Higher Dimensional Quantum Gravity Model

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Abstract

In this paper the constructive and consistent formulation of quantum gravity as a quantum field theory for the case of higher dimensional ADM space-times, which is based on the author previous works, is presented. The present model contains a certain new contribution which, however, do not change the general idea which leads to extraordinary simple treatment of quantum gravity in terms of fundamental notions of quantum field theory, like e.g. the Fock space, quantum correlations, etc. We discuss the way to establishing the dimension of space and the relation to string theory of the model.

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1 Introduction

Recently quantum gravity has been denoted increasing interesting of numerous theoreticians in seeking for the new. By this reason, diverse approaches to study this intriguing part of modern theoretical physics has been emerged. Both the most popularized and key models: perturbative quantum gravity (See e.g. the Refs. ['t Hooft & Veltman, 1973], [Veltman, 1976], and [Bern et al., 2010]), quantum geometrodynamics (See e.g. the Refs. [Giulini & Kiefer, 2007] and [Kiefer, 2009]), string theory (See e.g. the Refs. [Wadia, 2008], [Blau & Theisen, 2009], and [Giddings, 2011]), loop quantum gravity (See e.g. the Refs. [Ashtekar, 2007], [Perez, 2009], [Domagala et al., 2010] and [Rovelli, 2011]) have met both theoretical and technical difficulties in extraction of, possibly new, physical facts which could be putted through confrontation with experimental data. Such an embarrassing state of affairs leads to manifestly nonphysical character of quantum gravity, and results in the shift of its considerations from theoretical physics towards evidently nonphysical mathematics. Factually, such a situation is dominating and numerous theoretical approaches (For certain summary see e.g. the Ref. [Oriti, 2009]) are overcrowded by abstract mathematics, while extraction of physical facts is neglected or manifestly omitted.

In such a situation the tremendous problem, probably having epochal meaning in quantum gravity studies, is to take the step forward towards extraction of physics. The task problem of quantum gravity is, therefore, to find out empirically verifiable facts having physical significance. However, there are not known even the energy regions in which quantum gravity or its effects could be really existent. The phenomenological approach, based for example on quantum field theory, would be leading to transparent physical picture and empirical verification. The most popular models are, however, far from the successful implementation of this idea.

Quantum geometrodynamics (QGD), the pioneering approach to quantization of Einstein's General Relativity investigated by J.A. Wheeler [Wheeler, 1957-1970] and B.S. DeWitt [DeWitt, 1967], is based on the Dirac approach to constrained Hamiltonian systems [Dirac, 1949-1964] and the Arnowitt–Deser–Misner (ADM) Hamiltonian formulation of General Relativity [Arnowitt et al., 1961]. The well-known problem of QGD is the functional-differential Wheeler–DeWitt (WDW) equation. For a D-dimensional space embedded into a $D+1$-dimensional enveloping space-time the embarrassing functional differentiation is taken with respect to $D\times D$-dimensional symmetric matrix following from ADM decomposition. The complexity level of QGD is truly tremendous: one must solve $D(D+1)/2$ functional differential equations which from the formal point of view seem to have no sense. In this manner both physical and mathematical nature of a WDW wave function Ψ are not established. Crucial complications arise due to the Wheeler Superspace defining the structure of QGD. The only known class of solutions are the Wentzel–Kramers–Brillouin (WKB) wave functions determined by the Feynman path integral, which is called the Hartle–Hawking wave function [Hartle & Hawking, 1983]. WKB states are computable for simplest situations, have many applications in cosmology (See e.g. the Refs. [Halliwell & Hawking, 1985] and [Coleman et al., 1991]), and their applicability recently have been developed significantly in the new directions (See e.g. the Ref. [Hartle et al., 2008]).

In this paper the constructive and consistent formulation of quantum gravity as a quantum field theory is presented. We arrange the model of quantum gravity within quantum geometrodynamics based on the previous works [Glinka, 2007-2010, Glinka, 2011]. The present model contains a certain new contribution which, however, do not change the general idea which leads to extraordinary simple treatment of quantum gravity in terms of fundamental notions of quantum field theory, like e.g. the Fock space, quantum correlations, etc. The present model is applicable to any $D+1$ ADM space-time of General Relativity.

2 Higher Dimensional Quantum Geometrodynamics

Let a space-time be a $D+1$-dimensional pseudo-Riemannian manifold (M,g) equipped with $D+1$-volume form $g=\det g_{\mu\nu}$ defined by a metric $g_{\mu\nu}$ of the Lorentzian signature $(1,D)$, the Christoffel symbols $\Gamma^{\rho}_{\mu\nu}$, the Riemann–Christoffel curvature tensor $R^{\lambda}_{\mu\alpha\nu}$, the Ricci curvature tensor $R_{\mu\nu}$, and the Ricci scalar curvature ${}^{(D+1)}R$

$$\begin{aligned}
\Gamma^{\rho}_{\mu\nu} &= \frac{1}{2}g^{\rho\sigma}\left(g_{\mu\sigma,\nu}+g_{\sigma\nu,\mu}-g_{\mu\nu,\sigma}\right), && (1)\\
R^{\lambda}_{\mu\alpha\nu} &= \Gamma^{\lambda}_{\mu\nu,\alpha}-\Gamma^{\lambda}_{\mu\alpha,\nu}+\Gamma^{\lambda}_{\sigma\alpha}\Gamma^{\sigma}_{\mu\nu}-\Gamma^{\lambda}_{\sigma\nu}\Gamma^{\sigma}_{\mu\alpha}, && (2)\\
R_{\mu\nu} &= R^{\lambda}_{\mu\lambda\nu}=\Gamma^{\lambda}_{\mu\nu,\lambda}-\Gamma^{\lambda}_{\mu\lambda,\nu}+\Gamma^{\lambda}_{\sigma\lambda}\Gamma^{\sigma}_{\mu\nu}-\Gamma^{\lambda}_{\sigma\nu}\Gamma^{\sigma}_{\mu\lambda}, && (3)\\
{}^{(D+1)}R &= g^{\mu\nu}R_{\mu\nu}. && (4)
\end{aligned}$$

In General Relativity (See e.g. [Weinberg, 1972], [Misner et al., 1973], [Landau & Lifshitz, 1994], and [Carroll, 2004]) non-stationary Matter fields are reflected by nonzero stress-energy tensor $T_{\mu\nu}$. In such a situation the Einstein field equations hold

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}{}^{(D+1)}R + \Lambda g_{\mu\nu} = \kappa \ell_P^2 T_{\mu\nu}, \tag{5}$$

where $\kappa = 8\pi G/c^4 \approx 2.08 \cdot 10^{-43}\ \mathrm{N}^{-1}$ is the Einstein constant, and Λ is the cosmological constant. The RHS of (5) has been multiplied by $\ell_P^2 = \hbar G/c^3$ for dimensionlessness. The reciprocal $(\kappa \ell_P^2)^{-1} = \rho_P/6 \approx 1.15 \cdot 10^{35}\ \mathrm{YeV} \cdot \mathrm{ym}^{-3}$ is $1/6$ of the density $\rho_P = E_P/V_P$ of the Planck energy $E_P = \hbar c/\ell_P$ in the volume $V_P = (4/3)\pi \ell_P^3$ of the Planck sphere.

The Hamiltonian formulation of General Relativity for an arbitrary four-dimensional pseudo-Riemannian space-time (See e.g. the Refs. [Poisson, 2004] and [Glinka, 2011]) can be easy generalized to a $D+1$-dimensional space-time of Lorentzian signature $(1,D)$. For this it is necessary to foliate a space-time manifold M with slices, i.e. a family of space-like hypersurfaces. It can be done if M is globally hyperbolic manifold, and Lorentzian space-times belong to this class of manifolds. Let $t(x^\mu)$ be a scalar field on M such that the foliation $t = constans$ defines a family of nonintersecting space-like hypersurfaces $\Sigma(t)$. Let y^i be the coordinates on all Σs, and let us fix a concrete hypersurface Σ defined by a parametric equations $x^\mu = x^\mu(y^i)$, where $i = 1, \ldots, D$ labels coordinates intrinsic to Σ. Equivalently Σ can be restricted by $f(x^\mu) = 0$. Then $\partial_\mu f(x^\mu)$ is a normal to Σ which, if is not null, defines the unit normal vector field to Σ as future-directed time-like vector field $n_\mu \sim \partial_\mu t$ satisfying $n^\mu n_\mu = -1$

$$n_\mu = -\frac{\partial_\mu f}{\sqrt{|\partial_\mu f \partial^\mu f|}} \quad , \quad n^\mu \partial_\mu f > 0. \tag{6}$$

Let γ be a congruence of curves intersecting $\Sigma(t)$'s which in general are not geodesics nor orthogonal to $\Sigma(t)$. Let t be a parameter on γ, and a tangent vector t^μ to γ. Then

$$t^\mu \partial_\mu t = 1. \tag{7}$$

An arbitrary fixed curve γ_F is a mapping between points on all hypersurfaces $\Sigma(t)$

$$\gamma_F : P \in \Sigma(t) \mapsto P' \in \Sigma(t') \mapsto P'' \in \Sigma(t'') \mapsto \ldots P^{(n)} \in \Sigma(t^{(n)}) \quad , \quad n \in \mathbb{Z}_+ \tag{8}$$

and fixation of coordinates on any two Σs establishes constant coordinates y^i for each n. This determines the coordinate system (t, y^i) in M. If there is another coordinate system x^μ: $x^\mu = x^\mu(t, y^i)$ then t^μ and the tangent vectors e_i^μ on any $\Sigma(t)$ in (t, y^i) are

$$t^\mu = (\partial_t x^\mu)_{y^i} = \delta_t^\mu \quad , \quad e_i^\mu = (\partial_{y^i} x^\mu)_t = \delta_i^\mu \tag{9}$$

In any coordinates $\mathscr{L}_t e_i^\mu = 0$ holds. Let us consider the unit normal vector field n_μ to $\Sigma(t)$

$$n_\mu = -N\partial_\mu t \quad , \quad n_\mu e_i^\mu = 0, \tag{10}$$

where N, called the lapse scalar, normalizes n_μ. In general $t^\mu \nparallel n^\mu$ and therefore t^μ can be decomposed in the basis (n^μ, e_i^μ)

$$t^\mu = Nn^\mu + N^i e_i^\mu, \tag{11}$$

where N^i is called the shift vector. The transformation $x^\mu = x^\mu(t, y^i)$ allows to write in (t, y^i)

$$dx^\mu = t^\mu dt + e_i^\mu dy^i = (Nn^\mu + N^i e_i^\mu)dt + e_i^\mu dy^i = (Ndt)n^\mu + (dy^i + N^i dt)e_i^\mu, \tag{12}$$

and hence one can evaluate the space-time interval $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dx^\mu dx_\mu$ as

$$ds^2 = -\left(N^2 - N_i N^i\right)dt^2 + N_i dx^i dt + N_j dx^j dt + h_{ij}dx^i dx^j, \tag{13}$$

where h_{ij} is an induced metric on $\Sigma(t)$

$$h_{ij} = g_{\mu\nu}e_i^\mu e_j^\nu. \tag{14}$$

The decomposition (13), for the special case $D = 3$, was investigated in [Arnowitt et al., 1961]. A space-time metric $g_{\mu\nu}$ satisfying the Einstein field equations (5) is therefore

$$g_{\mu\nu} = \left[\begin{array}{cc} -N^2 + N_i N^i & N_j \\ N_i & h_{ij} \end{array}\right] \quad , \quad g^{\mu\nu} = \frac{1}{N^2}\left[\begin{array}{cc} -1 & N^j \\ N^i & N^2 h^{ij} - N^i N^j \end{array}\right], \tag{15}$$

where $N^j = h^{ij}N_i$ is the contravariant shift vector, and the spatial metric satisfies the orthogonality condition

$$h_{ik}h^{kj} = \delta_i^j. \tag{16}$$

Completeness relations are

$$g_{\mu\nu} = -n_\mu n_\nu + h_{ij}e_\mu^i e_\nu^j \quad , \quad g^{\mu\nu} = -n^\mu n^\nu + h^{ij}e_i^\mu e_j^\nu. \tag{17}$$

It can be verified straightforwardly that the transformation between the $D+1$-volume form and the D-volume form is

$$\sqrt{-g} = N\sqrt{h}. \tag{18}$$

The second fundamental form of a slice, called the extrinsic curvature tensor or induced curvature, is

$$K_{ij} = n_{\mu;\nu} e^{\mu}_{i} e^{\nu}_{j} = -\nabla_{(i} n_{j)} - n_{(i} n^{k} n_{j)|k}, \tag{19}$$

and its trace, called the intrinsic curvature, has the form

$$K = K^{i}_{i} = h^{ij} K_{ij} = n^{\mu}_{;\mu}. \tag{20}$$

Σ is convex when congruence is diverging ($K > 0$) and concave when the congruence is converging ($K < 0$). The tangent vectors to $\Sigma(t)$ satisfy the Gauss–Weingarten equations

$$e^{\alpha}_{i;\beta} e^{\beta}_{j} = \Gamma^{k}_{ij} e^{\alpha}_{k} + K_{ij} n^{\mu}, \tag{21}$$

and the Gauss–Codazzi equations can be derived straightforwardly

$$R_{\mu\nu\kappa\lambda} e^{\mu}_{i} e^{\nu}_{j} e^{\kappa}_{k} e^{\lambda}_{l} = R_{ijkl} - K_{il} K_{jk} + K_{ik} K_{jl}, \tag{22}$$

$$R_{\mu\nu\kappa\lambda} n^{\mu} e^{\nu}_{i} e^{\kappa}_{j} e^{\lambda}_{k} = K_{ij|k} - K_{ik|j}. \tag{23}$$

Applying the decomposition of the Ricci curvature tensor and Ricci scalar curvature

$$R_{\mu\nu} = -R_{\kappa\mu\lambda\nu} n^{\kappa} n^{\lambda} + h^{ij} R_{\kappa\mu\lambda\nu} e^{\kappa}_{i} e^{\lambda}_{j}, \tag{24}$$

$${}^{(D)}R = -2h^{kl} R_{\kappa\mu\lambda\nu} n^{\kappa} n^{\lambda} e^{\mu}_{k} e^{\nu}_{l} + h^{kl} h^{ij} R_{\kappa\mu\lambda\nu} e^{\kappa}_{i} e^{\lambda}_{j} e^{\mu}_{k} e^{\nu}_{l}, \tag{25}$$

one receives the projected Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} {}^{(D+1)}R$

$$2G_{\mu\nu} n^{\mu} n^{\nu} = {}^{(D)}R + K^{ij} K_{ij} + K^{2}, \tag{26}$$

$$G_{\mu\nu} e^{\mu}_{j} n^{\nu} = K^{i}_{j|i} - K_{,j}. \tag{27}$$

Another identity, called the Ricci equation

$$\mathscr{L}_{n} K_{ij} = n^{\mu} n^{\nu} e^{\kappa}_{i} e^{\lambda}_{j} R_{\mu\nu\kappa\lambda} - \frac{1}{N} N_{|ij} - K_{ik} K^{k}_{j}, \tag{28}$$

can be also easy derived. The stroke on the left of an index means the intrinsic covariant differentiation with respect to a coordinate labeled by this index. Two indices before the stroke means taking two times the intrinsic covariant derivative with respect to each of the indices. For instance for a vector V_i and a tensor T_{ij} one has

$$\begin{aligned} V_{i|j} &= \nabla_j V_i = \partial_j V_i - \Gamma^k_{ji} V_k, \quad (29)\\ T_{ij|k} &= \nabla_k T_{ij} = \partial_k T_{ij} - T_{lj}\Gamma^l_{ik} - T_{il}\Gamma^l_{jk}, \quad (30)\end{aligned}$$

where Γ^k_{ij} are the spatial Christoffel symbols

$$\Gamma^k_{ij} = \frac{1}{2} h^{kl}\left(h_{il,j} + h_{lj,i} - h_{ij,l}\right). \quad (31)$$

Applying the completeness relations (17) for the inverted metric and the fact

$$R_{\kappa\mu\lambda\nu} n^\kappa n^\mu n^\lambda n^\nu = 0, \quad (32)$$

one sees that the first term in (25) reduces to $-2R_{\mu\nu}n^\mu n^\nu$. Using of the relations

$$\begin{aligned} R_{\mu\nu}n^\mu n^\nu &= 2\left(n^\mu_{;[\nu} n^\nu\right)_{;\mu]} + 2 n^\mu_{;[\mu} n^\nu_{;\nu]}, \quad (33)\\ n^\mu_{;\nu} n^\nu_{;\mu} &= K^{ij}K_{ij}, \quad (34)\end{aligned}$$

to the reduced first term in (25), and the Gauss–Codazzi equations (22) to the second one

$$h^{kl}h^{ij}R_{\kappa\mu\lambda\nu}e^\kappa_i e^\lambda_j e^\mu_k e^\nu_l = h^{kl}h^{ij}\left(R_{ijkl} - K_{il}K_{jk} + K_{ik}K_{jl}\right) = {}^{(D)}R + K^2 - K^{ij}K_{ij}, \quad (35)$$

leads to the D-dimensional evaluation of the $D+1$-dimensional Ricci scalar curvature

$${}^{(D+1)}R = {}^{(D)}R + K^2 - K^{ij}K_{ij} - 2\left(n^\mu_{;\nu} n^\nu - n^\mu n^\nu_{;\nu}\right)_{;\mu}. \quad (36)$$

An induced metric h_{ij} and the extrinsic curvature K_{ij} are the dynamical variables describing geometry of a submanifold ∂M by the Einstein field equations in $D+1$ ADM decomposition. The pair (h_{ij}, K_{ij}) describes the local geometry of a single space-like hypersurface ∂M, and then the evolution of the global space-time geometry can be formulated in terms of the one-parameter family $(h_{ij}(t), K_{ij}(t))$ describing evolution of the local space geometry of the constant time hypersurfaces ∂M_t. For consistency the relation between the time evolution operator ∂_t and the vector field n normal to ∂M_t must be specified

$$\partial_t = Nn + N^i\partial_i. \quad (37)$$

Let an enveloping space-time manifold M be compact and possesses a space-like boundary $(\partial M, h)$ equipped with a D-volume form $h = \det h_{ij}$ defined by an induced metric h_{ij}, and the second fundamental form K_{ij}. Let topology of M be $\Sigma \times \mathbb{R}$, where Σ is an unrestricted topology of ∂M. Then the Einstein field equations (5) are the Euler-Lagrange equations of motion following from the Hilbert–Palatini action principle with respect to the fundamental field $g_{\mu\nu}$ supplemented by the suitable boundary condition

$$\frac{\delta S[g]}{\delta g_{\mu\nu}} = 0 \quad , \quad \delta g_{\mu\nu}|_{\partial M} = 0, \tag{38}$$

and applied to the Einstein–Hilbert action complemented by the suitable York–Gibbons–Hawking boundary action, i.e. the action of a $D+1$-geometry with fixed an induced D-geometry of a boundary

$$S[g] = \frac{1}{2\kappa c \ell_P^2} \int_M d^{D+1}x\sqrt{-g}\left(-{}^{(D+1)}R + 2\Lambda\right) - \frac{1}{\kappa c \ell_P^2} \int_{\partial M} d^D x \sqrt{h} K + S_\phi[g], \tag{39}$$

where $S_\phi[g]$ is the action of Matter fields

$$S_\phi[g] = \frac{1}{c}\int_M d^{D+1}x\sqrt{-g}L_\phi. \tag{40}$$

The Einstein field equations (5) result from vanishing of the variation $\delta S = \delta S_G + \delta S_\phi = 0$ on ∂M, where $S_G = S_{EH} + S_{YGH}$ and

$$\delta S_G = \frac{1}{2\kappa c \ell_P^2}\int_M d^{D+1}x\sqrt{-g}\left(G_{\mu\nu} + \Lambda g_{\mu\nu}\right)\delta g^{\mu\nu}, \tag{41}$$

$$\delta S_{YGH} = \frac{1}{2\kappa c \ell_P^2}\int_{\partial M} d^D y\sqrt{|h|}h^{\mu\nu}n^\rho \delta g_{\mu\nu,\rho}, \tag{42}$$

$$\delta S_\phi = -\frac{1}{2c}\int_M d^{D+1}x\sqrt{-g}T_{\mu\nu}\delta g^{\mu\nu}. \tag{43}$$

Moreover, the variational principle establishes the relationship

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}\left(\sqrt{-g}L_\phi\right). \tag{44}$$

In general, stationarity of Matter fields, i.e. $T_{\mu\nu} \equiv 0$, results in existence of a global time-like Killing vector field $\mathscr{K}_\mu$ for a metric tensor $g_{\mu\nu}$. Recall (See e.g. [Weinberg, 1972]) that such a field obeys the equation

$$\mathscr{L}_K g_{\mu\nu} = \lim_{\varepsilon\to 0}\frac{g_{\mu\nu}(\tilde{x}) - \tilde{g}_{\mu\nu}(\tilde{x})}{\varepsilon} = 0, \tag{45}$$

where $\tilde{g}_{\mu\nu}(\tilde{x})$ is a metric $g_{\mu\nu}(x)$ transformed under the isometric mapping

$$\tilde{x}^{\mu} = x^{\mu} + \varepsilon\mathscr{K}^{\mu}, \tag{46}$$

which is equivalent to the Killing equation

$$\nabla_{(\mu}\mathscr{K}_{\nu)}(x) = 0. \tag{47}$$

In other words, the Killing vector fields are the infinitesimal generators of isometries. One can specify a coordinate system such that $\mathscr{K}_{\mu} = [\partial_t, 0, 0, 0]$ and the foliation $t = constans$ is space-like. In such a situation $g_{\mu\nu}$ depends at most on a spatial coordinates x^i, and time t is a global coordinate [DeWitt, 2003]. Let us specify such a coordinate in such a way that an induced space is a $t = constans$ hypersurface. Then ∂M satisfies conditions of the Nash embedding theorem (See e.g. [Masahiro, 1987]). If $\Lambda = 0$ then a global time-like Killing vector field on M exists. When $\Lambda > 0$ then $\mathscr{K}_{\mu}$ does not exist, and space-like boundary ∂M only foliates an exterior to the horizons on geodesic lines. Then ADM decomposition (15) is a gauge of the fundamental field $g_{\mu\nu}$.

The action (39) evaluated for $D+1$ ADM metric (15) takes the Hamiltonian form

$$S[g] = \int dt L, \tag{48}$$

where L is the total Lagrangian in $D+1$ splitting. The Lagrangian related to the York–Gibbons–Hawking boundary action is total derivative, and therefore not essential in the analysis. Contributions due to Matter fields and cosmological constant are simple to analysis. The most complicated is the geometric part of the Einstein–Hilbert Lagrangian

$$\begin{aligned}
\sqrt{g}\left(-{}^{(D+1)}R + 2\Lambda\right) &= N\sqrt{h}\left(-K_{ij}K^{ij} + K^2 - {}^{(D)}R + 2\Lambda\right) && (49)\\
&+ 2\partial_0\left(\sqrt{h}K\right) - 2\partial_i\left(\sqrt{h}(KN^i - h^{ij}N_{|j})\right), && (50)
\end{aligned}$$

and because the last two terms are total derivatives they can be dropped when performing a canonical formulation. In result one obtains the following Lagrangian

$$L = \frac{1}{2\kappa\ell_P^2}\int_{\partial M} d^D x N\sqrt{h}\left(K^2 - K_{ij}K^{ij} - {}^{(D)}R + 2\Lambda + 2\kappa\ell_P^2\rho\right). \tag{51}$$

The Einstein field equations (5) can be decomposed in the $D+1$ splitting. In result one obtains the evolutionary equations for the induced metric h_{ij} and the intrinsic curvature K_{ij}

$$\begin{aligned}
\partial_t h_{ij} &= N_{i|j}+N_{j|i}-2NK_{ij}, \qquad (52)\\
\partial_t K_{ij} &= -N_{|ij}+N(R_{ij}+KK_{ij}-2K_{ik}K^k_j)+N^kK_{ij|k}+K_{ik}N^k_{|j}+K_{jk}N^k_{|i}\\
&- \kappa\ell_P^2 N\left[S_{ij}-\frac{S-\rho}{D-2}h_{ij}\right]. \qquad (53)
\end{aligned}$$

Here ρ, called the energy density, is double projection of the stress-energy tensor onto the normal vector field

$$\rho = T(n,n) = T_{\mu\nu}n^\mu n^\nu, \tag{54}$$

and n^μ is the normal vector field following from the $D+1$ splitting

$$n^\mu = \left[\frac{1}{N},-\frac{N^i}{N}\right] \quad , \quad n_\mu = [-N,0,\ldots,0]^T. \tag{55}$$

The tensor S_{ij}, called spatial stress tensor, is double projection of the stress-energy tensor onto the spatial metric, and S is its trace called spatial stress density

$$S_{ij} = T(h,h) = T_{\mu\nu}h^\mu_i h^\nu_j \quad , \quad S = h^{ij}S_{ij}, \tag{56}$$

where $h^\mu_\nu = \delta^\mu_\nu + n^\mu n_\nu$. The Lagrangian (51) leads to the Euler–Lagrange equations of motion

$$2c\kappa h^{-1}\left(h_{ik}h_{jl}-\frac{1}{2}h_{ij}h_{kl}\right)\frac{\delta S}{\delta h_{ij}}\frac{\delta S}{\delta h_{kl}} - \frac{\ell_P^2}{2c\kappa}\left({}^{(D)}R-2\Lambda-2\kappa\ell_P^2\rho\right)=0, \tag{57}$$

and

$$\frac{c}{\ell_P^2}\pi^{ij}_{|j}+J^i=0, \tag{58}$$

where π^{ij} is the momentum conjugated to the induced metric

$$\pi^{ij} = \frac{1}{\ell_P}\frac{\delta S[g]}{\delta h_{ij}} = \frac{1}{\ell_P}\frac{\delta L}{\delta\left(\partial_t h_{ij}\right)} = -\frac{\ell_P}{2c\kappa}\sqrt{h}\left(K^{ij}-h^{ij}K\right), \tag{59}$$

and J^i, called the momentum density, is the stress-energy tensor projected onto the normal vector field and the spatial metric

$$J^i = T(n,h) = T_{\mu\nu}n^\mu h^{\nu i}. \tag{60}$$

The equation (57) is the Hamilton–Jacobi equation of $D+1$-dimensional General Relativity and defines the classical geometrodynamics.

Let us analyze the Lagrangian (51). The canonical momenta of the theory are

$$\pi_\phi = \frac{\beta}{\ell_P}\frac{\delta L}{\delta\left(\partial_t\phi\right)}, \tag{61}$$

$$\pi = \frac{1}{\ell_P}\frac{\delta L}{\delta\left(\partial_t N\right)} = 0, \tag{62}$$

$$\pi^i = \frac{1}{\ell_P}\frac{\delta L}{\delta\left(\partial_t N_i\right)} = 0, \tag{63}$$

where β is a constant of dimension $[\phi]$ constructed from the Planck units. Then the Legendre transformation allows to rewrite the total Lagrangian in the form

$$L = \int_{\partial M} d^D x \left[\frac{1}{2\kappa\ell_P}\left(\pi_\phi\partial_t\phi + \pi\partial_t N + \pi^i\partial_t N_i + \pi^{ij}\partial_t h_{ij}\right) - NH - N_i H^i\right], \tag{64}$$

where the quantities H and H^i are defined as

$$H = \frac{\sqrt{h}}{2\kappa\ell_P^2}\left(K^2 - K_{ij}K^{ij} - {}^{(D)}R + 2\Lambda + 2\kappa\ell_P^2\rho\right), \tag{65}$$

$$H^i = -2\frac{c}{\ell_P^2}\pi^{ij}{}_{|j} - 2J^i = -2\frac{c}{\ell_P^2}\partial_j\pi^{ij} - \frac{c}{\ell_P^2}h^{il}\left(2h_{jl,k} - h_{jk,l}\right)\pi^{jk} - 2J^i, \tag{66}$$

Application of the time-preservation to the primary constraints

$$\pi \approx 0, \tag{67}$$

$$\pi^i \approx 0, \tag{68}$$

leads to the secondary constraints

$$H \approx 0, \tag{69}$$

$$H^i \approx 0, \tag{70}$$

called the Hamiltonian (scalar) constraint which yields the dynamics, and the diffeomorphism (vector) constraint which merely reflects the spatial diffeoinvariance. The quantities H^i generate the spatial diffeomorphisms $\widetilde{x}^i = x^i + \xi^i$

$$i\frac{\ell_P}{\hbar}\left[h_{ij}, \int_{\partial M} H_a\xi^a d^D x\right] = c\ell_P\left(-h_{ij,k}\xi^k - h_{kj}\xi^k{}_{,i} - h_{ik}\xi^k{}_{,j}\right), \tag{71}$$

$$i\frac{\ell_P}{\hbar}\left[\pi^{ij}, \int_{\partial M} H_a\xi^a d^D x\right] = c\ell_P\left[-\left(\pi^{ij}\xi^k\right)_{,k} + \pi^{kj}\xi^i{}_{,k} + \pi^{ik}\xi^j{}_{,k}\right], \tag{72}$$

where $H_i = h_{ij}H^j$. Application of the structure constants of the diffeomorphism group

$$c^a_{ij} = \delta^a_i \delta^b_j \delta^{(D)}_{,b}(x,z)\delta^{(D)}(y,z) - (x \to y), \tag{73}$$

to the relations (71) and (72) leads to the first-class constraints algebra

$$\begin{aligned} i\frac{\ell_P}{\hbar}\left[H_i(x), H_j(y)\right] &= \frac{c}{\ell_P^5}\int_{\partial M} H_a c^a_{ij} d^D z, & (74)\\ i\frac{\ell_P}{\hbar}\left[H(x), H_i(y)\right] &= \frac{c}{\ell_P^5} H \delta^{(D)}_{,i}(x,y), & (75)\end{aligned}$$

while involving of the elementary relation

$$\delta\left(\sqrt{h}^{(D)}R\right) = \sqrt{h}h^{ij}h^{kl}\left(\delta h_{ik,jl} - \delta h_{ij,kl}\right) - \sqrt{h}\left[R^{ij} - \frac{1}{2}h^{ij(D)}R\right]\delta h_{ij}, \tag{76}$$

allows to establish the third bracket

$$i\frac{\ell_P}{\hbar}\left[\int_{\partial M} H\xi_1 d^D x, \int_{\partial M} H\xi_2 d^D x\right] = c\ell_P \int_{\partial M} H^a\left(\xi_{1,a}\xi_2 - \xi_1\xi_{2,a}\right)d^D x. \tag{77}$$

Canonical primary quantization [Dirac, 1949-1964] gives the commutation relations

$$\begin{aligned} i\frac{\ell_P}{\hbar}\left[\pi^{ij}(x), h_{kl}(y)\right] &= \frac{1}{2}\left(\delta^i_k\delta^j_l + \delta^i_l\delta^j_k\right)\delta^{(D)}(x,y), & (78)\\ i\frac{\ell_P}{\hbar}\left[\pi^i(x), N_j(y)\right] &= \delta^i_j\delta^{(D)}(x,y), & (79)\\ i\frac{\ell_P}{\hbar}\left[\pi(x), N(y)\right] &= \delta^{(D)}(x,y). & (80)\end{aligned}$$

A representation of the momenta operators obeying (78)-(80) is a choice. Usually the Wheeler metric representation is taken into account. In such a representation

$$\pi = -i\frac{\hbar}{\ell_P}\frac{\delta}{\delta N} \quad , \quad \pi^i = -i\frac{\hbar}{\ell_P}\frac{\delta}{\delta N_i} \quad , \quad \pi^{ij} = i\frac{\hbar}{\ell_P}\frac{\delta}{\delta h_{ij}}, \tag{81}$$

which applied to the Hamiltonian constraint (65) yield the Wheeler–DeWitt equation

$$\left\{2c\kappa\frac{\hbar^2}{\ell_P^2}G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} + \frac{\ell_P^2}{2c\kappa}\sqrt{h}\left({}^{(D)}R - 2\Lambda - 2\kappa\ell_P^2\rho\right)\right\}\Psi[h_{ij},\phi] = 0, \tag{82}$$

where G_{ijkl} is the DeWitt supermetric on the configurational space of General Relativity called the Wheeler Superspace (See e.g. [Giulini, 2009])

$$G_{ijkl} = \frac{1}{2\sqrt{h}}\left(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}\right). \tag{83}$$

Other first-class constraints satisfy the canonical commutation relations

$$\left[\pi(x), \pi^i(y)\right] = 0 \quad , \quad \left[\pi(x), H^i(y)\right] = 0 \tag{84}$$

$$\left[\pi^i(x), H^j(y)\right] = 0 \quad , \quad \left[\pi^i(x), H(y)\right] = 0, \tag{85}$$

and after the canonical primary quantization are the supplementary conditions on a wave functional $\Psi[h_{ij}, \phi]$. The primary constraints lead to the equations

$$-i\frac{\hbar}{\ell_P}\frac{\delta\Psi[h_{ij}, \phi]}{\delta N} = 0 \quad , \quad -i\frac{\hbar}{\ell_P}\frac{\delta\Psi[h_{ij}, \phi]}{\delta N_i} = 0. \tag{86}$$

The diffeomorphism constraint also leads to such a condition

$$i\frac{E_P}{\ell_P^2}\left(\frac{\delta\Psi[h_{ij}, \phi]}{\delta h_{ij}}\right)_{|j} = J^i\Psi[h_{ij}, \phi], \tag{87}$$

which merely reflects diffeoinvariance.

3 Global Optimization in Higher Dimensions

Geometry of an enveloping space-time can be evaluated by an embedding geometry, which characteristics are functionals of an induced metric h_{ij}. Spatial stress density and cosmological constant are also functionals of h_{ij}, so energy density of Matter fields ρ must be a functional of h_{ij}. This situation implies the ramification – Matter fields are, in general, functionals of h_{ij}. In this manner in general the DeWitt supposition [DeWitt, 1967]

$$\Psi[h_{ij}, \phi] \equiv \Psi[\mathfrak{G}^D], \tag{88}$$

is true. The problem is to solve the Wheeler–DeWitt equation, what in general has never been made. It should be emphasized that the only WKB solutions [Hartle & Hawking, 1983] are known. Even $\Psi[\mathfrak{G}^D] = \Psi[h_{ij}]$ does not simplify the situation – there is still a functional dependence on $D \times D$ matrix h_{ij}, and it is not clear how to treat $\Psi[h_{ij}]$.

One should re-think the structure of a wave functional. In general, the Wheeler–DeWitt equation as the result of the primary canonical quantization is a kind of the Schrödinger/Klein–Gordon equation. In this manner $\Psi[h_{ij}]$ should be a scalar functional in h_{ij}. The Wheeler–DeWitt operator is $Diff(\partial M)$-invariant, so full invariance is assured iff $\Psi[h_{ij}]$ is a diffeoinvariant, i.e. is a function of another $Diff(\partial M)$-invariants which must be constructed from h_{ij} only! Let us apply such a strategy.

3.1 D-Dimensional Global One-Dimensionality Conjecture

By generalization of the DeWitt construction [DeWitt, 1967] $\Psi[h_{ij}]$ is a functional of the $D \times D$ symmetric matrix h_{ij}. Analogy to quantum mechanics suggests that $\Psi[h_{ij}]$ as a wave function is a classical scalar field, i.e. $\Psi[h_{ij}]$ is a single functional. It means that we shall to perform *global optimization of quantum geometrodynamics*. For realization of this point of view $\Psi[h_{ij}]$ should be dependent on diffeoinvariant matrix invariants of h_{ij}. Matrix invariants of $h_{ij} = \mathbf{h}$ are the coefficients

$$
\begin{aligned}
c_1 &= (-1)^D \mathrm{Tr}\mathbf{h}, && (89)\\
c_2 &= \frac{(\mathrm{Tr}\mathbf{h})^2 - \mathrm{Tr}\mathbf{h}^2}{2}, && (90)\\
\ldots, & && (91)\\
c_D &= (-1)^D \det \mathbf{h}, && (92)
\end{aligned}
$$

of the characteristic polynomial of h_{ij}, and according to the Cayley–Hamilton theorem

$$\mathbf{h}^D + c_1\mathbf{h}^{D-1} + c_2\mathbf{h}^{D-2} + \ldots + c_D\mathbf{I}_{D\times D} = 0. \tag{93}$$

A scalar valued matrix function $\Psi(h_{ij})$ that depends merely on the matrix invariants of $\mathbf{h}$

$$\Psi\left(h_{ij}\right) = \Psi\left(c_1, c_2, \ldots, c_D\right), \tag{94}$$

remains unchanged under rotations of a coordinate system, is called *objective function*. As in [Glinka, 2011] we call $h = \det h_{ij}$ *the global dimension* and *objective D-dimensional quantum gravity* the quantum gravity described by (94). In this manner we shall consider the theory:

$$\left\{-2c\kappa\frac{\hbar^2}{\ell_P^2}G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} - \frac{\ell_P^2}{2c\kappa}h^{1/2}\left({}^{(D)}R - 2\Lambda - 2\kappa\ell_P^2\rho[h]\right)\right\}\Psi\left(c_1, c_2, \ldots, c_D\right) = 0. \tag{95}$$

This conjecture describes isotropic spacetimes, and is related to the strata of the Wheeler superspace, called midisuperspace.

3.2 Reduction of the Wheeler–DeWitt equation

All the problem to solve is contained in evaluation of the operator

$$G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}}\Psi(c_1,c_2,\ldots,c_D), \tag{96}$$

which can be performed as follows. Let us consider the obvious identity

$$\frac{\delta}{\delta h_{ij}}\Psi(c_1,c_2,\ldots,c_D)=\sum_{n=1}^{D}\frac{\delta c_n}{\delta h_{ij}}\frac{\delta}{\delta c_n}\Psi(c_1,c_2,\ldots,c_D), \tag{97}$$

together with the elementary relation

$$\frac{\delta}{\delta c_n}\Psi(c_1,c_2,\ldots,c_D)=\frac{\delta h}{\delta c_n}\frac{\delta}{\delta h}\Psi(h). \tag{98}$$

Elementary calculation gives

$$\frac{\delta}{\delta h_{ij}}\Psi(c_1,c_2,\ldots,c_D)=Dhh^{ij}\frac{\delta}{\delta h}\Psi(h), \tag{99}$$

where we have used the identity

$$\delta h=hh^{ij}\delta h_{ij}. \tag{100}$$

following from the Jacobi formula for determinant of the space-time metric $\delta g=gg^{\mu\nu}\delta g_{\mu\nu}$ jointed with the $D+1$ ADM decomposition. Consequently, one receives the evaluation

$$G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}}=DG_{ijkl}h^{ij}h^{kl}h^2\frac{\delta^2}{\delta h^2}, \tag{101}$$

so that the reduction is given by the double projection of the DeWitt supermetric onto an induced metric, which can be established straightforwardly

$$G_{ijkl}h^{ij}h^{kl} = -\frac{D(D-2)}{2}h^{-1/2}, \tag{102}$$

where we have used the relations for D-dimensional embedded space $h^{ab}h_{bc} = \delta^a_c$, $\delta^a_a = D$. Jointing (101) and (102) one obtains finally the transformation

$$G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} = -\frac{D^2(D-2)}{2}h^{3/2}\frac{\delta^2}{\delta h^2}, \tag{103}$$

which leads to the quantum geometrodynamics

$$\left[2c\kappa\frac{\hbar^2}{\ell_P^2}\frac{D^2(D-2)}{2}h^{3/2}\frac{\delta^2}{\delta h^2} - \frac{\ell_P^2}{2c\kappa}h^{1/2}\left({}^{(D)}R - 2\Lambda - 2\kappa\ell_P^2\rho[h]\right)\right]\Psi(h) = 0, \tag{104}$$

or in simplified form

$$\left[\frac{\delta^2}{\delta h^2} - \frac{{}^{(D)}R - 2\Lambda - 12\rho[h]/\rho_P}{2(8\pi)^2 D^2(D-2)}\frac{1}{h}\right]\Psi(h) = 0, \tag{105}$$

which is correct for $D \neq 0, 2$.

3.3 Classical Field Theory

Quantum geometrodynamics (104) can be rewritten as the Klein–Gordon equation

$$\left(\frac{\delta^2}{\delta h^2} + \omega^2\right)\Psi = 0, \tag{106}$$

where ω^2 is squared *gravitational dimensionless frequency* of the field Ψ

$$\omega^2 = -\frac{{}^{(D)}R - 2\Lambda - 12\rho/\rho_P}{2(8\pi)^2 D^2(D-2)}\frac{1}{h} = \frac{K^2 - K_{ij}K^{ij}}{2(8\pi)^2 D^2(D-2)}\frac{1}{h}, \tag{107}$$

which can be positive, negative or vanishing identically. The equation (106) can be treated as the Euler–Lagrange equation of motion arising from stationarity of the action

$$S[\Psi] = \int \delta h L\left(\Psi, \frac{\delta\Psi}{\delta h}\right), \tag{108}$$

where L is the Lagrange function

$$L = \frac{1}{2}\Pi_\Psi^2 - \frac{\omega^2}{2}\Psi^2, \tag{109}$$

where Π_Ψ is the momentum conjugated to the classical scalar field Ψ

$$\Pi_\Psi = \frac{\partial L}{\partial\left(\dfrac{\delta\Psi}{\delta h}\right)} = \frac{\delta\Psi}{\delta h}. \tag{110}$$

$S[\Psi]$ is the action in the classical field Ψ, and therefore h behaves as a parameter in ω, i.e. is not essential in derivation of the Euler–Lagrange equations of motion

$$\frac{\partial L}{\partial\Psi} - \frac{\delta}{\delta h}\frac{\partial L}{\partial\left(\dfrac{\delta\Psi}{\delta h}\right)} = 0 \quad , \quad \int\delta\left(\frac{\partial L}{\partial\Psi}\delta\Psi\right) = \frac{\partial L}{\partial\Psi}\delta\Psi\bigg|_0 = 0 \tag{111}$$

where we have taken *ad hoc* the field theoretical condition of vanishing boundary term. It can be seen easy that the first equation in (111) coincides with (106). By application of the conjugate momentum Π_Ψ one rewrites the equation (106) as

$$\frac{\delta\Pi_\Psi}{\delta h} + \omega^2\Psi = 0, \tag{112}$$

so that the equations (110) and (112) are the canonical Hamilton equations of motion

$$\frac{\delta}{\delta h}\Psi = \frac{\delta}{\delta\Pi_\Psi}H(\Psi,\Pi_\Psi) \quad , \quad \frac{\delta}{\delta h}\Pi_\Psi = -\frac{\delta}{\delta\Psi}H(\Psi,\Pi_\Psi), \tag{113}$$

where the Hamilton function $H(\Psi,\Pi_\Psi)$ is obtained from the Lagrange function (109) by the Legendre transformation

$$H(\Psi,\Pi_\Psi) = \Pi_\Psi\frac{\delta\Psi}{\delta h} - L\left(\Psi,\frac{\delta\Psi}{\delta h}\right) = \frac{1}{2}\Pi_\Psi^2 - \frac{\omega^2}{2}\Psi^2. \tag{114}$$

If one introduces the classical two-component field $\Phi = [\Pi_\Psi \;\; \Psi]^T$ then the Hamilton canonical equations of motion (113) can be presented as the one-dimensional Dirac equation

$$\left(-i\sigma_y\frac{\delta}{\delta h} - M\right)\Phi = 0, \tag{115}$$

where M is the mass matrix of the field Φ and σ_y is the Pauli matrix

$$M = \begin{bmatrix} -1 & 0 \\ 0 & -\omega^2 \end{bmatrix} \quad , \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}. \tag{116}$$

The Pauli matrix σ_y satisfies the algebra

$$\sigma_y^2 = \mathbf{I}_2 \quad , \quad \{\sigma_y, \sigma_y\} = 2\mathbf{I}_2 \quad , \quad \mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{117}$$

which is the Clifford algebra over the complex vector space $\mathbb{C}^2$

$$\mathscr{C}\ell_2(\mathbb{C}) = \mathscr{C}\ell_0(\mathbb{C}) \otimes \mathrm{M}_2(\mathbb{C}) \cong \mathrm{M}_2(\mathbb{C}) = \mathbb{C} \oplus \mathbb{C}, \tag{118}$$

where $\mathscr{C}\ell_n \equiv \mathscr{C}\ell_{n,0}$, and $\mathrm{M}_2(\mathbb{C})$ denotes algebra of all 2×2 matrices over $\mathbb{C}$. The Clifford algebra $\mathscr{C}\ell_{2,0}(\mathbb{C})$ possesses a two-dimensional complex representation. Restriction to the pinor group $\mathrm{Pin}_{2,0}(\mathbb{R})$ yields a complex representation of two-dimensional pinor group, i.e. the two-dimensional spinor representation, whereas restriction to the spinor group $\mathrm{Spin}_{2,0}(\mathbb{R})$ splits $\mathscr{C}\ell_{1,1}(\mathbb{R})$ onto a sum of two half spin representations of dimension 1, i.e. the one dimensional Weyl representations. There is the isomorphism

$$\mathrm{Spin}_{2,0}(\mathbb{R}) \cong \mathrm{U}(1) \cong \mathrm{SO}(2), \tag{119}$$

and the spinor group $\mathrm{Spin}_{2,0}(\mathbb{R})$ acts on a 1-sphere S^1 in such a way that one has a fibre bundle with fibre $\mathrm{Spin}_{1,0}(\mathbb{R})$

$$\mathrm{Spin}_{1,0}(\mathbb{R}) \longrightarrow \mathrm{Spin}_{2,0}(\mathbb{R}) \longrightarrow S^1, \tag{120}$$

and the homotopy sequence is

$$\pi_1\left(\mathrm{Spin}_{1,0}(\mathbb{R})\right) \longrightarrow \pi_1\left(\mathrm{Spin}_{2,0}(\mathbb{R})\right) \longrightarrow \pi_1\left(S^1\right). \tag{121}$$

The Clifford algebra $\mathscr{C}\ell_2(\mathbb{C})$ can be generated by complexification

$$\mathscr{C}\ell_2(\mathbb{C}) \cong \mathscr{C}\ell_{1,1}(\mathbb{R}) \otimes \mathscr{C}\ell_0(\mathbb{C}), \tag{122}$$

where $\mathscr{C}\ell_{1,1}(\mathbb{R})$ is the four-dimensional Clifford algebra over the real vector space $\mathbb{R}^{2,0}$

$$\mathscr{C}\ell_{1,1}(\mathbb{R}) \cong \mathrm{M}_2(\mathbb{R}) \otimes \mathscr{C}\ell_0(\mathbb{R}) \cong \mathrm{M}_2(\mathbb{R}), \tag{123}$$

with $\mathrm{M}_2(\mathbb{R})$ being algebra of 2×2 matrices over $\mathbb{R}$, and

$$\mathscr{C}\ell_0(\mathbb{R}) = \mathbb{R} \quad , \quad \mathscr{C}\ell_0(\mathbb{C}) = \mathbb{C}. \tag{124}$$

The Clifford algebra (123) can be decomposed into a direct sum of central simple algebras isomorphic to matrix algebra over $\mathbb{R}$

$$\mathscr{C}\ell_{1,1}(\mathbb{R}) = \mathscr{C}\ell_{1,1}^{+}(\mathbb{R}) \oplus \mathscr{C}\ell_{1,1}^{-}(\mathbb{R}) \quad , \quad \mathscr{C}\ell_{1,1}^{\pm}(\mathbb{R}) = \frac{1\pm\gamma}{2}\mathscr{C}\ell_{1,1}(\mathbb{R}) \cong \mathbb{R}, \tag{125}$$

as well as into a tensor product

$$\mathscr{C}\ell_{1,1}(\mathbb{R}) = \mathscr{C}\ell_{2,0}(\mathbb{R}) \otimes \mathscr{C}\ell_{0,0}(\mathbb{R}) \quad , \quad \mathscr{C}\ell_{2,0}(\mathbb{R}) = \mathrm{M}_2(\mathbb{R}) \otimes \mathscr{C}\ell_{0,0}(\mathbb{R}) \cong \mathrm{M}_2(\mathbb{R}). \tag{126}$$

3.4 Field Quantization in Static Fock Space

The one-dimensional Dirac equation (115) can be canonically quantized

$$\left(-i\sigma_y\frac{\delta}{\delta h}-M\right)\hat{\Phi}=0, \tag{127}$$

according to the canonical commutation relations characteristic for bosonic fields

$$i\left[\hat{\Pi}_\Psi[h'],\hat{\Psi}[h]\right]=\delta(h'-h)\quad,\quad i\left[\hat{\Pi}_\Psi[h'],\hat{\Pi}_\Psi[h]\right]=0\quad,\quad i\left[\hat{\Psi}[h'],\hat{\Psi}[h]\right]=0. \tag{128}$$

The choice of the bosonic relations is argued by the one-dimensionality – there is no difference between bosons and fermions. Particles obeying one-dimensional statistics are called *axions*, so the quantum field theory (127) describes axions which are *gravitons* in our approach.

Let us apply the Fock space formalism, which give explicit decomposition

$$\hat{\Phi}=\mathbf{Q}\mathfrak{B}\quad,\quad\mathbf{Q}=\frac{1}{\sqrt{2}}\left[\begin{array}{cc}\sqrt{1/\omega}&\sqrt{1/\omega}\\-i\sqrt{\omega}&i\sqrt{\omega}\end{array}\right], \tag{129}$$

where $\mathfrak{B}=\mathfrak{B}[h]$ is a dynamical repère

$$\mathfrak{B}=\left\{\left[\begin{array}{c}\mathsf{G}[h]\\\mathsf{G}^\dagger[h]\end{array}\right]:\left[\mathsf{G}[h'],\mathsf{G}^\dagger[h]\right]=\delta\left(h'-h\right),\left[\mathsf{G}[h'],\mathsf{G}[h]\right]=0\right\}, \tag{130}$$

on the Fock space of creation and annihilation operators. Application of the decomposition (129) yields the modified Heisenberg equations of motion

$$\frac{\delta\mathfrak{B}}{\delta h}=\mathbf{X}\mathfrak{B}\quad,\quad\mathbf{X}=\left[\begin{array}{cc}-i\omega&\dfrac{1}{2\omega}\dfrac{\delta\omega}{\delta h}\\\dfrac{1}{2\omega}\dfrac{\delta\omega}{\delta h}&i\omega\end{array}\right]. \tag{131}$$

Let us suppose that there is another repère $\mathfrak{F}$ determined by the Bogoliubov transformation

$$\mathfrak{F}\;=\;\left[\begin{array}{cc}u&v\\v^*&u^*\end{array}\right]\mathfrak{B}\quad,\quad|u|^2-|v|^2=1. \tag{132}$$

Let us assume *ad hoc* that dynamics of the new repère is governed by the Heisenberg equations of motion

$$\frac{\delta\mathfrak{F}}{\delta h}=\left[\begin{array}{cc}-i\Omega&0\\0&i\Omega\end{array}\right]\mathfrak{F}. \tag{133}$$

Application of these assumptions to the equations (131) leads to the vector equation

$$\frac{\delta \mathbf{b}}{\delta h} = \mathbf{X}\mathbf{b} \quad , \quad \mathbf{b} = \left[\begin{array}{c} u \\ v \end{array} \right], \tag{134}$$

and gives trivial value of the unknown frequency $\Omega \equiv 0$. Therefore, the new repère $\mathfrak{F}$ becomes the static Fock repère related to initial data (I)

$$\mathfrak{F} = \left\{ \left[\begin{array}{c} \mathsf{G}_I \\ \mathsf{G}_I^\dagger \end{array} \right] : \left[\mathsf{G}_I, \mathsf{G}_I^\dagger\right] = 1, [\mathsf{G}_I, \mathsf{G}_I] = 0 \right\}, \tag{135}$$

and the vacuum state $|0\rangle$ is correctly defined

$$\mathsf{G}_I |0\rangle = 0 \quad , \quad \langle 0| \mathsf{G}_I^\dagger = 0. \tag{136}$$

Integrability of the equations (134) is the crucial element of the scheme presented above. The Bogoliubov transformation (132) suggests the superfluid parametrization

$$u = e^{i\theta} \cosh\phi \quad , \quad v = e^{i\theta} \sinh\phi, \tag{137}$$

with the angles

$$\theta = \pm i \int_{h_I}^{h} \omega' \delta h' \quad , \quad \phi = \ln \sqrt{\left|\frac{\omega_I}{\omega}\right|}, \tag{138}$$

where $\omega' = \omega(h')$ and ω_I is the initial datum of gravitational dimensionless frequency

$$\omega_I = -\frac{1}{8\sqrt{2}\pi D\sqrt{D-2}}, \tag{139}$$

which yield the Bogoliubov coefficients

$$u = \frac{\mu+1}{2\sqrt{\mu}} \exp\left\{ i \int_{h_I}^{h} \omega' \delta h' \right\} \quad , \quad v = \frac{\mu-1}{2\sqrt{\mu}} \exp\left\{ -i \int_{h_I}^{h} \omega' \delta h' \right\}, \tag{140}$$

where $\mu = \dfrac{\omega}{\omega_I}$ measures the relative gravitational dimensionless frequency. Consequently, the integrability problem is solved by the equation

$$\hat{\Phi} = \mathbf{Q}\mathbf{G}\mathfrak{F}, \tag{141}$$

where $\mathbf{G}$ is the monodromy matrix

$$\mathbf{G} = \left[\begin{array}{cc} \frac{1+\mu}{2\sqrt{\mu}} \exp\left\{ -i \int_{h_I}^{h} \omega' \delta h' \right\} & \frac{1-\mu}{2\sqrt{\mu}} \exp\left\{ i \int_{h_I}^{h} \omega' \delta h' \right\} \\ \frac{1-\mu}{2\sqrt{\mu}} \exp\left\{ -i \int_{h_I}^{h} \omega' \delta h' \right\} & \frac{1+\mu}{2\sqrt{\mu}} \exp\left\{ i \int_{h_I}^{h} \omega' \delta h' \right\} \end{array} \right]. \tag{142}$$

Now it is clear the presented version of quantum geometrodynamics formulates quantum gravity as a quantum field theory, where graviton is associated with configuration of embedded space and given by the decomposition (141) in the static Fock space.

The initial data condition $\omega = \omega_I$ generates the equation for the initial manifold

$$
{}^{(3)}R^{(I)} - 2\Lambda - 12\rho_I/\rho_P = h_I, \tag{143}
$$

or equivalently

$$
K_{ij}^{(I)} K^{(I)ij} - K^{(I)2} = h_I, \tag{144}
$$

where the superscript I means initial value of given quantity. The quantum evolution (106) in such a situation takes the form

$$
\left(\frac{\delta^2}{\delta h_I^2} - \frac{1}{2(8\pi)^2 D^2 (D-2)} \right) \Psi(h_I) = 0, \tag{145}
$$

and after taking into account the suitable boundary conditions

$$
\Psi(h_I = h_0) = \Psi_0 \quad , \quad \left. \frac{\delta \Psi(h_I)}{\delta h_I} \right|_{h_I = h_0} = \Pi_\Psi^0, \tag{146}
$$

can be solved straightforwardly

$$
\Psi(h_I) = \Psi_0 \cosh\left\{ \frac{h_I - h_0}{8\sqrt{2}\pi D\sqrt{D-2}} \right\} + 8\pi\sqrt{6}\ell_P^2 \Pi_\Psi^0 \sinh\left\{ \frac{h_I - h_0}{8\sqrt{2}\pi D\sqrt{D-2}} \right\}. \tag{147}
$$

4 Several Implications

Let us look on several implications of the quantum field theory of gravity within quantum geometrodynamics.

4.1 Quantum Correlations

With using of the matrices (142) and (129), and the relation (141) one derives the quantum field

$$
\hat{\Psi}(h) = \sqrt{\frac{\omega_I}{8}} \frac{1}{\omega} \left(\exp\left\{ -i \int_{h_I}^{h} \omega' \delta h' \right\} \mathsf{G}_I + \exp\left\{ i \int_{h_I}^{h} \omega' \delta h' \right\} \mathsf{G}_I^\dagger \right). \tag{148}
$$

Let us take into account the n-particle one-point quantum states determined as

$$|h,n\rangle \equiv \hat{\Psi}^n |0\rangle = \left(\sqrt{\frac{\omega_I}{8}}\frac{1}{\omega}\exp\left\{\int_{h_I}^{h}\omega'\delta h'\right\}\right)^n \mathsf{G}_I^{\dagger n}|0\rangle\,, \tag{149}$$

which yield two-point correlators $\mathrm{Cor}_{n'n}(h',h) \equiv \langle n',h'|h,n\rangle$ or explicitly

$$\mathrm{Cor}_{n'n}(h',h) = \left(\frac{\omega_I}{8}\right)^{(n'+n)/2}\exp\left\{i\left(n'\int_{h'}^{h_I}+n\int_{h_I}^{h}\right)\omega''\delta h''\right\}\frac{\langle 0|\mathsf{G}_I^{n'}\mathsf{G}_I^{\dagger n}|0\rangle}{\omega'^{n'}\omega^n}. \tag{150}$$

Basically one obtains

$$\mathrm{Cor}_{00}(h,h) = \mathrm{Cor}_{00}(h',h) = \mathrm{Cor}_{00}(h_I,h_I) = \langle 0|0\rangle\,, \tag{151}$$

$$\mathrm{Cor}_{11}(h_I,h_I) = \frac{1}{8\omega_I}, \tag{152}$$

$$\frac{\mathrm{Cor}_{n'n}(h_I,h_I)}{[\mathrm{Cor}_{11}(h_I,h_I)]^{(n'+n)/2}} = \langle 0|\mathsf{G}_I^{n'}\mathsf{G}_I^{\dagger n}|0\rangle\,, \tag{153}$$

and by elementary algebraic manipulations one receives

$$\mathrm{Cor}_{11}(h',h) = \frac{\sqrt{\mathrm{Cor}_{11}(h',h')\mathrm{Cor}_{11}(h,h)}}{\mathrm{Cor}_{11}(h_I,h_I)}\exp\left\{i\int_{h'}^{h}\omega''\delta h''\right\}, \tag{154}$$

$$\frac{\mathrm{Cor}_{nn}(h',h)}{\mathrm{Cor}_{00}(h_I,h_I)} = \left[\frac{\mathrm{Cor}_{11}(h',h)}{\mathrm{Cor}_{00}(h_I,h_I)}\right]^n, \tag{155}$$

$$\frac{\mathrm{Cor}_{11}(h,h)}{\mathrm{Cor}_{00}(h_I,h_I)} = \left(\frac{\omega_I}{\omega}\right)^2\mathrm{Cor}_{11}(h_I,h_I). \tag{156}$$

Another interesting relation is

$$\frac{\mathrm{Cor}_{n'n}(h,h)}{\mathrm{Cor}_{n'n}(h_I,h_I)} = \left(\frac{\mathrm{Cor}_{11}(h,h)}{\mathrm{Cor}_{11}(h_I,h_I)\mathrm{Cor}_{00}(h_I,h_I)}\right)^{(n'+n)/2}\exp\left\{-i(n'-n)\int_{h_I}^{h}\omega''\delta h''\right\}. \tag{157}$$

A whole information about the quantum gravity is contained in the parameters of the theory, i.e. frequency ω and its initial data ω_I. It is evident that the quantum correlations are strictly determined by these fundamental quantities only. In other words measurement of quantum correlations can be used for deduction of values of the fundamental parameters of the theory.

4.2 Dimension of Space. Relation to String Theory.

The quantum field-theoretic model of quantum gravity presented in this chapter can be used for determination of the dimension D of embedded space by its geometry, energy density of Matter fields, and the wave functional solving the reduced quantum geometrodynamics. It can be seen by straightforward and easy algebraic manipulation that the equation

$$\left[\frac{\delta^2}{\delta h^2}-\frac{{}^{(D)}R-2\Lambda-12\rho[h]/\rho_P}{2(8\pi)^2D^2(D-2)}\frac{1}{h}\right]\Psi(h)=0, \tag{158}$$

can be transformed into the equation for the dimension D

$$D^2(D-2)=\frac{1}{2(8\pi)^2}\left({}^{(D)}R-2\Lambda-12\frac{\rho}{\rho_P}\right)\left(\frac{h}{\Psi}\frac{\delta^2\Psi}{\delta h^2}\right)^{-1}\equiv\Delta(h), \tag{159}$$

which can be solved immediately. When $\Delta(h)>0$ then the real solution is

$$\begin{aligned} D &= \frac{2}{3}+\frac{1}{3}\left(8+\frac{27}{2}\Delta(h)-\frac{3}{2}\sqrt{3}\sqrt{32\Delta(h)+27\Delta^2(h)}\right)^{1/3}+\\ &+\frac{1}{3}\left(8+\frac{27}{2}\Delta(h)+\frac{3}{2}\sqrt{3}\sqrt{32\Delta(h)+27\Delta^2(h)}\right)^{1/3}, \end{aligned} \tag{160}$$

and therefore the physical values of the space dimension to this case are

$$D>2\longrightarrow D=3,4,5,\ldots \tag{161}$$

what is a relevant result and unambiguously establishes the lower bound

$$\Delta(h)\geqslant 9. \tag{162}$$

To the case $\Delta(h)<0$ one receives

$$\begin{aligned} D &= \frac{2}{3}+\frac{1}{3}\left(8-\frac{27}{2}|\Delta(h)|-\frac{3}{2}\sqrt{3}\sqrt{-32|\Delta(h)|+27\Delta^2(h)}\right)^{1/3}+\\ &+\frac{1}{3}\left(8-\frac{27}{2}|\Delta(h)|+\frac{3}{2}\sqrt{3}\sqrt{-32|\Delta(h)|+27\Delta^2(h)}\right)^{1/3}, \end{aligned} \tag{163}$$

and therefore the physical values of the space dimension are

$$D\in(-\infty,2)/\{0\}\longrightarrow D=1, \tag{164}$$

what unambiguously establishes $\Delta(h) = -1$, and is physically irrelevant result. In the case of the trivial situation $\Delta(h) = 0$ the space dimension has two possible values

$$D = 0, 2 \tag{165}$$

and the only $D = 2$ can be physically relevant value.

The question arises: what is the physical meaning of $\Delta(h)$? Because of the bound (162) one can suggest that $\Delta = D^3 - 2D^2$ is a dimension of a certain effective space. Then, in the light of string theory, one can suggest that string theory lives in such an effective space-time, and the upper bound for space-time dimensionality is 26 so that

$$\Delta_{String}(h) \leqslant 25. \tag{166}$$

If supersymmetry is involved then the upper bound for space-time dimensionality is 11, so

$$\Delta_{Superstring}(h) \leqslant 10. \tag{167}$$

It means that in such a situation the dimension D of space is 3 or more. Such a scenario establishes clear relationship between quantum gravity and string theory - string theory is a theory on the effective space-time, while quantum gravity should be existent in the classical space-time. The minimal dimension of the effective space-time following from the bound (162) would be 10. Interestingly, when $\Delta(h) \leqslant 25$ then

$$D \lessapprox 3.7643, \tag{168}$$

what in the light of the bound (161) and the fact that D should be an integer means that 4 is the stable dimension of space-time. Similarly when $\Delta(h) \leqslant 11$ then

$$D \lessapprox 3.0647. \tag{169}$$

Both the bounds (168) and (169) can be interpreted as the syndrome of the fluctuations of the dimension of space. It is consistent with the dimensional regularization ['t Hooft & Veltman, 1972], in which dimension of space-time is $4 + \varepsilon$, where ε is a fluctuation of space dimension which goes to 0^- in the regularization procedure. In our scenario one receives the following values of fluctuations of space dimension

$$\varepsilon_{String} \lessapprox 0.7643, \tag{170}$$

$$\varepsilon_{Superstring} \lessapprox 0.0647, \tag{171}$$

and therefore the conclusion is: supersymmetry significantly restricts fluctuations of the space dimensionality

$$\frac{\varepsilon_{Superstring}}{\varepsilon_{String}} \approx 0.0846. \tag{172}$$

Good qualitative question is: for which value of the effective dimension Δ the dimension of space changes diametrically? If one writes $D=3+\varepsilon$ and put this into the relation (160) then the fluctuation of space dimension is

$$\begin{aligned}\varepsilon &= -\frac{7}{3}+\frac{1}{3}\left(8+\frac{27}{2}\Delta(h)-\frac{3}{2}\sqrt{3}\sqrt{32\Delta(h)+27\Delta^2(h)}\right)^{1/3}+\\ &+ \frac{1}{3}\left(8+\frac{27}{2}\Delta(h)+\frac{3}{2}\sqrt{3}\sqrt{32\Delta(h)+27\Delta^2(h)}\right)^{1/3}. \end{aligned}\tag{173}$$

In this manner change of the space dimension by the value $\varepsilon=1$ corresponds to the following change in the dimension of an effective space

$$\Delta(h)=32. \tag{174}$$

It means that 5-dimensional classical space-time corresponds to 33-dimensional effective space-time. It means also that when the effective dimension is $\Delta(h)\gg 32$ then the fluctuation of space dimension is $\varepsilon\gg 1$. In this manner the fluctuation ε of space-time dimension is much more less than 1 if and only if the dimension $\Delta(h)$ of effective space-time is much more less than 32.

5 Discussion

We have presented the scenario in which global optimization of quantum geometrodynamics allows to formulate quantum gravity as a quantum field theory. In fact, this result to the case of 4-dimensional space-times has been established in [Glinka, 2011]. In this chapter we have generalized this result to $D+1$-dimensional space-times of Lorentzian signature $(1,D)$, and we have performed another reduction of the Wheeler–DeWitt equation which involved *all* matrix invariants of a metric of D-dimensional embedded space. Such a procedure has been changed the results by constant multipliers only, but possibly is more adequate.

It is clear that the presented construction in itself is nontrivial, because allows to express quantum gravity as the quantum field theory formulated in the Fock space. Moreover, the theory is extraordinary simple – this is one-dimensional Dirac equation. The gravitons in this approach are axions.

As has been shown in [Glinka, 2011] the global one-dimensional quantum gravity is solvable in general, and can be solved for a number of particular situations of the Einstein field equations. The same conclusion can be deduced for the present approach, because of this is the only minor generalization of the general idea propagated in [Glinka, 2011]. Particularly interesting solutions which could be studied in the further research are higher dimensional black holes, and higher dimensional cosmological solutions.

In this manner, the model to quantum gravity presented in this chapter possesses manifestly phenomenological meaning. Quantum field theory gives the possibilities to empirical verification, as well as gives the nontrivial physical nature for the quantum gravity. We hope for further development of the approach to quantum gravity based on the reductions within quantum geometrodynamics. In our opinion such a research line is highly prospective way to the consistent and constructive theory of quantum gravity.

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