Complement to Special Relativity at Superluminal Speeds: CERN Neutrinos Explained

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Abstract

The most recent notifications from OPERA Collaboration of CERN Geneva report highly probable existence of faster-than-light neutrinos. Such a state of affairs has been also detected earlier in radio galaxies, quasars and recently in microquasars. The usual scenario explaining superluminal speeds is based on a black hole contained in these sources producing the high speed mass ejection.

Superluminal speeds are, however, plainly and efficiently explainable within the framework of Special Relativity, in which the Einstein postulates, the Minkowski energy-momentum space, and both the Poincaré and the Lorentz symmetries remain unchanged, but the energy-momentum relation is deformed. In this paper superluminal deformations of Special Relativity, complementing the Einstein theory at faster-than-light speeds, are studied in the context of CERN neutrinos. For full consistency we propose to apply the non-parallelism hypothesis, the deformation derived ab initio, and the concept of measured speed of light which can be higher than c. We show that such a theory is able to explain both superluminal speed as well as mass of neutrino.

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1 Introduction

22nd September 2011 was truly revolutionary day for high energy particle physics. Then the OPERA Collaboration, experimental project of CERN Geneva, has announced that their results based on high-statistics experimental data demonstrate superluminally traveling neutrinos [OPERA Collaboration, 2011]. Similar situation is well-known in modern astronomy since the early 1980s, when faster-than-light motion was suggested to contradict the quasars having cosmological distances. Presently, superluminal travels are seen in radio galaxies, quasars and microquasars. It is usually believed that a black hole contained in these sources provokes the high speed mass ejection. Mostly believed opinion about superluminal speeds, which do not use explanations wrecking Special Relativity, are the optical illusions. By this reason the speculations about time travels and other hypothetical possibilities of Special Relativity arising from a faster-than-light motion, are cordially propagated. It should be noticed that even Einstein never excluded possibility of superluminal speeds. His account is the postulate of Special Relativity saying that the speed of light c is the maximal speed, what does not mean that measured speed of a moving object must be less c. In this manner the speed of light postulate is the only internal restriction of Special Relativity, but not the limitation of Nature, so the sceptics are forced to accept the state of affairs in which Special Relativity is a consistent physical theory.

In this paper we present the complement of Special Relativity for faster-than-light speeds. We show that superluminally moving objects can be efficiently described with using of Special Relativity. A whole mathematical structure of the Minkowski space, four-vector formalism, as well as the fundamental Einstein's postulates remain unchanged, while the energy-momentum relation expressing the Einstein equivalence principle is deformed due to certain additional arguments. It should be emphasized that the original Einstein's energy-momentum relation has nothing to the postulates of Special Relativity, and by this reason its modification does not lead to inconsistencies. The problem is, however, to complement Special Relativity in such a way that the Poincaré invariance, as its special case the Lorentz invariance, remain preserved. For creating of such an appropriate model we show general consequences due to deformed energy-momentum relations for Special Relativity, which include motion with superluminal speeds.

For consistency we propose to the non-parallelism hypothesis, expressing lack of parallelism between momentum and velocity vectors of a moving body. The deformation of Einstein's energy-momentum relation is derived ab initio from definition of the velocity. We introduce also the measured speed of light which can be higher than c. These assumptions taken together lead to superluminal speeds within Special Relativity and preserve both the Poincaré and Lorentz symmetries of the Minkowski energy-momentum space. Finally, we discuss this theory in the context of CERN neutrinos, and show that both faster-than-light speed and nonzero mass of neutrinos are consistently described within the our framework.

2 Deformed Energy-Momentum Relations

Let us consider Special Relativity, in which the Hamiltonian constraint called the Einstein energy-momentum relation is modified by the constituent Δ arising due to certain additional arguments. This extra-term has physical dimensionality of squared energy, and in general is a function of momentum vector and energy of a moving object, and also another non-dynamical parameters

$$E^2 = p^2c^2 + m^2c^4 + \Delta(E, p_i; m, \ldots). \tag{1}$$

We assume that energy is a function of momentum value, i.e. $E = E(p)$. It is obvious that for the trivial case $\Delta \equiv 0$ one has to deal with the situation described by Special Relativity and its all consequences among which the key one is the Standard Model of particles and fundamental interactions. In this manner, existence of a deformation Δ leads to a new physics which is manifestly beyond the Standard Model.

The origins of the deformation $\Delta(E, p; m, \ldots)$ can have diverse nature. In case of Doubly Special Relativity such a deformation is purely algebraic. Another particular situation is $\Delta = \dfrac{p^4c^4}{\varepsilon^2}$, where $\varepsilon = \dfrac{\hbar c}{\sqrt{\alpha}\ell}$ is a maximal energy constructed from a minimal scale ℓ, for example the Planck scale where $\ell = \ell_P = \sqrt{\dfrac{\hbar G}{c^3}}$, and a dimensionless small constant $\alpha \sim 1$ which value can be established as $1/(2\pi)^2$ [Glinka, 2011]. This deformation arises from the Snyder non-commutative geometry of phase-space (p,x) of a moving particle, and its consequences for physics at the Planck scale were primarily studied by Sidharth. Among many essential changes due to the Snyder–Sidharth deformation of Special Relativity, possibly the most essential one is its exceptional usefulness for generation of nonzero neutrino mass and the new physics, for instance consistent description of the Compton scattering and the Compton effect [Glinka, 2011]. All hitherto known models, including Doubly Special Relativity and the Snyder–Sidharth deformation, preserve the structure of the Minkowski space and the Einstein postulates, but manifestly violate both the Poincaré and the Lorentz (CP) symmetries.

In this manner, it is reasonable to suspect that the most general Hamiltonian constraint (1) also leads to consistent description of the new physics, including superluminal speeds. We shall call the extra-term Δ by *superluminal deformation*. The problem is to establish the concrete form of a superluminal deformation, and for the derived formula of Δ to deduce the physical content of the corresponding deformed Special Relativity. The first way for construction of Δ is the phenomenology of an experiment, and the second way is a theoretical proposal based on any reasonable arguments. In this paper we shall to apply the second way.

3 Superluminal Speeds

Let us examine velocity and speed of a particle characterized by any energy-momentum relation of the form (1). According to the usual definition, which is particularly valid for Special Relativity, classical mechanics and other well-known cases, the velocity vector is

$$v^i = \frac{\partial E}{\partial p_i}. \tag{2}$$

Using of (1) leads to the relation

$$\frac{\partial E}{\partial p_i} = \frac{1}{2E}\left(2p^i c^2 + \frac{\partial \Delta}{\partial p_i} + \frac{\partial \Delta}{\partial E} v^i\right). \tag{3}$$

For consistency Δ should be a scalar quantity, and therefore one can suggest that Δ depends on the powers of momentum value $p = \sqrt{p^i p_i}$. Hence the p_i-derivative can be transformed into the p-derivative

$$\frac{\partial \Delta}{\partial p_i} = \frac{\partial \Delta}{\partial p}\frac{\partial p}{\partial p_i} + \frac{\partial \Delta}{\partial E}\frac{\partial E}{\partial p_i} = \frac{\partial \Delta}{\partial p}\frac{p^i}{p} + \frac{\partial \Delta}{\partial E} v^i, \tag{4}$$

applied to the relation (3) leads to

$$v^i\left(1 - \frac{1}{E}\frac{\partial \Delta}{\partial E}\right) = \frac{1}{2E}\left(2p^i c^2 + \frac{\partial \Delta}{\partial p}\frac{p^i}{p}\right). \tag{5}$$

If $\frac{1}{E}\frac{\partial\Delta}{\partial E} \neq 1$ then the velocity of a moving object has the form

$$v^i = v\frac{p^i}{p}, \tag{6}$$

where $v = \sqrt{v^i v_i}$ is the speed of a moving object

$$v = \frac{pc^2 + \frac{1}{2}\frac{\partial\Delta}{\partial p}}{E - \frac{\partial\Delta}{\partial E}}. \tag{7}$$

Both sides of the relation (6) can be multiplied by v_i or by p_i what gives

$$vp = v_i p^i = v^i p_i. \tag{8}$$

Applying the definition of a scalar product one has $v^i p_i = vp\cos\alpha$, where $\alpha = (v^i, p_i)$ is the angle between the vectors v^i and p^i, and consequently (8) gives identically $\cos\alpha = 1$, i.e. $\alpha = 0$.

The speed formula (7) can be presented in the equivalent form

$$v = v_r + \delta v, \tag{9}$$

where v_r is the speed, which does not contain the derivatives of a superluminal deformation Δ and in the trivial situation $\Delta \equiv 0$ leads to the speed computed from standard Special Relativity

$$v_r = \frac{pc^2}{E}, \tag{10}$$

and δv is the speed correction due to a superluminal deformation

$$\delta v = v - v_r = \frac{v_r\frac{\partial\Delta}{\partial E} + \frac{1}{2}\frac{\partial\Delta}{\partial p}}{E - \frac{\partial\Delta}{\partial E}}. \tag{11}$$

In the usual sense of smallness of the speed correction must be

$$\left|\frac{\delta v}{v}\right| \ll 1, \tag{12}$$

what means that

$$-\frac{1}{2pc^2}\frac{\partial\Delta}{\partial p} \leqslant \frac{1}{E}\frac{\partial\Delta}{\partial E} \ll 1, \quad \text{for} \quad -\frac{1}{2pc^2}\frac{\partial\Delta}{\partial p} \leqslant 1, \tag{13}$$

$$1 \ll \frac{1}{E}\frac{\partial\Delta}{\partial E} \ll -1 - \frac{1}{pc^2}\frac{\partial\Delta}{\partial p}, \quad \text{for} \quad -\frac{1}{2pc^2}\frac{\partial\Delta}{\partial p} \gg 1. \tag{14}$$

Let us consider the approximation

$$\left|\frac{1}{E}\frac{\partial\Delta}{\partial E}\right| \ll 1, \tag{15}$$

which in the light of the relation (32) means that

$$\cos\alpha \gg 2\left|1-\frac{v_r}{v}\right| - 2, \tag{16}$$

which together with the condition $-1 \leqslant \cos\alpha \leqslant 1$ means that

$$v \in \left[\frac{2}{5}v_r, \frac{2}{3}v_r\right] \cup [2v_r, \infty]. \tag{17}$$

We shall call (15) *Ultrahigh Speed Approximation (USA)*. It can be seen straightforwardly that USA satisfies the smallness condition (12), i.e. is a particular case within the conditions

$$\left|\frac{1}{2pc^2}\frac{\partial\Delta}{\partial p}\right| \leqslant \left|\frac{1}{E}\frac{\partial\Delta}{\partial E}\right| \ll 1, \quad \text{for} \quad \left|\frac{1}{2pc^2}\frac{\partial\Delta}{\partial p}\right| \leqslant 1, \tag{18}$$

$$1 \ll \left|\frac{1}{E}\frac{\partial\Delta}{\partial E}\right| \ll \left|1+\frac{1}{pc^2}\frac{\partial\Delta}{\partial p}\right|, \quad \text{for} \quad \left|\frac{1}{2pc^2}\frac{\partial\Delta}{\partial p}\right| \gg 1. \tag{19}$$

In the region of speeds (17), which is defined by USA (15), the speed correction formula (11) becomes

$$\delta v = \frac{v_r\dfrac{1}{E}\dfrac{\partial\Delta}{\partial E} + \dfrac{1}{2E}\dfrac{\partial\Delta}{\partial p}}{1-\dfrac{1}{E}\dfrac{\partial\Delta}{\partial E}}. \tag{20}$$

Let us look on this formula for really small values of $\frac{1}{E}\frac{\partial\Delta}{\partial E}$. Then one can take into account the Taylor power series expansion of (20)

$$\delta v = \frac{1}{2E}\frac{\partial\Delta}{\partial p} + v_r\left(1+\frac{1}{2pc^2}\frac{\partial\Delta}{\partial p}\right)\sum_{n=1}^{\infty}\frac{1}{E^n}\left(\frac{\partial\Delta}{\partial E}\right)^n, \tag{21}$$

in which the leading terms are

$$\delta v \approx \frac{1}{2E}\frac{\partial\Delta}{\partial p} + \frac{pc^2}{E^2}\frac{\partial\Delta}{\partial E} + \frac{1}{2E^2}\frac{\partial\Delta}{\partial E}\frac{\partial\Delta}{\partial p} + \frac{pc^2}{E^3}\left(\frac{\partial\Delta}{\partial E}\right)^2 + \frac{1}{2E^3}\left(\frac{\partial\Delta}{\partial E}\right)^2\frac{\partial\Delta}{\partial p} + \dots \tag{22}$$

If, moreover, additionally the condition $\dfrac{\partial\Delta}{\partial E} \neq 0$ holds then the speed correction can be presented in the form

$$\delta v = \frac{v_r + \dfrac{1}{2}\dfrac{\partial E}{\partial p}}{\left(\dfrac{1}{E}\dfrac{\partial\Delta}{\partial E}\right)^{-1} - 1}, \tag{23}$$

and in the appropriate Taylor power series expansion

$$\delta v = \left(v_r + \frac{1}{2}\frac{\partial E}{\partial p}\right)\sum_{n=1}^{\infty}\frac{1}{E^n}\left(\frac{\partial\Delta}{\partial E}\right)^n, \tag{24}$$

the leading terms are

$$\delta v \approx \frac{pc^2}{E^2}\frac{\partial\Delta}{\partial E} + \frac{1}{2E}\frac{\partial E}{\partial p}\frac{\partial\Delta}{\partial E} + \frac{pc^2}{E^3}\left(\frac{\partial\Delta}{\partial E}\right)^2 + \frac{1}{2E^2}\frac{\partial E}{\partial p}\left(\frac{\partial\Delta}{\partial E}\right)^2 + \ldots \tag{25}$$

In the region of speeds opposite to USA

$$\left|\frac{1}{E}\frac{\partial\Delta}{\partial E}\right| \gg 1, \tag{26}$$

the expansions (22) as well as (24) are not still valid. Then the assumption $\dfrac{\partial\Delta}{\partial E} \neq 0$ is obligatory because of otherwise the region of speed opposite to USA does not exist. Dividing both sides of the fraction in the speed formula (11) by $\dfrac{\partial\Delta}{\partial E}$ one obtains the speed correction formula

$$\delta v = -\frac{v_r + \dfrac{1}{2}\dfrac{\partial E}{\partial p}}{1 - \left(\dfrac{1}{E}\dfrac{\partial\Delta}{\partial E}\right)^{-1}}, \tag{27}$$

which shows that in such a case the speed correction contributes purely negative constituents, so that the speed is always lower than v_r. In such a situation the appropriate Taylor power series expansion

$$\delta v = -\left(v_r + \frac{1}{2}\frac{\partial E}{\partial p}\right)\sum_{n=1}^{\infty}E^n\left(\frac{\partial\Delta}{\partial E}\right)^{-n}, \tag{28}$$

has the leading terms as follows

$$\delta v \approx v_r + \frac{1}{2}\frac{\partial E}{\partial p} + v_r E\left(\frac{\partial \Delta}{\partial E}\right)^{-1} + v_r E^2\left(\frac{\partial \Delta}{\partial E}\right)^{-2} + \frac{E}{2}\frac{\partial E}{\partial p}\left(\frac{\partial \Delta}{\partial E}\right)^{-1} + \ldots \quad (29)$$

In this manner, any deformation Δ of Special Relativity can be used for generation of the complement for superluminally moving objects. In other words, within the presented approach for nontrivial Δ's the speed of a moving object can be higher than the speed of light c.

4 The Non-Parallelism Hypothesis

Let us consider the elementary formula

$$\frac{\partial E}{\partial p} = \frac{\partial E}{\partial p_i}\frac{\partial p_i}{\partial p} = v^i \frac{p_i}{p} = v\cos\alpha, \quad (30)$$

where $\alpha = (p^i, v^i)$ is the angle between momentum and velocity vectors of an moving object. In the light of the relation (6) this angle is identically 0, i.e. $\cos\alpha = 0$. However, such a state of affairs must not be sufficient for description of a motion with superluminal speeds. By this reason we propose

Proposition (The Non-Parallelism Hypothesis)**.** *In a motion with superluminal speed parallelism of the momentum and the velocity vectors of a faster-than-light traveling object is lost.*

Applying ad hoc the non-parallelism hypothesis within the formula (27) one receives the speed correction in the form

$$\delta v = \frac{v_r + \frac{1}{2} v\cos\alpha}{\left(\frac{1}{E}\frac{\partial \Delta}{\partial E}\right)^{-1} - 1}. \quad (31)$$

In such a situation, using of the definition (9) leads to another form of the speed formula

$$v = \frac{v_r}{1 - \left(\frac{1}{E}\frac{\partial \Delta}{\partial E}\right)\left(1 + \frac{1}{2}\cos\alpha\right)}, \quad (32)$$

which can be expanded around the non-deformed case, i.e. $\cos\alpha = 1$ and $\frac{1}{E}\frac{\partial\Delta}{\partial E} = 0$, as follows

$$\frac{v}{v_r} = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{k=0}^{m}\frac{(-1)^k 2^{-n}3^{n-m}\Gamma(m-n)\Gamma(n+1)(\cos\alpha)^k\left(\frac{1}{E}\frac{\partial\Delta}{\partial E}\right)^n}{\Gamma(k+1)\Gamma(m+1)\Gamma(m-k+1)\Gamma(-n)\Gamma(n-m+1)}, \tag{33}$$

or around $\alpha = 0$

$$\frac{v}{v_r} = \frac{1}{1-\frac{3}{2E}\frac{\partial\Delta}{\partial E}} - \frac{\frac{1}{E}\frac{\partial\Delta}{\partial E}}{4\left(1-\frac{3}{2E}\frac{\partial\Delta}{\partial E}\right)^2}\alpha^2 + \frac{\frac{1}{E}\frac{\partial\Delta}{\partial E}\left(1+\frac{3}{2E}\frac{\partial\Delta}{\partial E}\right)}{48\left(1-\frac{3}{2E}\frac{\partial\Delta}{\partial E}\right)^3}\alpha^4 + O[\alpha^6]. \tag{34}$$

The essential effect is that in absence of the deformation, i.e. in the trivial case $\Delta \equiv 0$ which corresponds to Special Relativity, the corrections due to the Non-Parallelism Hypothesis vanish identically. In this manner, the momentum-velocity non-parallelism has a sense if and only if Special Relativity is superluminally deformed.

Calculating $\cos\alpha$ from the relation (32)

$$\cos\alpha = 2\left(\frac{1}{E}\frac{\partial\Delta}{\partial E}\right)^{-1}\left(1-\frac{v_r}{v}\right) - 2, \tag{35}$$

and taking into account the fact that $-1 \leqslant \cos\alpha \leqslant 1$ one obtains the upper and the lower bounds for the speed v

$$\frac{v_r}{1-\frac{1}{2E}\frac{\partial\Delta}{\partial E}} \leqslant v \leqslant \frac{v_r}{1-\frac{3}{2E}\frac{\partial\Delta}{\partial E}}. \tag{36}$$

5 Ab Initio Deformation

Usually the deformation Δ os the energy-momentum relation follows from an algebraic deformation or non-commutative geometry of phase space. However, also usually in such scenarios deformations are high-order polynomials in momentum value and energy violating the Poincaré symmetry of the Minkowski energy-momentum space, which is the crucial and fundamental feature of Special Relativity. In this section we shall to deduce the deformation which, as it will be shown in the next parts of this paper, allows to preserve the Poincaré symmetry.

Let us consider a superluminal deformations which are functions of time t hidden within the dynamical parameters E and p, i.e.

$$\Delta = \Delta(E(t), p(t)) = \Delta(t). \tag{37}$$

In such a situation the partial derivatives with respect to p and E can be transformed as follows

$$\frac{\partial\Delta}{\partial p} = \frac{\partial\Delta}{\partial t}\frac{\partial t}{\partial p} = \frac{\dot{\Delta}}{\dot{p}} = \frac{\dot{\Delta}}{F}, \tag{38}$$

$$\frac{\partial\Delta}{\partial E} = \frac{\partial\Delta}{\partial t}\frac{\partial t}{\partial E} = \frac{\dot{\Delta}}{\dot{E}} = \frac{\dot{\Delta}}{P}, \tag{39}$$

where $F = \dot{p}$ is a value of force acting on a particle, and P is a power. It is easy to see that application of the relations (38) and (39) within the formula (7) lead to the following speed formula

$$v = \frac{P}{F}\frac{pc^2F + \frac{1}{2}\dot{\Delta}}{EP - \dot{\Delta}}, \tag{40}$$

which allows to establish the differential equation for the deformation

$$\dot{\Delta} = 2FP\frac{Ev - pc^2}{P + 2Fv}. \tag{41}$$

This equation is not hard to straightforward integration

$$\begin{aligned}
\Delta &= 2\int_{t_0}^{t} dt' F(t')P(t')\frac{E(t')v(t') - p(t')c^2}{P(t') + 2F(t')v(t')} \qquad (42)\\
&= 2\int_{t_0}^{t} dt' \frac{dp}{dt'}\frac{dE}{dt'}\frac{E(t')v(t') - p(t')c^2}{\frac{dE}{dt'} + 2\frac{dp}{dt'}v(t')}\\
&= 2\int_{t_0}^{t} dt' \frac{dp}{dt'}\frac{dE}{dt'}\frac{E(t')v(t')}{\frac{dE}{dt'} + 2\frac{dp}{dt'}v(t')} - 2c^2\int_{t_0}^{t} dt' \frac{dp}{dt'}\frac{dE}{dt'}\frac{p(t')}{\frac{dE}{dt'} + 2\frac{dp}{dt'}v(t')}\\
&= 2\int_{E(t_0)}^{E(t)} dE(t')\frac{dp}{dt'}\frac{E(t')v(t')}{\frac{dE}{dt'} + 2\frac{dp}{dt'}v(t')} - 2c^2\int_{p(t_0)}^{p(t)} dp(t')\frac{dE}{dt'}\frac{p(t')}{\frac{dE}{dt'} + 2\frac{dp}{dt'}v(t')}\\
&= 2\int_{E(t_0)}^{E(t)} dE(t')\frac{E(t')v(t')}{\frac{dE(t')}{dp(t')} + 2v(t')} - 2c^2\int_{p(t_0)}^{p(t)} dp(t')\frac{p(t')}{1 + 2\left(\frac{dE(t')}{dp(t')}\right)^{-1}v(t')},
\end{aligned} \tag{43}$$

and with using of the non-parallelism hypothesis

$$\frac{dE(t')}{dp(t')}=\frac{dE(t')}{dp_i(t')}\frac{dp_i(t')}{dp(t')}=v^i(t')\frac{p_i(t')}{p(t')}=v(t')\cos\alpha(t'), \tag{44}$$

its solution takes the following form

$$\Delta=2\int_{E(t_0)}^{E(t)}dE(t')\frac{E(t')}{2+\cos\alpha(t')}-2c^2\int_{p(t_0)}^{p(t)}dp(t')\frac{p(t')\cos\alpha(t')}{2+\cos\alpha(t')}. \tag{45}$$

Let us evaluate the deformation for the case when the angle α and the energy E, as well as the angle α and the momentum value p are independent variables. In fact, such a state of affairs is generally true, because of angle α is the only free parameter. In such a situation one obtains the formula

$$\Delta=\frac{2}{2+\cos\alpha(t')}\int_{E(t_0)}^{E(t)}dE(t')E(t')-\frac{2c^2\cos\alpha(t')}{2+\cos\alpha(t')}\int_{p(t_0)}^{p(t)}dp(t')p(t'), \tag{46}$$

in which the integrals can be derived straightforwardly, and consequently the superluminal deformation derived ab initio has is a second-order polynomial in the momentum value and the energy

$$\Delta=\frac{E^2-E_0^2}{2+\cos\alpha}-\frac{(p^2-p_0^2)c^2\cos\alpha}{2+\cos\alpha}, \tag{47}$$

where $E_0=E(t_0)$ and $p_0=p(t_0)$ are the initial data.

6 Ab Initio Energy-Momentum Relation

Applying the ab initio deformation (69) within the energy-momentum relation (1), one receives the ab initio energy-momentum relation

$$E^2=p^2c^2+m^2c^4+\frac{1}{2+\cos\alpha}E^2-\frac{\cos\alpha}{2+\cos\alpha}p^2c^2-\Delta_0, \tag{48}$$

where Δ_0 is the constant deformation

$$\Delta_0=\frac{1}{2+\cos\alpha}E_0^2-\frac{\cos\alpha}{2+\cos\alpha}p_0^2c^2, \tag{49}$$

which after minor algebraic manipulations becomes

$$E^2=\frac{2p^2c^2}{1+\cos\alpha}+m_{eff}^2c^4, \tag{50}$$

where m_{eff} is the effective mass arising from the ab initio superluminal deformation

$$m_{eff}=\sqrt{m^2+\frac{1}{c^4(1+\cos\alpha)}\left(m^2c^4+\cos\alpha\, p_0^2c^2-E_0^2\right)}. \tag{51}$$

In this manner, the necessary and the sufficient conditions for existence of the new mass in the deformed theory are

$$\cos\alpha\neq 1, \tag{52}$$

$$\cos\alpha\neq\frac{E_0^2-m^2c^4}{p_0^2c^2}. \tag{53}$$

The first condition is the only the non-parallelism hypothesis, while the second one is exclusion of the specific value of the angle.

The effective mass (51) must not be a real number. It is a real number if and only if $m_{eff}^2\geqslant 0$, i.e. the expression under the square root is positive. It leads to the condition

$$E_0^2\leqslant(2+\cos\alpha)m^2c^4+p_0^2c^2\cos\alpha, \tag{54}$$

which can be understood as the lower bound for the mass parameter

$$m\geqslant\frac{E_0^2-p_0^2c^2\cos\alpha}{(2+\cos\alpha)c^4}, \tag{55}$$

or equivalently as the lower bound for the angle α

$$\cos\alpha\geqslant\frac{E_0^2-2m^2c^4}{p_0^2c^2+m^2c^4}. \tag{56}$$

Let us focus on the case when the effective mass is identically equal to zero, i.e. $m_{eff}=0$. In such a situation if α is known from experiment then the mass parameter m is established as

$$m=\frac{E_0^2-p_0^2c^2\cos\alpha}{(2+\cos\alpha)c^4}, \tag{57}$$

or if m is known from experimental data then

$$\cos\alpha=\frac{E_0^2-2m^2c^4}{p_0^2c^2+m^2c^4}. \tag{58}$$

If, moreover, the initial data are constrained by the Special Relativity energy-momentum relation

$$E_0^2 = p_0^2c^2 + m^2c^4, \tag{59}$$

then one receives

$$\cos\alpha = 1 - \frac{2m^2c^2}{E_0^2} = \frac{2p_0^2c^2}{E_0^2} - 1. \tag{60}$$

Because of the bounds $-1 \leqslant \cos\alpha \leqslant 1$ one obtains the bounds

$$E_0^2 \geqslant m^2c^4 \geqslant 0 \ \text{ and } \ E_0^2 \geqslant p_0^2c^2 \geqslant 0, \tag{61}$$

what is consistent with (59).

Using of the ab initio deformed energy-momentum relation (50) for calculation of the velocity of a moving object gives the result

$$v^i = \frac{\partial E}{\partial p_i} = \frac{2pc^2}{E(1+\cos\alpha)}\frac{p^i}{p}, \tag{62}$$

and consequently the speed of such an object is

$$v = \frac{2pc^2}{E(1+\cos\alpha)}. \tag{63}$$

It is visible that for broken non-parallelism hypothesis, i.e. $\cos\alpha = 1$, the speed of a moving object (63) becomes

$$v = \frac{pc^2}{E}, \tag{64}$$

where energy is usual relativistic energy

$$E^2 = p^2c^2 + m_{eff}^2c^4, \tag{65}$$

with the appropriate effective mass

$$m_{eff} = \sqrt{m^2 + \frac{1}{c^4}\left(m^2c^4 + p_0^2c^2 - E_0^2\right)}. \tag{66}$$

which together with the Special Relativity condition for initial data

$$E_0^2 = p_0^2c^2 + m^2c^4, \tag{67}$$

becomes the non-deformed mass of Special Relativity

$$m_{eff} = m. \tag{68}$$

In this manner, the case of $\cos\alpha = 1$ under specific condition (67) for the initial data can be resulting in Special Relativity despite the fact that than the ab initio superluminal deformation (69) is still present

$$\Delta = \frac{E^2 - p^2c^2}{3} - \frac{E_0^2 - p_0^2c^2}{3}. \tag{69}$$

One sees that the most important result of the non-parallelism hypothesis is the deformed speed (63). It is easy to check that this formula can be presented in the form

$$v = \frac{2c}{\sqrt{1+\cos\alpha + \dfrac{1}{2}\left(\dfrac{m_{eff}c}{p}\right)^2(1+\cos\alpha)^2}}. \tag{70}$$

If the effective mass vanishes $m_{eff} = 0$ then

$$v = \frac{2c}{\sqrt{1+\cos\alpha}}, \tag{71}$$

and in this manner in such a situation the speed of a moving has a superluminal value if and only if

$$\frac{2}{\sqrt{1+\cos\alpha}} \geqslant 1 \longrightarrow \cos\alpha \leqslant 3, \tag{72}$$

what is always true because of the natural bounds of cosinus function. The case of non-vanishing effective mass if a little bit difficult. In such a situation, the speed of a moving object is superluminal if and only if

$$\left(\frac{m_{eff}c}{p}\right)^2 \cos^4\frac{\alpha}{2} + \cos^2\frac{\alpha}{2} - 2 \leqslant 0, \tag{73}$$

where of course $\cos^2\dfrac{\alpha}{2} = \dfrac{1+\cos\alpha}{2} \in [0,1]$. This inequality can be easy solved straightforwardly

$$\cos^2\frac{\alpha}{2} \geqslant \frac{1}{2}\left(\frac{p}{m_{eff}c}\right)^2\left(\sqrt{1+8\left(\frac{m_{eff}c}{p}\right)^2} - 1\right), \tag{74}$$

and gives the lower bound

$$\cos\alpha \geqslant \left(\frac{p}{m_{eff}c}\right)^2 \left(\sqrt{1+8\left(\frac{m_{eff}c}{p}\right)^2}-1\right)-1 \tag{75}$$

and because of the bounds $\cos^2\frac{\alpha}{2} \in [0,1]$

$$m_{eff} \geqslant \frac{p}{c}. \tag{76}$$

In the light of the energy-momentum relation (50) the result (76) leads to the lower energetic bound

$$E \geqslant \sqrt{\frac{3+\cos\alpha}{1+\cos\alpha}}pc, \tag{77}$$

or equivalently the lower bound for the cosinus

$$\cos\alpha \geqslant \frac{\left(\frac{E}{pc}\right)^2-3}{1-\left(\frac{E}{pc}\right)^2}. \tag{78}$$

Because of the bound $\cos\alpha \leqslant 1$ one receives the upper energetic bound

$$E \leqslant \sqrt{2}pc. \tag{79}$$

In this manner the deformation jointed with the non-parallelism hypothesis formally enables the values higher than the speed of light c.

7 Energy-Momentum Interval

Let us focus our attention on the energy-momentum interval

$$s^2 = \eta_{\mu\nu}p^\mu p^\nu = -\frac{E^2}{c^2}+p^2, \tag{80}$$

where $p^\mu = \left[\frac{E}{c}, p^i\right]$ is a momentum four-vector and $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$ is the metric of the Minkowski space. Direct calculation gives

$$s^2 = \frac{\cos\alpha-1}{\cos\alpha+1}p^2 - m_{eff}^2c^2. \tag{81}$$

Let us perform the sign analysis of the energy-momentum interval (81).

1. The case $m_{eff}^2 > 0$ for which

$$\cos\alpha > \frac{E_0^2 - 2m^2c^4}{p_0^2c^2 + m^2c^4}. \tag{82}$$

   Then the energy-momentum interval (81) is

   (a) light-like if and only if $s^2 = 0$, i.e.

$$p = \sqrt{\frac{\cos\alpha + 1}{\cos\alpha - 1}} m_{eff} c, \tag{83}$$

   and $E = pc$.

   (b) energy-like if and only if $s^2 < 0$, i.e.

$$p < \sqrt{\frac{\cos\alpha + 1}{\cos\alpha - 1}} m_{eff} c, \tag{84}$$

   and $E > pc$.

   (c) momentum-like if and only if $s^2 > 0$, i.e.

$$p > \sqrt{\frac{\cos\alpha + 1}{\cos\alpha - 1}} m_{eff} c, \tag{85}$$

   and $E < pc$.

   For consistency of all these cases, however, must be

$$\cos\alpha \leqslant -1 \quad or \quad \cos\alpha > 1, \tag{86}$$

   what is manifestly wrong condition.

2. The case $m_{eff}^2 < 0$ for which

$$\cos\alpha < \frac{E_0^2 - 2m^2c^4}{p_0^2c^2 + m^2c^4}. \tag{87}$$

   Then the energy-momentum interval (81) is

   (a) light-like if and only if $s^2 = 0$, i.e.

$$p = \sqrt{\frac{1 + \cos\alpha}{1 - \cos\alpha}} |m_{eff}| c, \tag{88}$$

   and $E = pc$.

(b) energy-like if and only if $s^2 < 0$, i.e.

$$p < \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}|m_{eff}|c, \tag{89}$$

and $E > pc$.

(c) momentum-like if and only if $s^2 > 0$, i.e.

$$p > \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}|m_{eff}|c, \tag{90}$$

and $E < pc$.

All these cases are consistent if and only if

$$-1 \leqslant \cos\alpha < 1, \tag{91}$$

what is equivalent to

$$\frac{1}{3}\left(\frac{E_0^2}{c^4} - \frac{p_0^2}{c^2}\right) < m^2 \leqslant \frac{E_0^2}{c^4} + \frac{p_0^2}{c^2}. \tag{92}$$

Otherwise the momentum value are purely imaginary, i.e. non-physical.

3. The case $m_{eff}^2 = 0$ for which

$$\cos\alpha = \frac{E_0^2 - 2m^2c^4}{p_0^2c^2 + m^2c^4}. \tag{93}$$

We assume that always $p^2 \geqslant 0$. Then the energy-momentum interval (81) is

(a) light-like if and only if $s^2 = 0$, i.e.

$$\cos\alpha = 1, \tag{94}$$

and $E = pc$. In this case

$$m^2 = \frac{1}{3}\left(\frac{E_0^2}{c^4} - \frac{p_0^2}{c^2}\right). \tag{95}$$

However, for consistency with the non-parallelism hypothesis must be $p = 0$ and $E = 0$.

(b) energy-like if and only if $s^2 < 0$, i.e.

$$\cos\alpha < -1 \quad or \quad \cos\alpha \geqslant -1, \tag{96}$$

and $E > pc$. This case is manifestly nonphysical.

(c) momentum-like if and only if $s^2 > 0$, i.e.

$$-1 < \cos\alpha \leqslant 1, \tag{97}$$

and $E < pc$. In this case

$$\frac{1}{3}\left(\frac{E_0^2}{c^4} - \frac{p_0^2}{c^2}\right) \leqslant m^2 < \frac{E_0^2}{c^4} + \frac{p_0^2}{c^2}. \tag{98}$$

8 Violated Poincaré Invariance

The problem is that the ab initio deformed energy-momentum relation (50) leads to breakdown of the Poincaré symmetry, which is the fundamental feature of Special Relativity. Such a state of affairs can be seen by the straightforward calculation. The Poincaré invariance

$$s'^2 = s^2, \tag{99}$$

demands preservation of the energy-momentum interval (81) under action of the Poincaré transformation in the energy-momentum space

$$p'^\mu = \Lambda^\mu_\nu p^\nu + P^\mu, \tag{100}$$

where $p'^\mu = \left[\frac{E'}{c}, p'^i\right]$, P^μ is a constant momentum four-vector, and Λ^μ_ν is a constant matrix satisfying the condition

$$\eta_{\mu\nu}\Lambda^\mu_\kappa\Lambda^\nu_\lambda = \eta_{\kappa\lambda}, \tag{101}$$

which after contraction of both sides with $\eta^{\mu\nu}\eta^{\kappa\lambda}$ leads to

$$\Lambda^{\alpha\mu}\Lambda^\nu_\lambda = \eta^\alpha_\lambda\eta^{\mu\nu}, \tag{102}$$

and the particular relation is

$$\Lambda^{\alpha\mu}\Lambda^\nu_\alpha = \eta^{\mu\nu}. \tag{103}$$

Contracting of the equation (102) with $\eta_{\alpha\beta}\eta_{\mu\kappa}$ leads to

$$\Lambda_{\kappa\beta}\Lambda^\nu_\lambda = \eta_{\beta\lambda}\eta^\nu_\kappa, \tag{104}$$

and particular case $\nu = \kappa$ gives

$$\Lambda_{\alpha\beta}\Lambda^{\alpha}_{\lambda} = \eta_{\beta\lambda}. \tag{105}$$

Application of the Poincaré transformation (100) to the interval (81) gives the result

$$\begin{aligned} s'^2 &= \frac{\cos\alpha - 1}{\cos\alpha + 1}\left(\eta_{ik}\Lambda^i_j\Lambda^k_i p^j p^i + P^i P_i + P^i\eta_{ik}\Lambda^k_i p^i + P_i\Lambda^i_j p^j\right) + m^2_{eff}c^2 & (106)\\ &= \frac{\cos\alpha - 1}{\cos\alpha + 1}\left(\eta_{ji}p^j p^i + P^i P_i + 2P_i\Lambda^i_j p^j\right) + m^2_{eff}c^2 & (107)\\ &= s^2 + \frac{\cos\alpha - 1}{\cos\alpha + 1}P_i\left(P^i + 2\Lambda^i_j p^j\right), & (108) \end{aligned}$$

which breaks the Poincaré invariance (99) manifestly. The situations preserving the Poincaré symmetry are defined by the trivial momentum vector $P_i = 0$ or by the following vector

$$P^i = -2\Lambda^i_j p^j. \tag{109}$$

Contracting both sides of the relation (109) with $\delta_{ki}\Lambda^{jk}$ and taking into account the equation (103) for all spatial indexes one receives the result

$$p^j = -\frac{1}{2}\Lambda^j_i P^i, \tag{110}$$

which allows to establish the form of spatial part of the Lorentz matrix

$$\Lambda^j_i = -2\frac{p^j P_i}{P^2}. \tag{111}$$

The constant vector P^μ can be identified with the initial data vector

$$P^\mu = \left[\frac{E_0}{c}, p^i_0\right]. \tag{112}$$

In such a situation $P^i P_i = p_0^2$, and by this reason the matrix Λ^j_i can be presented in following the form

$$\Lambda^j_i = -2\frac{p^j p_{0i}}{p_0^2}. \tag{113}$$

The covariant momentum vector can be established by contraction of the contravariant momentum vector (110) with the unit matrix

$$p_j = \delta_{jk}p^k = -\frac{1}{2}\Lambda_{ji}P^i, \tag{114}$$

and in this manner one can calculate the square of momentum vector

$$p^2 \equiv p^j p_j = \frac{1}{4}\Lambda_{jk}\Lambda_i^j P^i P^k = \frac{1}{4}\delta_{ik}P^i P^k = \frac{1}{4}p_0^2, \tag{115}$$

where we have applied the relation (105), what gives

$$p = \frac{p_0}{2}. \tag{116}$$

In this manner, with using of the result (116) the matrix (113) becomes

$$\Lambda_i^j = -\frac{p^j}{p}\frac{p_{0i}}{p_0}, \tag{117}$$

or after using the constraint (116)

$$\Lambda_i^j = -\frac{p^j p_{0i}}{2p^2}. \tag{118}$$

In this manner the matrix Λ_i^j possesses three representations (113), (117) and (118) which are nonequivalent from the point of view of dependence on the momentum vector p^j.

Finally one can apply the matrix (118) to the fundamental relation (101) for the Lorentz transformation. The result is nontrivial

$$p_{0i}p_{0j} = p_0^2\delta_{ij} = 4p^2\delta_{ij} = 2pp_0\delta_{ij}, \tag{119}$$

and allows to find three nonequivalent forms of the unit matrix

$$\delta_{ij} = \frac{p_{0i}}{p_0}\frac{p_{0j}}{p_0} = \frac{1}{2}\frac{p_{0i}}{p}\frac{p_{0j}}{p_0} = \frac{1}{4}\frac{p_{0i}}{p}\frac{p_{0j}}{p}. \tag{120}$$

Because, however, the unit matrix should be a constant matrix the representation involving the only initial data p_0^j is the only justified form. Another two representations of the unit matrix can not be used in the analysis because of they were obtained by involving of the constraint (116) relating values of the momentum vector p^j and the initial data momentum vector p_0^j.

Let us consider the Taylor power series expansion around the initial data point $p^j = p_0^j$ of the matrix Λ_i^j in all the representations (113), (117), and (118). Such a power series can be presented in the form

$$\Lambda_i^j = \sum_{n=0}^{\infty} a_i^j(n)\left(1 - \frac{p^j p_{0j}}{p_0^2}\right)^n = \sum_{n=0}^{\infty} a_i^j(n)\left(1 + \frac{1}{2}\Lambda_j^j\right)^n, \tag{121}$$

where the coefficients $a_i^j(n)$ are constant matrices dependent on n and initial data momentum vector p_0^j

$$a_i^j(n) = \frac{(-1)^n (p_0^j)^n}{n!} \frac{\partial^n \Lambda_i^j}{\partial p^{j n}} (p^j = p_0^j). \tag{122}$$

For simplicity we shall apply also the notation

$$\Lambda_i^j (p^j = p_0^j) = \Lambda^{(0)}{}_i^j, \tag{123}$$

and this quantity depends on the choice of the representation.

The matrix Λ_i^j in the representation (113) is linear in p^j

$$\Lambda_i^j = -2 \frac{p_{0i}}{p_0^2} p^j. \tag{124}$$

so that its analysis is simple. The only non-vanishing are $n = 0$ and $n = 1$ coefficients of the Taylor series, and in this manner

$$\Lambda_i^j = \Lambda^{(0)}{}_i^j - 2 \frac{p_{0i}}{p_0^2} \left(p^j - p_0^j \right), \tag{125}$$

where $\Lambda^{(0)}{}_i^j = -2\delta_i^j = -2\frac{p_{0i} p_0^j}{p_0^2}$.

The representation (118) of the matrix Λ_i^j is more difficult in the analysis. This formula can be presented in more convenient form

$$\Lambda_i^j = -\frac{p^j p_{0i}}{2\delta_{jk} p^j p^k} = -\frac{p_{0i}}{2\delta_{jk}} \frac{1}{p^k}, \tag{126}$$

which shows that now one has to deal with non-linear (reciprocal) dependence on p^j. In such a situation the nth derivative of the matrix Λ_i^j can be established straightforwardly

$$\frac{\partial^n \Lambda_i^j}{\partial p^{j n}} = \Lambda_i^j \frac{(-1)^n n!}{(p^j)^n}, \tag{127}$$

so that the coefficients of the series are

$$a_i^j(n) = \Lambda^{(0)}{}_i^j, \tag{128}$$

where now $\Lambda^{(0)}{}_i^j$ has the form

$$\Lambda^{(0)}{}_i^j = -\frac{1}{2} \delta_i^j. \tag{129}$$

In this manner, the corresponding Taylor series is

$$\Lambda_i^j = \Lambda^{(0)}{}_i^j \sum_{n=0}^{\infty} \left(1 + \frac{1}{2}\Lambda_j^j\right)^n. \tag{130}$$

This series, having the form of a geometric series, can be summed up straightforwardly to the result

$$\Lambda_i^j = -2\frac{\Lambda^{(0)}{}_i^j}{\Lambda_j^j}, \tag{131}$$

which after using of (118) leads to (119), what confirms its correctness.

Similar procedure can be performed in the case of the representation (117) of the matrix Λ_i^j. Then the matrix Λ_i^j

$$\begin{aligned}
\Lambda_i^j &= -\frac{p^j}{\sqrt{p^j p_j}}\frac{p_{0i}}{p_0} = -\frac{p^j}{\sqrt{\delta_{jk}p^j p^k}}\frac{p_{0i}}{p_0} && (132)\\
&= -\frac{1}{\sqrt{\delta_{jk}}}\sqrt{\frac{p^j}{p^k}}\frac{p_{0i}}{p_0} = -\frac{1}{\sqrt{\delta_{jk}}}\frac{p_{0i}}{p_0}(p^j)^{1/2}(p^k)^{-1/2}, && (133)
\end{aligned}$$

is also nonlinear in p^j, and in such a situation one has

$$\Lambda^{(0)}{}_i^j = -\delta_i^j. \tag{134}$$

Applying the Leibniz product formula one obtains

$$\frac{\partial^n \Lambda_i^j}{\partial p^{j^n}} = -\frac{1}{\sqrt{\delta_{jk}}}\frac{p_{0i}}{p_0}\sum_{l=0}^{n}\binom{n}{l}\frac{d^{n-l}}{dp^{j^{n-l}}}(p^j)^{1/2}\frac{d^l}{dp^{j^l}}(p^k)^{-1/2}. \tag{135}$$

Derivation of the derivatives can be done immediately

$$\begin{aligned}
\frac{d^{n-l}}{dp^{j^{n-l}}}(p^j)^{1/2} &= \frac{1}{2^{n-l}}(p^j)^{1/2}\frac{(-1)^{n-l-1}}{(p^j)^n}(p^j)^l(2(n-l)-1)!!, && (136)\\
\frac{d^l}{dp^{j^l}}(p^k)^{-1/2} &= \frac{\delta_j^k}{2^l}(p^k)^{-1/2}(-1)^l(p^k)^{-l}(2l+1)!!, && (137)
\end{aligned}$$

and by this reason one receives

$$\frac{\partial^n \Lambda_i^j}{\partial p^{j^n}} = \Lambda_i^j\frac{(-1)^{n-1}}{2^n}\frac{1}{(p^j)^n}\sum_{l=0}^{n}\binom{n}{l}(2(n-l)-1)!!(2l+1)!!. \tag{138}$$

Using of the relations for the double factorials

$$(2k+1)!! = \frac{(2k+1)!}{2^k k!} = \frac{2^{k+1}}{\sqrt{\pi}}\Gamma\left(k+\frac{3}{2}\right), \tag{139}$$

$$(2k-1)!! = \frac{(2k)!}{2^k k!} = \frac{2^k}{\sqrt{\pi}}\Gamma\left(k+\frac{1}{2}\right), \tag{140}$$

and the Newton binomial symbols

$$\binom{n}{l} = \frac{n!}{l!(n-l)!} = \frac{\Gamma(n+1)}{\Gamma(l+1)\Gamma(n-l+1)}, \tag{141}$$

one receives

$$\sum_{l=0}^{n}\binom{n}{l}(2(n-l)-1)!!(2l+1)!! = \frac{2^{n+1}n!}{\pi}\sum_{l=0}^{n}\frac{\Gamma\left(l+\frac{3}{2}\right)}{\Gamma(l+1)}\frac{\Gamma\left(n-l+\frac{1}{2}\right)}{\Gamma(n-l+1)}, \tag{142}$$

or after summation

$$\sum_{l=0}^{n}\binom{n}{l}(2(n-l)-1)!!(2l+1)!! = \frac{2^{n+1}n!}{\pi}\frac{\pi(n+1)}{2} = 2^n n!(n+1). \tag{143}$$

In this manner one obtains finally

$$\frac{\partial^n \Lambda_i^j}{\partial p^{jn}} = \Lambda_i^j \frac{1}{(p^j)^n}(-1)^{n-1}n!(n+1), \tag{144}$$

and by this reason the series coefficients are

$$a_i^j(n) = -\Lambda^{(0)}{}_i^j(n+1). \tag{145}$$

The Taylor power series for this case takes the form

$$\Lambda_i^j = -\Lambda^{(0)}{}_i^j \sum_{n=0}^{\infty}(n+1)\left(1+\frac{1}{2}\Lambda_j^j\right)^n, \tag{146}$$

and can be summed up immediately

$$\Lambda_i^j = -\Lambda^{(0)}{}_i^j \frac{1}{\left(1+\frac{1}{2}\Lambda_j^j - 1\right)^2} = -\frac{4\Lambda^{(0)}{}_i^j}{\left(\Lambda_j^j\right)^2}. \tag{147}$$

It is easy to check by direct using of the representation (117) that the formula (147) leads to the result

$$p^j = -\frac{1}{2}p_0^j, \tag{148}$$

which is relevant in the light of the constraint (116).

9 Restoration of Poincaré Invariance

In the light of the fact that P^i is a constant vector and Λ^i_j is a constant matrix, the solution (110) means that the Pincare invariance in the ab initio deformed Special Relativity is preserved if and only if $p^j = -\frac{1}{2}p_0^j$ is a constant vector. Such a state of affairs is in general against the dynamical nature of motion because means that an object is at rest. on the other hand the alternative trivial solution $P_i = 0$, which is equivalent to $p_{0i} = 0$, only in the case $P_0 = 0$, which is equivalent to $E_0 = 0$, allows to restore the Lorentz (CP) symmetry while then the Poincaré symmetry is not the symmetry of the theory. By this reason in general the ab initio energy-momentum relation violates both the Poincaré symmetry and the Lorentz symmetry. The only standard case $\cos\alpha = 1$, which does not assure superluminal values of speeds and is against the assumptions of the proposed theory, automatically leads to preserved both the Poincaré invariance and the Lorentz invariance.

However, both the Poincaré invariance and the Lorentz invariance can be restored straightforwardly by application of the appropriate reinterpretation of the speed of light. Interestingly, it is easy to see that the ab initio energy-momentum relation (50) can be transformed into the Einsteinian form

$$E^2 = p^2 c_N^2 + (m_{eff}^N)^2 c_N^4, \tag{149}$$

which naturally preserves both the Poincaré symmetry and the Lorentz symmetry with exchanged speeds of light $c \to c_N$. Such a situation has the place if one introduces ad hoc the new (N) speed of light as the effect of a motion with superluminal speeds

$$c \to c_N = \sqrt{\frac{2}{1+\cos\alpha}}c = \frac{c}{\cos\frac{\alpha}{2}}, \tag{150}$$

which is superluminal, i.e. higher than c, if

$$\cos\frac{\alpha}{2} \in (0,1] \longrightarrow \alpha \in [0,\pi). \tag{151}$$

For this one must express the effective mass m_{eff} in terms of c_I instead of c and obtain the new effective mass m_{eff}^N in the form

$$m_{eff}^N = \sqrt{m^2 + \frac{1}{1+\cos\alpha}\left(m^2 + \frac{2p_0^2\cos\alpha}{(1+\cos\alpha)c_N^2} - \frac{4E_0^2}{(1+\cos\alpha)^2 c_N^4}\right)}. \tag{152}$$

The new energy-momentum interval

$$s_N^2 = -\frac{E^2}{c_N^2} + p^2 = -(m_{eff}^N)^2 c_N^4, \tag{153}$$

is independent on energy and momentum values. In this manner if one considers the energy-momentum space of the four-vectors

$$p^\mu = \left[\frac{E}{c_N}, p^i\right], \tag{154}$$

then action of both the Poincaré transformation as well as its special case the Lorentz transformation remains unchanged the new energy-momentum interval (153), and consequently both the Poincaré group and the Lorentz group are symmetries of the theory.

Let us analyze the new energy-momentum interval (153) in some detail with respect to its sign. The interval is

1. light-like when $s_N^2 = 0$, i.e.

$$m^2 = 2\frac{2E_0^2 - p_0^2 c_N^2 \cos\alpha(1+\cos\alpha)}{c_N^4(2+\cos\alpha)(1+\cos\alpha)^2}. \tag{155}$$

   This parameter must be a real number. For this must be satisfied the inequality

$$2\cos^4\frac{\alpha}{2} - \cos^2\frac{\alpha}{2} - \left(\frac{E_0}{p_0 c_N}\right)^2 \leqslant 0, \tag{156}$$

   which solution

$$\cos^2\frac{\alpha}{2} \in \left[\frac{1}{4}\left(1 - \sqrt{1 + 8\left(\frac{E_0}{p_0 c_N}\right)^2}\right), \frac{1}{4}\left(1 + \sqrt{1 + 8\left(\frac{E_0}{p_0 c_N}\right)^2}\right)\right], \tag{157}$$

   must satisfy the natural bounds $\cos^2\frac{\alpha}{2} \in [0,1]$, i.e.

$$\frac{1}{4}\left(1 - \sqrt{1 + 8\left(\frac{E_0}{p_0 c_N}\right)^2}\right) \leqslant 0, \tag{158}$$

$$\frac{1}{4}\left(1 + \sqrt{1 + 8\left(\frac{E_0}{p_0 c_N}\right)^2}\right) \leqslant 1, \tag{159}$$

   having the unique solution

$$0 \leqslant E_0 \leqslant p_0 c_N. \tag{160}$$

2. energy-like when $s_N^2 < 0$, i.e.

$$m^2 > 2\frac{2E_0^2 - p_0^2 c_N^2 \cos\alpha(1+\cos\alpha)}{c_N^4(2+\cos\alpha)(1+\cos\alpha)^2}. \tag{161}$$

3. momentum-like when $s_N^2 > 0$, i.e.

$$m^2 < 2\frac{2E_0^2 - p_0^2 c_N^2 \cos\alpha(1+\cos\alpha)}{c_N^4(2+\cos\alpha)(1+\cos\alpha)^2}. \tag{162}$$

In the cases 2 and 3 the bounds (157) and (160) are still valid.

10 Vanishing Effective Mass

Among many situations within the proposed complement to Special Relativity, the particular and interesting for this paper considerations case is the vanishing effective mass, i.e.

$$m_{eff} = 0. \tag{163}$$

In such a situation the energy of a moving object is

$$E = pc_N = \frac{pc}{\cos\frac{\alpha}{2}}, \tag{164}$$

so that the value of the cosinus is

$$\cos\frac{\alpha}{2} = \frac{pc}{E} \in (0,1]. \tag{165}$$

Then the energy-momentum interval s_N^2 is light-like, and the mass parameter m is

$$m = \frac{p_0^2}{E_0}\sqrt{\frac{1-\left(\frac{p_0 c}{E_0}\right)^2\left(\frac{pc}{E}\right)^2\left(2\left(\frac{pc}{E}\right)^2-1\right)}{\left(2\left(\frac{pc}{E}\right)^2+1\right)\left(\frac{pc}{E}\right)^4}}. \tag{166}$$

When $\cos\alpha = 1$ then also $\cos\frac{\alpha}{2} = 1$, and in this manner it is useful to find the Taylor power series around the point $\frac{pc}{E} = 1$. It is not difficult to see that for $p_0 c \neq E_0$ the leading terms are

$$m = \frac{p_0^2}{\sqrt{3}E_0}\left(\sqrt{1-\left(\frac{p_0 c}{E_0}\right)^2} - \frac{8+\left(\frac{p_0 c}{E_0}\right)^2}{3\sqrt{1-\left(\frac{p_0 c}{E_0}\right)^2}}\left(\frac{pc}{E}-1\right)+\right. \tag{167}$$

$$\left. + \frac{\left(7+2\left(\frac{p_0 c}{E_0}\right)^2\right)\left(7\left(\frac{p_0 c}{E_0}\right)^2-4\right)}{6\left(1-\left(\frac{p_0 c}{E_0}\right)^2\right)^{3/2}}\left(\frac{pc}{E}-1\right)^2\right)+O\left[\left(\frac{pc}{E}-1\right)^3\right],$$

while for $p_0 c = E_0$ the series has purely imaginary coefficients, so that it is better to show the behavior around the point $\frac{pc}{E} = 0$. In such a situation the series is

$$m = \frac{E_0}{c^2}\left(\left(\frac{E}{pc}\right)^2 - \frac{1}{2} - \frac{1}{8}\left(\frac{pc}{E}\right)^2 - \frac{1}{16}\left(\frac{pc}{E}\right)^2 + O\left[\left(\frac{pc}{E}\right)^5\right]\right), \tag{168}$$

and all the contributions after the first term are negative, so that

$$m < \frac{E_0}{c^2}\left(\frac{E}{pc}\right)^2, \tag{169}$$

or in terms of the momentum value

$$p < \sqrt{\frac{p_0}{mc}}\frac{E}{c}. \tag{170}$$

In the light of the energy-momentum relation (164) one obtains the lower bound for the initial datum p_0

$$p_0 > mc\cos^2\frac{\alpha}{2}, \tag{171}$$

or in terms of the new speed of light c_N

$$p_0 > mc_N\cos^3\frac{\alpha}{2}. \tag{172}$$

The case of vanishing effective mass has another nontrivial consequence. Namely, in such a situation the speed of a moving object

$$v = c_N = \frac{c}{\cos\frac{\alpha}{2}}, \tag{173}$$

in the region $\alpha \in [0,\pi)$ is always higher than the speed of light c, but equal to the new speed of light c_N.

11 Conclusion: CERN Neutrinos Explained

In September 2011 the OPERA Collaboration of CERN publicly disclosed the computations which showed that in their experiments speed of 17 GeV and 28 GeV neutrinos is higher than the speed of light [OPERA Collaboration, 2011]. This result was absent in the previous attempts, and manifestly contradicts to another ones. The OPERA result is that neutrino speed is $1.0000248(28)c$. Let us accept this result as the verified one. Its explanation is impossible within the frameworks of modern high energy physics, i.e. the Standard Model, its extensions, SUSY, etc. However, it is possible to explain the OPERA result with using of the Superluminally Deformed Special Relativity with the ab initio deformation calculated in this paper.

From the point of view of the theory proposed in this paper we know that the new speed of light is $c_N = \frac{c}{\sqrt{\cos\alpha}}$ and the speed of the moving neutrino is $v = \frac{pc_N^2}{E}$, where α is the angle between the momentum vector and the velocity vector of a moving neutrino, p is the momentum value and $E = \sqrt{p^2c_N^2 + (m_{eff}^N)^2c_N^4}$ is the energy of the neutrino. For consistency there is needed the value of the momentum of neutrino and the value of the effective mass m_{eff}^N.

Let us involve the Standard Model point of view, which says that neutrinos are massless, i.e. are the Weyl neutrinos. This is, of course, incorrect point of view because of the nonzero neutrino mass has been detected in several experiments. However, we want to give the hypothesis claiming that the superluminal neutrinos are massless, i.e. are the Weyl neutrinos, i.e. the effective mass of such neutrinos is exactly equal to zero. In this manner, in our point of view the Standard Model is the effective theory. In such a situation the momentum value is determined as $p = \frac{E}{c_N}$. Such a hypothesis leads to

$$\cos\alpha = 2\frac{c^2}{v^2} - 1 = 0.9999006(92), \tag{174}$$

and in this manner one obtains the unique value of the angle between velocity and momentum vectors of a superluminal neutrino

$$\alpha \approx 0.01409325(69)\text{rad} \approx 0.80748414(10)^\circ. \tag{175}$$

This value is a little bit more than 0° - the value of the angle in Special Relativity.

Finally, let us summarize the our approach to explain neutrinos. We have build the theory on base of

1. Deformed energy-momentum relation within Special Relativity,
2. Ab initio derivation of the deformation,
3. The non-parallelism hypothesis,
4. New speed of light restoring the Poincaré invariance,
5. Vanishing effective mass.

Such a collection consistently complements Special Relativity for the case of superluminally moving neutrinos and, possibly, with the last point removed can be useful for description of any objects moving with superluminal speeds.

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