PROBLEMS WITH WARP DRIVE EXAMINED

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Abstract: In a short examination of some of the major problems raised as objections to Doctor Alcubierre's original proposal of warp drive within General Relativity(1) by many author's in both peer review publication and archive articles one discovers that solutions to these problems do exist if one is willing to consider a modified version of that original proposal. It is the findings of this Author that Warp Drive cannot be properly ruled out at this time at least as a possible future method of sub-light propulsion with the possible added benefit of working as a superluminal field propulsion drive.

PROBLEM ONE: The Weak Energy Condition Violation

Objection originally raised by Doctor Forward and others(2).

When Kerr Spactime is set so angular momentum is set to a=0 the result is the Schwarzchild space. Mathematically this space is defined with the following equation:

$$ds^{2=-a2dt2+v2(df-wdt)+(r2/D)dr2+r2dq2} \qquad [1]$$

If graphed the geometry of the Kerr geometry would somewhat resemble Figure 1 as seen below.

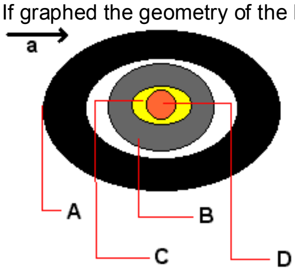

Figure 1

Generic representation of Kerr Spacetime in one dimension as viewed from the z plane, also note the horizons and singularity are not drawn to scale. a: represents the direction of angular momentum, A: outer horizon, B: inner horizon, C: singularity, D: "wormhole throat."

If the Schwarzchlid geometry were graphed in the similar fashion one would have only horizon, and the ring singularity would be replaced with a point singularity. Now before we get a head of ourselves to much let us look at the Schwarzchild Warp Drive as seen in the previous section:

$$ds2=\{1-(2GM)'A'(r'_{s})[vsr'\ 1-f'(r's)]2\}-\{dr'2/1-(2GM/r)'\}-r'2\ 2df'2-dz'2. \qquad [2]$$

For brevity we will now assume that this also the solution of the Kerr spacetime when a=0, and the egrosphere has minimal frame dragging so that it corresponds to a photon sphere of Schwarzchild black hole. From the last section we know if the Kerr spacetime is set with a=0 we may conclude that

$$0=R00-T00=(1/2)[T11+T22+T33] \qquad [3]$$

However if R00-T00>0, for the proposed spacetime that would imply that it is given by a Kerr geometry. This is also reveals a problem with the Kerr spacetime the geodesics of photons no longer remain symmetric. However under ideal cases it is somewhat easy to calculate the stress-energy associated with angular momentum tensors by

a=GJ/Mc3

which would alter the noted energy density from the previous section of

T00=-[1/A2]2[vs2/4][sinq2][dg(rs)/drs]2. [4]

The problem with this method is that it will only yield the total angular momentum of the system.

Dynamic Frame Dragging

Now for the interesting part the inner horizon of the Kerr Spacetime would have an inner egrosphere, which would act to frame drag the space located between the outer and inner horizons, resulting from the fact that inner horizon has a higher angular momentum via conservation laws. This is a simple supposition to make when one understand the properties of the Kerr spacetime and that of angular momentum (to aid in further exploration refer to figure 2 below through this discussion).

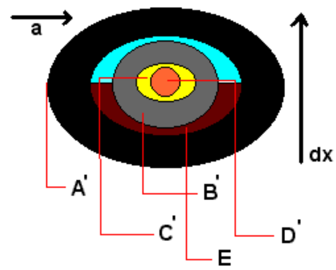

Figure 2

The Kerr Spacetime moving through space.

However, what becomes interesting is when one considers the Kerr Spacetime moving in the the x coordinate, as depicted with dx in figure 2. The region between the outer horizon (A') and inner horizon (B'), which is labeled as E, in region E the inner frame dragging egrosphere (not depicted), would be forced to fold on itself in the direction of travel. Thus the frame dragging effect begins to compress in front of the inner horizon, and from A' is forced to expand in the opposite direction, this is the property associated with the warp drive spacetime. From the graph it is seen that if a particle travels in region B' it is allowed to escape back into region E through the warp drive space, and from quantum theory has to potential to interact with the outer egrosphere again (possibly generating a new kind of quantum pressure), but this is speculation. One potential problem as seen from figure 2 is that there appears to exist a double coordinate transformation when one the compares Kerr metric with the Schwarzchild Warp Drive. The first being r(vs2*1-f(rs))1/2-->r' and the second coordinate transformation corresponding to x(vh21-f(x))1/2-->x', before I go into that problem I would like to discuss how the Kerr spacetime effect to geodesics such that light traveling to or from a Kerr horizon is manipulated by a lapse function. As light enters the horizon the light becomes ever more blue shifted, while a light beam attempting to leave becomes ever more red shifted the magnitude of this lapse function is given by:

a=(r/S)(D)1/2.

Manipulation of Intertia

Let's consider the case where T^{00} is zero. Since T^{00} has the interpretation of ship frame energy density and so there is zero ship frame energy requirement. The matter shell that could physically be constructed will have mass, but this is a requirement of materials, not of the inertia manipulation characteristics of the spacetime.

Taken as it, is has negative energy density. $T_{mn}U^{m}U^{n}$ is less than zero for some time-like vectors, but there is a way around the WEC violation. Consider the weak field approximation

$$ds^2 = (1 + 2f/c^2)dct^2 - ds^{2} \qquad [5]$$

The exact gravitational potential for the above spacetime is f = a fcosq . The addition of enough spherically symmetric mass into the T^{00} term will eliminate any WEC violation altogether, while in the weak field approximation it will merely add a term to the potential that is a function of r.

$$f_{new} = afcosq + V(r) \qquad [6]$$

This potential still has the inertia manipulating characteristics of the old potential, but has the addition of a central attraction.

The crew of such a ship would not be crushed against the hull of the ship for any amount of acceleration, but would instead accelerate with the ship in a weightless state of free fall. All fuel requirements would also then be eliminated.

Consider the spacetime interval given by

$$ds^2 = [1 + y/c^2]^2[1 + 2F/c^2]dct^2 - dr^2/[1 - 2r(dF/dr)/c^2] - r^2dq^2 - r^2sin^2qdf^2 \qquad [7]$$

where y is a function of z and F is a function of r.

y and F correspond to Newtonian gravitational potentials.

In the case that y = arcosq and F = - GM/r, where a is a function of time, the interval is a vacuum field solution to first order in the potentials. If either a or M is zero as well, it exactly reduces to a vacuum field solution.

One can use this spacetime as a means of gravitational self propulsion in the following way. Let y and F vary in the matter shell according toy = afcosq and $Ñ^{2}$ F = 4pGr. In the hollow interior, y = 0 and F = constant. The hollow interior will be the a vacuum field solution with a zero Riemann tensor and to first order in the potentials, the stress energy tensor of the shell will be

$$T^{00} = rc^2/g_{00}$$

$$T^{rr} = \left(\frac{c^4}{8\pi G}\right)\frac{2\alpha\cos\theta\left(r\frac{df}{dr} - f\right)}{c^2 r^2\sqrt{g_{00}}}$$

$$T^{r\theta} = T^{\theta r} = \left(\frac{c^4}{8\pi G}\right)\frac{\alpha\sin\theta\left(r\frac{df}{dr} - f\right)}{c^2 r^3\sqrt{g_{00}}}$$

$$T^{\theta\theta} = \left(\frac{c^4}{8\pi G}\right)\frac{\alpha\cos\theta\left(r^2\frac{d^2f}{dr^2} + r\frac{df}{dr} - f\right)}{c^2 r^4\sqrt{g_{00}}}$$

$$T^{\phi\phi} = \left(\frac{c^4}{8\pi G}\right)\frac{\alpha\cos\theta\left(r^2\frac{d^2f}{dr^2} + r\frac{df}{dr} - f\right)}{c^2 r^4\sin^2\theta\sqrt{g_{00}}}$$

all other $T^{mn} = 0$. [8]

This means that if this stress-energy is physically produced, to first order in the potentials it will have the desired spacetime interval above as its solution. (a) will be the proper acceleration of the ship and the crew will accelerate with the ship in a weightless state of free fall. Given a large enough r for a given a , this stress-

energy does not violate the weak energy condition.
Neglecting the df/dr and f terms in examining the source for the behavior of d^2f/dr^2 it turns out that one can physically produce those terms with a simple pressure dipole approximated by

$$\text{pressure} = \pm (c^4/8pG)(a/c^2)d^2f/dr^2 \qquad [9]$$

The + sign is taken for the half of the shell in the direction of the acceleration and the minus sign is taken for the other half of the shell. What you have here then is only the requirement that a pressure difference exists for the vacuum when front to back of the field is considered. This pressure difference does not have to actual create an exotic energy condition where the overall vacuum energy ever goes lower than normal.

Interesting enough there exists according to the findings of modern Cosmology just such a pressure difference in the form of the exotic energy or dark energy as it is often termed. This dark energy produces an anti-gravity like effect via a pressure difference that is according to theory responsible for the accelerated expansion of the cosmos. There also existed at one time according to modern theory a similar condition during the inflationary stage of the cosmos when the pressure difference resulted in the Universe inflating in size faster than C. Given all this if Nature can produce such a condition then duplicating such in a controlled area should be possible. Thus one of the major objections to warp drive involving the requirement of negative energy can be eliminated.

PROBLEM TWO: Control of the Forward Field Area

In Special relativity the vacuum speed of light is globally c (Eqn. 1.1.7 c = 299792458m/s exact by definition). Therefor in special relativity there is no faster than c travel. However, in general relativity the vacuum speed of light is only *locally invariant* and so the remote coordinate speed of light is not always c, and in some cases it is greater than c. There are metrics as shown below for which the coordinate speed of light c' can be increased above c. Because of this there are situations in general relativity where objects travel faster than c.

Lets consider for a moment the invariant interval for the world line of a particle moving in the x^1 direction, and dx^1 is a coordinate *distance* displacement.

$$ds^2 = g_{00}dct^2 + 2g_{01}dctdx^1 + g_{11}dx^1dx^1 \qquad [10]$$

If this is a photon, then $ds^2 = 0$.

$$0 = g_{00}dct^2 + 2g_{01}dctdx^1 + g_{11}dx^1dx^1$$

$$0 = g_{00} + 2g_{01}(dx^1/dct) + g_{11}(dx^1/dct)^2$$

$$0 = (1/2)g_{00} + g_{01}\beta + (1/2)g_{11}\beta^2$$

$$\beta = \{-g_{01} \pm [(g_{01})^2 - g_{00}g_{11}]^{1/2}\}/g_{11} \qquad [11]$$

So the remote coordinate speed of light is given by,

$$c' = \{-g_{01} \pm [(g_{01})^2 - g_{00}g_{11}]^{1/2}\}c/g_{11} \quad [12]$$

So we see that objects can travel faster than c in the $+ x^1$ direction if

$$\{-g_{01} + [(g_{01})^2 - g_{00}g_{11}]^{1/2}\}/g_{11} > 1 \quad [13]$$

But the question remains what about the control signal for a warp driven craft? Can such a control signal or more properly field forming signal be made to itself travel faster than C?

The proper answer to this question is found in the vacuum state in question and in the original implications of Einstein's relativity itself. As mentioned before according to special relativity the vacuum speed of light is globally c which is based upon certain properties of that vacuum itself. The implication being that if one where to alter any or all of those properties the local speed of light would not remain a constant. Generally, we term more properly Einstein's vacuum as a Minkowski spacetime. Again, the Minkowski spacetime has certain property values any of which if altered make for a different local velocity of light.

So, we must know determine if the vacuum state within a warp drive field still remains the same as that vacuum state found locally. For sake of argument here we will ignore for the moment the at rest field state around the craft itself since by definition that region would have the same vacuum condition as we globally normally encounter.

To begin to understand the warp field's own vacuum state we must consider the actual local energy level of that vacuum. Normally, we say that the local energy condition of the vacuum is given in GR by

$T_{\mu\nu}=0$ [14]

However, we already know that certain regions of a warp field have energy conditions that on the surface seem to violate the weak energy condition and as such would in theory be less than zero. If this was not the case none of the original problems with warp drive involving energy conditions would have been raised. Even in the before mentioned solution to that problem the energy condition of the warped region never equals zero except in the one region we've already elected to ignore and external to the warp field itself. So right from the start we are dealing with an altered from normal vacuum condition. We must then determine if the local velocity of light has remained a constant. To do this we must take a look at what exactly defines the local velocity of light in a vacuum.

The velocity of light in a medium such as the vacuum is determined by the electromagnetic (e.g. dielectric) properties of the medium. Effective path length on the other hand is increased by an increase in the density and hence it leads to higher refractive index and lower velocity. With relativity it's the energy density that we have already shown varies within a warp field. Any increase in energy density will result in a decrease in the local speed of light while any decrease in energy density will result in an increase.

In the forward region of the warp field we have a region with an increased energy density that creates the region towards which space-time and the craft itself embedded in the warp field, though at rest locally, is moving. By theory the local velocity of light should be lower in that region. If we follow the before mentioned solution to the energy condition problem one either has to view that region as having a decreasing energy density or a less than normal energy density if the normal path used by Alcubierre is followed. The result either way is a region where the local velocity of light is not a constant. If C is no longer a constant then in essence we can no longer treat a warp field as a normal Minkowski spacetime which to date most of the articles written on the problems of warp drive all tend to not take into account.

But this only partially deals with the problem. If we had only this information to go on it would be totally correct that one would have a forward control problem. The reasoning would be simple since the forward region has a velocity by theory less than C any control signal sent forward is going to move slower than the field itself once that field is established an in motion. In general, and again this is based upon only the information thus far given, you might could create such a field only to find oneself forever trapped in such a field with no means of actually shutting the field off. This is usually where most of the arguments against warp drive tend to leave the story off. But this is only part of the actual case involved here.

To the extent that we restrict our physics to special relativity and demand that the traditional order of causation is always correct, no information can travel faster than the speed c. However, special relativity is only a special case of physics within a more general theory known as general relativity.

In general relativity the vacuum speed of light is locally invariant, but not globally invariant. The vacuum speed of light remote from a given observer need not be c. It can be greater than c and can vary with direction depending on the gravitational field involved. Because this is allowed starships that travel between the stars faster than c are not ruled out a-priori within the physics of general relativity.

The first major modification to Alcubierre's original warp drive is given by the Alcubierre-Broeck Warp Drive equations(3). A case of the space-time geometry that he derived can be represented in the following equation:

$$ds^2 = dct^2 - (dx - bfdct)^2 - dy^2 - dz^2 \qquad [15]$$

f can be any function of the coordinates that is one at the location of the starship and zero far from it. Transforming coordinates to a particular choice of the starship coordinate frame and allowing it to travel faster than c (b > 1), coordinate singularities crop up in the equation corresponding to event horizons in front of and behind the ship enclosing it in a mathematical bubble. This is called the warp bubble. Even though the event horizons remain for any choice of ship frame, there is a choice for star ship coordinates for which the singular nature is transformed away.

In June 1999 Chris Van Den Broeck of the Institute for Theoretical Physics at the Catholic University of Leuven, Belgium, came up with an alteration for this space-time geometry that would retain all of the desired warp drive qualities but reduced the negative energy requirements down to the order of a transversable wormhole. see- http://xxx.lanl.gov/abs/gr-qc/9905084/ His space-time geometry can be represented by this equation:

$$ds^2 = dct^2 - B^2[(dx - b\ fdct)^2 + dy^2 + dz^2] \quad [16]$$

B can be any function that is large near the starship and one far from it though he used a specific top hat function for his example calculations. That not only brought the negative energy requirements down to a hopefully one day reachable goal but it also solved one of the quantum mechanics energy condition violations.

The Van Den Broeck Warp Drive was actually a case of a more general type of Alcubierre-Broeck Warp Drive SpaceTime. A more general case of an Alcubierre-Broeck Type Warp Drive SpaceTime can be written:

$$ds^2 = \left(A(x^\mu)^2 + g_{11}\beta^2 \cdot f^2\right)dct^2 - 2g_{11}\beta \cdot fdctdx^1 + g_{ij}dx^i dx^j \quad [17]$$

Where the Metric tensor reduces to that of special relativity far from the ship, and dx^1 represents a coordinate distance displacement.
The transformation of coordinates to the ship frame and the interval in the ship frame were given by Hiscock for the case of constant velocity in the consideration of only two dimensions. see - http://xxx.lanl.gov/abs/gr-qc/9707024. We can then use this to show something interesting here where his transformation extends to four dimensional spacetime with arbitrarily time dependent acceleration. We also present the ship frame energy density T^{00} from a four dimensional calculation and note that the 4d classical calculation is everywhere finite.

Consider an Alcubierre interval given according to a remote frame's cylindrical coordinates by:

$$ds^2 = (1 - b^2f^2)dct^2 + 2bfdctdz - dz^2 - dr^2 - r^2df^2 \quad [18]$$

Where f is a function that is 1 at the location of the ship and zero far from it.

Using

$$ct' = ct$$
$$z' = z - \int^{ct}\beta dct$$
$$r' = r$$
$$\phi' = \phi \qquad [19]$$

As the starting equations we find first that:

$$z' = z - \text{ουτβαχκ} \qquad [20]$$

Where b is first expressed here as a function of time ct. With some algebra for simplification this results in:

$$ds^2 = [1 - b^2(1 - f)^2]dct^2 - 2b(1 - f)dctdz' - dz'^2 - dr^2 - r^2df^2 \qquad [21]$$

Let g = 1 - f and this becomes:

$$ds^2 = (1 - b^2g^2)dct^2 - 2bgdctdz' - dz'^2 - dr^2 - r^2df^2 \qquad [22]$$

Notice that this returned the original intervals form with a reversal on the sign of b and a reversal of the boundary conditions for g.

Now we notice that at r = 0, this interval becomes the interval for special relativity transformed to cylindrical coordinates. Thus, we have found a transformation to a frame based local to the ship. One can also verify that in these coordinates the relevant affine connections vanish at r = 0.

Following the same general form of warp field at superluminal speeds, there are event horizons for this spacetime where $g_{00} = 0$, even in the case that we have transformed away the singularities there. These two horizons can be constructed as two half spheres enclosing the ship in a warp bubble. Information can not be sent from behind the ship outside the bubble to the inside. Also, information can not be sent from inside the ship to the region to outside, in front. For the warp drive, part of the matter region producing the warp, or the warp shell extends across the horizons. A signal sent from inside the ship can not reach the matter extending in front of the horizon that is in front of the ship. This piece of the warp shell can not then be turned off. This lead to the concern over the ability to control the warp at superluminal speeds.

We can work around the problem as follows. The matter controlling the behavior of $A(ct,x^i)$ can be arranged prior to the matter controlling the behavior of b(ct)g(r).

It is the behavior of b(ct)g(r) and not $A(ct,x^i)$ that controls the speed of the ship.

The horizon occurs where

$A(ct,x^i) = b(ct)g(r)$. [23]

The function g(r) is then manipulated so that it goes to 1 at a smaller distance from the ship then where

$A(ct,x^i) = b(ct)$. [24]

In other words, function $A(ct,x^i)$ is larger than b(ct) for an interval extending beyond the place where g becomes 1. Now the portion of the matter shell that was formed to control the behavior of $A(ct,x^i)$ that is in front of the horizon will not be accessible by a signal from the ship once superluminal speeds have been reached, but as far as controlling the speed of the ship goes, this does not matter. What matters is that the portion of the matter shell that controls the behavior of b(ct)g(r) is totally contained inside the horizons ensuring that the ship speed is controllable even after the ship has gone superluminal. So there does remain a solution to the control problem of a warp driven craft that has almost to the letter been ignored in all the published treatments of the subject to date.

Another issue here often overlooked is that the ship issuing the control signal in the first place is also moving forward superlumanaly once the field is established. If the craft itself is moving forward at 2C and it sends a signal forward into a region where C has already been established to be less than C in a normal vacuum state then its actual control signal will arrive at a velocity by timing that is equal to its velocity minus the local velocity of light in that region. The reason for this becomes apperant once one considers the boost the entire field itself privides. Yes, the control signal is slower than the field's velocity. But the region it has to actually reach has already been shown to be casually connected if we follow certain modified Alcubierre type field models. As such it only has to reach that velocity irrespective of the actual velocity the entire field is moving at. In short, you never actually lose control of the field once it is established.

PROBLEM THREE: Objects in the path of the Craft and Radiation.

While Jose Natario(4) is often credited as being the first Author to raise this problem he actually only raised one problem directly which concerned the effects of incoming blue shifted photons on the craft and crew as well as the warp bubble itself. The first problem in and of itself was actually first raised and a solution given by myself and several other Author's in an article that appeared first in Lanl (see: http://arxiv.org/PS_cache/gr-qc/pdf/0207/0207109.pdf) in poor rushed format with many mistakes and later in Peer Reviewed publication after much editing(see: http://65.108.189.168/Docs/The%20Shield%20Proposal.pdf for a copy of such that appeared in Volume Two of the Journal of Advanced Propulsion Methods (http://www.transtatorindustries.org/JOATP.html ISSN 1543-2661) This solution deals with both objects in the path of a warp driven craft and the energy level of incoming photons to the warped region and the craft.

Basically, again using the Broeck metric for warp drive we developed theory whereby a region is engineered into the warp field where not only can the actual energy of incoming photons be lowered to zero if one so chose, incoming larger matter in the path of the craft can be disrupted by gravitational tidal effects into very much smaller particles or even fully deflected out of the path of the craft in question. So in general objects in the path and incoming radiation are not to be considered a major stumbling block to a warp field development at some future date.

PROBLEM FOUR: Being able to navigate and see ahead while in flight.

This problem is perhaps the only real solid major objection out there to warp drive that does not offer at the current time a simple solution. However, there is a solution that was long ago proposed interesting enough in Science Fiction by Author's like Norton and others with their fictional hyperspace drives. That solution is simply to make short jumps via warp drive superlumanly and then slow down for a period to make observations ahead of the craft to determine if any major objects exist in the path of the craft and if the craft is actually still on course. I am not certain if those old Author's actually thought about the actual physics involved or not. But their solution actually does make sence when it comes to this type of flight into the unknown. If and when we actually go out there irrespective of the method of propulsion we use the first explores will have to rely upon something similar to find there actual way out there. Eventually some form of marker system could be established and even flight safe paths determined. But the first explores out there will generally be as limited as the first explores of the New World once where on earth.

CONCLUSIONS

In all by examining all the commonly raised objections to warp drive being valid we have found that there does exist solutions to those problems. It is then the conclusion of this Author and many of the actual researchers that have spent the time over the last few years since Alcubierre's original article appeared that one can simply not at this time fully rule out this form of field propulsion drive.

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