Rigorous testing of fair sampling assumption

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Abstract

Fair sampling assumption is used in photonic tests of Bell inequalities. However, rigorous testing of this assumption is still to be performed. Here it is argued that without rigorous testing bias can be introduced that would mask indications of unfair sampling. For purpose of argument local realistic model for polarization entangled photons is outlined. According to model coincidence rate and correlation visibility are complementary.

Entangled state of quantum particles is one of the most popular prediction that is used to illustrate non-classical nature of quantum mechanics. History of this prediction can be traced back to famous Einstein-Podolsky-Rosen paradox that questioned completeness of quantum mechanics [1]. After Bell formulated testable prediction on behalf of local realism [2] many experiments have tested correlations produced by entangled states. Most popular way to test Bell inequalities is based on polarization entangled photons.

However, all photon based experiments that test local realism suffer from fair sampling assumption [3]. Fair sampling means that sample of registered pairs is faithful representative of the whole ensemble emitted.[1] If this assumption is wrong correlations can result from postselection of coincidences from detected photons. It means that detection events for pairs of photons would be correlated for some configurations of analyzer settings leading to increased coincidence rate and anticorrelated for other configurations leading to decreased coincidence rate.

To avoid necessity for fair sampling assumption high efficiency photon detectors are needed. But general attitude toward Bell inequality tests with high efficiency detectors is that no surprises are expected [4]. The reason for that attitude is that quantum efficiency of photon detectors have improved over the years since first Bell inequality tests and Bell inequalities are still violated in modern versions of experiments with higher efficiency detectors (with quantum efficiency over 50%) without any sign that nature of entanglement correlations would tend toward reduction of Bell inequality violation.

However, attitude that is based on such everyday type observations instead of rigorous testing can be rather biased. Aim of this paper is to show how such a bias can appear and what guidelines need to be followed in experiments to

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[1] Here we should talk about dual channel analyzers i.e. polarization beam splitters that allow analysis of whole ensemble.

perform rigorous testing of fair sampling assumption.

First we should note that higher efficiency detectors by themselves do not guarantee reduction of postselection consequences. If there is any equipment between analyzer and detector that reduce photon ensemble unfair sampling can take place. But it is common practice to put bandwidth filters right before photon detectors. So if we want to test fair sampling assumption rigorously we would want to look at parameter that characterizes experimental setup as a whole. That parameter is coincident detection rate to single detection rate. Indeed if we reduce unpaired single detections we reduce consequences of post-selection. This coincidence to single rate is defined as follows:

$$n = \frac{N_C}{\sqrt{N_1 N_2}} \tag{1}$$

where N_C is count of coincident detections between detectors 1 and 2 but N_1 and N_2 are counts of detections in detectors 1 and 2 respectively. So if we compare two identical experimental setups except for different detector efficiencies we should check that we have higher value for parameter n with higher efficiency detectors.

Second we can't use high visibility of correlations as primary criterion for quality of experimental setup. To explain this consideration hypothetical local realistic model will be presented here.

Bell developed his inequalities based on argument that from perspective of local realism measurements of spin component should be predetermined by hidden variables attributed to individual particles. This is required to reproduce prediction of quantum mechanics about perfect correlations for matching measurements. Thus fair sampling assumption is quite naturally applicable to hypothesis of Bell. So we can consider that this hypothesis about hidden variables attributed to individual particles is falsified experimentally by current experiments that relay on fair sampling assumption.

However, modern Bell tests are aimed at testing any possible local realistic theory. Even theories where some correlations might result from emergent properties of photon ensemble.

And so I will describe here alternative hypothesis for photon polarization entanglement that can naturally incorporate unfair sampling. In building that hypothesis prediction of quantum mechanics about perfect correlations for matching measurements will be viewed as false.

As a starting point I will write straightforward local realistic prediction for parametric down-conversion (PDC) source of polarization entangled photons as described here [5]. This PDC source uses two nonlinear crystals with optical axes aligned orthogonally. With only one crystal and linearly polarized pump beam down-converted photons are produced with matching polarizations and they appear to be in product state [10]. And so using Malus law we arrive at correlations described by this equation:

$$P = \cos^2\alpha\cos^2\beta \tag{2}$$

with α and β being polarizer angles for two respective analyzers.

As we add to source second crystal with optical axis orthogonal in respect to optical axis of first crystal we can expect that equation describing correlations will become like that (for convenience normalization factor 1/2 is omitted here and in the following equations):

$$P = \cos^2\alpha\cos^2\beta + \sin^2\alpha\sin^2\beta \tag{3}$$

Here the first term describes expected correlations for pairs of photons that both are horizontally polarized and second term for vertically polarized photon pairs. Now we will compare this equation (3) with quantum mechanics description of this PDC source [5], that is:

$$P = |\,|HH\rangle + e^{i\phi}|VV\rangle|^2 \tag{4}$$

or expanded in sine and cosine form [6]:

$$P = \cos^2\alpha\cos^2\beta + \sin^2\alpha\sin^2\beta + \frac{1}{2}\sin 2\alpha\sin 2\beta\cos\phi \tag{5}$$

where ϕ is relative phase between two polarization modes. For $\phi = 0$ this equation reduces to relation of entangled state $P = \cos^2(\alpha - \beta)$. However, for $\phi = \pi/2$ this equation reduces to classical product state described by equation (3). So according to [5] we can reproduce classical product state experimentally.

Now we have to add something to local realistic model that can account for this third term (interference term) in equation (5). In theoretical treatment measurements of photon entangled state at different bases are all regarded as polarization measurements. However, experimental treatment differentiates measurements in $+45°/-45°$ basis — here they are described as interference between different polarization modes of photon pairs. So we will take this second approach as a guideline.

For that second measurement is introduced in local realistic model that measures some property of photon which is defined in respect to photon ensemble instead of some external reference. For example photon that was initially horizontally polarized after polarizer acquires property that is well defined in respect to photon subensemble that where vertically polarized before encountering polarizer.

Now for that second measurement to be regarded as unfair sampling in respect to polarization measurement we have to assume that polarizer changes this unknown property of photons. In addition it should change this property in different manner for horizontally and vertically polarized photons.

It should be noted that according to model second measurement physically does not happen at polarization analyzer. As an example where interference is recovered *after* photons have taken one of the two paths we can view experimental observation of Hong-Ou-Mandel effect with "postponed compensation".[7]

One more question that can appear about such a model is what physical mechanism can produce hypothesized effect of second measurement if photons are arriving one by one. As an answer hypothesis is proposed that photons change dynamical state of measurement equipment so that next photons arriving can be in different phases in respect to this dynamical state.

Based on presented local realistic model for polarization entangled photons we would expect that with increase in detection efficiency correlation law between entangled photons should approach eqn. (3). Or to put it differently we would expect reduced visibility for correlation measurements in $+45°/-45°$ basis.

Now we can return to considerations about primary criterion for quality of experimental setup. We should note that requirement of high visibility is exactly opposite to what we would expect to see in case of described local realistic model.

So the experimenter would be motivated to make whatever adjustments to setup that he considers reasonable and that increase visibility of correlations. And without rigorous monitoring how these changes affect coincidence to single rate introduction of unintended bias can go unnoticed.

From experimenter's side there is additional obstacle for recognition of any possible anomalies with lowered visibility as low visibility for measurements in $+45°/-45°$ basis is associated with longitudinal walk-off and distinguishability of different modes [8, 9].

In summary it was argued that coincidence to single rate should be used as indicator of increased photon pair detection efficiency. Secondly it was proposed that coincidence to single rate should be used as a primary criterion for quality of experimental setup with visibility as a secondary criterion. For purpose of argument local realistic model was outlined that predicts lowering of correlation visibility for measurements in +45/-45 basis as coincidence to single rate increases.

With those considerations on mind I propose that rigorous testing of fair sampling assumption is performed. For testing purposes null hypothesis can be made on behalf of quantum mechanics that different detection efficiency (characterized as coincidence to single rate) does not affect correlation visibility of polarization entangled photons.

While there is report about photon detectors with detection efficiency over 99% [10] Bell inequality tests attempting to close detection loophole will have no presentable results if high correlation visibility and high coincidence to single rate are mutually exclusive as it was suggested here. Situation like that can lead to publication bias.

References

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It says: “The visibilities for the polarization correlations are about 98.1% for $|H\rangle/|V\rangle$ basis and 92.6% for $|+45°\rangle/|-45°\rangle$ basis, without the help of narrow bandwidth interference filters.” When I inquired about lower visibility in +/- basis as compared to H/V basis Jin comment was: “Experimentally, we can not get so ideal condition, that means H1V2 and V1H2 are partially distinguishable. As a result, the entanglement visibility is limitted, this induce that we can not observe perfect correlation at +/- basis.

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