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The Black Hole Catastrophe: A Short Reply to J. J. Sharples

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A recent Letter to the Editor (Sharples J. J., Coordinate transformations and metric extension: a rebuttal to the relativistic claims of Stephen J. Crothers, *Progress in Physics*, v.1, 2010) has analysed a number of my papers. Dr. Sharples has committed errors in both mathematics and physics. His notion that $r = 0$ in the so-called "Schwarzschild solution" marks the point at the centre of the related manifold is false, as is his related claim that Schwarzschild's actual solution describes a manifold that is extendible. His *post hoc* introduction of Newtonian concepts and related mathematical expressions into the "Schwarzschild solution" is invalid; for instance, Newtonian two-body relations into what is alleged to be a one-body problem. Each of the objections are treated in turn and their invalidity fully demonstrated. Black hole theory is riddled with contradictions. This article provides definitive proof that black holes do not exist.

Keywords: Schwarzschild solution; singularity; black hole; Gaussian curvature; Riemannian curvature.

1 Introduction

A number of criticisms have been levelled in [1] against the arguments I have adduced to show that the black hole is not predicted by General Relativity. In reality, the black hole is a meaningless entity, without basis in any theory or in observation (nobody has ever found a black hole).

In the usual interpretation of Hilbert's [2–5] corrupted version of Schwarzschild's solution, the quantity r has *never* been properly identified by astrophysics. It has been variously and vaguely called a "*distance*" [6, 7], "*the radius*" [6, 8–22], the "*radius of a 2-sphere*" [1, 22, 23], the "*coordinate radius*" [24], the "*radial coordinate*" [1, 11, 16, 25–28], the "*Schwarzschild r-coordinate*" [26] , the "*radial space coordinate*" [29], the "*areal radius*" [24,25,27,30,31], the "*reduced circumference*" [28], and even "*a gauge choice: it defines the coordinate r*" [32]. In the particular case of $r = 2m = 2GM/c^2$ it is invariably referred to as the "*Schwarzschild radius*" or the "*gravitational radius*" [26]. However, it is *irrefutable* that this r is the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section [33–35]. It *does not* itself determine any distance in Hilbert's manifold. The correct identification of r in Hilbert's solution *completely subverts* all claims for black holes. Thus, $0 \leq r < 2m$ is meaningless for Hilbert's solution. It must also be emphasized that a geometry is completely determined by the *form* of its line-element [36, 37].

The usual *post hoc* inclusion of mass is invalid because it involves the arbitrary insertion of Newton's expression for escape velocity, which is a two-body relation (one body escapes from another), into what is alleged to be an expression for one body in an otherwise completely empty universe. All alleged black hole solutions pertain to a universe that contains only one mass. Since the 'Principle of Superposition' does not apply in General Relativity, Newton's expression for escape velocity cannot rightly appear in an expression that is claimed to pertain to a universe containing only one mass.

2 Concerning spherically symmetric metric spaces

In section 3 of [1] it is claimed that I write the general *static* spherically symmetric line-element in [35] as

$$ds^2 = A(r)dt^2 + B(r)dr^2 + C(r)d\Omega^2. \quad (1)$$

(where $d\Omega^2 = (d\theta^2 + \sin^2\theta d\varphi^2)$). This is incorrect, as I clearly wrote the *static* metric in all my relevant papers as

$$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \quad (2)$$

$A, B, C > 0$, in order to maintain Minkowski spacetime signature $(+, -, -, -)$, and hence the time-like character of the quantity t and the space-like character of the quantities r, θ, φ. Maintenance of correct signature is an important issue [7,36]. The following metric is then adduced in [1],

$$ds^2 = A^*(\rho)dt^2 + B^*(\rho)d\rho^2 + \rho^2 d\Omega^2. \quad (3)$$

Now, the actual metric I wrote in relation to eq. (3) is

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \quad (4)$$

where $A, B > 0$ once again to maintain signature. Besides the important matter of the required fixed signature $(+,-,-,-)$, Sharples has misunderstood my point: that the usual effective writing of $C(r) = r^2$ to get eq. (4) from eq. (2) *preempts* the form of the *a priori* unknown analytic function $C(r)$, and in that sense alone is eq. (4) not most general. Thus, eqs. (2) and (4) are geometrically equivalent, since the geometry is fully determined by the *form* of the line-element [36,37], not by the labelling of variables in the line-element. Moreover, in [38] I developed the relevant geometry from first principles, and explicitly stated that the most general 3-dimensional metric having spherical symmetry about an arbitrary point is [33],

$$ds^2 = A^2(R)dR^2 + R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right),$$

wherein R can be a real-valued function of some real parameter. This is a positive definite quadratic form. The objective in my relevant papers is determination of the admissible form of the function $C(r)$, bearing in mind the equivalent solutions due to Schwarzschild [39], Droste [40], and Brillouin [41]. Here is Schwarzschild's actual solution:

$$ds^2 = \left(1 - \frac{\alpha}{R}\right)dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{5}$$

$$R = R(r) = \left(r^3 + \alpha^3\right)^{\frac{1}{3}}, \quad 0 < r < \infty, \quad \alpha = const.$$

The r in Hilbert's solution (see eq. (8) below) can be replaced by *any* analytic function of r without disturbing spherical symmetry and without violation of $R_{\mu\nu} = 0$ [4, 35, 42]. The quantity r appearing in the solution for Schwarzschild spacetime acts as a parameter for all the components of the metric tensor. But any analytic function will not do: for instance, $C(r) = \exp(2r)$ does not satisfy *all* the required conditions. That there must be an infinite number of geometrically equivalent metrics is clear, as Eddington [42] has also noted. In the case of Schwarzschild, $C(r) = \left(r^3 + \alpha^3\right)^{2/3}$, $0 < r < \infty$, the metric singular only at $r = 0$; for Droste, $C(r) = r^2$, $2m < r < \infty$, m a constant, the metric singular only at $r = 2m$; and for Brillouin, $C(r) = (r + \alpha)^2$, $0 < r < \infty$, singular only at $r = 0$. The claim in [1] that I maintain that "*solutions of the gravitational field equations that are derived from the metric ansatz (9) are particular solutions rather than general solutions*" is inaccurate. Such solutions differ only by the specific assignment of $C(r)$. Clearly, the parameter r is not a distance in Schwarzschild space.

The usual derivation of the "Schwarzschild solution" begins with that for Minkowski spacetime [35, 38, 43–46], i. e.

$$ds^2 = c^2dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right).$$

The speed of light c is usually set to unity, to give,

$$ds^2 = dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \tag{6}$$

A generalisation thereof is proposed as, or equivalent to [2, 7, 12, 16, 22, 24–26, 29, 33, 36, 42, 47–57],

$$ds^2 = e^{2\lambda}dt^2 - e^{2\beta}dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{7}$$

$$0 \le r < \infty,$$

the real-valued exponential functions of r being introduced to emphasise the required fixed signature $(+,-,-,-)$. It is then required that $e^{2\lambda(r)}$ and $e^{2\beta(r)}$ be determined such as to satisfy $R_{\mu\nu} = 0$. Note that in going from eq. (6) to eq. (7), it is *assumed* $0 \le r < \infty$. Also note that eq. (7) not only retains the signature -2, but *also retains the signature* $(+,-,-,-)$, because $e^{2\lambda} > 0$ and $e^{2\beta} > 0$ *by construction* [7, 16, 36, 50, 57], since there is no possibility for Minkowski spacetime (eq. (6)) to change signature from $(+,-,-,-)$ to, for example, $(-,+,-,-)$. The astrophysics community then obtains the following "Schwarzschild solution",

$$ds^2 = \left(1 - \frac{2m}{r}\right)dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{8}$$

wherein the constant m is assigned to the mass causing the alleged associated gravitational field "outside" m. By inspection of eq. (8), it is asserted that there are two singularities; a removable coordinate singularity at $r = 2m$ and a physical singularity at $r = 0$. It is also asserted that $r = 2m$ gives the event horizon (the "Schwarzschild radius") of a black hole and that $r = 0$ is the position of the infinitely dense point-mass singularity of a black hole. Since neither $e^{2\lambda}$ nor $e^{2\beta}$ can change sign, the claim that r in metric (8) can take values less than $2m$ is false. In addition, the true nature of r in both eqs. (7) and (8) is entirely overlooked, and the geometric relations between the components of the metric tensor, fixed by the *form* of the line-element, are not applied.

Notwithstanding the fixing of the spacetime signature to $(+,-,-,-)$ in the writing of eq. (7), the astrophysics community permits $0 < r < 2m$ in eq. (8), changing the signature to $(-,+,-,-)$, and then admits that t becomes spacelike and r becomes timelike. When $2m < r < \infty$ the signature of eq. (8) is $(+,-,-,-)$; but if $0 < r < 2m$ in eq. (8), then

$$g_{00} = \left(1 - \frac{2m}{r}\right) < 0 \quad \text{and} \quad g_{11} = -\left(1 - \frac{2m}{r}\right)^{-1} > 0.$$

Therefore, the signature of metric (8) changes to $(-,+,-,-)$. According to Misner, Thorne and Wheeler [26], who use the spacetime signature $(-,+,+,+)$ instead of $(+,-,-,-)$,

> *"The most obvious pathology at $r = 2M$ is the reversal there of the roles of t and r as timelike and spacelike coordinates. In the region $r > 2M$, the t direction, $\partial/\partial t$, is timelike ($g_{tt} < 0$) and the r direction, $\partial/\partial r$, is spacelike ($g_{rr} > 0$); but in the region $r < 2M$, $\partial/\partial t$, is spacelike ($g_{tt} > 0$) and $\partial/\partial r$, is timelike ($g_{rr} < 0$).*

"At $r = 2M$, where r and t exchange roles as space and time coordinates, g_{tt} vanishes while g_{rr} is infinite."

Set $t = r^*$ and $r = t^*$; then, for $0 < r < 2m$, eq. (8) becomes,

$$ds^2 = \left(1 - \frac{2m}{t^*}\right) dr^{*2} - \left(1 - \frac{2m}{t^*}\right)^{-1} dt^{*2} - t^{*2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

$$0 < t^* < 2m.$$

It is now evident that this is a *non-static metric* since all the components of the metric tensor are functions of the timelike t^*, and thus this metric *bears no relationship to the original static problem to be solved* [4, 40, 41].

Conclusions:

1. In [1] the play on the words "particular solutions" and "general solution" is not a scientific argument.
2. All Schwarzschild metrics adduced in my papers are *geometrically equivalent* - I have never claimed otherwise.
3. The change of signature from $(+, -, -, -)$ to $(-, +, -, -)$ in eq. (8) violates the required fixed Minkowski spacetime signature $(+, -, -, -)$.
4. The range $0 < r < 2m$ on eq. (8) produces a *non-static* 'solution' to a *static* problem, and is therefore invalid.

3 The five additional criticisms

It is alleged in [1] that I have erred in holding the following five points true:

1. "*The coordinate 'ρ' appearing in (9), is not a proper radius,*
2. *The "Schwarzschild" solution as espoused by Hilbert and others is different to the Schwarzschild solution obtained originally by Schwarzschild,*
3. *The original Schwarzschild solution is a complete (i.e. inextendible) metric,*
4. *There are an infinite number of solutions to the static, spherically symmetric solutions to the field equations corresponding to a point mass,*
5. *For line-elements of the Schwarzschild form, the scalar curvature f remains bounded everywhere, and hence there is no 'black hole'.*"

Claim 1. This 'criticism' involves a change of meaning for *nowhere* in my writings have I ever asserted that the quantity ρ which appears in eq. (9) of [1] (i. e. eq. (3) above) cannot be "*a proper radius*" in some set of circumstances, such as by embedding into Euclidean 3-space the spherically symmetric geodesic surface in the spatial section of Hilbert's metric. I have, in fact, repeatedly proven that this ρ (r in eq. (8) above) is *not even a distance*, let alone a radius, in the Hilbert manifold. Here again is the proof. A line-element in spherical coordinates for 3-dimensional Euclidean space is,

$$ds^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (9)$$

$$0 \leq r < \infty.$$

The proper radius R_p of the associated sphere, for which $\theta = const.$ and $\varphi = const.$, is given by,

$$R_p = \int_0^r dr = r.$$

The equation of a sphere of radius ρ centered at the point C located at the extremity of the fixed vector $\mathbf{r}_o$ in Euclidean 3-space, is given by the inner product,

$$(\mathbf{r} - \mathbf{r}_o) \cdot (\mathbf{r} - \mathbf{r}_o) = \rho^2.$$

If $\mathbf{r}$ and $\mathbf{r}_o$ are collinear, the vector notation can be dropped, and this expression becomes,

$$\left|r - r_o\right| = \rho,$$

where $r = |\mathbf{r}|$ and $r_o = \left|\mathbf{r}_o\right|$, and the common direction of $\mathbf{r}$ and $\mathbf{r}_o$ becomes entirely immaterial. This scalar expression for a shift of the centre of spherical symmetry away from the origin of the coordinate system plays a significant rôle in the equivalent line-elements for Schwarzschild spacetime [2, 35].

Consider next the generalisation of eq. (9) to,

$$ds^2 = dR_p^2 + R_c^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) =$$

$$= \Psi\left(R_c\right) dR_c^2 + R_c^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (10)$$

$$R_c = R_c(r), \quad \Psi\left(R_c\right) > 0,$$

$$R_c(0) \leq R_c(r) < \infty,$$

where both $\Psi(R_c)$ and $R_c(r)$ are *a priori* unknown analytic functions. Since neither $\Psi(R_c)$ nor $R_c(r)$ are known, eq. (10) may or may not be well-defined at $R_c(0)$: one cannot know until $\Psi(R_c)$ and $R_c(r)$ are somehow specified. There is a one-to-one point-wise correspondence between the manifolds described by metrics (9) and (10), as those versed in differential geometry have explained [33]. If $R_c(r)$ is constant, eq. (10) reduces to a 2-dimensional spherically symmetric geodesic surface described by the first fundamental quadratic form,

$$ds^2 = R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \qquad (11)$$

Similarly, if r is constant, eq. (9) reduces to,

$$ds^2 = r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \qquad (12)$$

In the case of eqs. (11) and (12) the Gaussian curvature K (which depends only upon position), is given by [16, 33, 38, 58, 59, 61–64],

$$K = \frac{R_{1212}}{g}, \tag{13}$$

where R_{1212} is a component of the Riemann tensor of the 1st kind and $g = g_{11}g_{22} = g_{\theta\theta}g_{\varphi\varphi}$ (because the metric tensor of eq. (11) is diagonal). Gaussian curvature is an intrinsic geometric property of a surface (Theorema Egregium*); independent of any embedding space. Recall that

$$R_{\mu\nu\rho\sigma} = g_{\mu\gamma}R^{\gamma}_{.\,\nu\rho\sigma}$$

$$R^{1}_{.\,212} = \frac{\partial\Gamma^{1}_{22}}{\partial x^{1}} - \frac{\partial\Gamma^{1}_{21}}{\partial x^{2}} + \Gamma^{k}_{22}\Gamma^{1}_{k1} - \Gamma^{k}_{21}\Gamma^{1}_{k2}$$

$$\Gamma^{i}_{ij} = \Gamma^{i}_{ji} = \frac{\partial\left(\frac{1}{2}\ln\left|g_{ii}\right|\right)}{\partial x^{j}}$$

$$\Gamma^{i}_{jj} = -\frac{1}{2g_{ii}}\frac{\partial g_{jj}}{\partial x^{i}}, \quad (i \neq j) \tag{14}$$

and all other Γ^{i}_{jk} vanish. In the above, $i, j, k = 1, 2$, $x^1 = \theta$, $x^2 = \varphi$. Applying eqs. (13) and (14) to expression (11) gives,

$$K = \frac{1}{R_c^2} \tag{15}$$

by which $R_c(r)$ is the inverse square root of the Gaussian curvature. Hence, in eq. (8) the quantity r is the inverse square root of the related Gaussian curvature. This Gaussian curvature is intrinsic to all geometric surfaces having the form of eq. (11) [33]. This simple geometric fact also completely subverts all claims that General Relativity predicts black holes.

Sharples [1] objects to my use of the proper radius in Schwarzschild spacetime because this "*...does not take into account the effect of coordinate transformations*" [1]. This objection is groundless. Concerning Hilbert's metric, Carroll and Ostlie remark [25]:

> *"The 'curvature of space' resides in the radial term. The radial distance measured simultaneously ($dt = 0$) between two nearby points on the same radial line $d\theta = d\varphi = 0$ is just the proper distance*
>
> $$dL = \sqrt{-(ds)^2} = \frac{dr}{\sqrt{1 - 2GM/rc^2}}.$$
>
> *..."The factor of $1/\sqrt{1 - 2GM/rc^2}$ must be included in any calculation of spatial distances."*[†]

*i.e. Gauss' Most Excellent Theorem.

[†]Nonetheless, Carroll and Ostlie fail to apply this mathematical fact.

The solution for $R_{\mu\nu} = 0$ is [35],

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right)dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

$$R_c(r) = \left(\left|r - r_o\right|^n + \alpha^n\right)^{\frac{1}{n}} = \frac{1}{\sqrt{K(r)}},$$

$$r \in \Re, \quad n \in \Re^+, \quad r \neq r_o. \tag{16}$$

The proper radius is given by [35],

$$R_p = \int_0^{R_p} dR_p = \int_{R_c(r_o)}^{R_c(r)} \frac{dR_c(r)}{\sqrt{1 - \frac{\alpha}{R_c(r)}}} =$$

$$= \sqrt{R_c(r)\left(R_c(r) - \alpha\right)} + \alpha \ln\left(\frac{\sqrt{R_c(r)} + \sqrt{R_c(r) - \alpha}}{\sqrt{\alpha}}\right).$$

Then, by eqs. (16), if $r_o = 0$, $r \to r_o^+$, $n = 1$, Brillouin's solution results. If $r_o = 0$, $r \to r_o^+$, $n = 3$, then Schwarzschild's actual solution results. If $r_o = 2m$, $r \to r_o^+$, $n = 1$, then Droste's solution results, which is the correct solution in terms of the line-element of eq. (8). In addition, the required infinite set of equivalent metrics is obtained. The point r_o in the auxiliary manifold (Minkowski spacetime) is mapped into the point $R_p(r_o) = 0$ in Schwarzschild space, $\forall\, r_o\ \forall\, n$. The arbitrary point r_o is also mapped to $R_c(r_o) = \alpha\ \forall\, r_o\ \forall\, n$.

In [1] it is asserted, in relation to Schwarzschild's actual solution, that

> *...the manifold...is foliated by 2-spheres of radius greater than $\alpha = 2m$ – the spacetime has a hole in its centre!*

This is incorrect. The 2-spheres referred to in [1] are *not* in the Schwarzschild manifold because the said 2-spheres relate to a Euclidean 3-space in which the spherical surface described by $ds^2 = R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$ is considered to be embedded; making this 3-space an auxiliary manifold. Radial distance in Schwarzschild spacetime is given by $R_p(r)$. Consequently, distances between two points (one fixed at r_o) in the spatial section of Minkowski spacetime (which is precisely Euclidean 3-space entire) are mapped into distances in the Euclidean 3-space involving Schwarzschild's R, where the point at the centre of spherical symmetry is at $R(0) = \alpha$, *not* at $R = 0$. In general, according to eqs. (16) herein, $R_c(r)$ maps Euclidean 3-space into itself and thence maps distances therein into all the components of the metric tensor for Schwarzschild spacetime, as depicted in figure 1.

It is clear from expressions (16) that there is only one singularity, at the arbitrary constant r_o, where $R_c(r_o) = \alpha\ \forall\, r_o\ \forall\, n$ and $R_p(r_o) = 0\ \forall\, r_o\ \forall\, n$. Hence, the "removal" of the singularity at $r = 2m$ in eq. (8) is fallacious because, according to expressions (16), in eq. (8) $0 \leq r < 2m$ is meaningless [2–4, 7, 35, 38–40, 60, 64, 65]. There is no black hole anywhere.

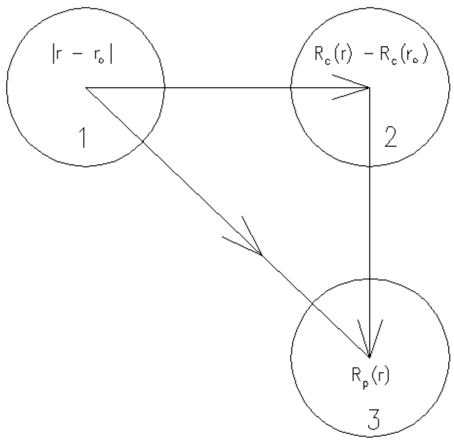

Fig. 1: Distances between two points (one fixed arbitrarily at the centre of spherical symmetry r_o) in the spatial section of Minkowski spacetime 1 (Euclidean 3-space) are mapped by $R_c(r)$ into Euclidean 3-space 2 (where the relevant centre of spherical symmetry is at the point $R_c(r_o) = \alpha \ \forall r_o \ \forall n$) and thence into all the components of the metric tensor for Schwarzschild space 3 where the point at the centre of spherical symmetry is located at $R_p(r_o) = 0 \ \forall r_o \ \forall n$. There are no holes in any of the manifolds.

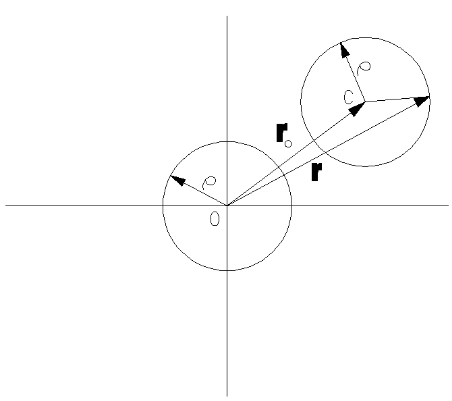

Fig. 2: The intrinsic geometry of the sphere is independent of its position in 3-space. The centre of the sphere is translated with the sphere - its centre point is not left at the origin of the coordinate system.

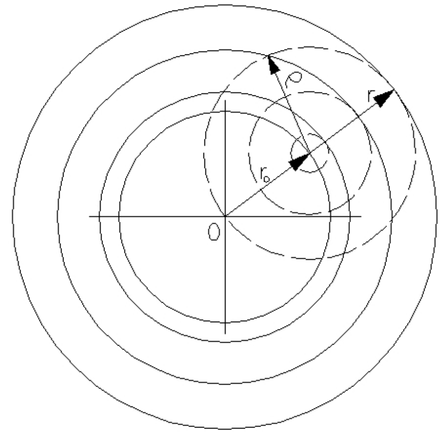

Fig. 3: The meaning of r_o – it is the relevant point at the centre of spherical symmetry in the spatial section of Minkowski spacetime. This point is arbitrary and need not be coincident with the origin of the coordinate system. The equation of a sphere of radius ρ centre C at the extremity of the fixed vector $\mathbf{r}_o$ is $(\mathbf{r} - \mathbf{r}_o) \cdot (\mathbf{r} - \mathbf{r}_o) = \rho^2$. If the vectors are collinear the equation is $|r - r_o| = \rho$; as depicted.

Consider the sphere in figure 2, radius ρ, centred at the origin of coordinates O. Shift the sphere to some arbitrary point C in the space, as depicted. The centre of the shifted sphere is now located at the extremity of the fixed vector $\mathbf{r}_o$ relative to the origin of coordinates. The surface of the shifted sphere is the locus of points at the extremity of the variable vector $\mathbf{r}$, and the radius of the sphere is $\rho = \left|\mathbf{r} - \mathbf{r}_o\right|$. Consider a point on the surface of the translated sphere. Let this point approach the centre of the sphere along any radius of the sphere, i. e. $\rho \to 0$. It would be a misconception to suggest that although the sphere has been shifted away from the origin of the coordinate system, the centre of the sphere is still located at the origin of the coordinate system. The black hole is precisely a result of this misconception.

Consider again the situation in figure 2, except that the vectors $\mathbf{r}$ and $\mathbf{r}_o$ are always collinear. In this case, the vector notation can be dropped, so that $\rho = \left|r - r_o\right|$ where $r = |\mathbf{r}|$ and $r_o = \left|\mathbf{r}_o\right|$. Now consider figure 3. As a point on the surface of the shifted sphere approaches the centre of the sphere along the collinear radial line, so that $r \to r_o^{\pm}$, in the case of the black hole this has been misinterpreted as the said point approaching the surface of a sphere of radius r_o (circle through the tip of r_o in figure 3), and hence the spherical space contained at that radius is misinterpreted as the interior of a black hole, with the event horizon at radius r_o. It is evident from figure 3 that as $r \to r_o^{\pm}$, $\rho \to 0^{+}$, and when $r = 0$, $\rho = r_o$.

Minkowski spacetime plays the rôle of a parametric space for Schwarzschild spacetime. There is a mapping of distance between two arbitrary points, one fixed, in the spatial section of Minkowski space into all the components of the metric tensor of eq. (16). What the astrophysicists have unknowingly done by writing metric (8), is to shift the parametric sphere in the spatial section of Minkowski spacetime away from the origin of coordinates of Minkowski spacetime to a *point* at distance $r_o = 2m$ whilst thinking that the centre of the shifted parametric sphere is still located at $r = 0$. This is compounded by misinterpretation of r in eq. (8) as the proper radius in the spatial section thereof. With that, the astrophysicists think that $0 \leq r < \infty$ in the eq. (8). The shift of the location of the centre of spherical symmetry was pointed out explicitly by Abrams [2], and implicitly by Schwarzschild [39]. Note in figure 3 that, as $\rho \to \infty$, the whole of the spatial section of Minkowski spacetime is accounted for – *there is no hole in the manifold*.

According to [1], the manifold associated with Schwarzschild's actual solution is extendible:

> *"Indeed, in deriving this form of the line-element, Schwarzschild imposed a very specific boundary condition, namely that the line-element is continuous everywhere except at $r = 0$, where $r \in (0, \infty)$ is the standard spherical radial coordinate. Imposition of this boundary condition has significant implications for the solution obtained. In particular, as a consequence of the boundary condition the coordinate R is shifted away from the origin. Indeed, if $r \in (0, \infty)$ then $R \in (\alpha, \infty)$. Hence . . . the spacetime has a hole in its centre!*

However, the argument is specious. First and foremost, Schwarzschild's R is *not even a distance* let alone a radial one in his manifold. Second, the erroneous argument is similar to that adduced by G. Szekeres [66] in that it is not recognised that the shifting of R away from the origin is a shifting of the relevant *point* at the centre of spherical symmetry away from the point at the origin in a corresponding Euclidean 3-space, as depicted in the preceeding figures. Consider the spatial section of Minkowski spacetime,

$$ds^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right),$$

$$0 \leq r < \infty.$$

Make the substitution $r = \overline{r} - 2m$, m a positive number. Then, the metric becomes

$$ds^2 = d\overline{r}^2 + (\overline{r} - 2m)^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right).$$

According to Szekeres [66], there is now *"an apparent singularity on the sphere $\overline{r} = 2m$, due to a spreading out of the origin over a sphere of radius 2m."* The claim is easily proven false, as follows:

$$\begin{aligned} ds^2 &= d\overline{r}^2 + (\overline{r} - 2m)^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \\ &= \frac{(\overline{r} - 2m)^2}{|\overline{r} - 2m|^2} d\overline{r}^2 + |\overline{r} - 2m|^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \\ &= (d\,|\overline{r} - 2m|)^2 + |\overline{r} - 2m|^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \\ &= d\rho^2 + \rho^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \\ \rho &= |\overline{r} - 2m| \geq 0, \end{aligned}$$

which describes the *whole* of Euclidean 3-space [38] and is therefore inextendible. There is no *"hole in its centre"*, and no separate manifold; the relevant centre has simply been shifted away from the origin of coordinates to some point at distance $2m$ from it, the direction of the translation being immaterial, as figures 2 and 3 illustrate.

Sharples has not understood Schwarzschild's argument for fixing his value of r_o to zero. In his paper, Schwarzschild [39] obtained a constant of integration ρ relating to his r_o and his function $R(r)$, thus

$$r_o^3 = \alpha^3 - \rho, \quad \text{and} \quad R(r) = \left(r^3 + \rho\right)^{\frac{1}{3}}.$$

He began his analysis with a generalisation that located the parametric point at the centre of spherical symmetry by construction at $r_o = 0$, thus [39]

$$ds^2 = F dt^2 - \left(G + Hr^2\right) dr^2 - Gr^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right),$$

where F, G, H are all functions of $r = \sqrt{x^2 + y^2 + z^2}$. Therefore, when x, y and z are all zero, r is zero. To make the origin $R_p = 0$ of his solution coincide with $r_o = 0$ of the auxiliary manifold, he chose his constant of integration at $\rho = \alpha^3$ to get $R_p(0) = 0$. However, Schwarzschild was at liberty to choose *any real value* for his constant. Indeed, if he set $\rho = 0$ he would have obtained $R = r$ and the line-element (8) above, with the range $\alpha < r < \infty$, with $R_p(\alpha) = 0$, the metric being singular only at $r = \alpha$, as Droste [40] determined independently. If Schwarzschild chose $\rho = 0$ he would have *moved* the centre of spherical symmetry along a radial line from the point $r_o = 0$ in the auxiliary manifold to the point $r_o = \alpha$ (direction of translation being immaterial); and as demonstrated above, this does not make a *"hole"* appear in the auxiliary manifold or in the Schwarzschild manifold.

The arbitrary point r_o of the (Euclidean) spatial section of Minkowski spacetime corresponds to the point $R_c(r_o) = \alpha$ $\forall r_o$ $\forall n$ in the Euclidean R_c space and thence to the point $R_p(r_o) = 0$ $\forall r_o$ $\forall n$ in the (non-Euclidean) spatial section of Schwarzschild spacetime. Then, as $r \to r_o^{\pm}$, $R_c(r) \to \alpha^+$ and $R_p(r) \to 0^+$, as depicted in figure 4. In Schwarzschild spacetime, the quantity R_c is not a distance of any sort therein. According to figure 4,

$$\rho_c = R_c(r) - R_c(r_o) = R_c(r) - \alpha$$

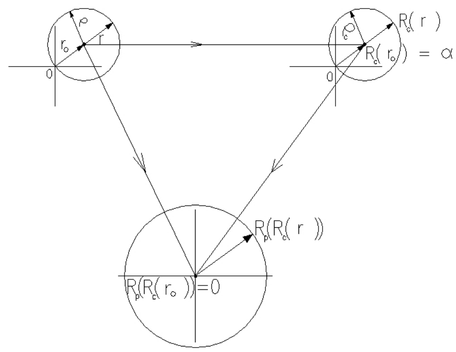

Fig. 4: This is a schematic representation of the relations between the three manifolds (Minkowski spacetime, the R_c intermediary manifold, and Schwarzschild spacetime). The following implications are apparent: $r \to \pm\infty \Rightarrow \rho \to \infty$, then $R_c(r) \to \infty \Rightarrow \rho_c \to \infty$, and then $R_p(r) \to \infty$. Similarly, $r \to r_o^{\pm} \Rightarrow \rho \to 0^+$, and so $R_c(r) \to \alpha^+ \Rightarrow \rho_c \to 0^+$, then $R_p(r) \to 0^+$. $R_c(r_o) = \alpha\ \forall\ r_o\ \forall\ n$ and $R_p(r_o) = 0\ \forall\ r_o\ \forall\ n$. Each manifold is inextendible.

since by eqs. (16) $R_c(r_o) = \alpha\ \forall\ r_o\ \forall\ n$. Also by eqs. (16),

$$\rho_c = \left(\left|r - r_o\right|^n + \alpha^n\right)^{\frac{1}{n}} - \alpha = (\rho^n + \alpha^n)^{\frac{1}{n}} - \alpha,$$

since $\rho = \left|r - r_o\right|$. Then, the proper radius for Schwarzschild spacetime can be written as

$$R_p(\rho_c) = \sqrt{\rho_c(\rho_c + \alpha)} + \alpha \ln\left(\frac{\sqrt{\rho_c + \alpha} + \sqrt{\rho_c}}{\sqrt{\alpha}}\right).$$

Therefore, $\rho \to 0^+ \Rightarrow \rho_c \to 0^+ \Rightarrow R_p \to 0^+$ and $\rho \to \infty \Rightarrow \rho_c \to \infty \Rightarrow R_p \to \infty$. Hence, all three manifolds are inextendible, as Abrams [2] proved by a different method. Also of importance is the fact that Hagihara [67] proved, in 1931, that all geodesics that do not run into the boundary of the "Schwarzschild" metric at $r = 2m$ (i. e. at $R_p(r_o = 2m) = 0$) are complete, which therefore holds for eqs. (16) as well.

The following remark [1],

> *"In fact it is well-known that there exist coordinates in which the difficulty at $R = 2m$ can be removed, resulting in a single manifold that satisfies the field equations."*

is apparently an allusion to Eddington-Finkelstein [42,68] coordinates and Kruskal-Szekeres [66, 69] coordinates. I have shown elsewhere [70] that the Eddington-Finkelstein coordinates are without scientific merit and the invalidity of the Kruskal-Szekeres method as well [70, 71].

Sharples' claim that

> "*...the proper radius does not depend on the form of the line-element*" [1]

is patently false, as proven above, because the *form* of the line-element fully determines the geometry [36, 37]. One must not confuse labels with the *form* of the metric, as Sharples has done. The claim that I have asserted that the contested quantity denoted by ρ in [1] is "*not a proper radius*" is false - it can be a proper radius if related to a 3-D Euclidean embedding space, but it is certainly *not* the proper radius in the "Schwarzschild" manifold, which is what I have actually argued and proven.

Conclusions:

1. Sharples' use of the words "*not a proper radius*", by substituting the article "a" for the article "the" in relation to Hilbert's metric is not a scientific argument.

2. The quantity r in the "Schwarzschild" solution is *not* even a distance in "Schwarzschild" spacetime.

3. My use of the proper radius *is valid*.

Claims 2 and 3. One can plainly see that Schwarzschild's solution is different to that of Hilbert.

It is asserted in [1] that

> "*...by imposing the additional boundary condition at infinity, that the solution be consistent with the predictions of Newtonian gravitational theory, it is found that the constant $\alpha = 2m$, where m is the mass at the origin*".

This reveals once again Sharples' erroneous notion that $r = 0$ marks the point at the centre of spherical symmetry of "Schwarzschild" spacetime (where in fact $\rho = r_o$ as in figure 3). The "Schwarzschild" solution pertains to a universe that allegedly contains only one mass. But Newtonian gravitational potential is a *two-body* concept; it is defined as the work done per unit mass against Newton's gravitational field. There is no meaning to a Newtonian potential for a single mass in an otherwise empty Universe. Newton's theory of gravitation is defined in terms of the *a priori* interaction of *two* masses in a space for which the 'Principle of Superposition' applies. In General Relativity, spacetime and matter are causally linked, and the Principle of Superposition *does not apply*. It is impossible for "Schwarzschild" spacetime to reduce to, or otherwise contain, an expression that is defined in terms of the *a priori* interaction of *two* masses. Writing eq. (8) in terms of c and G explicitly gives,

$$ds^2 = \left(c^2 - \frac{2GM}{r}\right)dt^2 - c^2\left(c^2 - \frac{2GM}{r}\right)^{-1} dr^2 - \\ - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right).$$

The term $2GM/r$ is immediately recognised as the square of the Newtonian escape velocity of a body from a mass M. From this arbitrary insertion of Newton's expression for escape velocity it is claimed that when the "escape velocity" is that of light in vacuum, there is an event horizon and hence a black hole. Yet escape velocity is a concept that involves *two* *bodies* - one body escapes from another body. Even though one mass appears in Newton's expression for escape velocity, it cannot be determined without recourse to a fundamental two-body gravitational interaction. Recall that Newton's Universal Law of Gravitation is,

$$F_g = -G\frac{mM}{r^2},$$

where G is the gravitational constant and r is the distance between the centres of mass for m and M. Newton's gravitational potential Φ is defined mathematically as,

$$\Phi = \lim_{\sigma\to\infty}\int_\sigma^r -\frac{F_g}{m}dr = -G\frac{M}{r},$$

This is a two-body concept. The potential energy P in the gravitational field due to masses m and M is given by,

$$P = m\Phi = -G\frac{mM}{r},$$

which is a two-body concept. Similarly, the velocity required by a mass m to escape from the gravitational field due to masses m and M is determined by,

$$F_g = -G\frac{mM}{r^2} = ma = m\frac{dv}{dt} = mv\frac{dv}{dr}.$$

Separating variables and integrating gives,

$$\int_v^0 mv\,dv = \lim_{r_f\to\infty}\int_R^{r_f} -GmM\,\frac{dr}{r^2} \;\Rightarrow\; v = \sqrt{\frac{2GM}{R}},$$

where R is the radius of the mass M. Thus, escape velocity necessarily involves *two bodies*. The denominator in the Newtonian expressions is the *proper radius*. But it is not even a distance in the "Schwarzschild" manifold.

That Schwarzschild spacetime cannot be extended is reaffirmed by the Riemannian (or Sectional) curvature K_s of the spatial section of Schwarzschild spacetime, given by

$$K_s = \frac{-\frac{\alpha}{2}W_{1212} - \frac{\alpha}{2}W_{1313}\sin^2\theta + \alpha R_c\left(R_c - \alpha\right)W_{2323}}{R_c^3 W_{1212} + R_c^3 W_{1313}\sin^2\theta + R_c^4\sin^2\theta\left(R_c - \alpha\right)W_{2323}}$$

$$R_c = \left(\left|r - r_o\right|^n + \alpha^n\right)^{\frac{1}{n}}, \quad r \in \Re, \quad n \in \Re^+,$$

where

$$W_{ijkl} = \begin{vmatrix} U^i & U^j \\ V^i & V^j \end{vmatrix}\begin{vmatrix} U^k & U^l \\ V^k & V^l \end{vmatrix}$$

and $< U^i >$ and $< V^i >$ are two arbitrary non-zero contravariant vectors at any point in the space. Thus, in general, the Riemannian curvature is dependent upon both position and direction (i. e. the direction of the contravariant vectors). Now,

$$K_s\left(r_o\right) = -\frac{1}{2\alpha^2} = -\frac{1}{2R_c^2\left(r_o\right)} = -\frac{1}{2}K\left(r_o\right)$$

which is *entirely independent* of the contravariant vectors (and hence independent of direction). This is a scalar invariant that corresponds to $R_c(r_o) = \alpha\ \forall r_o\ \forall n$ and $R_p(r_o) = 0\ \forall r_o\ \forall n$; thereby marking the true singularity.

Doughty [72] has shown that the radial geodesic acceleration a of a point in a manifold described by a line-element with the *form* of eq. (8) is given by,

$$a = \frac{\sqrt{-g_{11}}\left(-g^{11}\right)\left|g_{00,1}\right|}{2g_{00}}.$$

In the case of eq. (8), for which $r_o = 2m$,

$$a = \frac{2m}{r^{\frac{3}{2}}\sqrt{r-2m}},$$

and so $a \to \infty$ as $r \to 2m^+$, where, according to the astrophysicists, *there is no matter*!

Conclusions:

1. Hilbert's solution is different to Schwarzschild's solution.
2. The introduction of Newtonian two-body relations and concepts into Hilbert's solution is inadmissible.
3. Schwarzschild spacetime cannot be extended.

Claim 4. Sharples [1] essentially reproduces variations in the notation of the line-element that already occur in my papers in order to claim that,

> *"... what appears to be an infinitude of particular solutions are actually just different coordinate expressions of the same solution ... "*

Yet, in my papers, I have repeatedly remarked that *all* the line-elements I adduce via eqs. (16) *are geomerically equivalent*. None contain a black hole. In the abstract of [60] I wrote:

> *"It is proved herein that the metric in the so-called 'isotropic coordinates' for Einstein's gravitational field is a particular case of an infinite class of equivalent metrics."*

In the abstracts of my conference papers [45, 46] I wrote:

> *"With the correct identification of the associated Gaussian curvature it is also easily proven that there is only one singularity associated with all Schwarzschild metrics, of which there is an infinite number that are equivalent."*

Sharples appeals to the so-called 'Birkoff's Theorem':

> *"This theorem establishes, with mathematical certainty, that the Schwarzschild solution (exterior, interior or both) is the only solution of the spherically symmetric vacuum field equations."*

Abrams [2] pointed out that Birkoff's Theorem only establishes the *form* of the line-element, not the range on the related Gaussian curvature. Nikias Stavroulakis [73] has argued that Birkoff's Theorem is not even a theorem.

Conclusions:

1. Sharples' claim that I have asserted that there is an infinite number of geometrically inequivalent solutions for $R_{\mu\nu} = 0$ is patently false.

2. Birkoff's "theorem" establishes only the *form* of the line-element; if it is a theorem at all.

Claim 5. Sharples [1] appeals to the Kretschmann scalar $f = R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$, reiterating the usual but unproven argument that a singularity must occur where this invariant is unbounded [22, 24, 26, 29, 51]. The Kretschmann scalar for Schwarzschild spacetime is not an independent curvature invariant because it is a function of the related Gaussian curvature K, and K is constrained by the proper radius to the range $0 < K < \alpha^{-2}$, not $0 < K < \infty$. The Kretschmann scalar is given incorrectly by Sharples [1] as $f = 12\alpha^2/R^3$. For Schwarzschild spacetime the Kretschmann scalar is actually given by

$$f = 12\alpha^2 K^3 = \frac{12\alpha^2}{R_c^6} = \frac{12\alpha^2}{\left(|r - r_o|^n + \alpha^n\right)^{\frac{6}{n}}}.$$

Then,

$$f\left(r_o\right) = \frac{12}{\alpha^4} \quad \forall\, r_o \;\forall\, n,$$

which is a scalar invariant that corresponds to the scalar invariants $R_p\left(r_o\right) = 0$, $R_c\left(r_o\right) = \alpha$, and $K_s\left(r_o\right) = -\frac{1}{2}\alpha^{-2}$.

Conclusion:

1. The Kretschmann scalar is everywhere finite in Schwarzschild spacetime.

4 Epilogue

The theoretician D. Rabounski [74] has reaffirmed my arguments that a black hole cannot form in Schwarzschild space.

A more detailed version of this paper is given in [75].

Dedication

I dedicate this paper to my late brother,

Paul Raymond Crothers

12th May 1968 – 25th December 2008.

24 December 2010

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