Duality of the quantum competing systems

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Abstract

In superconducting systems, Josephson junction device known as a quantum effect devices which operates by using the **quantum flux** tunneling. Mesoscopic Josephson junction is a quantum effect device which operates by using the single Cooper-pair tunneling created by a **Coulomb blockade**. The **quantum flux** and the **Cooper-pair** is known that the duality relation to each other. On the other hand, in **ferromagnet** system that competes with superconductivity, **spontaneous magnetization** and **domain wall** are known to have a dual relationship.

As shown above, based on dual systems of two types, we considered competing system of two types. One type of these competing system is composed of the sandwich structure by "**supercnductor**-**superinsulator** –**supercnductor**" junction, and its dual junction is consists of the sandwich structure by "**superinsulator** -**supercnductor**- **superinsulator** " junction. Another one type of these system is consists of the sandwich structure by "**supercnductor**-**ferromagnet** –**supercnductor**" junction. and its dual junction is consists of the sandwich structure by "**ferromagnet** -**supercnductor**-**ferromagnet**" junction (**spin Josephson junctions**).

In this paper, we consider mechanisms that lie behind the Junction system to compete with each other, and theoretical conditions to control these devices.

0. Introduction

Recent advances in nano-fabrication technology enables us to confine electric charge or spin in nano-structures such quantum wires or quantum dots. In such systems, by the **strong quantum fluctuations** due to confinement of particles, it cannot be ignore the quantum effects. Thus the effects of quantum fluctuations, in the condensed matter systems of **original particles**, **topological defects** are spontaneously generated. It can be treated as a **dual particles**. The duality of **spontaneous magnetization** and **domain wall** or the electric charge and magnetic flux be become pronounced. Our objective is to build the theory of quantum nano devices where the freedom of a dual particle play an important role, and it is to build the duality theory of competitive systems.

1.1. The Duality Conditions

The original particles field (order field) $\psi(x)$ and the dual particles field (disorder field) $\tilde{\psi}(x)$ are defined by the following relations, respectively.

$$\psi(x) \equiv \sqrt{N_c(x)} \exp\left[i\theta(x)\right] \qquad (1-1a)$$

$$\tilde{\psi}(x) \equiv \sqrt{N_v(x)} \exp\left[i\tilde{\theta}(x)\right] \qquad (1-1b)$$

In the superconducting system, $N_v(x)$, $N_c(x)$ describe number operator of vortex and number operator of Cooper pair respectively, and $\theta(x)$, $\tilde{\theta}(x)$ are phase of Cooper pair and phase of vortex respectively.

Commutation relations between them:

$$\left[N_c(x), \theta(x')\right] = i\delta_{xx'}, \qquad (1-2a)$$

$$\left[N_v(x), \tilde{\theta}(x')\right] = i\delta_{xx'}, \qquad (1-2b)$$

In order to study the various quantum condensed system,We consider the following duality conditions,

$$V(i) = \tilde{I}(i), \qquad (1-3a)$$

$$\tilde{V}(i) = I(i). \qquad (1-3b)$$

Where V and I describe voltage and current of **original particles** respectively, $\tilde{V}$ and $\tilde{I}$ describe voltage and current of **dual particles** respectively.

In this case, From the conditions of eq.(1-3a) and (1-3b) we derived the next two type relationship. Relationship between the phase of Cooper pair $\theta(x)$ and vortex number $N_v(x)$ are as follows

$$N_v(x) = \frac{-1}{2\pi} \sin\theta(x) \qquad (1-4a)$$

Relationship between the phase of vortex field $\tilde{\theta}(x)$ and Cooper pair number $N_c(x)$ are as follows:

$$N_c(x) = \frac{1}{2\pi} \sin\tilde{\theta}(x) \qquad (1-4b)$$

1.2. SC/SI/SC Junction

We propose the single quantum flux device which operates by using the single quantum flux tunneling,The single flux transistor consists of the sandwich structure of the " **supercnductor**- **superinsulator** –**supercnductor** " junction.

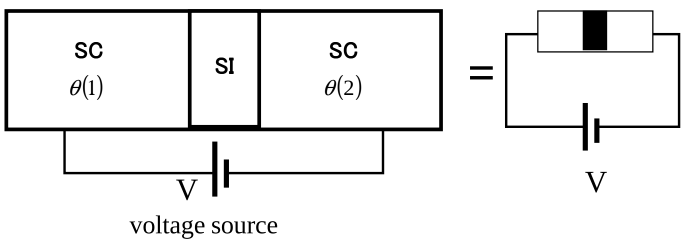

FIG.1 Schematic of superconductor(SC)/superinsulator(SI)/superconductor(SC) junction and its equivalent circuit.

1.3. The relation of original particle (Cooper pair) for SC/SI/SC junction

Hamiltonian: $H_c = 4E_c N_c^{\,2} + E_J\left(1-\cos\theta\right),$ (1−5)

Josephson coupling energy $E_J \equiv \dfrac{\hbar I_c}{2e} > 0,$ Critical current: $I_c,$

Unit Coulomb energy: $E_c \equiv \dfrac{e^2}{2C},$ Total charge $Q \equiv (2e)N_c,$

Josephson's equation

voltage: $$V = \frac{\hbar}{2e}\frac{d\theta}{dt} = \frac{4N_c}{e}E_c, \qquad (1-6a)$$

current: $$I = (2e)\frac{dN_c}{dt} = -I_c\sin\theta. \qquad (1-6b)$$

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1.4. SI/SC/SI Junction

We propose the single electron device which operates by using the single electron tunneling,The single electron transistor consists of the sandwich structure by the "**superinsulator** –**supercnductor**- **superinsulator** " junction.

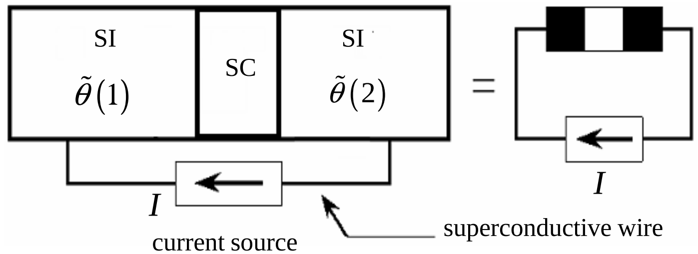

FIG.2 Schematic of superinsulator(SI)/superconductor(SC)/ superinsulator(SI) junction and its equivalent circuit.

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1.5. The reration of dual particle(superinsulator particle density = vortex density) for SI/SC/SI junction

Dual Hamiltonian: $H_v = 2\pi^2 E_J N_v^{\,2} + \dfrac{2E_c}{\pi^2}\left(1-\cos\tilde{\theta}\right),$ (1−7)

Magnetic energy: $E_v \equiv 2\pi^2 E_J = \dfrac{\Phi_0^{\,2}}{2L_c},$ Dual critical current: $\tilde{I}_c = \dfrac{2E_c}{\pi e},$

Critical Inductance: $L_c \equiv \dfrac{\hbar}{2eI_c},$ Total magnetic flux: $\Phi \equiv \Phi_0 N_v$,

Dual Josephson's equation

Dual Junction voltage: $$\tilde{V} = \frac{\hbar}{\Phi_0}\frac{d\tilde{\theta}}{dt} = 2\pi I_c N_v, \qquad (1-8a)$$

Dual Junction current: $$\tilde{I} = \Phi_0\frac{dN_v}{dt} = \tilde{I}_c\sin\tilde{\theta} \qquad (1-8b)$$

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	Cooper pair system	Superinsulator system
Quanta	$q_0 = 2e$	$\Phi_0 = \dfrac{h}{2e}$
Tunnel current	$I = I_c\sin\theta$	$\tilde{I} = \tilde{I}_c\sin\tilde{\theta}$
Phase-number relations	$N_v = -\sin\theta/2\pi$	$N_c = \sin\tilde{\theta}/2\pi$
Phase slip	$V = \dfrac{\hbar}{2e}\dfrac{d\theta}{dt} = \dfrac{4N_c}{e}E_c$	$\tilde{V} = \dfrac{\hbar}{\Phi_0}\dfrac{d\tilde{\theta}}{dt} = 2\pi I_c N_v$
Energy difference for one quanta tunneling	Coulomb blockade $\Delta E_c = \dfrac{2e}{C}(e \pm Q)$	Flux blockade $\Delta E_v = \dfrac{\Phi_0}{L}\left(\dfrac{\Phi_0}{2} \pm \Phi\right)$

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From the duality conditions of eq.(1-3a) and (1-3b) , we derived the quantum resistance by the next two ways method,

Quantum resistance

$$R=\frac{\tilde{I}}{I}=\frac{\Phi_0 \dfrac{dN_v}{dt}}{(2e)\dfrac{dN_c}{dt}}=\frac{h}{(2e)^2}\frac{\Delta N_v}{\Delta N_c}, \qquad (1-9a)$$

$$R=\frac{V}{\tilde{V}}=\frac{\dfrac{\hbar}{2e}\dfrac{d\theta}{dt}}{\dfrac{\hbar}{\Phi_0}\dfrac{d\tilde{\theta}}{dt}}=\frac{h}{(2e)^2}\frac{\Delta\theta}{\Delta\tilde{\theta}}. \qquad (1-9b)$$

(Ⅰ) Case of $\Delta N_v \gg \Delta N_c$ or $\Delta\theta \gg \Delta\tilde{\theta}$

In this extreme case $R \to \infty$:
it becomes **superinsulator** state .

(Ⅱ) Case of $\Delta N_v \cong \Delta N_c$ or $\Delta\theta \cong \Delta\tilde{\theta}$

In this case $R \to R_Q$: where R_Q is ***quantum resistance***,
it becomes **self dual** state.

$$R_Q \equiv \frac{h}{(2e)^2}$$

(Ⅲ) Case of $\Delta N_v \ll \Delta N_c$ or $\Delta\theta \ll \Delta\tilde{\theta}$

In this extreme case $R \to 0$:
it becomes **superconductor** state .

2. FM/SC/FM Junction (Spin Josephson Junction)

We propose the single quantum spin device which operates by using the single quantum spin tunneling,The single spin transistor consists of the sandwich structure of the ferromagnet / supercnductor/ ferromagnet junction.

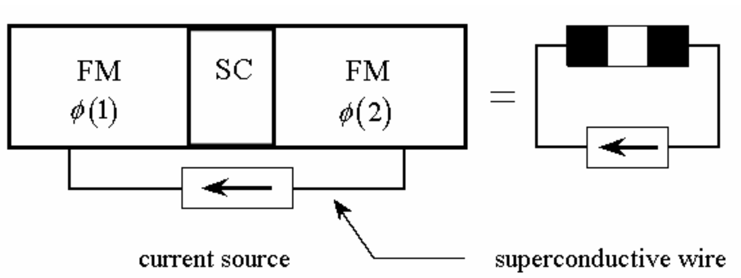

FIG.3 Schematic of ferromagnet(FM)－superconductor(SC)－ferromagnet(FM) junction and its equivalent circuit.

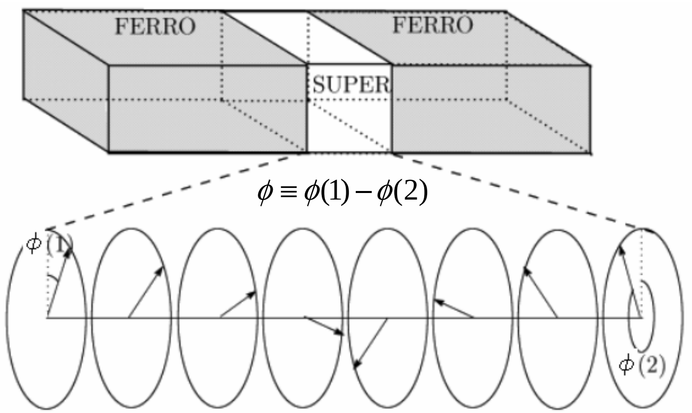

FIG. 4 The superconducting layer play a role as a domain wall.

2.1 The duality of FM/SC/FM junction

From analogy with Josephson junction, FM / SC / FM junction can be thought ferromagnetic junction with a superconducting thin film barrier. In this case, the superconducting thin film role as a **spin capacitor**. As a model for such ferromagnetic junction systems, we consider Hamiltonian of the Heisenberg XXZ spin models as follows:

$$H_{FM} = \sum_{j} J_z \left[S_z(j) \right]^2 - \sum_{\langle i,j \rangle} J_{xy} \left[S_x(i) S_x(j) + S_y(i) S_y(j) \right]. \qquad (2-1)$$

We introduce the Hamiltonian from an analogy of a single Josephson junctions as follows:

$$H_{FM} = E_{xy}^0 N_{XY}{}^2 + E_{xy}\left(1-\cos\phi\right), \qquad (2-2a)$$

$$S_z \equiv S_z^0 \left| N_{XY} \right|, \qquad S_z^0 \equiv \frac{\hbar}{2}. \qquad (2-2b)$$

The first term describe the Ising spin energy of XXZ spin models, here E_{xy}^0 is spin charging energy per single spin :

$$E_{xy}^0 \equiv \frac{\left(S_z^0\right)^2}{2C_s}, \qquad (2-3a)$$

where $C_s \equiv 1/2J_z$ is spin capacitance, The second term in (2-1) describe the XY ferromagnetic spin junction energy, where $\phi \equiv \phi(1) - \phi(2)$ is phase difference across the junction, and E_{xy} is XY ferromagnetic spin junction energy per single-spin that is defined as follows:

$$E_{xy} \equiv J_{xy} S^2 \equiv \frac{I_s^c \hbar}{S_Z^0}, \qquad (2-3b)$$

where $I_s^c \equiv S_z^0 E_{xy} / \hbar$ is critical spin current. Approximate commutation relations between N_{XY} and ϕ as follows:

$$\left[N_{XY}(x), \phi(x') \right] \approx i\delta_{xx'}, \qquad (2-4)$$

Spin Josephson's equation

Spin voltage $$V_s \equiv \frac{\hbar}{S_z^0}\frac{\partial\phi}{\partial t} = \frac{2N_{XY}}{S_z^0} E_{xy}^0, \qquad (2-5a)$$

Spin current $$I_s \equiv \frac{dS_z}{dt} = S_z^0 \frac{dN_{XY}}{dt} = -I_s^c \sin\phi, \qquad (2-5b)$$

The dual Hamiltonian of eq.(2-1) and (2-2a) respectively are given by:

$$H_{DW} = \sum_{j} \tilde{J}_z \left[\tilde{S}_z(j) \right]^2 - \sum_{\langle i,j \rangle} \tilde{J}_{xy} \left[\tilde{S}_x(i) \tilde{S}_x(j) + \tilde{S}_y(i) \tilde{S}_y(j) \right]. \qquad (2-6)$$

$$H_{DW} = \tilde{E}_{xy}^0 N_{DW}{}^2 + \tilde{E}_{xy}\left(1-\cos\tilde{\phi}\right), \qquad (2-7)$$

Where $\tilde{S}_z$, $\tilde{E}_{xy}^0$ and $\tilde{E}_{xy}$ are defined by

$$\tilde{S}_z \equiv \tilde{S}_z^0 \left| \tilde{N}_{XY} \right| \equiv \Phi_{dw}^0 \left| N_{DW} \right|, \qquad \tilde{S}_z^0 \equiv \Phi_{dw}^0 = 4\pi,$$

$$\tilde{E}_{xy}^0 \equiv 2\pi^2 E_{xy}, \qquad \tilde{E}_{xy} \equiv \frac{E_{xy}^0}{2\pi^2}. \qquad (2-8)$$

Dual Spin Josephson's equation

Dual Spin voltage $$\tilde{V}_s \equiv \frac{\hbar}{\Phi_{dw}^0}\frac{\partial\tilde{\phi}}{\partial t} = I_s^c 2\pi N_{DW}, \qquad (2-9a)$$

Dual Spin current $$\tilde{I}_s \equiv -\Phi_{dw}^0 \frac{\partial N_{DW}}{\partial t} = \tilde{I}_s^c \sin\tilde{\phi}. \qquad (2-9b)$$

Also, in this case, From the conditions of eq.(1-3a) and (1-3b) we derived the next two type relationship.

Relationship between the phase of XY-ferromagnet $\phi(x)$ and number density of domain wall $N_{DW}(x)$ are as follows:

$$N_{DW}(x) = \frac{-1}{2\pi}\sin\phi(x), \qquad (2-10a)$$

Relationship between the phase of domain wall $\tilde{\phi}(x)$ and number density of XY-ferromagnet $N_{XY}(x)$ are as follows:

$$N_{XY}(x) = \frac{1}{2\pi}\sin\tilde{\phi}(x), \qquad (2-10b)$$

Approximate commutation relations between N_{DW} and $\tilde{\phi}$ as follows:

$$\left[N_{DW}(x), \tilde{\phi}(x')\right] \approx -i\delta_{xx'}. \qquad (2-11)$$

	Ferromagnetic system	Domain wall system
Quanta	$S_z^0 = \hbar/2$	$\Phi_{dw}^0 = 4\pi$
Tunnel current	$I_s = -I_s^c \sin\phi$	$\tilde{I}_s = \tilde{I}_s^c \sin\tilde{\phi}$
Phase-number relations	$N_{DW}(x) = \frac{-1}{2\pi}\sin\phi(x)$	$N_{XY}(x) = \frac{1}{2\pi}\sin\tilde{\phi}(x)$
Phase slip	$V_s \equiv \frac{\hbar}{S_z^0}\frac{\partial\phi}{\partial t} = \frac{2N_{XY}}{S_z^0}E_{xy}^0$	$\tilde{V}_s \equiv \frac{\hbar}{\Phi_{dw}^0}\frac{\partial\tilde{\phi}}{\partial t} = I_s^c 2\pi N_{DW}$
Energy difference for one quanta tunneling	$\Delta E_{FM} = 2J_z S_0^z\left(S^z \pm \frac{S_0^z}{2}\right)^2$	$\Delta E_{xy} = \frac{1}{2\tilde{L}_c^s}\Phi_{dw}^0\left(\Phi_{dw} \pm \frac{\Phi_{dw}^0}{2}\right)^2$

From the duality conditions of eq.(1-3a) and (1-3b) , we derived the **quantum spin resistance** by the next two ways method,

Quantum spin resistance

$$R_s = \frac{\tilde{I}_s}{I_s} = \frac{-\Phi_{dw}^0 \frac{\partial N_{DW}}{\partial t}}{S_z^0 \frac{dN_{XY}}{dt}} = \frac{-8\pi}{\hbar}\frac{\Delta N_{DW}}{\Delta N_{XY}}, \qquad (2-12a)$$

$$R_s = \frac{V_s}{\tilde{V}_s} = \frac{\frac{\hbar}{S_z^0}\frac{\partial\phi}{\partial t}}{\frac{\hbar}{\Phi_{dw}^0}\frac{\partial\tilde{\phi}}{\partial t}} = \frac{8\pi}{\hbar}\frac{\Delta\phi}{\Delta\tilde{\phi}}. \qquad (2-12b)$$

(I) Case of $\Delta N_{DW} \gg \Delta N_{XY}$ or $\Delta\phi \gg \Delta\tilde{\phi}$

In this extreme case $R_s \to \infty$:
it becomes **super** **spin** **insulator** state .

(II) Case of $\Delta N_{DW} \cong \Delta N_{XY}$ or $\Delta\phi \cong \Delta\tilde{\phi}$

In this case $R_s \to R_s^Q$:
where R_s^Q is ***quantum spin resistance***,
it becomes **self dual** state.

$$R_s^Q \equiv \frac{8\pi}{\hbar}$$

(III) Case of $\Delta N_{DW} \ll \Delta N_c$ or $\Delta\phi \ll \Delta\tilde{\phi}$

In this extreme case $R_s \to 0$:
it becomes **super** **spin** **conductor** state .

3. The quantum spin transistor

Now, using the following assumptions $\phi \propto \tilde{\theta}$ and $\tilde{\phi} \propto -\theta$,

Spin voltage:
$$V_s = \frac{2\pi}{e} I = 2\pi f_s, \qquad (3-1a)$$

Spin current:
$$I_s = \frac{e}{2\pi} V , \qquad (3-1b)$$

Where, f_s is the single-electron-tunneling oscilltions frequency.

In FIG.6 as a application of FM / SC / FM junction, we have devised a spin transistor, where L is inductance of FM / SC / FM junction defined by

$$L \equiv \frac{2\pi^2}{J_z e^2}, \qquad (3-2)$$

L_1 and L_2 are inductance of each junction1 and junction2 respectively,

I_g is current of gate current source, L_g is inductance of gate current source.

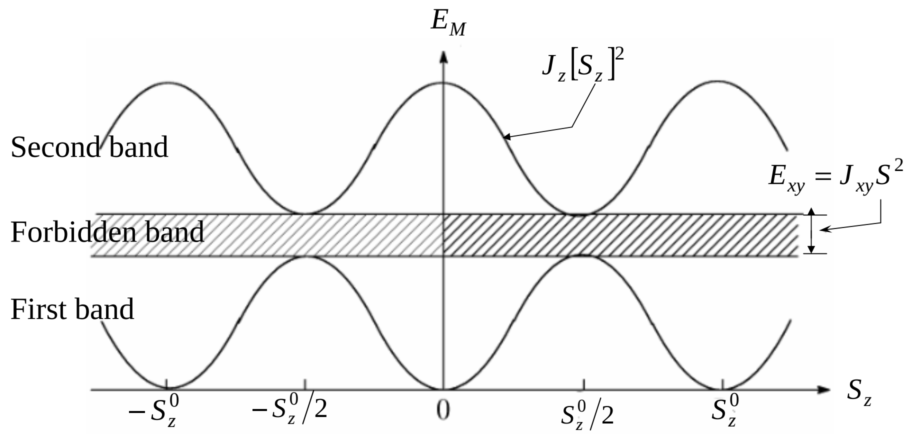

FIG.5. The energy band to the ferromagnetic system.

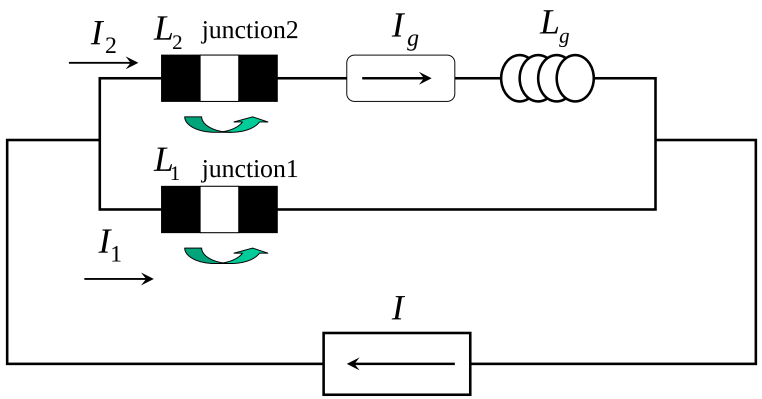

FIG.6 Schematic of quantum spin transistor

In each junction1 and junction2, forbidden condition of one quantum spin tunneling is given by the following equation respectively,

$$I = \frac{1}{e\left(L_2^s + L_g^s\right)}\left[\pm \pi S_z^0 + 2\pi S_z^0 (N_1 - N_2) + eL_g I_g\right], \qquad (3-3a)$$

$$I = \frac{1}{eL_1^s}\left[\pm \pi S_z^0 - 2\pi S_z^0 (N_1 - N_2) - eL_g I_g\right]. \qquad (3-3b)$$

The above conditions, FIG.7 shows the operating characteristics of FM / SC / FM junction .

Area inside the diamond, spin tunneling is blocked.

This means the **<u>” real ” spin blockade</u>.**

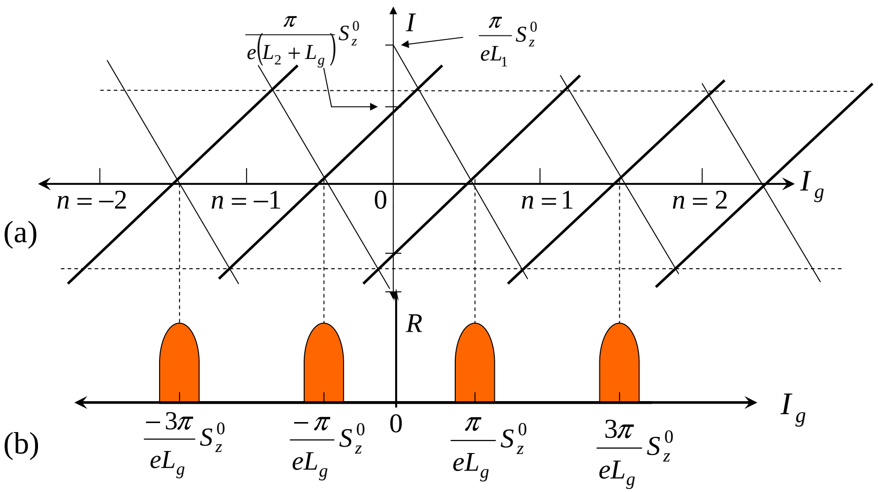

FIG.7 (a) Current I as a function of I_g .
(b) Resistans R as a function I_g of for quantum spin transistor .

4. The Superconductor – superinsulator(or ferromagnetic) network systems of compete with each other

Until the previous chapter, we had been dealt with models of **<u>single junctions</u>** in order to simplify the problem. In this chapter, we had extended the theory from single junctions to **<u>periodic quantum dot network systems</u>**. Such a system, we considered **<u>four type</u>** competing systems.

Two of those systems are superconducting dots systems (as shown in FIG.8(a))in superinsulator (or ferromagnetic) host.

The remaining two systems are ferromagnetic(or superinsulator) dots systems (as shown in FIG.8(b))in superconducting host.

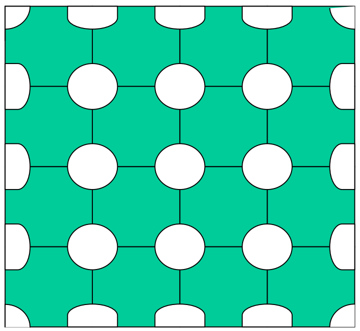

FIG.8(a) Superconducting dots in superinsulator (or ferromagnetic) host.

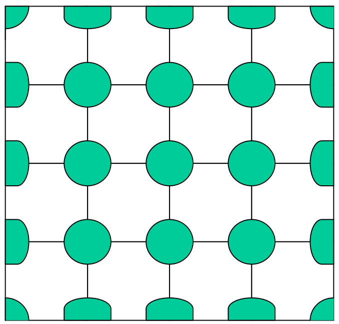

FIG.8(b) Ferromagnetic (or superinsulator) dots in superconducting host

4.1 The Josephson network systems

The Hamiltonian(space 2d+imaginary time) of Josephson network systems (lattice systems) by superinsulator dots in superconducting host is written as,

$$H_c(\theta,N_c)=2e\sum_{x,\tau}V(x,\tau)N_c(x,\tau)+4E_c\sum_{x}\left[N_c(x,\tau)\right]^2$$
$$+E_J\sum_{x,j}\left[1-\cos\left(\nabla_j\theta-q\alpha_j\right)(x,\tau)\right]+\frac{\sigma}{2\mu}\sum_{x,\tau}\left[\varepsilon^{ijk}\bar{\nabla}_j\alpha_k(x,\tau)\right]^2, \qquad (4-1)$$
$$q\equiv\frac{2\pi}{\Phi_0}[\mathrm{C}],\quad \alpha_j(x,\tau)[\mathrm{Wb}]\equiv -iA_j(x,\tau)\Delta x_j,$$
$$\nabla_j\theta(x,\tau)\equiv\theta(x+j,\tau)-\theta(x,\tau),$$

where A_j is vector potential, α_j is vector potential in the phase, μ is magnetic permeability, $\sigma\left[\mathrm{m}^{-3}\right]$ is inverse volume parameters

4.2 The dual Josephson network systems

The Hamiltonian(space 2d+imaginary time) of dual Josephson network systems (lattice systems) by superconducting dots in superinsulator host is written as,

$$H_v\left(\tilde{\theta},N_v\right)=\Phi_0\sum_{x,\tau}I(x,\tau)N_v(x,\tau)+2\pi^2E_J\sum_j\left[N_v(x,\tau)\right]^2$$
$$+\frac{2}{\pi^2}E_c\sum_{x,j}\left[1-\cos\left(\nabla_j\tilde{\theta}-\tilde{q}\tilde{\alpha}_j\right)(x,\tau)\right]+\frac{\sigma}{2\varepsilon}\sum_{x,\tau}\left[\varepsilon^{ijk}\overline{\nabla}_j\tilde{\alpha}_k(x,\tau)\right]^2, \qquad (4-2)$$
$$\tilde{q}\equiv\frac{2\pi}{2e}[\mathrm{Wb}], \qquad \tilde{\alpha}_j(x,\tau)[\mathrm{C}]\equiv -i\tilde{A}_j(x,\tau)\Delta x_j,$$

where I is electric current, $\tilde{A}_j[C/m]$ is dual vector potential, $\tilde{\alpha}_j$ is dual vector potential in the phase, ε is permittivity .

4.3 The spin Josephson network systems

The Hamiltonian of spin Josephson network systems by superconducting dots in ferromagnetic host is written as,

$$H_{FM}\left(\phi,N_{XY}\right)=S_z^0\sum_{x,\tau}V_s(x,\tau)N_{XY}(x,\tau)+4E_c\sum_x\left[N_{XY}(x,\tau)\right]^2$$
$$+E_{xy}\sum_{x,j}\left[1-\cos\left(\nabla_j\phi-q_s m_j\right)(x,\tau)\right]+\frac{\sigma}{2\mu_s}\sum_{x,\tau}\left[\varepsilon^{ijk}\overline{\nabla}_j m_k(x,\tau)\right]^2, \qquad (4-3)$$
$$q_s\equiv\frac{2\pi}{\Phi_{dw}^0}=\frac{1}{2}, \qquad m_j(x,\tau)\equiv -iM_j(x,\tau)\Delta x_j,$$

where $V_s\left[s^{-1}\right]$ is spin voltage, $M_j\left[\mathrm{m}^{-1}\right]$ is spin vector potential, m_j is vector potential in the phase, $\mu_s\left[\mathrm{J}^{-1}\cdot\mathrm{m}^{-1}\right]$ is spin permeability, $\sigma\left[\mathrm{m}^{-3}\right]$ is inverse volume parameters .

4.4 The dual spin Josephson network systems

The Hamiltonian of dual spin Josephson network systems by ferromagnetic dots in superconducting host is written as,

$$H_{DW}\left(\tilde{\phi},N_{DW}\right)=\Phi_{dw}^0\sum_{x,\tau}I_s(x,\tau)N_{DW}(x,\tau)+2\pi^2E_{xy}\sum_x\left[N_{DW}(x,\tau)\right]^2$$
$$+\frac{E_Z}{2\pi^2}\sum_{x,j}\left[1-\cos\left(\nabla_j\tilde{\phi}-\tilde{q}_s\tilde{m}_j\right)(x,\tau)\right]+\frac{\sigma}{2\varepsilon_s}\sum_{x,\tau}\left[\varepsilon^{ijk}\overline{\nabla}_j\tilde{m}_k(x,\tau)\right]^2, \qquad (4-4)$$
$$\tilde{q}_s\equiv\frac{2\pi}{S_z^0}=\frac{4\pi}{\hbar}, \qquad \tilde{m}_j(x,\tau)\equiv -i\tilde{M}_j(x,\tau)\Delta x_j,$$

where $I_s[\mathrm{J}]$ is spin current, $\tilde{M}_j[\mathrm{J\cdot s/m}]$ is dual spin vector potential, $\tilde{m}_j[\mathrm{J\cdot s}]$ is vector potential in the phase, $\varepsilon_s\left[\mathrm{J\cdot s^2\cdot m^{-1}}\right]$ is spin permittivity.

5 The dual transformations

Between of eq.(4-1) and eq.(4-1) or eq.(4-3) and eq.(4-3) respectively, the Villain approximation to the XY model can be mutually dual transformation. Here, as an example the dual transformation from eq.(4-1) to eq.(4-2) introduce our algorithm.

5.1 The anisotropic space- time XY model

We rewrote the Hamiltonian of eq.(4-1) , we have derived Lagrangian of the anisotropic space-time XY model. as follows:

$$L_{AXY}\left(\theta,\alpha_j\right)=\frac{-\sigma}{2\mu}\sum_{x,x'}\sum_{j=1}^{3}\alpha_j(x,\tau)\left[-\delta_{jm}\overline{\nabla}_l\nabla_l+\overline{\nabla}_m\nabla_j\right]\alpha_m(x,\tau)$$
$$-E_J^0\sum_x\left[1-\cos\left(\nabla_\tau\theta-q\alpha_0\right)(x,\tau)\right]-E_J\sum_x\sum_{j=1}^{2}\left[1-\cos\left(\nabla_j\theta-q\alpha_j\right)(x,\tau)\right], \qquad (5-1)$$
$$E_J^0\equiv\frac{\hbar^2}{8E_c(\Delta\tau)^2}, \qquad \Delta\tau\equiv f_J^{-1}, \qquad \omega_J=2\pi f_J=\frac{2eV}{\hbar}, \qquad (5-2)$$

where f_J is Josephson frequency.

5.2 The order field (GL) theory by mean field approximation

$$L_{GL}\left(\psi,\psi^*,\alpha_j\right)\cong\frac{-\sigma}{2\mu}\sum_{x,x'}\sum_{j=1}^{3}\alpha_j(x,\tau)\left[-\delta_{jm}\bar{\nabla}_l\nabla_l+\bar{\nabla}_m\nabla_j\right]\alpha_m(x,\tau)$$

$$+\sum_x\left\{\frac{\kappa_1}{8d}\left|D_j\psi(x,\tau)\right|^2+\frac{\kappa_2}{8}\left|D_0\psi(x,\tau)\right|^2+\frac{1}{4}\left(\frac{1}{d'E_J}-1\right)\left|\psi(x,\tau)\right|^2+\frac{1}{64}\left|\psi(x,\tau)\right|^4\right\},\qquad(5-3)$$

where D_j and D_0 are covariant difference defined by

$$D_j\psi(x,\tau)\equiv\psi\left(x+\mathbf{e}_j,\tau\right)\exp\left[-iq\alpha_j(x,\tau)\right]-\psi(x,\tau),\qquad j=0,1,2$$

where d' , κ_1 and κ_2 are defined by

$$d'\equiv d+E_J^0/E_J,\qquad \kappa_1\equiv\frac{E_Jd}{E_J^0+E_Jd},\qquad \kappa_2\equiv\frac{E_J^0}{E_J^0+E_Jd},\qquad(5-4)$$

where d=2 is space dimension.

5.3 The dual anisotropic space- time XY model

Applying the Villain approximation to the partition function of eq .(5-1),then, by applying the dual transformation, we derived the dual anisotropic space-time XY model.

$$L_{DAXY}\left(\tilde{\theta},\tilde{\alpha}_j\right)\cong\frac{-1}{2}\left(\frac{\tilde{q}}{2\pi}\right)^2\sum_{x,x'}\tilde{\alpha}_i(x,\tau)\mathbf{V}'^{-1}_m(x-x')\tilde{\alpha}_j(x',\tau')$$

$$-\gamma_0\sum_x\cos\left[\nabla_0\tilde{\theta}(x,\tau)-\tilde{q}\tilde{\alpha}_0(x,\tau)\right]-\gamma\sum_x\sum_{j=1}^{2}\cos\left[\nabla_j\tilde{\theta}(x,\tau)-\tilde{q}\tilde{\alpha}_j(x,\tau)\right],\qquad(5-5)$$

$$\mathbf{V}_m^{-1}(x-x')\equiv\left[-\delta_{ij}\bar{\nabla}'_l\nabla'_l+m_{ij}\right],\qquad \gamma\cong\frac{2(\Delta\tau)^2}{\pi^2\hbar^2\mathbf{V}_m(0)}E_c,\qquad \gamma_0\simeq\frac{1}{(2\pi)^2E_J\mathbf{V}_m(0)},$$

In eq. (5-5), if you ignore the massive gauge field, eq. (4-2) and eq. (5-5) are approximately matched.

5.4 The disorder field theory by mean field approximation

Disorder field theory by the disorder parameter , We was derived the dual representation for eq(5-5).

$$L_{DAXY}\left(\tilde{\psi},\tilde{\psi}^*,\tilde{\alpha}_j\right)\cong\frac{-1}{2}\left(\frac{\tilde{q}}{2\pi}\right)^2\sum_{x,x'}\tilde{\alpha}_i(x)\mathbf{V}'^{-1}_m(x-x')\tilde{\alpha}_j(x')$$

$$+\sum_x\left\{\frac{\tilde{\kappa}_1}{8d}\left|\tilde{D}_j\tilde{\psi}(x,\tau)\right|^2+\frac{\tilde{\kappa}_2}{8}\left|\tilde{D}_0\tilde{\psi}(x,\tau)\right|^2+\frac{1}{4}\left(\frac{1}{d'\gamma}-1\right)\left|\tilde{\psi}(x,\tau)\right|^2+\frac{1}{64}\left|\tilde{\psi}(x,\tau)\right|^4\right\},\qquad(5-6)$$

where $\tilde{D}_j$ and $\tilde{D}_0$ are covariant difference defined by

$$\tilde{D}_j\tilde{\psi}(x,\tau)\equiv\tilde{\psi}\left(x+\mathbf{e}_j,\tau\right)\exp\left[-i\tilde{q}\tilde{\alpha}_j(x,\tau)\right]-\tilde{\psi}(x,\tau),\qquad j=0,1,2$$

where d' , $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ are defined by

$$d'\equiv d+\gamma_0/\gamma,\qquad \tilde{\kappa}_1\equiv\frac{\gamma d}{\gamma_0+\gamma d},\qquad \tilde{\kappa}_2\equiv\frac{\gamma_0}{\gamma_0+\gamma d}.\qquad(5-7)$$

6. Summary and Conclusion

1. We considered competing dual junction systems of two types, One of them is S/I/S (or I/S/I) junction, Another one is S/F/S (or F/S/F) junction.
2. On competing dual junction systems of two types, we derived **four types Josephson equations** in each system.
3. In analogy of coulomb blockade, we devised quantum **spin transistor** consists of parallel connection of F/S/F junction.
4. On the quantum spin transistor, we have derive the **forbidden condition** of single spin tunneling.
5. We derived Ginzburg-Landau field theory and dual Ginzburg-Landau field theory in quantum competing system

7. References

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