted subset of loops are allowed classically by preferred extremals. The massless on mass shell property for wormhole throat momenta indeed allows very restricted phase space for loops. If all particles are massive bound states of massless wormhole throats intermediate virtual particles states with positive energies can contain only a finite number of particles so that the situation simplifies dramatically. The already mentioned collinear many-fermion states with propagator behaving like $1/p^n$, $n > 2$ are also present. Hence on can ask whether a more appropriate identification of generalized Feynman diagrams might be as counterparts of twistor diagrams.

3 What is the relationship of generalized Feynman diagrams to twistor diagrams?

The general idea about the construction of U-matrix gives strong support for the existing heuristics and provides a connection with category theoretical ideas (planar operads and generalized Feynman diagramatics [11]) and also suggests a generalization of twistor diagrammatics. Many questions of course remain unanswered. The basic question is the relationship of generalized Feynman diagrams with twistor diagrams. There are arguments favoring also the interpretation as direct counterparts of twistor diagrams. The following series of arguments however favors Feynman diagram interpretation and leads to a precise connection between the two diagrammatics.

3.1 What is the correct identification of pseudo-momenta

The modified Dirac equation gives as generalized eigenvalues the quantities $\lambda^k\gamma_k$. I have christen λ as f pseudo-momentum and proposed number theoretic quantization rules for the values of pseudo-momenta [9] The physical interpretation of pseudo-momenta is still open as is also their relationship to massless on mass shell momenta propagating in wormhole throats associated with virtual particles. It is convenient to consider wormhole contact with two wormhole throats as a representation of incoming or virtual particle. The questions are following.

1. Is there a summation over pseudo-momenta for wormhole throats such that the sum of pseudo-momenta equals to the total exchanged real momentum associated with the wormhole contact. The real momenta on virtual line would be massless and give strong kinematic conditions on phase space allowed in loops.

   Physical propagators from wormhole contacts would result as self energy loops for pseudo-momenta and there is the danger of getting divergences unless one uses the number theoretic conditions to reduce the summation as proposed. This picture would realize the idea about the emergence of bosonic propagators as fermionic radiative corrections and also more general propagators. Coupling constants would be predicted and appear in the normalization of bosonic propagators. Note that also the integration over WCW degrees of freedom affects the values of coupling constants.

   The question is how strong additional conditions the number theoretic quantization of pseudo-momenta poses on the exchanged massless real momenta depends on the strength of number theoretical conditions. Are these conditions sensible?

2. Can one really identify pseudo-momenta really identifiable as region momenta of the twistor approach as I have cautiously suggested? The above line of arguments does not encourage this interpretation. Whether the identification makes sense can be tested immediately by looking for the dependence of Grassmannian twistor amplitudes on pseudo-momenta. If it is of standard propagator form one can consider this interpretation.

3.2 Connection between generalized Feynman diagrams and generalized twistor diagrams

The connection between generalized Feynman diagrams and generalized twistor diagrams should be understood.

1. The natural manner to identify twistor diagrams for a given CD without radiative corrections given by the addition of sub-CDs would be as the diagrams obtained by connecting the points or upper and lower boundaries of CD to form a polygon. There are several manners to do this. The differences of region-momenta would give the massless momenta for each external wormhole throat. Region momenta would have nothing to do with pseudo-momenta.

2. Twistor diagrams would represent sum for a subset of allowed generalized Feynman diagrams with massless particles in internal lines. On mass shell condition for massless wormhole throats restricts dramatically the number of contributing diagrams and the assumption that all particles have at

least small mass means that particle numbers in intermediate states are finite. One however obtains infinite number of diagrams obtained as series of allowed diagrams. The problem is that although individual diagrams give rational functions, an infinite sum of them leads out from the algebraic extensions of p-adic numbers and rationals. This does not conform with number theoretic universality.

Therefore only irreducible diagrams not decomposing to series of allowed scatterings are allowed. As a consequence only finite number of diagrams are possible. The sum of these diagrams would correspond to a given basic twistor diagram. One could consider also the condition that at given length scaled determined by CD only tree diagrams are allowed. but this option looks ad hoc.

The addition of sub-CD:s would give radiative corrections from shorter length scales and the depth of the hierarchy of CDs within CDs hierarchy defines the IR and UV cutoffs and measurement resolution. If one accepts the assumption that the sizes of CD come as octaves of CP_2 time scale, there would be natural IR and UV cutoffs on the values of pseudo-momenta from p-adic length scale hypothesis so that the amplitudes should remain finite and there would no fear about the loss of number theoretic universality. Note that in zero energy ontology cutoffs would characterize physical states themselves rather than restrictions of physicist only.

3.3 Diagrammatics based on gluing of twistor amplitudes

Radiative corrections n shorter scales than that of CD would result from the gluing of basic amplitudes for CDs within CDs.

1. Radiative corrections could be organized in terms of twistor diagrams. The rule transforming twistor polygons to simplest Feynman diagrams is standard duality replacing polygon with external lines at vertices with a bundle of lines obtained by connecting external lines to same point in the interior of the polygon. For triangle this gives three vertex. For n-polygon this would give n-vertex which corresponds to tree diagram as a Feynman diagram.

   For instance, one can understand self energy corrections in this framework in terms of two twistorial triangles with two edges of both connected by two lines. Again on mass shell massless holds true for the throats. Vertex correction corresponds to triangle triangle within triangle with vertices of the inner triangle connected to the vertices of the outer triangle.. One obtains radiative corrections from this picture.

2. Also now one can have loops obtained as a closed ring of polygons connected to each other. There are also much more complex configurations of polygons. Unless one allow splitting of wormhole contacts the wormhole lines associated with a given wormhole throat end up to single CD.

3. For an outgoing pair of wormhole lines from given CD the wormhole throats should have same sign of energy: this would mean that only time-like momenta can propagate between CDs so that space-like loop momenta would be possible only for the fundamental radiative corrections. This would a further strong restriction on the amplitudes and space-like momentum exchanges would come from the fundamental level involving only a finite number of diagrams.

   Is this good or bad? If bad, should one be ready to assign independent CDs with the two wormhole throats? Or should the interpretation be that the wormhole contact is split and wormhole throats propagate in two different time directions? But is it possible to speak about single space-like momentum exchange if the wormhole contact is split. Note that pseudo-momentum propagator for wormhole throat would still make sense. This line of thought does not look attractive.

4. Massless particles assigned with wormhole lines connecting the polygons and net pseudo-momenta are not on mass shell. Apart from time-likeness of net momenta, the rules for the propagators seem exactly the same as for polygons. These rules would summarize how radiative corrections from shorter scales are obtained.

3.4 The generalization of the recursion formula to TGD framework

The great victory of twistor approach is the recursion formula for the amplitudes [22] (see also the representation in TGD framework [10]) applying to all planar diagrams of $N = 4$ SYM becoming an exact formula at the large N limit for gauge group $SU(N)$. In the recent case the infinite-dimensional character of the Yangian symmetry algebra of S-matrix could be correlate for large N limit so that the planar limit should make sense. Also the fact that string worlds sheets are an essential aspect of TGD approach suggests that stringy picture deduced by t'Hooft for gauge theories at this limit implies planarity. What is relevant in the recent case is the general structure of the reduction formula, not the details which as such are of course extremely interesting also in TGD framework since Grassmannian amplitudes are claimed to provide a universal representation of Yangian invariants.

The recursive formula expresses scattering amplitude with n external particles with k negative helicities up to l loops is expressible as a sum of two terms. The first term-referred to as classical contribution-involves a fusion of twistor amplitudes with smaller number of particles and with the number of loops not larger than l by a procedure used already for tree diagrams. Second term - called quantum contribution-involves l loops and is irreducible in the sense that it is not expressible as a fusion of lower amplitudes and is obtained from $n + 2$ particle by a process eliminating two particles. The identification of the TGD counterparts of these terms is obvious. The "classical" term corresponds to the proposed fusion of the lower level amplitudes associated with polygons for sub-CDs. The "quantum" term corresponds to the contribution appearing at the level of CD itself and involves genuine loops in Feynman sense but only a finite number of them.

Since zero energy states correspond to generators of Yangian algebra or rather- its Kac-Moody variant with integer power of phase factor identified as integer power of S, the recursion formula might allow an interpretation as a direct counterpart for the recursive definition of Yangian algebra in terms of relations allowing the construction of generators labeled by non-negative integers.

3.5 TGD counterpart for the duality of Feynman diagrams for twistors and Wilson loops for momentum twistors

One of the fascinating discoveries of twistor Grassmannian approach is that conformal invariance and its dual correspond in twistor approach to descriptions in terms of twistors in ordinary Minkowski space by starting from Feynman diagrams and in terms of momentum twistors in its dual by starting from Wilson loops. Also this duality has counterpart in TGD.

String world sheets are an essential part of quantum TGD and the translation of Witten's work with knots to TGD context led to a precise identification of string world sheets and a deep connection between TGD and the theory of knots, braids, braid cobordisms, and 2-knots emerges [18].

Amusingly, the basic idea of this connection emerged from the model of DNA as topological quantum computer [19] developed for few years ago. The braiding defining the quantum computation is time-like and can be illustrated using dance metaphor: the world lines of dancers define the running topological computation program. If you connect the feet of dancers to a wall with threads (dancers are lipids at cell membrane forming 2-D liquid, wall is represented by DNA nucleotide sequence, and threads are magnetic flux tubes), the threads entangle during dance and give rise to a space-like braiding and code the computer program to memory: a fundamental mechanism of memory. These braidings are clearly dual and this duality relates closely to the duality just to the duality between Feynman graphs and Wilson loops! The time evolution of this space-like braiding defines braid cobordism and also a 2-knot.

The natural implication of strong form of holography made possible by preferred extremal (Bohr orbit in generalized sense) property of space-time surfaces is that the descriptions in terms of string world sheets and partonic 2-surfaces are dual. The twistorial representation of this duality is as the duality of descriptions in terms of Feynman diagrammatics in ordinary space-time and Wilson sheets-rather than loops- in the dual space-time assigned with region momenta.

4 Generalized twistor diagrams and planar operads

The resulting diagrams would have very close resemblance to structures known as planar operads [21, 20] associated with both topological quantum field theories and subfactors of von Neumann algebras. Planar operads provide a graphic representation of these structures. Since TGD corresponds to almost topological QFT and since WCW ("world of classical worlds") Clifford algebras correspond to von Neumann algebras known as hyper-finite factors of type II_1 [15], the natural expectation is that generalized Feynman diagrams correspond to planar operads. This is indeed what I proposed for three years ago in [11] but with disks replaced with CDs so that a the recent view unifies several earlier visions.

An additional motivation for the operad picture came from the notion of super-symplectic analog of super-conformal field theory motivated by the assumption that the symplectic transformations of $\delta M^4_\pm \times CP_2$ act as isometries of WCW. The fusion rules of super-symplectic QFT lead to an infinite hierarchy of algebras forming an operad.

The basic structure of planar operad is very much reminiscent of generalized twistor diagrams.

1. One has essentially disks within disks connected by lines. The modification is obvious. Replace disks within disks disks with CDs within CDs and assign to the upper *resp.* lower boundaries of CDs positive *resp.* negative energy states. Many-sheeted space-time allows locally two CDs above each other corresponding to the identification of particles as wormhole contacts.

2. The planarity of the operad would be an obvious correlate for the absence of non-planar loops in twistor approach to $N = 4$ SUSY making it problematic. Stringy picture actually suggests the absence of non-planar diagrams. The proposed generalization of twistor diagrammatics allowing arbitrary polygons within polygons structure might be enough to compensate for the absence of non-planar diagrams.

To sum up, the recent view generalizes considerably twistor diagrammatics and gives a connection with hyper-finite factors of type II_1 and with planar operads. The identification of virtual states as composites of massless states is extremely natural in this framework. The construction is also consistent with the heuristic picture about generalized Feynman diagrams and with the earlier proposal about role of the planar operad. For these reasons I dare to expect that a big step towards precise form of the rules has been made

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