The Incredibly Shrinking Proton

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Abstract

The recent discovery that the charge radius of proton deduced from quantum average of nuclear charge density from the muonic version of hydrogen atom is 4 per cent smaller than the radius deduced from hydrogen atom challenges either QED or the view about proton or both. In TGD framework topological quantization leads to the notion of field body as a characteristic of any system. Field body is expected to contain substructures with sizes given by the primary and secondary p-adic length scales at at least. u and d quarks would have field bodies with size much larger than proton itself. In muonic atom the p-adic size scale of the field body of u quark having mass of 2 MeV according to the last estimates would be roughly twice the Boh radius so that the anomaly might be understood as a signature of field body.

Contents

1 Introduction

The discovery that the charge radius of proton deduced from the muonic version of hydrogen atom is about 4 per cent smaller than from the radius deduced from hydrogen atom [9, 8] is in complete conflict with the cherished belief that atomic physics belongs to the museum of science. The title of the article *Quantum electrodynamics-a chink in the armour?* of the article published in Nature [7] expresses well the possible implications, which might actually go well extend beyond QED.

The finding is a problem of QED or to the standard view about what proton is. Lamb shift [10] is the effect distinguishing between the states hydrogen atom having otherwise the same energy but different angular momentum. The effect is due to the quantum fluctuations of the electromagnetic field. The energy shift factorizes to a product of two expressions. The first one describes the effect of these zero point fluctuations on the position of electron or muon and the second one characterizes the average of nuclear charge density as "seen" by electron or muon. The latter one should be same as in the case of ordinary hydrogen atom but it is not. Does this mean that the presence of muon reduces the charge radius of proton as determined from muon wave function? This of course looks implausible since the radius of proton is so small. Note that the compression of the muon's wave function has the same effect.

Before continuing it is good to recall that QED and quantum field theories in general have difficulties with the description of bound states: something which has not received too much attention. For instance, van der Waals force at molecular scales is a problem. A possible TGD based explanation and a possible solution of difficulties proposed for two decades ago is that for bound states the two charged particles (say nucleus and electron or two atoms) correspond to two 3-D surfaces glued by flux tubes rather than being idealized to points of Minkowski space. This would make the non-relativistic description based on Schrödinger amplitude natural and replace the description based on Bethe-Salpeter equation having horrible mathematical properties.

2 Basic facts and notions

In this section the basic TGD inspired ideas and notions - in particular the notion of field body- are introduced and the general mechanism possibly explaining the reduction of the effective charge radius relying on the leakage of muon wave function to the flux tubes associated with u quarks is introduced. After this the value of leakage probability is estimated from the standard formula for the Lamb shift in the experimental situation considered.

2.1 Basic notions of TGD which might be relevant for the problem

Can one say anything interesting about the possible mechanism behind the anomaly if one accepts TGD framework? How the presence of muon could reduce the charge radius of proton? Let us first list the basic facts and notions.

1. One can say that the size of muonic hydrogen characterized by Bohr radius is by factor $m_e/m_\mu = 1/211.4 = 4.7 \times 10^{-4}$ smaller than for hydrogen atom and equals to 250 fm. Hydrogen atom Bohr radius is .53 Angstroms.

2. Proton contains 2 quarks with charge 2e/3 and one d quark which charge -e/3. These quarks are light. The last determination of u and d quark masses[12] gives masses, which are $m_u = 2$ MeV and $m_d = 5$ MeV (I leave out the error bars). The standard view is that the contribution of quarks to proton mass is of same order of magnitude. This would mean that quarks are not too relativistic meaning that one can assign to them a size of order Compton wave length of order $4 \times r_e \simeq 600$ fm in the case of u quark (roughly twice the Bohr radius of muonic hydrogen) and $10 \times r_e \simeq 24$ fm in the case of d quark. These wavelengths are much longer than the proton charge radius and for u quark more than twice longer than the Bohr radius of the muonic hydrogen. That parts of proton would be hundreds of times larger than proton itself sounds a rather weird idea. One could of course argue that the scales in question do not correspond to anything geometric. In TGD framework this is not the way out since quantum classical correspondence requires this geometric correlate.

3. There is also the notion of classical radius of electron and quark. It is given by $r = \alpha\hbar/m$ and is in the case of electron this radius is 2.8 fm whereas proton charge radius is .877 fm and smaller. The dependence on Planck constant is only apparent as it should be since classical radius is in question. For u quark the classical radius is .52 fm and smaller than proton charge radius. The constraint that the classical radii of quarks are smaller than proton charge radius gives a lower bound of quark masses: p-adic scaling of u quark mass by $2^{-1/2}$ would give classical radius .73 fm which still satisfies the bound. TGD framework the proper generalization would be $r = \alpha_K\hbar/m$, where α_K is Kähler coupling strength defining the fundamental coupling constant of the theory and quantized from quantum criticality. Its value is very near or equal to fine structure constant in electron length scale.

4. The intuitive picture is that light-like 3-surfaces assignable to quarks describe random motion of partonic 2-surfaces with light-velocity. This is analogous to zitterbewegung assigned classically to the ordinary Dirac equation. The notion of braid emerging from Chern-Simons Dirac equation via periodic boundary conditions means that the orbits of partonic 2-surface effectively reduces to braids carrying fermionic quantum numbers. These braids in turn define higher level

braids which would move inside a structure characterizing the particle geometrically. Internal consistency suggests that the classical radius $r = \alpha_K \hbar/m$ characterizes the size scale of the zitterbewegung orbits of quarks.

I cannot resist the temptation to emphasize the fact that Bohr orbitology is now reasonably well understood. The solutions of field equations with higher than 3-D CP_2 projection describing radiation fields allow only generalizations of plane waves but not their superpositions in accordance with the fact it is these modes that are observed. For massless extremals with 2-D CP_2 projection superposition is possible only for parallel light-like wave vectors. Furthermore, the restriction of the solutions of the Chern-Simons Dirac equation at light-like 3-surfaces to braid strands gives the analogs of Bohr orbits. Wave functions of -say electron in atom- are wave functions for the position of wormhole throat and thus for braid strands so that Bohr's theory becomes part of quantum theory.

5. In TGD framework quantum classical correspondence requires -or at least strongly suggests- that also the p-adic length scales assignable to u and d quarks have geometrical correlates. That quarks would have sizes much larger than proton itself how sounds rather paradoxical and could be used as an objection against p-adic length scale hypothesis. Topological field quantization however leads to the notion of field body as a structure consisting of flux tubes and and the identification of this geometric correlate would be in terms of Kähler (or color-, or electro-) magnetic body of proton consisting of color flux tubes beginning from space-time sheets of valence quarks and having length scale of order Compton wavelength much longer than the size of proton itself. Magnetic loops and electric flux tubes would be in question. Also secondary p-adic length scale characterizes field body. For instance, in the case of electron the causal diamond assigned to electron would correspond to the time scale of .1 seconds defining an important bio-rhythm.

2.2 Could the notion of field body explain the anomaly?

The large Compton radii of quarks and the notion of field body encourage the attempt to imagine a mechanism affecting the charge radius of proton as determined from electron's or muon's wave function.

1. Muon's wave function is compressed to a volume which is about 8 million times smaller than the corresponding volume in the case of electron. The Compton radius of u quark more that twice larger than the Bohr radius of muonic hydrogen so that muon should interact directly with the field body of u quark. The field body of d quark would have size 24 fm which is about ten times smaller than the Bohr radius so that one can say that the volume in which muons sees the field body of d quark is only one thousandth of the total volume. The main effect would be therefore due to the two u quarks having total charge of 4e/3.

   One can say that muon begins to "see" the field bodies of u quarks and interacts directly with u quarks rather than with proton via its elecromagnetic field body. With d quarks it would still interact via protons field body to which d quark should feed its electromagnetic flux. This could be quite enough to explain why the charge radius of proton determined from the expectation value defined by its wave function wave function is smaller than for electron. One must of course notice that this brings in also direct magnetic interactions with u quarks.

2. What could be the basic mechanism for the reduction of charge radius? Could it be that the electron is caught with some probability into the flux tubes of u quarks and that Schrödinger amplitude for this kind state vanishes near the origin? If so, this portion of state would not contribute to the charge radius and the since the portion ordinary state would smaller, this would imply an effective reduction of the charge radius determined from experimental data using the standard theory since the reduction of the norm of the standard part of the state would be erratically interpreted as a reduction of the charge radius.

3. This effect would be of course present also in the case of electron but in this case the u quarks correspond to a volume which million times smaller than the volume defined by Bohr radius so that electron does not in practice "see" the quark sub-structure of proton. The probability P

for getting caught would be in a good approximation proportional to the value of $|\Psi(r_u)|^2$ and in the first approximation one would have

$$\frac{P_e}{P_\mu} \sim (a_\mu/a_e)^3 = (m_e/m_\mu)^3 \sim 10^{-7} \ .$$

from the proportionality $\Psi_i \propto 1/a_i^{3/2}$, i=e,$\mu$.

2.3 A general formula for Lamb shift in terms of proton charge radius

The charge radius of proton is determined from the Lamb shift between 2S- and 2P states of muonic hydrogen. Without this effect resulting from vacuum polarization of photon Dirac equation for hydrogoen would predict identical energies for these states. The calculation reduces to the calculation of vacuum polarization of photon inducing to the Coulomb potential and an additional vacuum polarization term. Besides this effect one must also take into account the finite size of the proton which can be coded in terms of the form factor deducible from scattering data. It is just this correction which makes it possible to determine the charge radius of proton from the Lamb shift.

1. In the article [11] the basic theoretical results related to the Lamb shift in terms of the vacuum polarization of photon are discussed. Proton's charge density is in this representation is expressed in terms of proton form factor in principle deducible from the scattering data. Two special cases can be distinguished corresponding to the point like proton for which Lamb shift is non-vanishing only for S wave states and non-point like proton for which energy shift is present also for other states. The theoretical expression for the Lamb shift involves very refined calculations. Between 2P and 2S states the expression for the Lamb shift is of form

$$\Delta E(2P_{3/2}^{F=2} 2S_{1/2}^{F=1} = a - br_p^2 + cr_p^3 = 209.968(5)5.2248 \times r_p^2 + 0.0347 \times r_p^3 \ meV \ . \tag{2.1}$$

   where the charge radius $r_p = .8750$ is expressed in femtometers and energy in meVs.

2. The general expression of Lamb shift is given in terms of the form factor by

$$E(2P - 2S) = \int \frac{d^3q}{2\pi)^3} \times (-4\pi\alpha) \frac{F(q^2)}{q^2} \frac{\Pi(q^2)}{q^2} \times \int (|\Psi_{2P}(r)|^2 - |\Psi_{2S}(r)|^2) exp(iq \cdot r) dV \ . \tag{2.2}$$

   Here Π is is a scalar representing vacuum polarization due to decay of photon to virtual pairs.

The model to be discussed predicts that the effect is due to a leakage from "standard" state to what I call flux tube state. This means a multiplication of $|\Psi_{2P}|^2$ with the normalization factor $1/N$ of the standard state orthogonalized with respect to flux tube state. It is essential that $1/N$ is larger than unity so that the effect is a genuine quantum effect not understandable in terms of classical probability.

The modification of the formula is due to the normalization of the 2P and 2S states. These are in general different. The normalization factor $1/N$ is same for all terms in the expression of Lamb shift for a given state but in general different for 2S and 2P states. Since the lowest order term dominates by a factor of ~ 40 over the second one, one one can conclude that the modification should affect the lowest order term by about 4 per cent. Since the second term is negative and the modification of the first term is interpreted as a modification of the second term when r_p is estimated from the standard formula, the first term must increase by about 4 per cent. This is achieved if this state is orthogonalized with respect to the flux tube state. For states Ψ_0 and Ψ_{tube} with unit norm this means the modification

$$\begin{aligned} \Psi_0 &\rightarrow \frac{1}{1-|C|^2} \times (\Psi_i - C\Psi_{tube}) \ , \\ C &= \langle \Psi_{tube} | \Psi_0 \rangle \ . \end{aligned} \tag{2.3}$$

In the lowest order approximation one obtains

$$a - br_p^2 + cr_p^3 \quad \rightarrow \quad (1+|C|^2)a - br_p^2 + cr_p^3 \ . \tag{2.4}$$

Using instead of this expression the standard formula gives a wrong estimate r_p from the condition

$$a - b\hat{r}_p^2 + c\hat{r}_p^3 \rightarrow (1+|C|^2)a - br_p^2 + cr_p^3 \ . \tag{2.5}$$

This gives the equivalent conditions

$$\begin{aligned} \hat{r}_p^2 &= r_p^2 - \frac{|C|^2 a}{b} \ , \\ P_{tube} &\equiv |C|^2 \simeq 2\frac{b}{a} \times r_p^2 \times \frac{(r_p - \hat{r_p})}{r_p}) \ . \end{aligned} \tag{2.6}$$

The resulting estimate for the leakage probability is $P_{tube} \simeq .0015$. The model should be able to reproduce this probability.

3 A model for the coupling between standard states and flux tube states

Just for fun one can look whether the idea about confinement of muon to quark flux tube carrying electric flux could make sense.

1. Assume that the quark is accompanied by a flux tube carrying electric flux $\int EdS = -\int \nabla\Phi \cdot dS = q$, where $q = 2e/3 = ke$ is the u quark charge. The potential created by the u quark at the proton end of the flux tube with transversal area $S = \pi R^2$ idealized as effectively 1-D structure is

$$\Phi = -\frac{ke}{\pi R^2}|x| + \Phi_0 \ . \tag{3.1}$$

The normalization factor comes from the condition that the total electric flux is q. The value of the additive constant V_0 is fixed by the condition that the potential coincides with Coulomb potential at $r = r_u$, where r_u is u quark Compton length. This gives

$$e\Phi_0 = \frac{e^2}{r_u} + Kr_u \ , \ K = \frac{ke^2}{\pi R^2} \ . \tag{3.2}$$

2. Parameter R should be of order of magnitude of charge radius $\alpha_K r_u$ of u quark is free parameter in some limits. $\alpha_K = \alpha$ is expected to hold true in excellent approximation. Therefore a convenient parametrization is

$$R = z\alpha r_u \ . \tag{3.3}$$

This gives

$$K = \frac{4k}{\alpha r_u^2} \ , \ e\Phi_0 = 4(\pi\alpha + \frac{k}{\alpha})\frac{1}{r_u} \ . \tag{3.4}$$

3. The requirement that electron with four times larger charge radius that u quark can topologically condensed inside the flux tube without a change in the average radius of the flux tube (and thus in a reduction in p-adic length scale increasing its mass by a factor 4!) suggests that $z \geq 4$ holds true at least far away from proton. Near proton the condition that the radius of the flux tube is smaller than electron's charge radius is satisfied for $z = 1$.

3.1 Reduction of Schrödinger equation at flux tube to Airy equation

The 1-D Schrödinger equation at flux tube has as its solutions Airy functions and the related functions known as "Bairy" functions.

1. What one has is a one-dimensional Schrödinger equation of general form

$$-\frac{\hbar^2}{2m_\mu}\frac{d^2\Psi}{dx^2} + (Kx - e\Phi_0)\Psi = E\Psi \ , \ K = \frac{ke^2}{\pi R^2} \ . \tag{3.5}$$

By performing a linear coordinate change

$$u = (\frac{2m_\mu K}{\hbar^2})^{1/3}(x - x_E) \ , \ x_E = \frac{-|E| + e\Phi_0}{K} \ , \tag{3.6}$$

one obtains

$$\frac{d^2\Psi}{du^2} - u\Psi = 0 \ . \tag{3.7}$$

This differential equation is known as Airy equation (or Stokes equation) and defines special functions $Ai(x)$ known as Airy functions and related functions $Bi(x)$ referred to as "Bairy" functions [5]. Airy functions characterize the intensity near an optical directional caustic such as that of rainbow.

2. The explicit expressions for $A_i(u)$ and $Bi(u)$ are is given by

$$\begin{aligned} Ai(u) &= \frac{1}{\pi}\int_0^\infty cos(\frac{1}{3}t^3 + ut)dt \ , \\ Bi(u) &= \frac{1}{\pi}\int_0^\infty \left[exp(-\frac{1}{3}t^3) + sin(\frac{1}{3}t^3 + ut)dt\right] \ . \end{aligned} \tag{3.8}$$

$Ai(u)$ oscillates rapidly for negative values of u having interpretation in terms of real wave vector and goes exponentially to zero for $u > 0$. $Bi(u)$ oscillates also for negative values of x but increases exponentially for positive values of u. The oscillatory behavior and its character become obvious by noticing that stationary phase approximation is possible for $x < 0$.

The approximate expressions of $Ai(u)$ and $Bi(u)$ for $u > 0$ are given by

$$\begin{aligned} Ai(u) &\sim \frac{1}{2\pi^{1/2}}exp(-\frac{2}{3}u^{3/2})u^{-1/4} \ , \\ Bi(u) &\sim \frac{1}{\pi^{1/2}}exp(\frac{2}{3}u^{3/2})u^{-1/4} \ . \end{aligned} \tag{3.9}$$

For $u < 0$ one has

$$\begin{aligned} Ai(u) &\sim \frac{1}{\pi^{1/2}}sin(\frac{2}{3}(-u)^{3/2})(-u)^{-1/4} \ , \\ Bi(u) &\sim \frac{1}{\pi^{1/2}}cos(\frac{2}{3}(-u)^{3/2})(-u)^{-1/4} \ . \end{aligned} \tag{3.10}$$

3. $u=0$ corresponds to the turning point of the classical motion where the kinetic energy changes sign. $x=0$ and $x=r_u$ correspond to the points

$$\begin{aligned} u_{min} \equiv u(0) &= -(\frac{2m_\mu K}{\hbar^2})^{1/3} x_E \ , \\ u_{max} \equiv u(r_u) &= (\frac{2m_\mu K}{\hbar^2})^{1/3}(r_u - x_E) \ , \\ x_E &= \frac{-|E|+e\Phi_0}{K} \ . \end{aligned} \tag{3.11}$$

4. The general solution is

$$\Psi = aAi(u)+bBi(u) \ . \tag{3.12}$$

The natural boundary condition is the vanishing of Ψ at the lower end of the flux tube giving

$$\frac{b}{a} = -\frac{Ai(u(0)}{Bi(u(0))} \ . \tag{3.13}$$

A non-vanishing value of b implies that the solution increases exponentially for positive values of the argument and the solution can be regarded as being concentrated in an excellent approximation near the upper end of the flux tube.

Second boundary condition is perhaps most naturally the condition that the energy is same for the flux tube amplitude as for the standard solution. Alternative boundary conditions would require the vanishing of the solution at both ends of the flux tube and in this case one obtains very large number of solutions as WKB approximation demonstrates. The normalization of the state so that it has a unit norm fixes the magnitude of the coefficients a and b since one can choose them to be real.

3.2 Estimate for the probability that muon is caught to the flux tube

The simplest estimate for the muon to be caught to the flux tube state characterized by the same energy as standard state is the overlap integral of the ordinary hydrogen wave function of muon and of the effectively one-dimensional flux tube. What one means with overlap integral is however not quite obvious.

1. The basic condition is that the modified "standard" state is orthogonal to the flux tube state. One can write the expression of a general state as

$$\begin{aligned} \Psi_{nlm} &\rightarrow N\times(\Psi_{nlm} - C(E,nlm)\Phi_{nlm}) \ , \\ \Phi_{nlm} &= Y_{lm}\Psi_E \ , \\ C(E,nlm) &= \langle \Psi_E|\Psi_{nlm}\rangle \ . \end{aligned} \tag{3.14}$$

Here Φ_{nlm} depends a flux tube state in which spherical harmonics is wave function in the space of orientations of the flux tube and Ψ_E is flux tube state with same energy as standard state. Here an inner product between standard states and flux tube states is introduced.

2. Assuming same energy for flux tube state and standard state, the expression for the total total probability for ending up to single flux tube would be determined from the orthogonality condition as

$$P_{nlm} = \frac{|C(E,nlm)|^2}{1-|C(E,lmn)|^2} \ . \tag{3.15}$$

Here E refers to the common energy of flux tube state and standard state. The fact that flux tube states vanish at the lower end of the flux tube implies that they do not contribute to the expression for average charge density. The reduced contribution of the standard part implies that the attempt to interpret the experimental results in "standard model" gives a reduced value of the charge radius. The size of the contribution is given by P_{nlm} whose value should be about 4 per cent.

One can consider two alternative forms for the inner product between standard states and flux tube states. Intuitively it is clear that an overlap between the two wave functions must be in question.

1. The simplest possibility is that one takes only overlap at the upper end of the flux tube which defines 2-D surface. Second possibility is that that the overlap is over entire flux tube projection at the space-time sheet of atom.

$$\begin{aligned} \langle \Psi_E | \Psi_{nlm} \rangle &= \int_{end} \overline{\Psi}_r \Psi_{nlm} dS \quad \text{(Option I)} \quad , \\ \langle \Psi_E | \Psi_{nlm} \rangle &= \int_{tube} \overline{\Psi}_r \Psi_{nlm} dV \quad \text{(Option II)} \quad . \end{aligned} \tag{3.16}$$

2. For option I the inner product is non-vanishing only if Ψ_E is non-vanishing at the end of the flux tube. This would mean that electron ends up to the flux tube through its end. The inner product is dimensionless without introduction of a dimensional coupling parameter if the inner product for flux tube states is defined by 1-dimensional integral: one might criticize this assumption as illogical. Unitarity might be a problem since the local behaviour of the flux tube wave function at the end of the flux tube could imply that the contribution of the flux tube state in the quantum state dominates and this does not look plausible. One can of course consider the introduction to the inner product a coefficient representing coupling constant but this would mean loss of predictivity. Schrödinger equation at the end of the flux tubes guarantees the conservation of the probability current only if the energy of flux tube state is same as that of standard state or if the flux tube Schrödinger amplitude vanishes at the end of the flux tube.

3. For option II there are no problems with unitary since the overlap probability is always smaller than unity. Option II however involves overlap between standard states and flux tube states even when the wave function at the upper end of the flux tube vanishes. One can however consider the possibility that the possible flux tube states are orthogonalized with respect to standard states with leakage to flux tubes. The interpretation for the overlap integral would be that electron ends up to the flux tube via the formation of wormhole contact.

3.3 Option I fails

The considerations will be first restricted to the simpler option I. The generalization of the results of calculation to option II is rather straighforward. It turns out that option II gives correct order of magnitude for the reduction of charge radius for reasonable parameter values.

1. In a good approximation one can express the overlap integrals over the flux tube end (option I) as

$$\begin{aligned} C(E,nlm) &= \int_{tube} \overline{\Psi}_E \Psi_{nlm} dS \simeq \pi R^2 \times Y_{lm} \times C(E,nl) \quad , \\ C(E,nl) &= \overline{\Psi}_E(r_u) R_{nl}(r_u) \quad . \end{aligned} \tag{3.17}$$

An explicit expression for the coefficients can be deduced by using expression for Ψ_E as a superposition of Airy and Bairy functions. This gives

$$\begin{aligned}
C(E,nl) &= \overline{\Psi}_E(r_u)R_{nl}(r_u) \ , \\
\Psi_E(x) &= a_E Ai(u_E) + bBi(u_E) \ , \ \frac{a_E}{b_E} = -\frac{Bi(u_E(0))}{Ai(u_E(0)} \ , \\
u_E(x) &= (\frac{2m_\mu K}{\hbar^2})^{1/3}(x - x_E) \ , \ x_E = \frac{|E| - e\Phi_0}{K} \ , \\
K &= \frac{ke^2}{\pi R^2} \ , \ R = z\alpha_K r_u \ , \ k = \frac{2}{3} \ .
\end{aligned} \tag{3.18}$$

The normalization of the coefficients is fixed from the condition that a and b chosen in such a manner that Ψ has unit norm. For these boundary conditions Bi is expected to dominate completely in the sum and the solution can be regarded as exponentially decreasing function concentrated around the upper end of the flux tube.

In order to get a quantitative view about the situation one can express the parameters u_{min} and u_{max} in terms of the basic dimensionless parameters of the problem.

1. One obtains

$$\begin{aligned}
u_{min} \equiv u(0) &= -2(\frac{k}{z\alpha})^{1/3}\left[1 + \pi\frac{z}{k}\alpha^2(1 - \frac{1}{2}\alpha r)\right] \times r^{1/3} \ , \\
u_{max} \equiv u(r_u) &= u(0) + 2\frac{k}{z\alpha} \times r^{1/3} \ , \\
r &= \frac{m_\mu}{m_u} \ , \ R = z\alpha r_u \ .
\end{aligned} \tag{3.19}$$

Using the numerical values of the parameters one obtains for $z = 1$ and $\alpha = 1/137$ the values $u_{min} = -33.807$ and $u_{max} = 651.69$. The value of u_{max} is so large that the normalization is in practice fixed by the exponential behavior of Bi for the suggested boundary conditions.

2. The normalization constant is in good approximation defined by the integral of the approximate form of Bi^2 over positive values of u and one has

$$N^2 \simeq \frac{dx}{du} \times \int_{u_{min}}^{u_{max}} Bi(u)^2 du \ , \ \frac{dx}{du} = \frac{1}{2}(\frac{z^2\alpha}{k})^{1/3} \times r^{1/3} r_u \ , \tag{3.20}$$

By taking $t = exp(\frac{4}{3}u^{3/2})$ as integration variable one obtains

$$\begin{aligned}
\int_{u_{min}}^{u_{max}} Bi(u)^2 du &\simeq \pi^{-1}\int_{u_{min}}^{u_{max}} exp(\frac{4}{3}u^{3/2})u^{-1/2}du \\
&= (\frac{4}{3})^{2/3}\pi^{-1}\int_{t_{min}}^{t_{max}} \frac{dt}{log(t)^{2/3}} \simeq \frac{1}{\pi}\frac{exp(\frac{4}{3}u_{max}^{3/2})}{u_{max}} \ .
\end{aligned} \tag{3.21}$$

This gives for the normalization factor the expression

$$N \simeq \frac{1}{2}(\frac{z^2\alpha}{k})^{2/3} r^{1/3} r_u^{1/2} exp(\frac{2}{3}u_{max}^{3/2}) \ . \tag{3.22}$$

3. One obtains for the value of Ψ_E at the end of the flux tube the estimate

$$\Psi_E(r_u) \quad = \frac{Bi(u_{max})}{N} \simeq \quad 2\pi^{-1/2}\times(\frac{k}{z^2\alpha})^{2/3}r^{1/3}r_u^{-1/2} \quad , \quad r=\frac{r_u}{r_\mu} \quad . \tag{3.23}$$

4. The inner product defined as overlap integral gives for the ground state

$$\begin{aligned} C_{E,00} &= \Psi_E(r_u)\times\Psi_{1,0,0}(r_u)\times\pi R^2 \\ &= 2\pi^{-1/2}(\frac{k}{z^2\alpha})^{2/3}r^{1/3}r_u^{-1/2}\times(\frac{1}{\pi a(\mu)^3})^{1/2}\times exp(-\alpha r)\times\pi z^2\alpha^2r_u^2 \\ &= 2\pi^{1/2}k^{2/3}z^{2/3}r^{11/6}\alpha^{17/6}exp(-\alpha r) \ . \end{aligned} \tag{3.24}$$

The relative reduction of charge radius equals to $P = C^2_{E,00}$. For $z = 1$ one obtains $P = C^2_{E,00} = 5.5\times10^{-6}$, which is by three orders of magnitude smaller than the value needed for $P_{tube} = C^2_{E,20} = .0015$. The obvious explanation for the smallness is the α^2 factor coming from the area of flux tube in the inner product.

3.4 Option II could work

The failure of the simplest model is essentially due to the inner product. For option II the inner product for the flux tube states involves the integral over the area of flux tube so that the normalization factor for the state is obtained from the previous one by the replacement $N\rightarrow N/\sqrt{\pi R^2}$. In the integral over the flux tube the exponent function is is in the first approximation equal to constant since the wave function for ground state is at the end of the flux tube only by a factor .678 smaller than at the origin and the wave function is strongly concentrated near the end of the flux tube. The inner product defined by the overlap integral over the flux tube implies $N\rightarrow NS^{1/2}$, $S=\pi R^2 = z^2\alpha^2r_u^2$. In good approximation the inner product for option II means the replacement

$$\begin{aligned} C_{E,n0} &\rightarrow A\times B\times C_{E,n0} \ , \\ A &= \frac{\frac{dx}{du}}{\sqrt{\pi R^2}} = \frac{1}{2\sqrt{\pi}}z^{-1/3}k^{-1/3}\alpha^{-2/3}r^{1/3} \ , \\ B &= \frac{\int Bi(u)du}{\sqrt{Bi(u_{max})}} = u_{max}^{-1/4} = 2^{-1/4}z^{1/2}k^{-1/4}\alpha^{1/4}r^{-1/12} \ . \end{aligned} \tag{3.25}$$

Using the expression

$$R_{20}(r_u) \quad = \quad \frac{1}{2\sqrt{2}}\times(\frac{1}{a_\mu})^{3/2}\times(2-r\alpha)\times exp(-r\alpha) \ , \quad r=\frac{r_u}{r_\mu} \tag{3.26}$$

one obtains for $C_{E,20}$ the expression

$$C_{E,20} \quad = \quad 2^{-3/4}z^{5/6}k^{1/12}\alpha^{29/12}r^{25/12}\times(2-r\alpha)\times exp(-r\alpha) \ . \tag{3.27}$$

By the earlier general argument one should have $P_{tube} = |C_{E,20}|^2 \simeq .0015$. $P_{tube} = .0015$ is obtained for $z = 1$ and $N = 2$ corresponding to single flux tube per u quark. If the flux tubes are in opposite directions, the leakage into 2P state vanishes. Note that this leakage does not affect the value of the coefficient a in the general formula for the Lamb shift. The radius of the flux tube is by a factor 1/4 smaller than the classical radius of electron and one could argue that this makes it impossible for electron to topologically condense at the flux tube. For $z = 4$ one would have $P_{tube} = .015$ which is 10 times too large a value. Note that the nucleus possess a wave function for the orientation of the flux tube. If this corresponds to S-wave state then only the leakage beween S-wave states and standard states is possible.

4 Are exotic flux tube bound states possible?

There seems to be no deep reason forbidding the possibility of genuine flux tube states decoupling from the standard states completely. To get some idea about the energy eigenvalues one can apply WKB approximation. This approach should work now: in fact, the study on WKB approximation near turning point by using linearization of the the potential leads always to Airy equation so that the linear potential represents an ideal situation for WKB approximation. As noticed these states do not seem to be directly relevant for the recent situation. The fact that these states have larger binding energies than the ordinary states of hydrogen atom might make possible to liberate energy by inducing transitions to these states.

1. Assume that a bound state with a negative energy E is formed inside the flux tube. This means that the condition $p^2 = 2m(E-V) \geq 0$, $V = -e\Phi$, holds true in the region $x \leq x_{max} < r_u$ and $p^2 = 2m(E-V) < 0$ in the region $r_u > x \geq x_{max}$. The expression for x_{max} is

$$x_{max} = \frac{\pi R^2}{k}(-\frac{|E|}{e^2} + \frac{1}{r_u} + \frac{kr_u}{\pi R^2})\hbar \ . \tag{4.1}$$

$x_{max} < r_u$ holds true if one has

$$|E| < \frac{e^2}{r_u} = E_{max} \ . \tag{4.2}$$

The ratio of this energy to the ground state energy of muonic hydrogen is from $E(1) = e^2/2a(\mu)$ and $a = \hbar/\alpha m$ given by

$$\frac{E_{max}}{E(n=1)} = \frac{2m_u}{\alpha m_\mu} \simeq 5.185 \ . \tag{4.3}$$

This encourages to think that the ground state energy could be reduced by the formation of this kind of bound state if it is possible to find a value of n in the allowed range. The physical state would of course contain only a small fraction of this state. In the case of electron the increase of the binding energy is even more dramatic since one has

$$\frac{E_{max}}{E(n=1)} = \frac{2m_u}{\alpha m_e} = \frac{8}{\alpha} \simeq 1096 \ . \tag{4.4}$$

Obviously the formation of this kind of states could provide a new source of energy. There have been claims about anomalous energy production in hydrogen [13]. I have discussed these claims from TGD viewpoint in [3]

2. One can apply WKB quantization in the region where the momentum is real to get the condition

$$I = \int_0^{x_{max}} \sqrt{2m(E+e\Phi)}\frac{dx}{\hbar} = n + \frac{1}{2} \ . \tag{4.5}$$

By performing the integral one obtains the quantization condition

$$\begin{aligned} I &= k^{-1}(8\pi\alpha)^{1/2} \times \frac{R^2}{r_u^{3/2} r_\mu} \times A^{3/2} = n + \frac{1}{2} \ , \\ A &= 1 + kx^2 - \frac{|E| r_u}{e^2} \ , \\ x &= \frac{r_u}{R} \ , \ k = \frac{2}{3\pi} \ , \ r_i = \frac{\hbar}{m_i} \ . \end{aligned} \tag{4.6}$$

3. Parameter R should be of order of magnitude of charge radius $\alpha_K r_u$ of u quark is free parameter in some limits. $\alpha_K = \alpha$ is expected to hold true in excellent approximation. Therefore a convenient parametrization is

$$R = z\alpha r_u \ . \tag{4.7}$$

This gives for the binding energy the general expression in terms of the ground state binging energy $E(1,\mu)$ of muonic hydrogen as

$$\begin{aligned} |E| &= C \times E(1,\mu) \ , \\ C &= D \times (1 + Kz^{-2}\alpha^{-2} - (\frac{y}{z^2})^{2/3} \times (n+1/2)^{2/3}) \ , \\ D &= 2y \times (\frac{K^2}{8\pi\alpha})^{1/3} \ , \\ y &= \frac{m_u}{m_\mu} \ , \ K = \frac{2}{3\pi} \ . \end{aligned} \tag{4.8}$$

4. There is a finite number of bound states. The above mentioned consistency conditions coming from $0 < x_{max} < r_\mu$ give $0 < C < C_{max} = 5.185$ restricting the allowed value of n to some interval. One obtains the estimates

$$\begin{aligned} n_{min} &\simeq \frac{z^2}{y}(1 + Kz^{-2}\alpha^{-2} - \frac{C_{max}}{D})^{3/2} - \frac{1}{2} \ , \\ n_{max} &= \frac{z^2}{y}(1 + Kz^{-2}\alpha^{-2})^{3/2} - \frac{1}{2} \ . \end{aligned} \tag{4.9}$$

Very large value of n is required by the consistency condition. The calculation gives $n_{min} \in \{1.22\times 10^7, 4.59\times 10^6, 1.48\times 10^5\}$ and $n_{max} \in \{1.33\times 10^7, 6.66\times 10^6, 3.34\times 10^6\}$ for $z \in \{1,2,4\}$. This would be a very large number of allowed bound states -about 3.2×10^6 for $z = 1$.

The WKB state behaves as a plane wave below x_{max} and sum of exponentially decaying and increasing amplitudes above x_{max}:

$$\begin{aligned} &\frac{1}{\sqrt{k(x)}}\left[Aexp(i\int_0^x k(y)dy) + Bexp(-i\int_0^x k(y)dy)\right] \ , \\ &\frac{1}{\sqrt{\kappa(x)}}\left[Cexp(-\int_{x_{max}}^x \kappa(y)dy + Dexp(\int_{x_{max}}^x \kappa(y)dy\right] \ , \\ &k(x) = \sqrt{2m(-|E| + e\Phi} \ , \ \kappa(x)\sqrt{2m(|E| - e\Phi} \ . \end{aligned} \tag{4.10}$$

At the classical turning point these two amplitudes must be identical.

The next task is to decide about natural boundary conditions. Two types of boundary conditions must be considered. The basic condition is that genuine flux tube states are in question. This requires that the inner product between flux tube states and standard states defined by the integral over flux tube ends vanishes. This is guaranteed if the Schrödinger amplitude for the flux tube state vanishes at the ends of the flux tube so that flux tube behaves like an infinite potential well. The condition $\Psi(0) = 0$ at the lower end of the flux tube would give $A = -B$. Combined with the continuity condition at the turning point these conditions imply that Ψ can be assumed to be real. The $\Psi(r_u) = 0$ gives a condition leading to the quantization of energy.

The wave function over the directions of flux tube with a given value of n is given by the spherical harmonics assigned to the state (n,l,m).

References

[1] M. Pitkänen (2006), *p-Adic length Scale Hypothesis and Dark Matter Hierarchy.* http://tgdtheory.com/public_html/paddark/paddark.html.

[2] M. Pitkänen (2006), *TGD and Fringe Physics.* http://tgdtheory.com/public_html/freenergy/freenergy.html.

[3] The chapter *The Notion of Free Energy and Many-Sheeted Space-Time Concept* of [2]. http://tgdtheory.com/public_html/freenergy/freenergy.html#freenergy.

[4] The chapter *p-Adic Particle Massivation: Hadron Masses* of [1]. http://www.helsinki.fi/~matpitka/paddark/paddark.html#mass4.

[5] *Airy functions.* http://en.wikipedia.org/wiki/Airy_function.

[6] *First images of heavy electrons in action.* Energy Daily. http://www.energy-daily.com/reports/First_Images_Of_Heavy_Electrons_In_Action_999.html.

[7] R. Pohl *et al* (2010), *The size of proton.* Nature 466. 213-216. http://www.nature.com/nature/journal/v466/n7303/full/nature09250.html.

[8] J. Flowers (2010), *Quantum electrodynamics: A chink in the armour?.* Nature 466. 195-196. http://www.nature.com/nature/journal/v466/n7303/pdf/466195a.pdf.

[9] K. McAlpine (2010), *Incredible shrinking proton raises eyebrows.*http://www.newscientist.com/article/dn19141-incredible-shrinking-proton-raises-eyebrows.html.

[10] *Lamb shift.* http://en.wikipedia.org/wiki/Lamb_shift.

[11] A. S. Antognini (2005), *The Lamb shift Experiment in Muonic Hydrogen* . http://edoc.ub.uni-muenchen.de/5044/1/Antognini_Aldo.pdf.

[12] D. T. H. Davies *et al* (2010), *Precise Charm to Strange Mass Ratio and Light Quark Masses from Full Lattice QCD.* Phys. Rev. Lett. 104, 132003. http://prl.aps.org/abstract/PRL/v104/i13/e132003.
A. Cho (2010), *Mass of the common quark finally nailed down.* http://news.sciencemag.org/sciencenow/2010/04/mass-of-the-common-quark-finally.html.

[13] R. Mills *et al*(2003), *Spectroscopic and NMR identification of novel hybrid ions in fractional quantum energy states formed by an exothermic reaction of atomic hydrogen with certain catalysts.* http://www.blacklightpower.com/techpapers.html .