INVALID INTERVALS OF X' CALCULATED BY THE LORENTZ TRANSFORMATIONS

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Abstract

Under Special Relativity (SR), as a consequence of the light postulate, a spherical light wave (SLW) propagates away from the light emission point in the frame in all directions at a constant velocity c. Further, assuming the standard configuration, since the Lorentz transformations (LT) are required to preserve the truth of all aspects of the SLW, then on any time interval, LT must preserve the propagation the SLW away from the origin of the primed frame. In short, since there is only one SLW under SR assuming one light pulse and given some time interval for the unprimed frame, since the set of vectors for that time interval demonstrates the propagation of the SLW away from the unprimed origin at c, then the corresponding set of LT mapped vectors must also demonstrate the SLW propagates away from the primed origin at c. More specifically, SLW expansion in one frame must translate using LT to SLW expansion for any other frame. However, it will be shown, using some arbitrary $y_g = y > 0$ with $z = 0$, on the interval $0 < x < \frac{\gamma v y_g}{c}$, if the SLW propagates away from the unprimed origin along the line $y = y_g$, then the output of LT produces an interval of x' that demonstrates the LT mapped SLW propagates toward the primed origin along that same line. Hence, such an interval of x' is invalid under LT.

In addition it will be proven, if two SR frames agree the SLW is located in between the origins of the two SR frames along some line $y = y_g > 0$ and SR is in a valid logical state, then any further propagation of the SLW along the line $y = y_g > 0$ in the unprimed frame forces SR into logically invalid state such that it will be shown some principle of SR will be proven to be logically contradicted by any further propagation of the SLW.

I. Introduction

Under SR, LT implements the conjunction of the light postulate and the relativity postulate. Therefore, LT is required to preserve the truth of all possible aspects of the SLW. In particular, the invariance of the light like space-time interval proves LT preserves the constant speed of light c across frames for statically defined light beams. This method of proof simply selects an arbitrary frame space coordinate and demonstrates if the corresponding light beam to the coordinate measures c, then the LT mapped light beam measures c. However, the light postulate is more general than that simple description. More specifically, the light postulate is a dynamic definition and states that the SLW propagates away from the light emission point in the frame in all directions at c. Therefore, a more general proof for LT would show, assuming the SLW propagates away from the light emission point in all directions at c in the unprimed frame, then the LT mapped SLW propagates away from the light emission point in all directions at c in the primed frame. This methodology requires calculus for proof because of its dynamic nature as opposed to statically defined light beams as in the case of proving the invariance of the light like space-time interval. The following argument will focus on the dynamic behavior of the SLW and the application of the derivative to model this behavior of the SLW under the calculations of LT. It will be shown, however, there are cases in which the output of LT demonstrates the SLW propagates toward the primed origin.

II. Method

Using the motivation above, given the equation for a SLW, $c^2t^2 = x^2 + y^2 + z^2$, it is the case that $t = \sqrt{x^2 + y^2 + z^2}\big/c$. Assume the standard configuration. Hence, to translate the x coordinates for the propagating SLW from the context of the unprimed frame to x' in the context of the primed frame, LT is applied and thus,

$$x' = (x - vt)\gamma = \left(x - v\sqrt{x^2 + y^2 + z^2}\big/c\right)\gamma$$

where $\gamma = 1\big/\sqrt{1 - v^2/c^2}$. Finally the partial derivative is calculated,

$$\frac{\partial x'}{\partial x} = \left(1 - \frac{vx}{c\sqrt{x^2 + y^2 + z^2}}\right)\gamma .$$

Now, in order for LT to preserve the truth of the light postulate across frames, it must be true for all possible cases, that if the SLW propagates away from the unprimed origin at c, then the corresponding set of LT mapped coordinates proves that the SLW propagates away from the primed origin at c. So, set $x > 0$, $y = y_g > 0$ where y_g is fixed, and $z = 0$. Then $\frac{\partial x'}{\partial x} = \left(1 - \frac{vx}{c\sqrt{x^2 + {y_g}^2}}\right)\gamma$. It is a trivial matter to show for these restrictions, $\frac{\partial x'}{\partial x} > 0$. Therefore, given $\frac{\partial x'}{\partial x} > 0$, if x increases, then x' increases. However, it will be shown it is possible that $x' < 0$ for the above restrictions. If it is possible there exists an interval of x in which the set of x' consists exclusively of negative numbers, then an increase of x requires an increase of x' which means $|x'|$ decreases and thus, the SLW propagates toward the primed origin.

Using the restrictions, $x' = \left(x - v\sqrt{x^2 + {y_g}^2}\big/c\right)\gamma$, and assume $x < \frac{\gamma v y_g}{c}$. Then, since $x > 0$,

$$x^2 < \frac{\gamma^2 v^2 {y_g}^2}{c^2}$$

$$x^2/\gamma^2 < \frac{v^2 {y_g}^2}{c^2}$$

$$\frac{x^2(c^2 - v^2)}{c^2} < \frac{v^2 {y_g}^2}{c^2}$$

$$x^2 < \frac{v^2(x^2 + {y_g}^2)}{c^2}$$

$$x' = \left(x - v\sqrt{x^2 + {y_g}^2}\big/c\right)\gamma < 0 .$$

So, for $0 < x < \frac{\gamma v y_g}{c}$, it is the case that $x' < 0$ and $\frac{\partial x'}{\partial x} > 0$. Therefore, as the SLW propagates away from the unprimed origin along the line $y = y_g$, given these restrictions, x increases and thus x' increases since $\frac{\partial x'}{\partial x} > 0$. But, since x' is negative and increasing, then the LT mapped SLW moves closer to the primed origin as it propagates since $|x'|$ decreases and y and z are fixed. Hence, the LT mapped subset of the SLW is not propagating away from the primed origin as necessitated by the light postulate for the primed frame. Consequently, LT does not preserve the full truth of the light postulate across frames for the above restrictions as required. So, based on the above, the everywhere increasing interval of x' generated by the applying LT to the propagating SLW in the context of the unprimed frame consists exclusively of negative numbers. Assuming a fixed $y_g > 0$, $z = 0$ and $0 < x < \frac{\gamma v y_g}{c}$, such an LT generated interval of x' is inconsistent with the light postulate in the primed frame since the interval implies the LT mapped SLW propagates toward the primed origin contrary to the requirements of the light postulate in the primed frame. Hence, such an interval of x' calculated by LT is invalid.

It is also the case that LT calculates impossibilities using the above example when translating time frame to frame. Using the same restrictions, if $0 < x < \frac{\gamma v y_g}{c}$, $y = y_g > 0$ and $z = 0$, then $y_g / c < t < y_g \gamma / c$. Next, from $c^2 t^2 = x^2 + y^2 + z^2$, deduce $x = \pm\sqrt{c^2 t^2 - y^2 - z^2}$. So, using LT,

$$t' = \left(t - vx/c^2\right)\gamma = \left(t - v\left(\pm\sqrt{c^2 t^2 - y^2 - z^2}\right)\Big/c^2\right)\gamma$$

Apply the above restrictions and,

$$t' = \left(t - v\left(\sqrt{c^2 t^2 - {y_g}^2}\right)\Big/c^2\right)\gamma = \left(t - \frac{v}{c}\left(\sqrt{t^2 - {y_g}^2/c^2}\right)\right)\gamma .$$

Calculate the partial derivative,

$$\frac{\partial t'}{\partial t} = \left(1 - \frac{vt}{c\sqrt{t^2 - {y_g}^2/c^2}}\right)\gamma .$$

This partial derivative is easily shown to be negative on the interval $y_g / c < t < y_g \gamma / c$ meaning $\frac{\partial t'}{\partial t} < 0$. Therefore, as t increases based on the propagation of the SLW in the context of the unprimed frame, t' decreases for the above restrictions. Since the output of the LT equation for t', based on the input x and t, is supposed to be the actual current time when the SLW is at that location for the primed frame, then with $y_g / c < t < y_g \gamma / c$ along the line $y = y_g$, as time elapses forward for the unprimed frame, time elapses backward for the primed frame based on the dynamics of the propagating SLW. However, along the line $y = y_g$ in the context of the primed frame, according to the light postulate, time only elapses forward at the intercepts for the propagating SLW. So, again, the output of LT based on the propagating SLW in the unprimed frame is not matching the conditions of the light postulate and the SLW in the primed frame as required.

IV. Discussion

The logic established above can easily be shown with a set of coordinate systems by allowing the SLW to acquire the coordinate $(0, y_g, 0)$ in the context of the unprimed frame. Given the standard configuration, the primed origin must be located in the positive x direction along the x-axis relative to the unprimed origin. Necessarily, once $(0, y_g, 0)$ is acquired, as the SLW propagates further from the unprimed origin, the SLW is located between the two origins of the frames. Therefore, as the SLW expands and intersects the line $y = y_g$ in the context of the unprimed frame it acquires increasing x intercepts. Since the SLW is in between the origins, increasing x intercepts translate to increasing x' intercepts with $x' < 0$, thus $|x'|$ decreases. Therefore, the SLW expansion in the unprimed frame forces the SLW to move closer to the primed origin when measured from its intersection with the line $y = y_g$. Hence, using the simple view of geometry with this example, if the SLW propagates away from the unprimed origin, then the SLW cannot propagate away from the primed origin as confirmed by the LT based derivatives above. See figure 1.

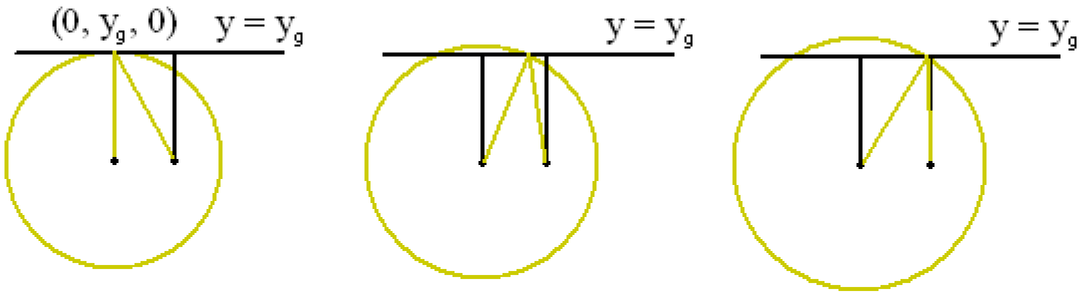

Figure 1.

The light path for a frame along the line $y = y_g$ is measured from the light emission point in the frame, the origin for this example, to the intersection of the SLW with the line $y = y_g$. According to SR, however, the actual length of this light path for the primed frame is longer since in the view of the unprimed frame, it is length contracted in the x direction and that is what is shown above. From the figure, as the SLW propagates away from the origin of the unprimed frame, the distance from the intersection of the SLW with the line $y = y_g$ to the primed origin decreases. If there is only one SLW for any light pulse, then any propagation of the SLW away from the light emission point in one frame, must translate, using LT, to the propagation of the SLW away from the light emission point of any other frame. As can be seen in the figure above, it is physically impossible for one SLW to propagate away from both origins of the frames along the line $y = y_g$ while maintaining the invariance of the light like space-time interval. (Maintaining the invariance of the light like space-time interval for this case means if you place a pencil point on the line and move it right to model the propagation of the SLW, the pencil cannot be picked up while moving and so the distance from the pencil point to the primed origin decreases.)

It should be also noted, as shown above, as the SLW propagates away from the unprimed origin while between the origins, both $|x'|$ and t' decrease along the line $y = y_g$ at the intercepts of the propagating SLW. However, each LT light beam mapped into the primed frame still satisfies the primed frame SLW equation $c^2t'^2 = x'^2 + y'^2 + z'^2 = x'^2 + y_g{}^2$, hence each light beam measures c. However, the SLW does not propagate away from the primed origin. Therefore, demonstrating the invariance of the light like space-time interval for all light beams is not sufficient to prove that LT preserves the full attributes of the light postulate and preserves the propagation of the SLW away from the primed origin.

It is a valid issue at this point to explain mathematically why it is impossible for the light postulate to be true in both frames along the line $y = y_g$ while the SLW is in between the origins. The explanation is simple. LT was derived by Einstein on the basis that space is homogeneous. *"In the first place it is clear that the equations must be* linear *on account of the properties of homogeneity which we attribute to space and time."* [1] For the purposes of this discussion, the homogeneity of space means the direction of motion of the SLW parallel to the x axis must be preserved by the transformations. LT's implementation of the homogeneity of space was proven above for the SLW with $\frac{\partial x'}{\partial x} > 0$ in the standard configuration. Thus, any increase in x translates to an increase in x'. Hence, the direction of motion of the unprimed SLW parallel to the x-axis is preserved by LT in the coordinate system of the primed frame.

So, by $\frac{\partial x'}{\partial x} > 0$, LT is only capable of preserving the direction of motion of the SLW from the context of the unprimed frame to coordinates of the primed frame. Next, for any frame, along a y line, from the coordinate $(0, y, 0)$, the SLW proceeds in a positive x-axis direction and in a negative x-axis direction. Again, since LT models homogeneous space, then this directional behavior of the SLW is translated directly to the coordinate system of the primed frame. However, while the SLW is in between the two origins of the frames, the light postulate in the unprimed frame requires the SLW to move in a positive x-axis direction while the light postulate in the primed frame requires the SLW to move in a negative x-axis direction. Hence, LT maps this positive x-axis motion of the SLW to the coordinate system of the primed frame and this is a different direction of motion compared to the SLW that satisfies the light postulate in the primed frame. Consequently, LT is mathematically incapable of mapping the truth of the light postulate of the unprimed frame to the truth of the light postulate of the primed frame while the SLW is in between the origins. Finally, since the direction of propagation of the LT mapped SLW is different from the SLW which satisfies the light postulate in the primed frame, then they cannot be the same SLW.

One may however, try to argue along the line $y = y_g$ on the interval $0 < x < \frac{\gamma v y_g}{c}$, the frames simply disagree on the order of events, and there is only one SLW, which is an application of the relativity of simultaneity. In particular, along the line $y = y_g$, the light postulate/SLW in the primed frame and the LT mapping of the light postulate/SLW of the unprimed frame produce the same set of 4-D vectors but in reverse order. Then, according to the argument, since the LT output has the time for the primed frame SLW intercepts proceeding backward and time only proceeds forward in the frame by definition, then the frames disagree on the order of events along the line $y = y_g$. More specifically, the frames disagree on the direction the SLW propagates along the line $y = y_g$. Note, this argument does not solve the fact that LT fails to preserve the truth of the light postulate across frames as proven with the derivatives above. Further, this argument requires one to scientifically ignore the ordered output of LT, which is the LT translated behavior of the SLW in the view of the unprimed frame in favor of a different order that is only postulated according to the light postulate in the primed frame. In short, the order logic postulates the direction of motion of the LT translated SLW is wrong making the argument immediately false.

Regardless, this order disagreement argument is easily shown to fail by a general *reductio ad absurdum* argument. To do this, it is assumed SR is in a valid logical state in the standard configuration where a light pulse is emitted from the origins of two frames in relative inertial motion when the origins are co-located. One SLW is assumed and by definition, time only proceeds forward for a frame. Next, assume some unprimed frame space-time coordinate has been acquired by the SLW $A = \left(t_A = \sqrt{x_g^{\ 2} + y_g^{\ 2}} \big/ c, x_g, y_g, 0 \right)$ where $0 < x_g < \frac{\gamma v y_g}{c}$. Both frames agree the SLW is located at this unprimed location since there is only one SLW by assumption. The primed frame space-time coordinate for the SLW located at A is $A' = \left(t'_A = (t_A - v x_g / c^2)\gamma, (x_g - v t_A)\gamma, y_g, 0 \right)$.

At this point, SR is in a valid logical state in that all the rules and conclusions of SR are assumed true. Finally, allow the SLW to propagate further in the context of the unprimed frame by some infinitesimal amount h where $0 < x_g < x_g + h < \frac{\gamma v y_g}{c}$ according to the light postulate in the unprimed frame. It will be shown, this further SLW propagation leaves SR in a logically invalid state in that rules of SR will be contradicted. Note that if SR is logically consistent, then any further propagation of the SLW at c in all directions in a frame should move SR from a logically valid state to a logically valid state given that the propagation of the SLW at c in all directions in a frame is based on the light postulate.

Therefore, in the context of the unprimed frame, the SLW has propagated to the space-time coordinate $B = \left(t_B = \sqrt{(x_g + h)^2 + y_g^{\ 2}} \Big/ c, x_g + h, y_g, 0 \right)$, which by LT, translates to $B' = \left(t'_B = \left(t_B - v(x_g + h)/c^2\right)\gamma, \left((x_g + h) - vt_B\right)\gamma, y_g, 0\right)$ for the primed frame. From the view of the primed frame, after this further SLW propagation, there are three possible cases:

- The LT space-time translation of B' for B is wrong.
- The LT space-time translation of B' for B is completely correct. (This is the standard interpretation). This means that an unprimed frame observer located at B, a primed frame observer located at B' and the SLW are all co-located at the same place. Also, the current elapsed time on the clock for the unprimed observer at B is t_B and the current elapsed time on the clock for the primed observer at B' is t'_B.
- The LT calculated time for a clock at the primed space coordinate for B' is less than the actual time on the primed clock after the SLW propagates and meets the primed space coordinate at B'. This is a hybrid of the relativity of simultaneity argument.

It will be demonstrated for all three cases above that SR is left in a logically invalid state after the further propagation of the SLW.

Case 1: The LT space-time translation for B is wrong.
If case 1 is true, then SR is not in a logically valid state since LT fails and hence, the argument is complete.

Case 2: **The LT space-time translation of B' for B is completely correct for the space-time location of the SLW.**
This case violates at least the four SR requirements listed below.

1. By assumption, time only proceeds forward for the primed frame when considering SLW intercept times. Since the primed frame agrees t'_A was the time on a clock at the initial state for the SLW intercept along the line $y = y_g$ and given the new intercept after the propagation of the SLW along the line $y = y_g$ is B, then the LT translated time is t'_B for B. However, given the derivative $\frac{\partial t'}{\partial t} < 0$ proven above, t'_B translates to a time in the primed frame that is less than t'_A for the SLW intercept. Therefore, if the LT translation for B is correct, then time proceeded backward for the propagating SLW intercepts along the line $y = y_g$ in the context of the primed frame contradicting the fact the time only proceeds forward for the propagating SLW intercepts in the primed frame.
2. At the initial state, the primed frame agrees the SLW was located at the unprimed space coordinate $(x_g, y_g, 0)$. In the view of the unprimed frame, the SLW propagates from $(x_g, y_g, 0)$ to $(x_g + h, y_g, 0)$. Using LT to translate $(x_g + h, y_g, 0)$, the LT translated space coordinate in primed coordinates is closer to the primed origin than is $(x_g, y_g, 0)$. Therefore, at the initial state, the primed frame agrees $(x_g, y_g, 0)$ and the SLW are co-located. Then, further propagation of the SLW occurred in the context of the unprimed frame. If the LT translated space coordinate for $(x_g + h, y_g, 0)$ is correct for the SLW, then after propagation, the SLW is closer to the primed origin than is $(x_g, y_g, 0)$. Therefore, this would imply during the SLW propagation, the space coordinate $(x_g, y_g, 0)$ moved faster than the SLW in the view of the

primed frame since it is further away from the primed origin than is the SLW after the propagation. This contradicts the SR assumption that objects cannot move at or above the speed of light. Therefore, if the LT translated space coordinate for the SLW at $(x_g + h, y_g, 0)$ is correct, the observer at $(x_g, y_g, 0)$ moved faster than the speed of light during the further propagation of the SLW.

3. By assumption, the frames disagree on the direction the SLW propagates along the line $y = y_g$. Hence, in the view of the primed frame, after any further propagation, the SLW must be located in the negative x direction relative to $(x_g, y_g, 0)$ whereas LT translated space coordinate for $(x_g + h, y_g, 0)$ is located in the positive x direction.
4. The LT translated motion for the SLW demonstrates that the SLW moves from A' to B'. If the SLW moves from A' to B' in the view of the primed frame, then the SLW exhibits a pattern that shows it propagating toward the primed origin, which contradicts the light postulate in the primed frame in that the SLW only propagates away from the primed origin. This was shown by the derivative $\frac{\partial x'}{\partial x} > 0$ above with $x > 0$ and $x' < 0$.

Therefore, after the further propagation of the SLW from $(x_g, y_g, 0)$, the unprimed frame places the SLW at $(x_g + h, y_g, 0)$. On the contrary, based on the four statements above, it is impossible for the primed frame to place the SLW at $(x_g + h, y_g, 0)$ after further propagation of the one SLW since the four conditions above demonstrate SR is forced into an invalid logical state. Hence, if the space-time coordinate of B' is the correct space and time coordinates for B after the above described propagation of the SLW, then SR is not in a valid logical state for case 2.

Case 3: The LT calculated time for a clock at the primed space coordinate for B' is less than the actual time on the primed clock after the SLW propagates and meets the primed space coordinate at B'.

This case is the relativity of simultaneity (ROS) argument in that the frames disagree on the order of the two events since $t_A < t_B$, $t'_A > t'_B$ and time proceeds forward in the frame at the intercepts of the SLW by definition. Thus, at the initial state, the primed frame contends that the event at the primed space coordinate of B' has already occurred. That space coordinate for B' is $\left(\left((x_g + h) - v\sqrt{(x_g + h)^2 + y_g^2} / c\right)\gamma, y_g, 0\right)$. Again, time only proceeds forward for the frame, and since the event at B' has already occurred in the view of the primed frame, then the time on the clock at the space coordinate for B' is greater than t'_B at the initial state. Hence, by ROS, the event, at the space coordinate for B' at the initial state, has occurred for the primed frame and has not yet occurred for the unprimed frame. Next, the SLW propagates further according to rules the above. Since the primed frame contends the event at B' has already occurred, $t'_A > t'_B$ and time only proceeds forward in the frame, then this further propagation of the SLW will meet the space coordinate for B' at a time later than t'_B for the clock at that primed position since t'_B already occurred. Otherwise, this further propagation of the SLW in the view of the unprimed frame does result in t'_B being the correct time on that primed clock which is case 2 above and that case has already been shown to result in an invalid logical SR state. Thus, t'_B is not the current time at the space coordinate for B' in the view of the primed frame after the further propagation of the SLW. However, the first problem with this interpretation is that LT calculates the current time on the clock at the space coordinate for B' incorrectly after the SLW propagates. This immediately leaves SR in an invalid logical state because LT does not function correctly by calculating the correct current time on the clock at B' given the location of the SLW in the view of the unprimed frame. In addition, this also implies the clock at the space coordinate for B' was struck at t'_B on the clock and then struck again at a different greater time on the clock after the SLW propagated. A space coordinate can only be struck twice by two SLW's contradicting the fact there is only one SLW. Therefore, case 3 leaves SR in an invalid logical state after further propagation of the SLW from the initial valid SR state.

Consequently, given the above initial valid SR logical state, any further propagation of the SLW in the view of the unprimed frame leaves SR in a logically invalid state.

V. Conclusions

It is a non-negotiable requirement under SR that all properties of the SLW are preserved by LT when mapped from the context of the unprimed frame to the context of the primed frame. Otherwise, given one light pulse, each frame would contain its own unique SLW centered at the light emission point in the frame and all the SLW's would be different from each other which contradicts nature. However, it was shown, for any fixed $y_g > 0$ with $z = 0$, the ordered domain $0 < x < \frac{\gamma v y_g}{c}$, which represents the propagation of the unprimed SLW away from the unprimed origin along the line $y = y_g$, supplied to the LT equation for x', produces a range of x' that consists exclusively of negative numbers and is everywhere increasing. The invariance of the light like space-time interval requires this range of x' to serve as the domain to the equation of the SLW for the primed frame. It was demonstrated, even though the light like space-time interval remains invariant, such an interval of x' does not preserve the propagation of the SLW away from the primed origin as required by the SR relativity principle and light postulate in the primed frame. Therefore, such an interval of x' calculated by LT is invalid under SR.

It was also demonstrated on the interval $0 < x < \frac{\gamma v y_g}{c}$ along the line $y = y_g$ with $z = 0$, that it was impossible for one SLW to propagate away from both the unprimed origin and the primed origin while maintaining the invariance of the light like space time interval. More specifically, based on the restrictions above, if the SLW propagates away from one origin along the line, and the light like space-time interval is invariant, then necessarily, the SLW must propagate closer to the other origin when measured from its intersection with the line $y = y_g$. Since the light postulate mandates that the SLW only propagates away from the light emission point in the frame in all directions at c, then, for these conditions, it is impossible for the light postulate to be true in both frames given only one SLW. Finally, since the LT mapped SLW does not satisfy the light postulate in the primed frame for all cases, then it is not the same as the SLW in the primed frame that does satisfy the light postulate in the primed frame for all cases. Hence, under SR, one light pulse emerges into an infinite number of unique SLW's one for each possible frame.

Finally, it was proven, if two SR frames agree the SLW is located in between the origins of the two frames along some line $y = y_g > 0$ and SR is in a valid logical state, then any further propagation of the SLW along the line $y = y_g > 0$ in the view of the unprimed frame forces SR into logically invalid state since some principle of SR was shown to be contradicted by the further propagation of the SLW.

VI. References

[1] Einstein A., in *The Principle of Relativity* (Dover, New York) 1952, p. 37.