Analysis of The Theory of Abstraction

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Abstract:

In this paper, a few more implications of the laws of physical transactions as per the Theory of Abstraction are dealt with. Analysis of those implications suggests the existence of 'hidden' mass and 'hidden' energy in a given physical transaction. Trajectory –examination of such possible transport is carried out. Relativistic cyclist phenomena are also dealt with in this paper.

Introduction:

A particle in an isolated box will tend to move in all possible directions. A bias towards any given direction indicates an imbalance of support towards its movement in that given direction and resistance against it. Considering the movement of an energy quantum in a particular direction, this difference between the concerned support and the concerned resistance must be al least the same of the given energy quantum, in accordance to the Theory of Abstraction. For a given quantum-state ($h\upsilon$),

$$S \sim R = h\upsilon \qquad \qquad \ldots(1)$$

Where, S and R represent the support and resistance respectively.

This means that at least one half of a total energy-quantum gives it its direction while the other part gives it its magnitude. The direction part remains 'hidden' while only the magnitude part shows up as the value of the given quantum state. Considering the direction part however may reduce quantum-transport to classical transport as we shall see here.

Let us consider a photon revolving around a given body of mass (M) and within a radius (R). Let m_n be the instantaneous mass of the given photon. Then, according to the classical theory of gravitation, the force of gravity on the total photon (magnitude + direction parts) due to the given body equals

$$\frac{GM(m_n + m_n)}{R^2}, i.e. \frac{2GMm_n}{R^2}.$$

In order that the photon does not move away from the body, the difference between support and resistance in that given direction must be zero. Thus, in order to bind the photon within a radius R, only its direction-part needs to be bound.

Now the outward force experienced by the direction-part of the photon equals $\frac{m_n c^2}{R}$; vaccum being the medium considered.

For light to be bound within the radius R,

$$\frac{m_n c^2}{R} - \frac{2GMm_n}{R^2} \leq 0$$

Choosing the equality sign,

$$m_n\left(c^2 - \frac{2GM}{R}\right) = 0$$

For the energy-quantum to exist,

$$m_n \neq 0$$

Therefore,

$$c^2 - \frac{2GM}{R} = 0$$

$$i.e. \frac{M}{R} = \frac{c^2}{2G} \approx 10^{27} kg/m \qquad \ldots (2)$$

Thus, if mass is concentrated within a radius R such that $\frac{M}{R} \approx 10^{27} kg/m$, then the body is a black-hole, in accordance with the Theory of Abstraction. This result is validated by existing data.

In this treatment, considering the direction-part of a given energy-quantum, classical transport merges with quantum-transport quite perfectly. Support towards the given direction of transport of a given energy-quantum comprises the direction-part of it while a given resistance may act upon the total energy-quantum i.e., the direction-part and the magnitude-part combined.

Chaos-Analysis of Quantum-Transport:

For a given transport of energy-quantum, between an initial and a final point, let the trajectory of the initial point $x_o = x(o)$ be denoted by:

$$x(t) = f^t(x_o)$$

Expanding $f^t(x_o + \delta x_o)$ to linear order, the evolution of the distance to a neighbouring trajectory $x_i(t) + \delta x_i(t)$ is given by the Jacobian matrix J:

$$\delta x_i(t) = \sum_{j=1}^{d} J^t(x_o)_{ij}\ \delta x_{oj},$$

$$J^t(x_o)_{ij} = \frac{\delta x_i(t)}{\delta x_{oj}} \qquad \ldots (3)$$

A trajectory of an energy-quantum as moving on a flat surface is specified by two position coordinates and the direction of motion. The Jacobian matrix describes the deformation of an infinitesimal neighbourhood of $x(t)$along the transport. Its eigenvectors and eigenvalues give the directions and the corresponding rates of expansion or contraction. The trajectories that start out in an infinitesimal neighbourhood separate along the unstable directions (those whose eigenvalues are greater than unity in magnitude), approach each other along the stable directions (those whose eigenvalues are less than unity in magnitude), and maintain their distance along the marginal directions (those whose eigenvalues equal unity in magnitude).

Holding the hyperbolicity assumption (i.e., for large n the prefactors a_i, reflecting the overall size of the system, are overwhelmed by the exponential growth of the unstable eigenvalues Λ_i, and may thus be neglected), to be justified, we may replace the magnitude of the area of the ith strip $|B_i|$ by $\frac{1}{|\Lambda_i|}$ and consider the sum,

$$\lceil n = \sum_{i}^{n} \frac{1}{|\Lambda_i|};$$

where the sum goes over all periodic points of period n. We now define a generating function for sums over all periodic orbits of all lengths:

$$\lceil z = \sum_{n=1}^{\infty} \lceil n \, z^n \qquad \ldots(4)$$

For large n, the nth level sum tends to the limit $\lceil n \rightarrow e^{-n\gamma}$, so the escape rate γ is determined by the smallest $z = e^{\gamma}$ for which equation (4) diverges:

$$\lceil z \approx \sum_{n=1}^{\infty} (ze^{-\gamma})^n = \frac{ze^{-\gamma}}{1 - ze^{-\gamma}} \qquad \ldots(5)$$

Making an analogy to the Riemann zeta-function, for periodic orbit cycles,

$$\lceil z = -z\frac{d}{dx}\sum_{p} \ln(1 - t_p);$$

$\lceil(z)$ is a logarithmic derivative of the infinite product

$$\frac{1}{\zeta(z)} = \prod_{p}(1 - t_p), t_p = \frac{z^{n_p}}{|\Lambda_p|}$$

This represents the dynamical zeta function for the escape rate of the trajectories of quantum-transport. The fraction of initial x whose trajectories remain within B at time t may decay exponentially

$$\lceil t = \frac{\int_s dx\, dy\, \delta[y - f^t(x)]}{\int_s dx} \rightarrow e^{-\gamma t}.$$

The integral over x starts a trajectory at every $x \epsilon B$. The integral over y tests if this trajectory still falls within limits of B at time t.

The Kernel of this integral is the evolution operator for a d-dimensional transport,

$$\mathcal{L}^t(y,x) = \delta[y - f^t(x)] \qquad \ldots(6)$$

Expressing the finite time Kernel $\mathcal{L}^t$ in terms of A, the generator of infinite time translations,

$$\mathcal{L}^t = e^{tA} \qquad \ldots(7)$$

This is very much similar to the way quantum evolution is generated by the Hamiltonian.

<u>Relativity of Cyclists and Symmetry Analysis:</u>

The component of the dynamics along the continuous symmetry directions of the trajectory behavior or 'drift' may be induced by the symmetries themselves. In presence of a continuous symmetry, an orbit explores the manifold swept by combined actions of the dynamics and the symmetry induced drifts. A group member can be parameterized by angle θ, with the group multiplication law $g(\theta')\, g(\theta) = g(\theta' + \theta)$ and its action on smooth periodic functions $u(\theta + 2\Pi) = u(\theta)$ generated by,

$$g(\theta') = e^{\theta' T}, T = \frac{d}{d\theta}$$

The differential operator T is reminiscent of the generator of spatial translations. The constant velocity field $v(x) = v = C.T$ acts on x_j by replacing it by the velocity vector C_j.

Let, G be a group and $gB \to B$ a group action on the state space B. The $[d\ x\ d]$ matrices g acting on vectors in the d-dimensional state space B from a linear representation of the group G. If the action of every element g of a group G commutes with the flow,

$$g\ v(x) = v(gx), g f^t(x) = f^t(gx)$$

G is a symmetry of the dynamics and is invariant under G or G-equivalent. For any $x \in B$, the group orbit B_x of x is the set of all group actions

$$B_x = [g\ x \mid g \in G] \qquad \ldots(8)$$

The time evolution and the continuous symmetries can be considered on the same Lie group footing. An element of a compact Lie group continuously connected to identity can be written as,

$$g(\theta) = e^{\theta T}, \theta.T = \sum \theta_a T_a\, , a = 1,2,3, \ldots, N;$$

where $\theta.T$ is a Lie element and θ_a are the parameters of the transformation.

Any representation of a compact Lie group G is fully reducible, and invariant tensors constructed by contractions of T_a are useful for identifying irreducible representations. The simplest such invariant is,

$$T^T.T = \sum_{\propto} C_2^{(\alpha)}\ \mathbb{I}^{(\alpha)}$$

equilibria satisfy $f^t(x) - x = 0$ and relative equilibria satisfy $f^t(x) - g(t)x = 0$ for any t. A relative periodic orbit is periodic in its mean velocity,$C_p = \theta_p/T_p$ comoving frame, but in the stationary frame its trajectory is quasiperiodic. A relative periodic orbit may be pre-periodic if it is equivariant under a discrete symmetry. Translational symmetry allows for relative equilibria characterized by a fixed profile Eulerian velocity $\mu_{TW}\ (x)$ moving with constant velocity C,i.e.,

$$\mu\ (x, t) = \mu_{TW}\ (x - Ct) \qquad \ldots (9)$$

A relative periodic solution is a solution that recurs at time T_p with exactly the same disposition of the Eulerian velocity fields over the entire cell, but shifted by a 2-dimensional (streamwise – spanwise) translational g_p. By discrete symmetries, these solutions come in counter-travelling pairs

$$u_q(x - Ct), -u_q(-x + Ct).$$

The Hilbert basis may be written as:

$$u_1 = x_1^2 + x_2^2, u_2 = y_1^2 + y_2^2, u_3 = x_1y_2 - x_2y_1, u_4 = x_1y_1 + x_2y_2, u_5 = z$$

This is invariant under the special orthogonal graph SO(2);i.e., a group of length-preserving rotations in a plane. The polynomials are linearly independent, but related through one syzygy,

$$u_1u_2 - u_3^2 - u_4^2 = 0 \qquad \ldots (10)$$

yielding a 4-dimensional B/ SO(2) reduced state-space.

The dynamical equations following from chain rule are,

$$\mathring{u}_i = \frac{\delta u_i}{\delta x_j} x_j \qquad \ldots (11).$$

Conclusion:

From the Theory of Abstraction, we arrive at ‘hidden’ direction part of an energy-quantum. Quantum dynamics is seen to merge with classical dynamics if this hidden direction-part of the quantum-states are taken into consideration, as validated by practical analysis and data. Moreover, this hidden part of an actual energy-quantum may explain the dark-energy problem. As a support towards transport comprises the direction-part only, and as the resistance against motion is offered against the whole of an energy-quantum (direction-part + magnitude-part), this hidden energy may very well affect a gravitational field. On the other hand, if matter is considered to be condensed energy, and as such condensed energy quanta in some form of orientation in space time, there ought to be some self-same hidden mass as the magnitude-parts of the constituent energy quanta, thereby indicating a hidden dark-matter. Thus the anomaly of the existence of dark-matter and dark-energy being hidden from us may be reconciled with in accordance with the Theory of Abstraction.