Expression for the Mathematical Constant e

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Abstract: Mathematical constant e can be expressed in logarithmic functions. There are six expressions for e. Five of them are step functions and another one is a constant function.

We can express mathematical constant e in terms of logarithmic function. The logarithmic function and hyperbolic sine and cosine function is a step function. Where as one of the hyperbolic cosine function is a constant function. All results have no proof. It can be easily verified by a calculator. The expressions for e is given below.

1) Consider the function $y_1 = \mathrm{mod}(\frac{y}{\log y})$, $y_2 = \mathrm{mod}(\frac{y_1}{\log y_1})$

For n terms $y_n = \mathrm{mod}(\frac{y_{n-1}}{\log y_{n-1}})$ Taking this way for infinite number of times ,

Limit as $n \to \infty$ we get the following result.

For $0 \le y < 1/e$ then $y_n = 0$, for $y = 1/e$ then $y_n = 1/e$,

For $y > 1/e$ then $y_n = e$

This function converges very fast.

2)Consider the function $y_1 = \mathrm{mod}(\frac{y}{\log\sinh y})$, $y_2 = \mathrm{mod}(\frac{y_1}{\log\sinh y_1})$

For n terms $y_n = \mathrm{mod}(\frac{y_{n-1}}{\log\sinh y_{n-1}})$, Taking limit as $n \to \infty$ we get the following result.

For $0 \le y < \sinh^{-1} 1/e$ then $y_n = 0$, for $y = \sinh^{-1} 1/e$ then $\sinh y_n = 1/e$,

For $y > \sinh^{-1} 1/e$ then $\sinh y_n = e$

3)Consider the function $y_1 = \mathrm{mod}(\dfrac{y}{\log\cosh y})$, $y_2 = \mathrm{mod}(\dfrac{y_1}{\log\cosh y_1})$

For n terms $y_n = \mathrm{mod}(\dfrac{y_{n-1}}{\log\cosh y_{n-1}})$, Taking limit as $n \to \infty$ we get the following result.

For $-\infty < y < \infty$ then $\cosh y_n = e$

4)Consider the function $y_1 = \mathrm{mod}(\dfrac{y}{\log\sinh^{-1} y})$, $y_2 = \mathrm{mod}(\dfrac{y_1}{\log\sinh^{-1} y_1})$

For n terms $y_n = \mathrm{mod}(\dfrac{y_{n-1}}{\log\sinh^{-1} y_{n-1}})$ Taking linit as $n \to \infty$ we get the following result.

For $0 \le y < \sinh 1/e$ then $y_n = 0$, for $y = \sinh 1/e$ then $\sinh^{-1} y_n = 1/e$

For $y > \sinh 1/e$ then $\sinh^{-1} y_n = e$

5)Consider the function $y_1 = \mathrm{mod}(\dfrac{y}{\log\cosh^{-1} y})$, $y_2 = \mathrm{mod}(\dfrac{y_1}{\log\cosh^{-1} y_1})$

For n terms $y_n = \mathrm{mod}(\dfrac{y_{n-1}}{\log\cosh^{-1} y_{n-1}})$ Taking limit as $n \to \infty$ we get the following result

For $y < \cosh 1/e$, The inverse of the hyperbolic cosine of a number less than one does not exist.

Hence y_n does not exist.

For $y = \cosh 1/e$ then $\cosh^{-1} y_n = 1/e$, for $y > \cosh 1/e$ then $\cosh^{-1} y_n = e$

6)Consider the function $y_1 = \mathrm{mod}(\dfrac{y}{\log\log y})$, $y_2 = \mathrm{mod}(\dfrac{y_1}{\log\log y_1})$

For n terms $y_n = \mathrm{mod}(\dfrac{y_{n-1}}{\log\log y_{n-1}})$ Taking limit as $n \to \infty$ we get the following result.

For $y < e^{1/e}$ Here the log of negative number does not exist. Hence y_n does not exist.

For $y = e^{1/e}$ then $y_n = e^{1/e}$, for $y > e^{1/e}$ then $y_n = e^e$

The hyperbolic sine and cosine functions converge very slowly.

References:

1)Wikipedia