

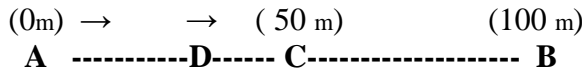
Straight line AB is not continuous unless a Common Dimensional Unit $AC > 0$ or $AC = ds \rightarrow AB$ is accepted, and since in this way,

- 1a. **Straight line AB is continuous with points as filling (Infinitively divisible).**
- 2a. **Straight line AB is discontinuous (discrete) with dimensional Units, ds, as filling** (that is made up of finite indivisible parts the Monads $ds \rightarrow AB/n$, where $n=1,2,\rightarrow\infty$)
- 3a. **Straight line AB is discontinuous (discrete) with dimensional Units ds, $ds = quantum = AB / n$ (where $n = 1,2,3 \rightarrow \infty, = [a + b.i] / n$, Infinitively divisible keeping the conservation of properties at end points A, B) as filling, and continuous with points as filling (for $n = \infty$ then $ds = 0$ i.e. a point). i.e.**

Monads $ds = 0 \rightarrow \infty$ are simultaneously (actual infinity) and (potential infinity) in Complex number form, and this defines infinity exists between all points which are not coinciding, and because **ds** comprises any two edge points with imaginary part then this property differs between the infinite points. **This is the Vector relation of Monads, ds, (or, as Complex Numbers in their general form $w = a + b.i$), and is the Dual Nature of lines (discrete and continuous).**

2. The dichotomy Paradox (Dichotomy) :

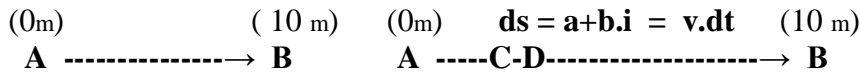
< *That which is in locomotion must arrive at the half-way stage before it arrives at the goal* >



As in 1a. Straight line AB is not continuous unless a Common Dimensional Unit $AC > 0$ or $ds = 0 \rightarrow AB/2 \rightarrow AB$ is accepted, Since point C is on line AB then $CA + CB = AB$ and since $CA = CB$ then $CD < CB$ Therefore point D,(AD) will pass through C,(AC) before it arrives at the goal B,(AB).

3. The Arrow Paradox :

< *If everything when it occupies an equal Space is at rest, and if that which is in locomotion is always occupying such a Space at any moment, the flying Arrow is therefore motionless* >



The Arrow Paradox is not a simple mathematical problem because is referred to ***motion in Absolute Euclidean Space*** i.e.in a Space where ***issues Geometry, Parallel Postulate the Squaring of circle etc, and Physics*** where Space [PNS] is not moving and because of its Duality (discrete and continuous as Complex numbers are), ***Time is not existing.*** This Paradox ***is not*** in metaphysical problem since [15] is proved that, Complex numbers and Quantum Mechanics Spring out of ***the Quantized Euclidean Geometry.***

As in(**3a**) Straight line AB is discontinuous (*discrete*) with dimensional Units , **ds=CD** as filling and *continuous with points* as filling (The Complex Numbers in the general form $w = a + b \cdot i$) , **which is the Dual Nature of lines** (*discrete and continuous*) . [15] It has been shown that Primary Neutral Space is not moving and Time is not existing so *Points* , in Primary Space cannot move to where they are because are already there and motion is impossible . Since *Points C , D* of Primary Neutral Space , PNS , are motionless ($v = 0$) at any Time (the composed instants are $dt = 0$) then motion is impossible .

i.e. issues [$ds = a + b \cdot i = v \cdot dt$] and for ,

$a = 0$ then $ds = b \cdot i = v \cdot dt$ and for $b \neq 0$, $dt = 0$ then $ds = Constant = v \cdot 0$

$b = 0$ then $ds = a = v \cdot dt$ and for $dt = 0$ then $\rightarrow ds = a = Constant = v \cdot 0$

therefore in PNS , $v = \infty$, $T = 0$, **meaning infinite velocity and Time not existing** , so **Since Arrow is moving from point A to point B** , then exists a **Numerical order $A \rightarrow B$** which is not valid for Temporal order (dt) . In case $dt = 0$ then motion from Point A to point B has not any concept , and distance CD and anywhere exist the Equal CD is unmovable , i.e. **Motion of points C, D of PNS is not existing because time (dt=0) and infinite velocity ($v = \infty$) exists** , while motion of the same points C, D exists in PNS out of a moving Sub-Space of AB (arrow CD is one of the ∞ roots of AB) ($ds = CD = Monad$ in PNS) [15].

FUNDAMENTAL PRINCIPLES .

1. The first dimensional Unit AB , of any two points A and B , is the geometrical Shape that has as Position the (direction \hat{AB} , \hat{BA}) and as Magnitude (the length $AB = 0 \rightarrow N \rightarrow \infty$) . (F2-3) .

ds = An Infinitely small increment of length AB in the direction AB , BA .

∞ = An Infinitely great magnitude AB in the direction AB , BA .

Any point C is on Straight line AB , only when exists equation $CA + CB = AB$,

i.e. the Whole AB is equal to the Parts CA and CB . (*equation*) . (F 2.6.)

In case $CA + CB > AB$ then point C is not on line AB , and this is the main difference between Euclidean and Non-Euclidean geometries . In Definition 2 (*a line AB is breathless length*) is altered as , for any point C on line AB exists $CA + CB = AB$. Edge points A , B not coinciding on Monad **AB** , keep the properties of Complex Numbers with Imaginary part which differs between the infinite points. Monads $ds = 0 \rightarrow \infty$ are simultaneously (*actual infinity*) and (*potential infinity*) in Complex number form , and this defines that infinity , is existing between all points which are not coinciding , and because **ds** comprises any two edge points with imaginary part then this property differs between the infinite points .

2. **Spaces of Unit AB** are (in Plane) the Infinite Regular Polygons inscribed in the circle *with AB as Side* , (repetition of Unit AB) , the Nth Space , the Nth Unit Tensor of the N equal finite Elements **ds** , and for the ∞ Spaces line $AB \leftrightarrow$. (F.3)
The diameter of this circle extends to infinity (*it is of potential nature*) .

3. **Anti-Spaces of Unit AB** are (in the three dimensional space) the Symmetrically Infinite Regular Solids inscribed in the Sphere *with AB as side* of the Solid , (Harmonic Repetition of Unit BA , symmetrical to AB) , the Nth Anti-Space , the Nth Unit Tensor of the N equal finite Anti-Elements and for the ∞ Spaces , line $BA \leftrightarrow$. (F2-6 , F3)
The diameter of this Sphere extends to infinity (*it is of potential nature*) .

4. **Sub-Spaces of Unit AB** are (in Plane) the Infinite Regular Polygons inscribed in the circle *with AB as diameter* , (Harmonic Repetition of the Roots in Unit AB) and in Nth Sub-Space , the Nth Unit Tensor of the N finite Roots and in case of ∞ Elements are the points on the circle) , (*actual infinity*) . (F.4)
The Superposition of Spaces , Anti - Spaces and Sub-Space Layers of Unit AB is shown in F.5 . *Remark* : (+) Spaces , (-) Anti-Spaces , (\pm) Sub-Spaces , of a unit AB are between magnitude (Point = 0 =Nothing) and the Infinite magnitude ($\leftrightarrow = \pm \infty =$ Infinite) which means that all Spaces are in one Space .
Because in Spaces and Anti-Spaces , the ∞ Spaces of **Unit AB** is line $AB \leftrightarrow$, and in Sub-Spaces , the ∞ Sub-Spaces of **Unit AB** are the points on the circle with AB as diameter , then this ordered continuum *for points on the circle of Unit AB and on line AB* shows the correlation of Spaces in Unit AB . i.e.

Monads ds = 0 \rightarrow ∞ are Simultaneously , actual infinity (because for $n = \infty$ then $ds = [AB / n = \infty] = 0$ i.e. a point) , and potential infinity , (because for $n = 0$ then $ds = [AB / n=0] = \infty$ i.e. the straight line through AB) .

5. **Algebraic Numbers** : According to F.5 *Monad AB = 0 \leftrightarrow AB \leftrightarrow $\pm \infty$* . Spaces , Anti-Spaces , Sub-Spaces of AB are **the Infinite Regular Polygons** , on circle with AB as Side ,and on circle with AB as diameter . According to De Moivre's formula the n-th roots on the unit circle AB are represented by the vertices of these Regular n-sided Polygon inscribed in the circle which are Complex numbers in the general form $w = a+b.i = r.e(i\phi)$, **a** and **b** = Real Numbers , $r = \sqrt{a^2+b^2}$, (\pm) **i** = Imaginary Unit. We will show that since Complex Numbers are *on Monad AB (any two points)* and it is the only manifold for Physical reality *then Euclidean Geometry is Quantized* (Fig.1)

- a. Exists ${}^2\sqrt{1} = \pm 1$ or $[-1 \leftrightarrow +1]$, therefore **xx** (axis) coordinate system represents the one-dimensional Space and Anti-Space . (the Straight line) , $1.1 = 1$, $(-1).(-1) = 1$
 $[+ i]$
- b. Exists ${}^2\sqrt{-1} = \pm i$ or $[\updownarrow]$, therefore **yy** (axis) coordinate system represents a perpendicular one-dimensional $[-i]$ to **xx** Space and Anti-Space . (the Straight line) ,
exists ,
 $(-i).(-i) = +i^2 = +(-1) = -1$, $(+i).(+i) = +i^2 = -1$

c. Exists $\sqrt[3]{1} = [1, -\frac{1}{2} + (\sqrt{3}i)/2, -\frac{1}{2} - (\sqrt{3}i)/2]$ therefore **xx-yy** coordinate system represents two - dimensional \pm Spaces and \pm Complex numbers .(the Plane)
 $1.1.1 = 1$, $[-\frac{1}{2} + (\sqrt{3}i)/2]^3 = 1$, $[-\frac{1}{2} - (\sqrt{3}i)/2]^3 = 1$
 $+ i$
d. Exists $\sqrt[4]{1} = \sqrt[2]{\sqrt[2]{1}} = \sqrt[2]{\pm 1} = [+1, -1], [\sqrt[2]{-1} = +i, -i]$ or $-1 \leftrightarrow +1, \uparrow$
therefore coordinate systems **xx - yy** represent all these Spaces , $-i$
(\pm Real and \pm Complex numbers), where Monad = 1 = (*that which is one*) ,
represents the three-dimensional Space and Anti-Space. (the Sphere) . $[\pm 1]^4 = [\pm i]^4 = 1$

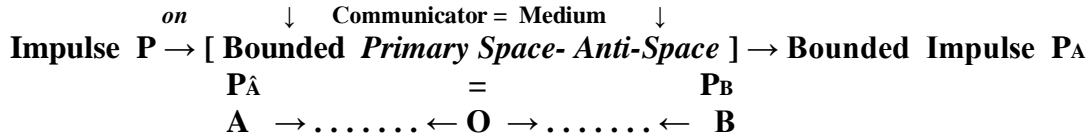
The fourth root of **1** *are the vertices of Square in circle* with **1** as diameter and since the Geometrical Visualization of Complex numbers , is formula $\sqrt[4]{1} = \pm 1, \pm i$ (d) and since ± 1 is the one-dimensional *real Space* (*the straight line*) , the vertical axis is the other one-dimensional *Imaginary Space* $\pm i$. Since for dimension are needed N-1 points then (d) is representing the Space with three dimensions (dx ,dy ,dz) which are Natural , Real and Complex . Monads (Entities = AB) are the Harmonic repetition of their roots , and since roots are the combinations of purely real and purely Imaginary numbers , *which is a similarity with Complex numbers* (*Real and Image*) , then , **Monads are composed of Real and Imaginary parts as Complex Numbers are** . i.e. Objective reality contains both aspects (Real and Imaginary , discrete , AB , and Continuous , Impulses **PA, PB**, etc) , i.e. , *Euclidean geometry is Quantized* . [15]

i.e. The Position and Dimension of all Points which are connected across Universe and that of Spaces exists , because of this Static Inner Impulse P, on the contrary should be one point only (Primary Point = Black Hole $\rightarrow ds = 0$) . [14-15] Impulse is ∞ and may be Vacuum , Momentum or Potential or Induced Potential.

Change (motion) and plurality are impossible in Absolute Space [PNS] and since is composed only of Points that consist an Unmovable Space , then neither Motion nor Time exists i.e. a constant distance $AB = ds$ anywhere existing is motionless . $ds = [AB / n] > 0 = quantum$, and for infinite continuous **n** , **ds** convergence to **0** . Even the smallest particle (say a photon) has mass [15] and any Bounded Space of $ds > 0$ is not a mass-less particle and occupies Momentum (motion) . Physical world is scale-variant and infinitely divisible , consisted of finite indivisible entities $ds = AB \rightarrow 0$ and infinite points ($ds = 0$) i.e. is Continuous with points and Discontinuous with $ds > 0$. In PNS $dt = 0$, so motion cannot exist at all .

Since points A,B of PNS coincide with the infinite Points , of the infinite Spaces Anti-Spaces and Sub-Spaces of PNS , and since Motion may occur at all Bounded Sub-Spaces then this *Relative motion* is happening between all points belonging to PNS and those belonging to other Spaces . *Time exists in Relative Motion and it is the numerical order of material change in PNS - Space , and is not a fundamental entity* . Infinity exists between all points not coinciding and because Monad **ds** comprises any two edge points with Imaginary part then this property differs between the points forming infinite Dipoles .

On Monad $AB = 0 \leftrightarrow AB \leftrightarrow \pm \infty$ exists < a bounded State for each of the Infinite Spaces and Anti-Spaces > and the [Dipole $AB = \text{Matter}$] is the communicator of Impulse [P] of Primary Space , with the Bounded Impulses (P_A, P_B) of Dipole or [15] [P] \leftrightarrow [FMD = $AB - P_A, P_B$] $\rightarrow P_A, P_B$.



Motion is Continuous and occurs on Dimensional Units , ds , and not on Points which are dimensionless , upon these Bounded States of [PNS] , Spaces and Anti-Spaces , and because of the different Impulses P_A, P_B of points A, B and that of Impulses $P_i A, P_i B$, of Sub-Spaces , are either on straight lines AB or on tracks of Spaces , Anti-Spaces , Sub-Spaces of AB . [14-15] . The range of Relative velocities is bounded according to the single slices of spaces (ds).

Remarks :

1. Spaces and Anti-Spaces are continuous and represent Real numbers , ${}^2\sqrt{1} = \pm 1$ A Continuous Function is a Static Completed Entity while ds is a quality existing Entity conveyed through PNS .
2. The Model of nature is not built on Complex numbers because Complex numbers spring out of Spaces , Anti-Spaces and Sub-Spaces of the FDU ($ds = 0 \rightarrow AB \rightarrow \infty$) and represent reality . The roots of Monads are the same Monads in Space and Anti-Space as well as Imaginary Monads in Sub-Space i.e. The Harmonic repetition of the roots (Principle of Equality) composes units with no need to be Image or real dimensions .

Image or Real dimensions exist in Euclidean Geometry as the vertices of the Regular Polygons (and Anti-Polygons) on any First dimensional unit AB . The geometrical Visualization of Complex numbers , springs from formula ${}^4\sqrt{1} = \pm 1, \pm i$ (d) and since ± 1 is the one dimensional **real Space** (the straight line) the vertical axis is on (Harmonic repetition of Spaces) the other one dimensional **Imaginary Space** which is conveyed . Since dimension needs (N-1) points then (d) is representing the Space with three dimensions (dx, dy, dz) which is Natural , Real and Complex numbers and it is not four dimensional Space as it is in “Space-Time” theory .

Position and Momentum are incompatible variables because any determination of either one of them , leaves the other completely undetermined i.e.

The Eigenvalues of Spation Position are Incompatible with the Eigenvalues of Momentum (motion) , and so any ds in PNS has a definite Position and Momentum simultaneously . This is the Relative motion of Spaces .

3. **Natural Numbers with their discrete nature Symbolize Discontinuity of Spaces , because Physical World is Continuous with Points (motion) and Discrete with Numbes = Monads = ds . The Dual property of Physical World. also exists ,**

- 1a. **Point** is nothing , Everything , it is Anywhere , without Position and Magnitude.
- 2b. **Straight line** is 0 and $\pm\infty$ and since is composed of infinite points which are filling line , then nature of line is that of Point (the all is one for Lines and Points).
- 3c. **Plane** is Positive , Negative , \pm Neutral and \pm Complex and since is composed of infinite Straight lines which are filling Plane then nature of Plane is that of Line and that of Points (the all is one for Planes , Lines and Points).
- 4d. **Space** is Positive , Negative , \pm Neutral and \pm Complex and since is composed of infinite Planes which are filling Space then nature of Space is that of Plane and that of Points (the all is one for Spaces , Planes , Lines and Points).

5e. **The Bounded Spaces , Anti-Spaces , Sub-Spaces , of the First dimensional Unit $AB = a + b.i$, are composed of the two Elements of the [Dipole $AB = \text{Matter}$] which is communicator AB , and Impulse [P] of Primary Space , with the Bounded Impulses (PA, PB) of Dipole or [P] \leftrightarrow [$FMD = AB - PA, PB$] $\rightarrow PA, PB$**

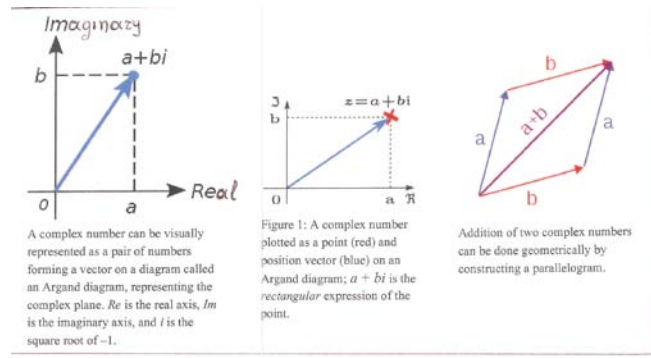


Fig.1

6f. **Achilles has to pass every point of line AB which is then as passing from the starting point A , $ds=0$, where Velocity of Achilles is $v(A) = ds/dt = 0$. The same happens for Tortoise at point B where Velocity $v(T) = ds/dt = 0$. On the contrary , Achilles passing AB on dimensional Units , ds , then Achilles velocity $v(A) = ds/dt(A)$ is greater than that of Tortoise $v(T) = ds / dt(T)$.**

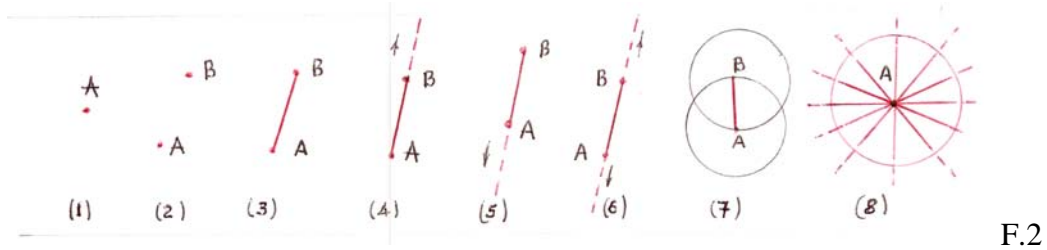
$A \quad ds = (AB / n = \infty) = 0 \quad B$ <i>Continuous (.)</i>	$A \quad ds = \rightarrow AB / n = 11 \quad B$ $\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow$ <i>Discrete (→)</i>
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Since in PNS , $v = \infty$, $T = 0$, meaning infinite velocity and Time not existing , then Arrow AB in [PNS] is constant because $AB = ds = \text{Constant} = u \cdot 0 = \infty \cdot 0$ Straight line AB is discontinuous (discrete) with dimensional Units $ds = AB / n$ where $n = 1 \rightarrow \infty$ and continuous with points [$n = \infty$] , (This is the Dual Nature of lines , and generally in geometry , discrete and continuous) .

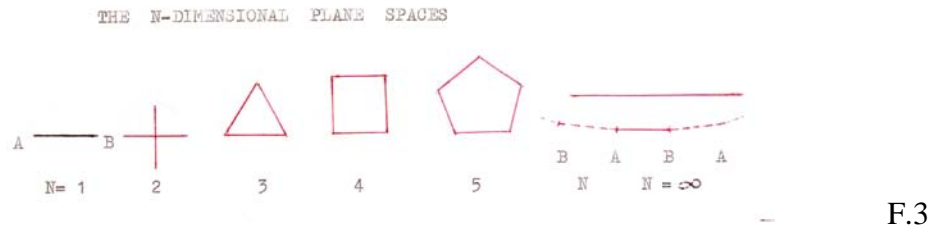
Spaces Anti-Spaces and Sub-Spaces are **Homogenous** because the Points of Monads are also. Since all Directions \vec{AB} , \vec{BA} are equivalent , then PNS is also **Isotropic** , i.e all Relative Natural sizes and Laws remain Inalterable with Displacement and Rotation.

Since [Spaces Anti-Spaces Sub-Spaces] are the Roots of Monads and since Monads are composed of **Real and Imaginary Parts** , as Complex numbers are , then are Real and Imaginary , Discrete and Continuous and is not needed to be separate in Visualization. Since velocity is ∞ and time is not existing , $T=0$, Curvature in PNS is zero and ∞ .

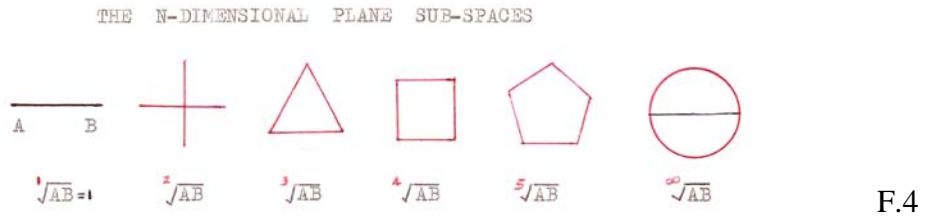
1. The Elements of Euclidean geometry are , Points , straight Lines , circle .etc,



2. The Space Layers (Regular Solids) with sides equal to $AB = 0 \rightarrow \infty$
The Increasing Plane Spaces by Repetition of the same Unit AB . [10]

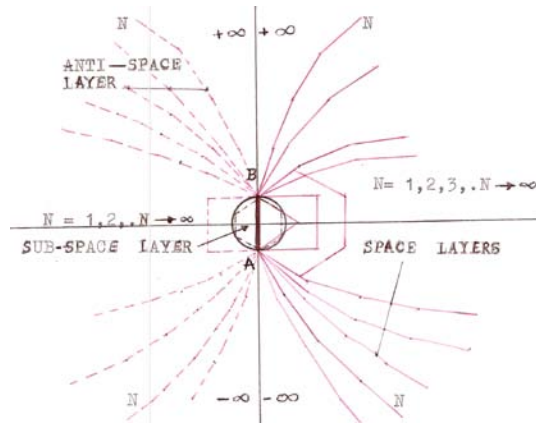


3. The Sub-Space Layers (Regular Solids on AB) as Roots of $AB = 0 \rightarrow \infty$
The Decreasing Plane Spaces by Repetition of the same Unit AB . [10]



4. The Superposition of Plane Space , Anti-Space Layers and Sub-Space Layers :

The simultaneously co-existence of Spaces , Anti-Spaces and Sub-Spaces of any **Unit $AB = 0 \rightarrow \infty$** , i.e. , Euclidean , Elliptic , Spherical , Parabolic , Hyperbolic, Geodesics , Metric and Non-metric geometries , exists in Euclidean Model as an Sub-case within .The Interconnection of Homogeneous and Heterogeneous bounded Spaces Anti-Spaces and Subspaces of the Universe . The Unity of Opposites.[10] This is also the *Quantized* property of Euclidean geometry <all is one> as it is, Discrete (for Monads AB) and Continuous (for Points A, B) . F.5



F.5

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