A simple numeric example of a contradiction in special relativity

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Abstract. Assume the standard configuration under Special Relativity (SR) and a light pulse is emitted when the origins of two coordinate systems are common. Further assume $v = .6c$ and that the spherical light wave (SLW) has attained the unprimed coordinated $(x_1 = 2ls, 10ls, 0)$ where ls is the distance light travels in 1 second. Then $t_1 = \sqrt{104}s$ and using Lorentz transformations (LT), $(x'_1 = 1.25(2 - .6\sqrt{104})ls, 10ls, 0)$. Since $x_1 > 0$ and $x'_1 < 0$, both frames agree along the line $y = 10$ the SLW is in between the two origins. According to nature, the SLW will propagate further. So, assume that condition. Both frames conclude, along the line $y = 10$, any further propagation of the SLW must place the SLW further from its own origin by assuming the light postulate in its frame. A valid question to propose is, by considering coordinates only with $y = 10$ and $z = 0$, where will the SLW move to after further propagation? If both frames agree the SLW must move further from the respective origin, and the SLW is in between the two origins, then the SLW must move two different directions along the line $y = 10$ to satisfy the SR conditions of each frame. Based on this fact, it will be proven in the context of either frame, after further propagation of the SLW, LT will contradict the light postulate in the target frame.

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I. METHOD

Assume the conditions in the abstract (see figure 1). All calculations below will be restricted to the line $y = 10$ such that only x intercepts at the intersections of the SLW with the line $y = 10$ will be considered. Therefore, let $11/2 > h > 0$ be some infinitesimally small further propagation of the SLW along the line $y = 10$ in the context of the unprimed frame. Hence, $(2 + h ls, 10ls, 0)$ is further from the unprimed origin and is therefore consistent with the further propagation of the SLW and the light postulate for the unprimed frame (see figure 2). Also, $t_2 = \sqrt{(2+h)^2 + 100}$ with $x_2 = 2 + h$. Next, LT is applied to this "new" coordinate. Note $\gamma = 1.25$ when $v = .6c$.

$$x'_2 = (x - vt)\gamma = \left((2+h)ls - .6c\sqrt{(2+h)^2 ls^2 + 100 ls^2}\right)1.25$$

$$y' = 10$$

$$z' = 0$$

Next, since the SLW propagated further, the context of the primed frame is considered. Assume $k > 0$ is the distance in the measurements of the primed frame for this further propagation of the SLW along the line $y = y' = 10$. It will be proven it is impossible for LT to match the light postulate of the primed frame and in fact, LT will contradict the direction of propagation for the SLW along the line $y = 10$ in the context of the primed frame.

Unprimed Frame View

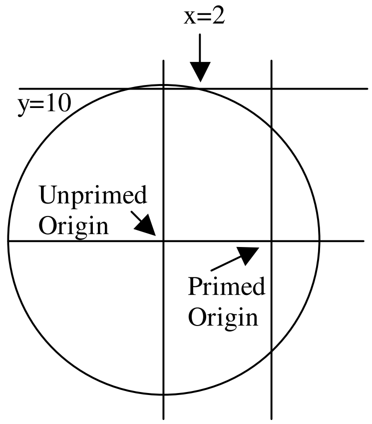

Figure 1

The initial condition is shown above as the location of the SLW at unprimed (2,10) along the line y=10.

Direction of intersection of the SLW with the line y=10

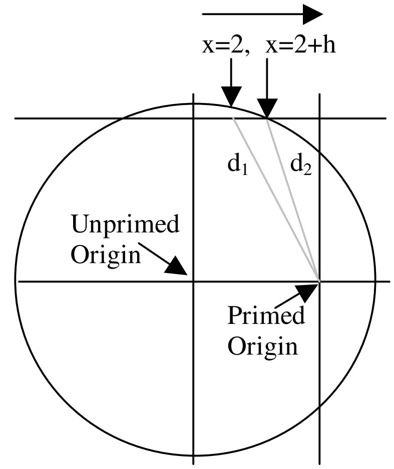

Figure 2

After further propagation, the SLW expands in all directions at c. This causes the larger SLW to intersect the line y=10 at (2+h,10) which is closer to the primed origin, i.e. $d_2 < d_1$.

So, from the view of the primed frame, after further propagation of the SLW, the SLW will be located at $(x'_2 = (x'_1 - k)ls, 10ls, 0)$ since $x'_1 < 0$. Clearly, since $k > 0$, $x'_1 = 1.25\left(2 - .6\sqrt{104}\right) < 0$ for the initial x' location of the SLW and x'_1 is negative, then $x'_2 = x'_1 - k < x'_1 < 0$. Since $(x'_2, 10, 0)$ is further from the primed origin than is $(x'_1, 10, 0)$, then this selection of $k > 0$ is consistent with the light postulate in the primed frame. However, LT performs the calculation as $x'_2 = \left((2+h)ls - .6c\sqrt{(2+h)^2 ls^2 + 100ls^2}\right)1.25$. It is now shown, LT calculates $x'_1 < x'_2 < 0$. If $x'_1 < x'_2 < 0$, this means LT claims the SLW moves closer to the primed origin after further propagation.

$$\frac{16}{9}h^2 + \frac{10\sqrt{104} - 12}{3}h > 0$$

$$\frac{16}{9}h^2 + \frac{10\sqrt{104}}{3}h > 4h$$

$$\frac{25}{9}h^2 + \frac{10\sqrt{104}}{3}h + 104 > h^2 + 4h + 104$$

$$\left(\frac{5}{3}h + \sqrt{104}\right)^2 > (2+h)^2 + 100$$

$$\frac{5}{3}h + \sqrt{104} > \sqrt{(2+h)^2 + 100}$$

$$\frac{5}{3}h - \sqrt{(2+h)^2 + 100} > -\sqrt{104}$$

$$(2+h) - .6\sqrt{(2+h)^2 + 100} > \left(2 - .6\sqrt{104}\right)$$

$$x'_2 = \left((2+h) - .6\sqrt{(2+h)^2 + 100}\right)1.25 > x'_1 = \left(2 - .6\sqrt{104}\right)1.25$$

But, the light postulate in the primed frame claims it is impossible that $x'_1 < x'_2 < 0$ after further propagation of the SLW even though that is exactly what LT calculates. And from above with $h > 0$, it is impossible for LT to match the light postulate in primed frame as it is required to do. In fact, since LT contends $x'_1 < x'_2 < 0$ after further propagation of the SLW and the primed frame light postulate contends $x'_2 < x'_1 < 0$ after further propagation of the SLW, there is no possible selection of $11/2 > h > 0$ that can satisfy the light postulate in the primed frame using LT. But, the very reason that $h > 0$ is because the light postulate in the unprimed frame requires it. Therefore, after further propagation of the SLW, LT contradicts the light postulate for the primed frame i.e. there is no possible selection of $11/2 > h > 0$ that can satisfy the light postulate for the primed frame.

One may try to argue that LT provides 4-D vectors and hence one must consider space-time and not just space to solve this contradiction. And, as the argument would go, when space-time is considered, the frames simply disagree on the order of events and hence LT does not actually arrive at the conclusion that the SLW moved closer to the primed origin. That argument is easily refuted. In the calculations of the unprimed frame, the unprimed frame absolutely agrees the SLW moves closer to the primed origin from $(2ls,10ls,0)$ to $((2+h)ls,10ls,0)$. By simple geometry, $d_1 = \sqrt{(vt_1 - x_1)^2 + y^2} = \sqrt{(.6\sqrt{104} - 2)^2 + 100}$ and $d_2 = \sqrt{(vt_2 - (x_1 + h))^2 + y^2} = \sqrt{(.6t_2 - (2+h))^2 + 100}$ are the unprimed frame calculations for the distances of the two unprimed coordinates from the SLW to the primed origin. It is an easy task to prove $d_2 < d_1$. Now, how does the unprimed frame convert the x component distance to the primed origin of $(vt_1 - x_1)$ into measurements in the primed coordinates? It must be recognized that primed frame measurements look length contracted in the view of the unprimed frame. Hence, the x' distance in the primed frame looks like x'/γ to the unprimed frame. So, for the unprimed frame to convert $(vt_1 - x_1)$ to a primed frame x' distance, $x'_1/\gamma = (vt_1 - x_1)$. Thus, $x'_1 = (vt_1 - x_1)\gamma$. Then by simply taking this length contraction into consideration, we have $d'_1 = \sqrt{(vt_1 - x_1)^2\gamma^2 + y^2}$ and $d'_2 = \sqrt{(vt_2 - (x_1 + h))^2\gamma^2 + y^2}$ which is exactly the LT calculation for the distance of the SLW to the primed origin. Therefore, by operating from the context of the unprimed frame at rest, the distance from the SLW to the primed origin at the initial condition was d'_1 when using the unprimed frame measurements of d_1, which are converted to primed frame measurements. After further propagation, it is d'_2. It was already proven above that $d'_2 < d'_1$ by showing $x'_1 < x'_2 < 0$ using LT above. Hence, the unprimed frame calculations conclude $d_2 < d_1$. The primed frame converted measurements using LT on d_1 and d_2 conclude $d'_2 < d'_1$. Thus, both calculations agree that the SLW moved closer to the primed origin after further propagation of the SLW away from $(2ls,10ls,0)$ along the line $y = 10$. Thus, a 4-D vector argument does not resolve this contradiction.

It is just as simple to show, using the calculations of LT from the view of the primed frame light postulate, that LT claims after further propagation of the SLW, $0 < x_2 < x_1$ for the unprimed frame which contradicts the light postulate for the unprimed frame since that relation implies the SLW would move closer to the unprimed origin.

Therefore, with $v = .6c$, after further propagation of the SLW from the unprimed coordinate $(2ls,10ls,0)$, SR calculates that the SLW moves in both the positive x direction and the negative x direction away from $(2ls,10ls,0)$ along the line $y = 10$ and SR contradicts its light postulate.

II. CONCLUSIONS

A specific example was provided under the rules of SR for two frames in relative motion in the standard configuration. It was assumed a light pulse was emitted when the origins of the two frames were common and that the SLW had acquired a specific unprimed frame coordinate. Next, it was assumed the SLW propagated further as in nature. It was proven, for this specific example, LT contradicted the light postulate in the primed frame and LT contradicted the light postulate in the unprimed frame. Any theory that contradicts its postulates is logically inconsistent.

III. REFERENCES

Einstein A., in *The Principle of Relativity* (Dover, New York) 1952, p. 37.