Fermion-Antifermion Asymmetry

G. Quznetsov
Chelyabinsk State University//
e-mail: gunn@mail.ru, quznets@yahoo.com

Abstract

An event with positive energy transfers this energy photons which carries it on recorders observers. Observers know that this event occurs, not before it happens. But event with negative energy should absorb this energy from observers. Consequently, observers know that this event happens before it happens. Since time is irreversible then only the events with positive energy can occur. In single-particle states events with a fermion have positive energy and occurences with an antifermion have negative energy. In double-particle states events with pair of antifermions have negative energy and events with pair of fermions and with fermion-antifermion pair have positive energy.

1 Introduction

Let t, x_1, x_2,x_3 be real numbers, and let $\mathbf{x} := \langle x_1, x_2, x_3 \rangle$.

Let $\mathcal{A}$ be some pointlike event.

Let $\varphi(t, \mathbf{x})$ be a 4×1-complex matrix such that

$$\varphi^{\dagger}(t, \mathbf{x}) \varphi(t, \mathbf{x}) = \rho(t, \mathbf{x}) \tag{1}$$

where $\rho(t, \mathbf{x})$ is the probability density of $\mathcal{A}$.

Let[1] $\rho(t, \mathbf{x}) = 0$ if $t > \frac{\pi \mathrm{c}}{\mathrm{h}}$ and/or $|\mathbf{x}| > \frac{\pi \mathrm{c}}{\mathrm{h}}$.

In that case $\varphi(t, \mathbf{x})$ obeys some generalization of the Dirac equation [1]. The Dirac equation for free fermion does have the following form:

$$\left(\frac{1}{\mathrm{c}} \frac{\partial}{\partial t} - \sum_{s=1}^{3} \beta^{[s]} \frac{\partial}{\partial x_s} - \mathrm{i} \frac{\mathrm{h}}{\mathrm{c}} n \gamma^{[0]} \right) \varphi(t, \mathbf{x}) = 0.$$

Here n is a natural number and

$$\beta^{[1]} \quad : \quad = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \beta^{[2]} := \begin{bmatrix} 0 & -\mathrm{i} & 0 & 0 \\ \mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathrm{i} \\ 0 & 0 & -\mathrm{i} & 0 \end{bmatrix},$$

[1] $\mathrm{c} := 299792458, \mathrm{h} := 6.6260755^{-34}$

$$\beta^{[3]} \quad : \quad = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \gamma^{[0]} := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

In this case operator $\widehat{H}_0$ is *the free Dirac Hamiltonian* if

$$\widehat{H}_0 := \mathrm{c}\left(\sum_{s=1}^{3} \beta^{[s]} \mathrm{i} \frac{\partial}{\partial x_s} + \frac{\mathrm{h}}{\mathrm{c}} n \gamma^{[0]}\right).$$

Let $\mathbf{k}$ be a vector $\langle k_1, k_2, k_3 \rangle$ where k_s are integer numbers and let

$$\omega\left(\mathbf{k}\right) := \sqrt{k_1^2 + k_2^2 + k_3^2 + n^2}$$

where n is a natural number.

Let

$$e_1\left(\mathbf{k}\right) := \frac{1}{2\sqrt{\omega\left(\mathbf{k}\right)\left(\omega\left(\mathbf{k}\right) + n\right)}} \begin{bmatrix} \omega\left(\mathbf{k}\right) + n + k_3 \\ k_1 + \mathrm{i}k_2 \\ \omega\left(\mathbf{k}\right) + n - k_3 \\ -k_1 - \mathrm{i}k_2 \end{bmatrix},$$

$$e_2\left(\mathbf{k}\right) := \frac{1}{2\sqrt{\omega\left(\mathbf{k}\right)\left(\omega\left(\mathbf{k}\right) + n\right)}} \begin{bmatrix} k_1 - \mathrm{i}k_2 \\ \omega\left(\mathbf{k}\right) + n - k_3 \\ -k_1 - \mathrm{i}k_2 \\ \omega\left(\mathbf{k}\right) + n + k_3 \end{bmatrix},$$

$$e_3\left(\mathbf{k}\right) := \frac{1}{2\sqrt{\omega\left(\mathbf{k}\right)\left(\omega\left(\mathbf{k}\right) + n\right)}} \begin{bmatrix} -\omega\left(\mathbf{k}\right) - n + k_3 \\ k_1 + \mathrm{i}k_2 \\ \omega\left(\mathbf{k}\right) + n + k_3 \\ k_1 + \mathrm{i}k_2 \end{bmatrix},$$

$$e_4\left(\mathbf{k}\right) := \frac{1}{2\sqrt{\omega\left(\mathbf{k}\right)\left(\omega\left(\mathbf{k}\right) + n\right)}} \begin{bmatrix} k_1 - \mathrm{i}k_2 \\ -\omega\left(\mathbf{k}\right) - n - k_3 \\ k_1 - \mathrm{i}k_2 \\ \omega\left(\mathbf{k}\right) + n - k_3 \end{bmatrix}.$$

In that case functions $e_1(\mathbf{k})(2\mathrm{c}/\mathrm{h})^{3/2}\exp(-\mathrm{i}(\mathrm{h}/\mathrm{c})\mathbf{kx})$ and $e_2(\mathbf{k})(2\mathrm{c}/\mathrm{h})^{3/2}\exp(-\mathrm{i}(\mathrm{h}/\mathrm{c})\mathbf{kx})$ are eigenvectors of $\widehat{H}_0$ with eigenvalues $(+\mathrm{h}\omega(\mathbf{k}))$, and functions $e_3(\mathbf{k})(2\mathrm{c}/\mathrm{h})^{3/2}\exp(-\mathrm{i}(\mathrm{h}/\mathrm{c})\mathbf{kx})$ and $e_4(\mathbf{k})(2\mathrm{c}/\mathrm{h})^{3/2}\exp(-\mathrm{i}(\mathrm{h}/\mathrm{c})\mathbf{kx})$ are eigenvectors of $\widehat{H}_0$ with eigenvalues $(-\mathrm{h}\omega(\mathbf{k}))$.

2 Single-Particle States

Let $\mathfrak{H}$ be some unitary space. Let $\widetilde{0}$ be the zero element of $\mathfrak{H}$. That is any element $\widetilde{F}$ of $\mathfrak{H}$ obeys to the following conditions:

$0\widetilde{F} = \widetilde{0}$, $\widetilde{0} + \widetilde{F} = \widetilde{F}$, $\widetilde{0}^{\dagger}\widetilde{F} = \widetilde{F}$, $\widetilde{0}^{\dagger} = \widetilde{0}$.

Let $\widehat{0}$ be the zero operator on $\mathfrak{H}$. That is any element $\widetilde{F}$ of $\mathfrak{H}$ obeys to the following condition:

$\widehat{0}\widetilde{F} = 0\widetilde{F}$, and if $\widehat{b}$ is any operator on $\mathfrak{H}$ then

$\widehat{0} + \widehat{b} = \widehat{b} + \widehat{0} = \widehat{b}$, $\widehat{0}\widehat{b} = \widehat{b}\widehat{0} = \widehat{0}$.

Let $\widehat{1}$ be the identy operator on $\mathfrak{H}$. That is any element $\widetilde{F}$ of $\mathfrak{H}$ obeys to the following condition:
$\widehat{1}\widetilde{F} = 1\widetilde{F} = \widetilde{F}$, and if $\widehat{b}$ is any operator on $\mathfrak{H}$ then
$\widehat{1}\widehat{b} = \widehat{b}\widehat{1} = \widehat{b}$.

Let linear operators $b_{s,\mathbf{k}}$ ($s \in \{1, 2, 3, 4\}$) act on all elements of this space. And let these operators fulfill the following conditions:

$$\left\{b_{s,\mathbf{k}}^{\dagger}, b_{s',\mathbf{k}'}\right\} := b_{s,\mathbf{k}}^{\dagger}b_{s',\mathbf{k}'} + b_{s',\mathbf{k}'}b_{s,\mathbf{k}}^{\dagger} = \left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{3}\delta_{\mathbf{k},\mathbf{k}'}\delta_{s,s'}\widehat{1},$$

$$\{b_{s,\mathbf{k}}, b_{s',\mathbf{k}'}\} = b_{s,\mathbf{k}}b_{s',\mathbf{k}'} + b_{s',\mathbf{k}'}b_{s,\mathbf{k}} = \left\{b_{s,\mathbf{k}}^{\dagger}, b_{s',\mathbf{k}'}^{\dagger}\right\} = \widehat{0}.$$

Hence,

$$b_{s,\mathbf{k}}b_{s,\mathbf{k}} = b_{s,\mathbf{k}}^{\dagger}b_{s,\mathbf{k}}^{\dagger} = \widehat{0}.$$

There exists element $\widetilde{F}_0$ of $\mathfrak{H}$ such that $\widetilde{F}_0^{\dagger}\widetilde{F}_0 = 1$ and for any $b_{s,\mathbf{k}}$: $b_{s,\mathbf{k}}\widetilde{F}_0 = \widetilde{0}$. Hence, $\widetilde{F}_0^{\dagger}b_{s,\mathbf{k}}^{\dagger} = \widetilde{0}$.

Let

$$\psi_s(\mathbf{x}) := \sum_{\mathbf{k}}\sum_{r=1}^{4} b_{r,\mathbf{k}}e_{r,s}(\mathbf{k})\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{k}\mathbf{x}\right).$$

Because

$$\sum_{r=1}^{4} e_{r,s}(\mathbf{k})\, e_{r,s'}(\mathbf{k}) = \delta_{s,s'}$$

and

$$\sum_{\mathbf{k}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{k}(\mathbf{x}-\mathbf{x}')\right) = \left(\frac{2\pi\mathrm{c}}{\mathrm{h}}\right)^{3}\delta(\mathbf{x}-\mathbf{x}')$$

then

$$\begin{aligned}\left\{\psi_s^{\dagger}(\mathbf{x}), \psi_{s'}(\mathbf{x}')\right\} &:= \psi_s^{\dagger}(\mathbf{x})\,\psi_{s'}(\mathbf{x}') + \psi_{s'}(\mathbf{x}')\,\psi_s^{\dagger}(\mathbf{x}) \\ &= \delta(\mathbf{x}-\mathbf{x}')\,\delta_{s,s'}\widehat{1}.\end{aligned}$$

And these operators obey the following conditions:

$\psi_s(\mathbf{x})\widetilde{F}_0 = \widetilde{0}$, $\{\psi_s(\mathbf{x}), \psi_{s'}(\mathbf{x}')\} = \left\{\psi_s^\dagger(\mathbf{x}), \psi_{s'}^\dagger(\mathbf{x}')\right\} = \widehat{0}$.

Hence,

$\psi_s(\mathbf{x})\psi_{s'}(\mathbf{x}') = \psi_s^\dagger(\mathbf{x})\psi_{s'}^\dagger(\mathbf{x}') = \widehat{0}$.

Let

$$\Psi(t,\mathbf{x}) := \sum_{s=1}^{4} \varphi_s(t,\mathbf{x})\psi_s^\dagger(\mathbf{x})\widetilde{F}_0.$$

These function obey the following condition:

$$\Psi^\dagger(t,\mathbf{x}')\Psi(t,\mathbf{x}) = \varphi^\dagger(t,\mathbf{x}')\varphi(t,\mathbf{x})\delta(\mathbf{x}-\mathbf{x}').$$

Hence,

$$\int d\mathbf{x}' \cdot \Psi^\dagger(t,\mathbf{x}')\Psi(t,\mathbf{x}) = \rho(t,\mathbf{x}).$$

Let a Fourier series of $\varphi_s(t,\mathbf{x})$ has the following form:

$$\varphi_s(t,\mathbf{x}) = \sum_{\mathbf{p}} \sum_{r=1}^{4} c_r(t,\mathbf{p}) e_{r,s}(\mathbf{p}) \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{p}\mathbf{x}\right).$$

In that case:

$$\underline{\Psi}(t,\mathbf{p}) := \left(\frac{2\pi\mathrm{c}}{\mathrm{h}}\right)^3 \sum_{r=1}^{4} c_r(t,\mathbf{p}) b_{r,\mathbf{p}}^\dagger \widetilde{F}_0.$$

If

$$\mathcal{H}_0(\mathbf{x}) := \psi^\dagger(\mathbf{x})\widehat{H}_0\psi(\mathbf{x}) \tag{2}$$

then $\mathcal{H}_0(\mathbf{x})$ is called a Hamiltonian $\widehat{H}_0$ density.

Because

$$\widehat{H}_0\varphi(t,\mathbf{x}) = \mathrm{i}\frac{\partial}{\partial t}\varphi(t,\mathbf{x})$$

then

$$\int d\mathbf{x}' \cdot \mathcal{H}_0(\mathbf{x}')\Psi(t,\mathbf{x}) = \mathrm{i}\frac{\partial}{\partial t}\Psi(t,\mathbf{x}). \tag{3}$$

Therefore, if

$$\widehat{\mathbb{H}} := \int d\mathbf{x}' \cdot \mathcal{H}_0(\mathbf{x}')$$

then $\widehat{\mathbb{H}}$ acts similar to the Hamiltonian on space $\mathfrak{H}$.

And if

$$E_\Psi\left(\widetilde{F}_0\right) := \sum_{\mathbf{p}} \underline{\Psi}^\dagger(t,\mathbf{p})\widehat{\mathbb{H}}\underline{\Psi}(t,\mathbf{p})$$

then $E_{\Psi}\left(\widetilde{F}_0\right)$ is an energy of Ψ on vacuum $\widetilde{F}_0$.

Operator $\widehat{\mathbb{H}}$ obeys the following condition:

$$\widehat{\mathbb{H}} = \left(\frac{2\pi \mathrm{c}}{\mathrm{h}}\right)^3 \sum_{\mathbf{k}} \mathrm{h}\omega\left(\mathbf{k}\right)\left(\sum_{r=1}^{2} b_{r,\mathbf{k}}^{\dagger} b_{r,\mathbf{k}} - \sum_{r=3}^{4} b_{r,\mathbf{k}}^{\dagger} b_{r,\mathbf{k}}\right).$$

This operator is not positive defined and in this case

$$E_{\Psi}\left(\widetilde{F}_0\right) = \left(\frac{2\pi \mathrm{c}}{\mathrm{h}}\right)^3 \sum_{\mathbf{p}} \mathrm{h}\omega\left(\mathbf{p}\right)\left(\sum_{r=1}^{2} \left|c_r\left(t,\mathbf{p}\right)\right|^2 - \sum_{r=3}^{4} \left|c_r\left(t,\mathbf{p}\right)\right|^2\right).$$

This problem is usually solved in the following way [2, p.54]:
Let:

$$\begin{aligned} v_1\left(\mathbf{k}\right) &:= \gamma^{[0]} e_3\left(\mathbf{k}\right), \\ v_2\left(\mathbf{k}\right) &:= \gamma^{[0]} e_4\left(\mathbf{k}\right), \\ d_{1,\mathbf{k}} &:= -b_{3,-\mathbf{k}}^{\dagger}, \\ d_{2,\mathbf{k}} &:= -b_{4,-\mathbf{k}}^{\dagger}. \end{aligned}$$

In that case:

$$\begin{aligned} e_3\left(\mathbf{k}\right) &= -v_1\left(-\mathbf{k}\right), \\ e_4\left(\mathbf{k}\right) &= -v_2\left(-\mathbf{k}\right), \\ b_{3,\mathbf{k}} &= -d_{1,-\mathbf{k}}^{\dagger}, \\ b_{4,\mathbf{k}} &= -d_{2,-\mathbf{k}}^{\dagger}. \end{aligned}$$

Therefore,

$$\begin{aligned} \psi_s\left(\mathbf{x}\right) :=& \sum_{\mathbf{k}} \sum_{r=1}^{2} \Big(b_{r,\mathbf{k}} e_{r,s}\left(\mathbf{k}\right) \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{k}\mathbf{x}\right) + \\ & + d_{r,\mathbf{k}}^{\dagger} v_{r,s}\left(\mathbf{k}\right) \exp\left(\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{k}\mathbf{x}\right)\Big) \end{aligned}$$

$$\begin{aligned} \widehat{\mathbb{H}} =& \left(\frac{2\pi \mathrm{c}}{\mathrm{h}}\right)^3 \sum_{\mathbf{k}} \mathrm{h}\omega\left(\mathbf{k}\right) \sum_{r=1}^{2} \left(b_{r,\mathbf{k}}^{\dagger} b_{r,\mathbf{k}} + d_{r,\mathbf{k}}^{\dagger} d_{r,\mathbf{k}}\right) \\ & -2 \sum_{\mathbf{k}} \mathrm{h}\omega\left(\mathbf{k}\right) \widehat{1}. \end{aligned}$$

The first term on the right side of this equality is positive defined. This term is taken as the desired Hamiltonian. The second term of this equality is infinity constant. And this infinity is deleted (?!) [2, p.58]

But in this case $d_{r,\mathbf{k}}\widetilde{F}_0 \neq \widetilde{0}$. In order to satisfy such condition, the vacuum element $\widetilde{F}_0$ must be replaced by the following:

$$\widetilde{F}_0 \rightarrow \widetilde{\Phi}_0 := \prod_{\mathbf{k}} \prod_{r=3}^{4} \left(\frac{2\pi \mathrm{c}}{\mathrm{h}}\right)^3 b_{r,\mathbf{k}}^{\dagger} \widetilde{F}_0.$$

But in this case:

$$\psi_s(\mathbf{x})\,\widetilde{\Phi}_0 \neq \widetilde{0}.$$

And condition (3) isn't carried out.

In order to satisfy such condition, operators $\psi_s(\mathbf{x})$ must be replaced by the following:

$$\psi_s(\mathbf{x}) \rightarrow \phi_s(\mathbf{x}) :=$$
$$:= \sum_{\mathbf{k}} \sum_{r=1}^{2} \left(b_{r,\mathbf{k}} e_{r,s}(\mathbf{k}) \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right) + d_{r,\mathbf{k}} v_r(\mathbf{k}) \exp\left(\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)\right).$$

Hence,

$$\begin{aligned} \widehat{\mathbb{H}} &= \int d\mathbf{x} \cdot \mathcal{H}(\mathbf{x}) = \int d\mathbf{x} \cdot \phi^{\dagger}(\mathbf{x})\, \widehat{H}_0 \phi(\mathbf{x}) = \\ &= \left(\frac{2\pi \mathrm{c}}{\mathrm{h}}\right)^3 \sum_{\mathbf{k}} \mathrm{h}\omega(\mathbf{k}) \sum_{r=1}^{2} \left(b_{r,\mathbf{k}}^{\dagger} b_{r,\mathbf{k}} - d_{r,\mathbf{k}}^{\dagger} d_{r,\mathbf{k}} \right). \end{aligned}$$

And again we get negative energy.

Let's consider the meaning of such energy: An event with positive energy transfers this energy photons which carries it on recorders observers. Observers know that this event occurs, not before it happens. But event with negative energy should absorb this energy from observers. Consequently, observers know that this event happens before it happens. This contradicts Theorem 3.4.2 [3]. Therefore, events with negative energy do not occur.

Hence, over vacuum $\widetilde{\Phi}_0$ single fermions can exist, but there is no single antifermions.

3 Two-Particle States

A two-particle state is defined the following field operator [4]:

$$\psi_{s_1,s_2}(\mathbf{x},\mathbf{y}) := \begin{vmatrix} \phi_{s_1}(\mathbf{x}) & \phi_{s_2}(\mathbf{x}) \\ \phi_{s_1}(\mathbf{y}) & \phi_{s_2}(\mathbf{y}) \end{vmatrix}.$$

In that case:

$$\widehat{\mathbb{H}} = 2\mathrm{h}\left(\frac{2\pi \mathrm{c}}{\mathrm{h}}\right)^6 \left(\widehat{\mathbb{H}}_a + \widehat{\mathbb{H}}_b\right)$$

where

$$\begin{aligned}\widehat{\mathbb{H}}_a \quad : \quad &= \sum_{\mathbf{k}} \sum_{\mathbf{p}} \left(\omega\left(\mathbf{k}\right) - \omega\left(\mathbf{p}\right)\right) \sum_{r=1}^{2} \sum_{j=1}^{2} \times \\ &\times \Big\{ v_j^{\dagger}\left(-\mathbf{k}\right) v_j\left(-\mathbf{p}\right) e_r^{\dagger}\left(\mathbf{p}\right) e_r\left(\mathbf{k}\right) \times \\ &\times \left(+b_{r,\mathbf{p}}^{\dagger} d_{j,-\mathbf{k}}^{\dagger} d_{j,-\mathbf{p}} b_{r,\mathbf{k}}\right) + \\ &+ \left(+d_{r,-\mathbf{p}}^{\dagger} b_{j,\mathbf{k}}^{\dagger} b_{j,\mathbf{k}} d_{r,-\mathbf{p}}\right) + \\ &+ v_j^{\dagger}\left(-\mathbf{p}\right) v_j\left(-\mathbf{k}\right) e_r^{\dagger}\left(\mathbf{k}\right) e_r\left(\mathbf{p}\right) \times \\ &\times \left(-b_{r,\mathbf{k}}^{\dagger} d_{j,-\mathbf{p}}^{\dagger} d_{j,-\mathbf{k}} b_{r,\mathbf{p}}\right) + \\ &+ \left(-b_{r,\mathbf{p}}^{\dagger} d_{j,-\mathbf{k}}^{\dagger} d_{j,-\mathbf{k}} b_{r,\mathbf{p}}\right) \Big\}\end{aligned}$$

and

$$\begin{aligned}\widehat{\mathbb{H}}_b \quad : \quad &= \sum_{\mathbf{k}} \sum_{\mathbf{p}} \left(\omega\left(\mathbf{k}\right) + \omega\left(\mathbf{p}\right)\right) \sum_{r=1}^{2} \sum_{j=1}^{2} \times \\ &\times \Big\{ v_j^{\dagger}\left(-\mathbf{p}\right) v_j\left(-\mathbf{k}\right) v_r^{\dagger}\left(-\mathbf{k}\right) v_r\left(-\mathbf{p}\right) \times \\ &\times \left(-d_{r,-\mathbf{k}}^{\dagger} d_{j,-\mathbf{p}}^{\dagger} d_{j,-\mathbf{k}} d_{r,-\mathbf{p}}\right) + \\ &+ \left(-d_{r,-\mathbf{p}}^{\dagger} d_{j,-\mathbf{k}}^{\dagger} d_{j,-\mathbf{k}} d_{r,-\mathbf{p}}\right) \\ &+ e_r^{\dagger}\left(\mathbf{k}\right) e_r\left(\mathbf{p}\right) e_j^{\dagger}\left(\mathbf{p}\right) e_j\left(\mathbf{k}\right) \times \\ &\times \left(+b_{r,\mathbf{k}}^{\dagger} b_{j,\mathbf{p}}^{\dagger} b_{j,\mathbf{k}} b_{r,\mathbf{p}}\right) + \\ &+ \left(+b_{r,\mathbf{p}}^{\dagger} b_{j,\mathbf{k}}^{\dagger} b_{j,\mathbf{k}} b_{r,\mathbf{p}}\right) \Big\}.\end{aligned}$$

If velosities are small then the following formula is fair.

$$\widehat{\mathbb{H}} = 4\mathrm{h} \left(\frac{2\pi \mathrm{c}}{\mathrm{h}}\right)^{6} \left(\widehat{\mathbb{H}}_a + \widehat{\mathbb{H}}_b\right)$$

where

$$\begin{aligned}\widehat{\mathbb{H}}_a \quad : \quad &= \sum_{\mathbf{k}} \sum_{\mathbf{p}} \left(\omega\left(\mathbf{k}\right) - \omega\left(\mathbf{p}\right)\right) \times \\ &\times \sum_{r=1}^{2} \sum_{j=1}^{2} \left(d_{j,-\mathbf{p}}^{\dagger} b_{r,\mathbf{k}}^{\dagger} b_{r,\mathbf{k}} d_{j,-\mathbf{p}} - b_{j,\mathbf{p}}^{\dagger} d_{r,-\mathbf{k}}^{\dagger} d_{r,-\mathbf{k}} b_{j,\mathbf{p}}\right)\end{aligned}$$

and

$$\widehat{\mathbb{H}}_b \quad : \quad = \sum_{\mathbf{k}} \sum_{\mathbf{p}} \left(\omega\left(\mathbf{k}\right) + \omega\left(\mathbf{p}\right)\right) \times$$

$$\times \sum_{j=1}^{2} \sum_{r=1}^{2} \left(b_{j,\mathbf{p}}^{\dagger} b_{r,\mathbf{k}}^{\dagger} b_{r,\mathbf{k}} b_{j,\mathbf{p}} - d_{j,-\mathbf{p}}^{\dagger} d_{r,-\mathbf{k}}^{\dagger} d_{r,-\mathbf{k}} d_{j,-\mathbf{p}} \right).$$

Therefore, in any case events with pairs of fermions and events with fermion-antifermion pairs can occur, but events with pairs of antiftrmions can not happen.

4 Conclusion

Therefore, an antifermion can exists only with a fermion.

References

[1] Quznetsov G. 4×1-Marix Functions and Dirac's Equation, *Progress in Physics*, 2009, v. 2, 96–106

[2] For instance, Peskin M. E., Schroeder D. V. An Introduction to Quantum Field Theory, Perseus Books Publishing, L.L.C., 1995.

[3] Quznetsov G. Logical Foundation of Theoretical Physics, Nova Sci. Publ., 2006, 34–35

[4] For instance, Ziman J. M. Elements of Advanced Quantum Theory, Cambridge, 1969, 32.