This book is a collection of interrelated papers. It takes its name from the main paper which is an interim report that tries to find the cracks in the fundaments of physics and that suggests repair procedures

This leads to unconventional solutions and a new model of Physics:

The Hilbert book model.

On the origin of physical dynamics

Report of a research project

Hans van Leunen

The Hilbert book model

ON THE ORIGIN OF PHYSICAL DYNAMICS

Ir J.A.J. van Leunen

ON THE ORIGIN OF PHYSICAL DYNAMICS

The main article is a report of a running research project.
That project started in 2009.

The continuing status of the project can be followed at
http://www.crypts-of-physics.eu
Software related subjects are treated at
http://www.scitech.nl/MyENHomepage.htm#software
The author's e-print site is:
http://vixra.org/author/Ir_J_A_J_van_Leunen .

*The nice thing about laws of physics is that they repeat themselves. Otherwise
they would not be noticed. The task of physicists is to notice the repetition.*

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Preface

This book contains a collection of interrelated papers. The title of the book equals the title of the main paper. The main paper (part 2) describes a research project whose target it is to uncover the origin of dynamics. As a starting action, cracks in the fundaments of physics are detected and suggestions are given for the repair of these deficiencies (part 1).

Part 3 concerns the origin of physical fields. It is essential for understanding why quantum logic must be extended.

Part 4 seeks the origin of mass in the presence geo-cavities. In that way minuscule geo-cavities may replace Higgs particles in bringing mass to elementary particles.

The Hilbert book model (part 10) is the name of a comprehensible paper that describes the new fundament in simple wording.

The last paper (part 11) describes in the form of a fairytale how the universe works.

The other parts have a lesser connection with the main subject of the book.

The paper that describes how the brain works (part 5) gives information on how the visual trajectory of vertebrates optimizes the perception of low light level signals. It indicates how this system during a billion years has helped vertebrates to survive low light level conditions. The observed radiation is generated by Poisson processes. The conclusion is that information comes to us in the form of clouds of quanta, rather than in waves.

The paper that introduces a new law of nature (part 6) explains that nature has a built-in tendency to reduce complexity via the procedure of modularization. As a result it establishes the construction of very complicated items that include intelligent species.

Two other papers (part 7 and 8) describe what happens when the merits of this new law are neglected and how that annoying situation can be cured.

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The next paper (part 9) treats the relation between physics and religion. The last two papers are light weight descriptions of the main subject.

The new model of physics that is introduced in this book is called "The Hilbert book model". It holds strictly to its fundament, which is traditional quantum logic. It extends that basic model such that physical fields and dynamics also fit. Despite its simplicity the model explains a large part of the results of current physics.

Logic model

It is always possible to build a potentially usable theory and a corresponding model by starting from a consistent set of axioms. Classical logic represents such a set. It is a theory on itself. However, in the realm of quantum physics nature cheats with one of the axioms of classical logic. So it is sensible to adapt classical logic and change the corresponding axiom such that this adapted logic fits better. The adapted version of the theory and the corresponding model build on this new logic. The adapted logic is known as traditional quantum logic.

Traditional quantum logic and as a consequence the new model still do not fit most of the features that we know from physical phenomena. The reason is that the new model cannot handle physical fields and it cannot treat dynamics. So, as a next step, the traditional quantum logic must be extended to a new version of quantum logic that can cope with fields. The trick required for this extension blurs a subset of the propositions. The blur represents the sticky resistance of these propositions against change. It can also be explained as a stochastically inaccurate coupling to the value domain of these propositions.

In practice this extension is achieved via the isomorphic companion of traditional quantum logic, which is the set of closed subspaces of an infinite dimensional separable Hilbert space for which the inner product is defined by the elements of the division ring of the quaternions. The new logic does not yet have a generally accepted name. So, we leave the name at extended quantum logic. The Hilbert space enables the application of mathematics.

Still the extended quantum logic and its isomorphic companion can only handle static situations. Thus the obtained model is not a dynamic model. This situation can be cured by taking a sequence of these extended quantum logics such that each subsequent element represents a static status quo of the dynamic universe that the final model is aimed to describe.

The result is called the Hilbert book model. In this model the progression is made in universe wide steps.

The attached fields are quaternionic probability amplitude distributions. The sign flavors of these fields will be used to explain the large diversity of particles that occur in nature.

The gravitation field will be treated as a descriptor of the local curvature rather than as the cause of that curvature. In this way a local geometric anomaly can also act as the cause of curvature.

Due to its foundation on adapted and extended classical logic and by including guidance from physical concepts the resulting model can be considered as an abstraction of physical reality.

Part one

Cracks of Fundamental Quantum Physics

Cracks of fundamental quantum physics

Abstract

The fundaments of quantum physics are still not well established. This paper tries to find the cracks in these fundaments and suggests repair procedures. This leads to unconventional solutions and a new model of physics. One of the innovations is the derivation of a curvature field from the cause of the curvature. The most revolutionary introduction is the representation of dynamics by a sequence of separable Hilbert spaces. Together, this embodies a repair of fundaments that does not affect the building.

History

In its first years the development of quantum physics occurred violently [1]. As a consequence some cracks sneaked into the fundaments of this branch of physics. A careful investigation brings these cracks to the foreground. The endeavor to repair these cracks delivers remarkable results.

In the early days of quantum physics much attention was given to equations of motion that were corrections of classical equations of motion. The Schrödinger approach was one and the Heisenberg approach was another. Schrödinger used a picture in which the state of a particle changes with time. The operators that act on these states are static. Heisenberg uses a picture in which the operators change with time, but the states are static. For the observables this difference in approach has no consequences. This fact is important. It shows that time is just a parameter

instead that it acts as a property of physical items[1]. Later Garret Birkhoff, John von Neumann and Constantin Piron found a more solid foundation that was based on quantum logic. They showed that the set of propositions of this logic is isomorphic with the set of closed subspaces of an infinite dimensional separable Hilbert space, whose inner product is defined with the numbers taken from a division ring. The ring can be the real numbers, the complex numbers or the quaternions. Since then many physicists do their quantum physics in the realm of a Hilbert space. However, the Hilbert space has no operator that delivers eigenvalues for parameter time.

Cracks in the fundaments

Fist scratches

These physicists quickly encountered the obstinate character of the separable Hilbert space. Its normal operators have countable eigenspaces. This can still correspond to a dense coverage of the corresponding hyper complex number space. However, this eigenspace is no continuum. Thus, functions defined using these eigenspaces as parameter domains cannot be differentiated. In order to cope with this defect, most physicists resorted to other types of Hilbert spaces than the separable Hilbert space, but in doing so they neglect that in this way the stringent relation with quantum logic gets broken.

Severe defects

Further, it appears that the separable Hilbert space cannot represent physical fields and cannot represent dynamics. This is a severe drawback and it looks as if the switch to the other Hilbert spaces becomes mandatory. For example quantum field theory represents fields as operators that reside in a non-separable Hilbert space. In this paper the

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[1] Later this fact is used in order to apply the progression step counter as a parameter that characterizes the members of a sequence of Hilbert spaces.

strategy is to hold strictly to the link with traditional quantum logic. So the road that is taken by quantum field theory is not followed.

Back to the future

On the other hand there are more and more signals that nature is fundamentally granular and the non-separable Hilbert spaces do not provide that feature. This guides backwards to the separable Hilbert space. But in that case we must learn to live with this separability. In addition we must find other ways to represent fields.

Dynamic way out

The non-separable Hilbert spaces including the rigged Hilbert space gave similar problems with representing dynamics as the separable Hilbert space does. There is no place for **time** as an eigenvalue of an operator neither in separable Hilbert space nor in the other Hilbert spaces. For that reason, it is better to accept that the separable Hilbert space can only represent a static status quo.

Granularity

Nature is fundamentally granular. The so called Planck units [2] are designed using dimension analyses, but it is generally accepted that below these limits (Planck-length, Planck-time, Planck constant = unit of action and Boltzmann's constant = unit of entropy) no discerning observation is possible. One could say that below these limits nature does not exist or that nature just steps over these regions. The Planck-length and Planck-time are related to the Planck constant, the speed of light c and the gravitational constant G. It is not said that nature's granularities have exactly these sizes. The Planck units are derived by dimensional analysis. The Planck unit sizes rather form an order of magnitude indication, but these measures are useful and we do not have a better estimate. The mutual relation between these units is important. For example, the ratio between the Planck-length and the Planck-time equals

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[2] http://en.wikipedia.org/wiki/Planck_units

the speed of light c. If you reckon that at every time step a physical item can at the utmost take one space step, then the maximum speed of all physical items is automatically set at the speed c.

This paper will not exploit the fact that eigenspaces are granular. We will rather start from the assumption that the eigenspace is not a continuum.

Coping with granularity

A solution must be found for the fact that GPS-like normal operators in separable Hilbert space do not possess continuum eigenspaces. A GPS operator with a granular eigenspace would have a lattice-like eigenspace of densely packed granules. The lattice would possess preferred directions. This does not correspond with physical reality. Such situations can occur in condensed matter, but that is an exceptional condition.

A dense packaging of granules may occur in horizons. For example, horizons of black holes appear to be covered by a dense package of granules.

Apart from these exceptions the exclusion holds for any multidimensional subset of eigenvalues, even if it contains a countable number of values that are taken from a continuum.

This consideration means that it is impossible to define in the separable Hilbert space a granular operator that acts like a proper global positioning system (GPS), which is required in the positioning of field values or when we want to relate Hilbert vectors with position.

The separable Hilbert space can provide a GPS-like operator that offers a dense coordinate system as its eigenspace. An eigenspace consisting from all rational quaternionic numbers would be countable and thus it can be an eigenspace of a normal operator in separable Hilbert space.

However this eigenspace is no continuum and as a consequence it does not support differentiation.

We can still maintain that the set of positions is a set of granules that have the size of the order of the Planck-length, which is $1.6 \cdot 10^{-35}$ m. However, this set does not have densely packed subsets that have a dimension larger than one.

The required separability is special. It is a granularity of differences rather than a granularity of values. This might guide the way to a solution.

Background coordinates

The separable Hilbert space $\mathbb{H}$ is connected with its Gelfand triple $\mathbb{H}$, which is called a rigged Hilbert space. The Gelfand triple is not a regular Hilbert space. In fact the rigged Hilbert space $\mathbb{H}$ is only named after its generating member $\mathbb{H}$.

A background coordinate system exists in rigged Hilbert space as the eigenspace of a GPS-like operator $\breve{Q}$, but it cannot be directly used in the separable Hilbert space in order to locate Hilbert vectors in a regular way. So, we must find an indirect way. This is delivered by the strand operator $\mathbb{Q}$[3], which resides in separable Hilbert space $\mathbb{H}$ and has an equivalent $\breve{\mathbb{Q}}$ in the rigged Hilbert space $\mathbb{H}$. There it can be **coupled** to the background coordinate system. Apart from horizons, the eigenspace of the strand operator does not contain multidimensional sets of eigenvalues. Instead, it contains chains of granules. Thus, in separable Hilbert space it avoids the mentioned problems.

Strand operator

The strand operator only makes sense for localizable particles. Pure plane wave "particles" are not localizable along the direction of the wave, but

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[3] The name strand operator is related to the strand model of Christoph Schiller that brought me the idea to use chains as a solution.

spherically oscillating particles are localizable. In a similar sense a wave package may be localizable.

The mentioned coupling between the eigenspace of the strand operator $\mathbb{Q}$ in separable Hilbert space and the eigenspace of the GPS-like operator $\check{Q}$ in the rigged Hilbert space is not precise. It cannot be so. It is like the situation that the number of observations in an experiment overwhelms the number of underlying variables. The usual way to solve such a situation is to suppose the presence of a stochastic inaccuracy. The observations are supposed to be blurred. The blur makes the coupling to the underlying variables inaccurate. Their spread of the observations has a minimal value. So the observations can be seen as granules.

Nature solves this problem in a similar way. However, it does not use a simple probability distribution. Nature gives the inaccuracy of the coupling the form of a quaternionic probability amplitude distribution (QPAD). The squared modulus of the QPAD is a probability density distribution. The real values of the QPAD can be interpreted as a charge density distribution and its imaginary value will then be the corresponding current density distribution. The eigenvector of $\mathbb{Q}$ that belongs to the resulting eigenvalue acts as the anchor point of the distribution. The charge represents a load of properties of the item for which the eigenvector provides the location. The granule can be considered as the "ground state" of the QPAD.

When the past and the future of the eigenvalues are kept in sight, then the eigenspace of the strand operator contains a set of chains that are put together from granules. In the chain the granules are ordered. In each chain one granule is exceptional. We call it the **current granule**. The part of the chain that ends just before the current granule is called the past sub-chain. The part that starts just after the current granule is the future sub-chain.

One could ask whether having only the current granule could be sufficient. For the model, the direct neighborhood of the current granule is the most relevant part of the chain. The rest of the chain is hardly used. It only gives a reflection of a possible past and a possible future which is derived from the current field configuration. However, the step to the next version of the "current granule" is taken inside the chain. **At a given progression step maximally one space step is allowed.** When that step is taken, then on the average that step has the size of one granule.

The step to the next granule is controlled by a probability density distribution (PDD). The extent of this PDD is set by the properties of the stochastic coupling between the background coordinate system and the position of the granule. **In its minimal format** the stochastic coupling has characteristics that to a certain extend are similar to the characteristics of the ground state of a quantum harmonic oscillator. This minimal extent is of the order of the Planck-length. However, the shape of the probability density distribution must be such that it is zero in a region of the size of the Planck length. This is why the granules appear to have a basic size of the order of the Planck-length and seem to be surrounded by a nonzero QPAD that can take a wider extent. The quantum harmonic oscillator is only mentioned as an example. The actual form of the wider extent of the QPAD may depend on the particle type. It depends on the characteristics of the particle that makes use of this granule as its anchor point. Due to the analogy we will call the central part of the QPAD its ground state.

At each actual step a space analog to the space covered by the ground state is inaccessible. Nature steps over this space and lands in the middle of a new current granule.

One might ask why this restriction exists. The reason must be sought in the combination of **stochastic inaccuracy** with the **atomicity** of quantum logic. This restriction goes further than countability.

Chains can split and they can merge. The corresponding creation and annihilation occurs during a progression step and is controlled by QPAD's that are attached to the current granules.

The chains in the eigenspace of the strand operator are causal chains.

Statistics

The QPAD is a constituent of the field that surrounds the granule. The creation and annihilation operators of fields have eigenfunctions that are Poisson distributions. Such distributions are produced by Poisson processes. A Poisson process can be combined with a subsequent binomial process in order to form a generalized Poisson process that has a lower efficiency than the original Poisson process. The efficiency is weakened by the weakening that is introduced by the binomial process. The spatial spread introduced by the QPAD can be interpreted as a binomial process with a spatially varying weakening factor. The spread function is equal to the squared modulus of the QPAD.

Canonical conjugate

Depending on the type of the particle that anchors on the granule there may be many types of QPAD's. Near the anchor point the basic shape of the QPAD's are all equal. Apart from a factor (1, i, -1 or $-i$) they are invariant under Fourier transformation. This means that near the anchor point the eigenspace of the canonical conjugate of the strand operator has the same basic format as the eigenspace of strand operator. It also anchors on similar granules.

Strand space

The strand operator has an outer horizon. Outside this horizon its eigenspace does not contain granules. It might also have inner horizons such that inside these inner horizons no granules exist.

Most inner horizons are borders of black holes. These horizons are bubbles that consist of densely packed granules. The QPAD's that are attached to these granules have taken their minimal possible size. Each

granule is connected to a Hilbert vector which is eigenvector of the strand operator. That Hilbert vector represents a quantum logical proposition. It carries a single bit of information that indicates its membership of the eigenspace of the strand operator. The inner horizons form an exception to the rule that the granules must not form a multidimensional subset.

Other horizons

Since light transports all information and has a limited speed, the eigenspace of the strand operator may feature information horizons. Every object in space has its own private information horizon. This horizon is in fact the image of a start horizon that occurred at the start of the universe. The start horizon is a special kind of inner horizon that was at the same time an outer horizon. It can be interpreted as a bubble that existed in empty space and that suddenly converted into matter[4]. From that moment the granules that formed this special horizon spread over space and their QPAD folds out, such that it takes more space than just the size of the granule. This occurrence must be unique or its probability must be very low. There is no indication that it happened more often during the lifetime of the universe.

Affine space

Since the unit sphere of the separable Hilbert space is an affine space and all eigenvectors of the strand operator are represented in that space, the strand operator can be considered to have no origin or the origin is just arbitrarily selected. The same consideration holds for the GPS-like operator in the rigged Hilbert space.

Types of chains

The chains may be closed or they start and end at a horizon. Further they may split and merge. This corresponds with creation and annihilation of particles that anchor on these chains. Actually, only the direct environment of the current granule of the chain is relevant. The granules

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[4] See Birth of the universe

in short closed chains may represent the anchors of virtual particles. These granules are virtual granules.

The generation and annihilation of particles occurs for example in field configurations that are locally invariant under Fourier transformation, such as linear and spherical harmonics.

The chains have things in common with the strands in Schiller's strand model. However, they are not the same.

Vacuum

The inaccuracy in the coupling between the background coordinate system and the granules also plays a role in the space where no current granules exist. In this space virtual granules may exist during a very short period, for example during a single progression step. These virtual granules form the content of vacuum.

Virtual granules only occur inside the outer horizon and outside the inner horizons of the strand operator. The virtual granules can be interpreted as the ground state of harmonic oscillators. This ground state corresponds with the minimal extend that the QPAD can take. The vacuum is supposed to have constant density ρ_{vac} of virtual granules.

In the Hilbert book model the space between horizons is supposed to be stochastically, but on the average uniformly covered with virtual granules. At every progression step these virtual granules are redistributed. The actual granules exist in between these virtual granules, but they possess a wider spread of the corresponding QPAD's. These wider QPAD's tend to last longer at (nearly) the same location.

Fields

Fields do not fit inside a separable Hilbert space. Any field would cover the whole Hilbert space. Every Hilbert vector would touch a value of the

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field. Which value is touched, depends on the functionality of this vector. When the vector is one of the eigenvectors of a normal operator and when the field can be expressed as a function of the eigenvalues of this operator taken as the parameter of the function, then the field value would correspond with the parameter value that equals the eigenvalue that corresponds to eigenvector. In that case, the considered field value will be connected to the considered vector.

In superposition, field values may compensate each other. That is possible when they have opposite sign.

Function of the field

The function of the physical fields is to take care of minimizing changes during dynamical steps. This function becomes evident when [dynamics](#) is implemented. Fields keep the shape of the chains of the strand operator smooth. In first instance the private fields influence the chain at their anchor point. Due to their extent, the fields also influence other chains.

Basic field constituent

A QPAD that is attached to the current granule takes care of the fact that the chain in the neighborhood of the current granule stays sufficiently smooth. This becomes important when dynamics is implemented because with each dynamic step the current granule either stays at its current position or it moves one place ahead in the chain.

It must be noticed that exactly this restriction is the reason why speed has a maximum! The ratio of the space step and the time step equals the speed of light.

The squared modulus of the QPAD is a probability density distribution (PDD). It determines the probability of the position of the current granule. The probability is large when the position is close to the position of the previous current granule.

Via its wave function a particle is identified with its private field. (They are one and the same thing). The notion of private field transfers quantum theory into quantum field theory.

Fields influence the chain

The private fields overlap and because they are all QPAD's their superposition causes an interaction between the particles that anchor on these fields.

Taken over a sequence of dynamic steps, the chain appears to fluctuate. The fluctuation determines the probability distribution and vice versa the dynamic changes of the probability density distribution determine the fluctuations of the chains. This relation is instantaneous. There is no causal relation. (The granules are ground states of field constituents).

If the chains would be observable, then the probability distribution could be determined by averaging the fluctuations over some period. However, neither the chains, nor the probability amplitude function are directly observable items. Only their effects become observable.

Particles

The Hilbert book model leaves open whether depending on its type, an elementary particle relates to one or more of these chains. In any way the current granules of these chains are related to the current section of the path of the particle.

Elementary particles can be identified by an ordered pair of coupled sign flavors of the same field. That field forms the private field of the particle. Four some particles the coupling factor is zero. The switch from one sign flavor to the other sign flavor can be considered as the charge of the field. The sign flavors determine the kind of charge that is involved. A continuity equation[5] describes the dynamics. In this equation the first member acts as the transported part of the field. The second member of the pair acts as source/drain. The event and location of the sign flavor switch is the observable of the field. It will be perceived as a quantum.

For massive particles the location of the sign flavor switch may go together with the location of a local spherical geometric anomaly (SGA).

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[5] Part two; Hilbert field equations; Continuity equation for charges

Strands, curvature, torsion and chirality

The idea to attach more than one strand to a particle is taken from **Christoph Schiller's strand model**[2].

In contrast to torsion, curvature relates to mass. For example, **according to Schiller's strand model**, the strand that represents a massless photon has a helix shape. The strands that represent the massive W bosons have the shape of an overhand knot. Since this knot shows chirality, it possesses electric charge. The strands that represent the massive Z bosons have the shape of a figure eight knot. Because the figure eight knot features no handedness, it does not possess electric charge. In a similar way Christoph Schiller attributes properties to all elementary particles.

The Hilbert book model does not use the strand concept of Schiller's strand model. Strands and chains are both one dimensional and both interact with fields. The Hilbert book model relates particles to ordered pairs of field sign flavors. The sign flavors decide how the particles are charged. That is how far the resemblance of the two models goes.

Extended Hilbert space

The addition of QPAD's to the Hilbert vectors that are attached to the current granules extends the separable Hilbert space to a new construct. For that reason we call this new construct an **extended separable Hilbert space**.

Extended quantum logic

Via the relation between the separable Hilbert space and traditional quantum logic we can extend quantum logic to an extended quantum logic that includes physical fields in a similar way as the extended separable Hilbert space model does. It means that a subset of the propositions is afflicted with a stochastic inaccuracy that can be characterized by a probability amplitude distribution.

Covering field

The QPAD that is connected to the current granule is a basic field constituent. The superposition of all these basic constituents forms a covering field. With respect to the dynamics of the picture, it must be reckoned that the elementary particles form a combination of two sign flavors of the same field in which one sign flavor acts as the transported part and the other acts as the source/drain part. Apart from that, the configuration of the covering field depends on the configuration of the elementary particles. When the configuration of chains changes, then the configuration of particles changes and the covering field changes accordingly.

Curvature field

According to Helmholtz decomposition theorem, the static version of the covering field decomposes into a rotation free part and one or two divergence free parts. The local decomposition depends on the local field configuration and in general it does not run along straight coordinate lines. The local decomposition into a one dimensional longitudinal part and a transverse part defines a local curvature. This curvature can be used to define a local metric. This metric is a tensor and on its turn it can be used to define a derived tensor field. We will call this the curvature field. It has all aspects of the gravitation field. When split back into curvature fields that are private to the particles the private curvature field can be used to attach the property "mass" to the corresponding particle.

What is curvature?

In order to comprehend quantum physics, it is sometimes sensible to step one dimension down. Optics is in many respects similar in 2D to quantum physics in 3D.

When optics is studied, then it is often done by following the live path of a point object. This can be done by ray tracing and it can be done by applying Fourier optics. When the quality of imaging equipment must be specified in an objective way, then it is often done in terms of the Optical

Transfer Function[6] (OTF). The OTF is defined as the Fourier transform of the 2D spatial spread of the point object. This definition supposes the presence of a projection surface. In practice the analyzed area is kept rather small. Further the energy contained in the point image is insufficient to activate the measuring equipment. For that reason the measurement is done by analyzing the image of a short thin slit object. Provided that the point image is spatially invariant in the area of the slit object, the analyzed image is the convolution of the Point Spread Function[7] (PSF) and the slit object. After taking the Fourier transform the analyzed image is the product of the OTF and the Fourier transform of the slit object. This last function is a two dimensional sinc function that extends in the direction across the slit in which the slit is small and is thin in the direction in the direction along the slit. The result corresponds closely to a vertical 1D cut through the OTF. When the PSF is rotationally symmetric, then the result is independent of the direction of the slit. The Modulation Transfer Function is the modulus of the MTF. Any vertical cut through the MTF is symmetric. Thus when the PSF is not rotationally symmetric usually two measurements of the MTF are specified. The first result is the one with maximal extent and the other has minimal extent. When the imaging system is a rotationally symmetric lens, then on-axis the PSF is rotationally symmetric, but off-axis the Seidel aberrations take their toll and the PSF is no longer rotationally symmetric. In that case a radial (longitudinal) OTF and a lateral (transverse) OTF are specified.

We only traced one ray. Actual images are constituted of the combined PSF's of an extended object. In this way the PSF is a constituent of a scalar field. **The divergence and the curl of that scalar field form a vector field**. According to Helmholtz theorem the vector field can be split in a rotation free component and a divergence free component. In the above situation these components are the longitudinal and the transverse components.

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[6] http://en.wikipedia.org/wiki/Optical_transfer_function
[7] http://en.wikipedia.org/wiki/Point_spread_function

Now exchange the lens against an arbitrary but smoothly shaped glass body. The direction of the longitudinal component no longer runs along a straight line. The curvature of the decomposition defines a local curvature. This 2D situation looks more like the situation that we have in 3D quantum physics.

In short: In optics the actual field configuration corresponds to a curvature of the coordinate system in which the PSF is spatially invariant.

Geometric anomalies

Spherical geometric anomalies (SGA's) are regions of coordinate space that are not occupied by physical objects and that are surrounded by a horizon such that information cannot enter that region. This means that the local curvature is such that information carrying particles cannot enter the region. Large black holes are examples of such geometric anomalies. However, such SGA's may also occur at very small sizes. Inside the region the value of the probability density distribution (PDD) of any particle is zero. The PDD is the modulus squared of the QPAD. If we take the covering field, then its modulus squared is zero inside a SGA. The center of an SGA acts as a center of virtual mass.

Combining the sources of curvature

Thus two sources of local curvature exist. One source is located in the local curvature that is due to the configuration of the covering field. The other source is located in the existence of local SGA's. We can also see the SGA's as special forms of local field configurations.

About the field concept

It is common practice to treat the EM fields and the gravitation field as different and independent subjects. In this interpretation, the gravitation field generates the curvature of the coordinate system in which the other fields must operate.

The Hilbert book model takes a different approach. It puts the reason for the curvature of the coordinates in the properties and configuration of the

covering field. This includes the existence of local SGA's. The curvature that exists in this way is used to derive the total curvature field. On its turn the curvature field determines the values and locations of actual or virtual masses. The wave function is also interpreted as a constituent of the covering field. In this way it also contributes to the curvature field. This picture unifies all fields.

The QPAD's can be seen as a reflection of the stochastic inaccuracy of the coupling between the eigenspace of strand operator and the eigenspace of the GPS-like operator that resides in rigged Hilbert space and acts as background coordinate system. In the same way the curvature field can be seen as an administrator of the deficiency of this coupling as is marked by the local curvature.

The start horizon

With this concept of the curvature field the field configuration near the origin of the expanding universe can be interpreted as to be generated completely by the curvature that corresponds with the local geometry. This curvature determines the field values of the local curvature field. This curvature field corresponds to a virtual mass that represents the influence of that local geometry. This virtual mass does not correspond to the presence of actual matter. It just represents the particular geometry that is present near the origin of the universe.

In the Hilbert book model the universe starts with a bubble shaped horizon, which is at the same time an inner horizon and an outer horizon. This start horizon[8] consists completely out of densely packed granules. At the start the size of these granules is quite large. These granules do not represent ground states of a corresponding QPAD. They represent a much higher state. Like the ground state this state offers the capability to form bubbles. The start horizon is instable. Its granules collapse into a new format whose size is many orders of magnitude smaller. As a

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[8]The idea of the existence of a start horizon is a speculation. Inner and outer horizons exist.

consequence the space that was taken by the start horizon gets filled with a diffused set of the smaller granules that can move around freely. As a consequence, part of these granules recombines into new smaller bubbles. These smaller bubbles are black holes. These new inner horizons contain a lower amount of granules and the granules are much smaller than in the start horizon. They represent the ground state of the QPAD. Other granules form loosely connected assemblies. Still others keep moving free. For the free and loosely packed granules the QPAD's unfold. This unfolding results in a large multitude of different private field types.

The result of this procedure is that the original geometry converts partly into matter. Some of it converted back into geometry. This happening is then the start of a new expanding universe. However, after this first implosion the expansion can be described as a metric expansion.

This also indicates what happens when a large mass collapses into a black hole. The matter disappears and converts into a strongly curved geometry.

The most important message is that the geometry determines the curvature field, rather than that the curvature field determines the geometry. This can go so far that the geometry not only determines a virtual mass, but under the proper circumstance it can also generate actual matter that corresponds to the virtual mass. What happens during the collapse of a large mass into a black hole is not only the generation of the horizon. It is also the folding together of the private fields that existed in the surround of the anchor points until they reach their smallest possible extent.

What also becomes clear is that the configuration of the anchor points in combination with the type of the private fields determines the curvature of the geometry.

In this picture the gravitation field only acts as an administrator. The real actors are the Hilbert vectors that correspond to the anchor points and the corresponding private fields.

Canonical coordinates

We start with the situation in which we can select ideal coordinates. What that means will become clear soon.

Ideal coordinates

The inner product of the Hilbert space can be used to relate two orthogonal bases that are each-other's canonical conjugate. In a quaternionic Hilbert space this is not a straightforward procedure. Luckier wise, the quaternionic number space can be divided into a series of complex number spaces. We just chose one imaginary direction and do as if we are in complex Hilbert space. However, this singles out that particular direction. We may choose the direction in which the local longitudinal direction of the covering field runs. The definition of longitudinal is in fact taken in the canonical coordinate space of the current configuration space. It can be any radial direction taken from the origin of that space. This may give problems when the original configuration space is curved, thus when the longitudinal direction changes with location.

The fact that space is curved follows from the fact that when this space is covered with shapes that should all have the same form; the form of the shape in fact changes with the location of that shape[9].

For the moment we assume that we have selected a coordinate system for which the selected longitudinal field direction runs along a straight line and stays that way. We do not bother about granularity, because we will base our investigations on fields that are specified using a continuum background Global Positioning System coordinates. In Fourier spaces we

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[9] See: What is curvature.

need the corresponding Global Momentum System coordinates. So we pick the eigenspace of a normal GPS-like operator $\breve{Q}$ that resides in rigged Hilbert space as our coordinate system. It has an equivalent GPS-like coordinate operator Q in separable Hilbert space whose eigenspace lays dense in the eigenspace of the rigged Hilbert space GPS. The operator $\breve{Q}$ is selected such that the selected longitudinal direction of the field runs along one of the imaginary base vectors of the eigenspace. The set of eigenvectors $\{|q>\}$ of operator Q forms an inner product with another normal operator P which is the canonical conjugate of Q. The eigenvector $|q>$ corresponds to an eigenvalue q and similarly the eigenvector $|p>$ of P corresponds to an eigenvalue p. The inner products are now given by:

$$< p|q > = \exp(\mathbf{i}\hbar qp) \tag{1}$$

The constant h in $\hbar = 2\pi h$ is Planck's constant. The imaginary 3D base vector $\mathbf{i}$ of the quaternionic number space is the imaginary base number of the selected complex number space.

This procedure can be performed for the two operators and three mutually perpendicular imaginary base vectors of the eigenspace. We have defined the procedure for the operators P and Q that reside in separable Hilbert space, but with respect to its application to Fourier transforms, it makes also sense for the equivalent operators $\breve{P}$ and $\breve{Q}$ in rigged Hilbert space. Their eigenspaces form a continuum.

Fourier transform

It can easily be seen that the specified inner product also defines a complex Fourier transform. We start with the separable Hilbert space. By taking all three dimensions the specified inner product defines the imaginary part of a quaternionic Fourier transform.

$$< q|f > = < f|q >^* = f^*(q) = \sum_{p}(< q|p >\cdot< p|f >) \tag{1}$$

And reversely:

$$< p|f > = \tilde{f}(p) = \sum_{q}(< p|q > \cdot < q|f >)$$ (2)

It must be reckoned that these are discrete transforms. Here the **Hilbert function**

$$f(q) = < f|q >$$ (3)

is a sampled function and is transformed in formula (2) into its Fourier partner $\tilde{f}(p)$.
In rigged Hilbert space the sum becomes an integral.

Use of the Fourier transform

In separable Hilbert space, Hilbert functions are sampled functions and are constructed from the eigenvectors and eigenvalues of a normal operator and a selected Hilbert vector. See formula (3).
The discrete transform and the Hilbert functions do not have many usages. In practice the Fourier transform is applied to <u>Hilbert fields</u>[10] rather than to Hilbert functions.
The Fourier transform of a quaternionic field must be performed with a quaternionic Fourier transform that acts in a continuous number space [3].
The Fourier transformation of <u>a private field</u>[11] of a particle does two things. It shifts from a GPS-like coordinate system to its canonical conjugate GMS-like coordinate system. Apart from that it transforms the private field from a quantum cloud into a wave package. This new probability distribution tells about momentum rather than about position.

Fourier transform habits

A Fourier transform keeps inner products invariant. Thus it is a unitary transformation. It has no eigenvectors and as a consequence it has no

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[10] Distributions and fields: Hilbert fields
[11] Distributions and fields: Hilbert fields: Private field

eigenvalues. However, in rigged Hilbert space functions exist that apart from a multiplication factor are invariant under Fourier transformation. Examples of these are the functions that describe linear and spherical quantum harmonic oscillators. The multiplication factor can be 1, i, -1, or -i.

In separable Hilbert space, the Fourier transform converts an orthogonal base into another orthogonal base, which is completely distinct from the original base. Any member of the second base is a linear combination of all members of the first base. The modulus of all coefficients in this linear combination is equal to unity. In rigged Hilbert space the function exp (i p q ℏ) and the Dirac delta function δ(q) form Fourier transform pairs. In separable Hilbert space the Kronecker delta replaces Dirac's delta function.

The existence of canonical conjugation is the reason of the weakening of the modular law that makes the difference between classical logic and quantum logic.

A very important property of Fourier transforms is that it transforms a differentiation into a multiplication with the canonical conjugated coordinate. This only works in rigged Hilbert space. In the Hilbert book model it is applied to Hilbert fields[12].

Actual coordinates

In practice the ideal conditions are seldom valid and if they are valid, they are only valid locally and with reduced accuracy. It means that the inner product that defines the canonical conjugate has only local validity and the same holds for the Fourier transforms that are specified with the aid of that inner product.

In actual situations depending on the field coordinates the coordinate system gets curved locally. Only an appropriate coordinate transformation can bring us back to the ideal situation. This is a purely mathematical activity and the required transform changes with the field

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[12] Distributions and fields: Hilbert fields

configuration that resides in the current static status quo. It does not affect physical reality. So if we know how to perform this coordinate transformation then physics in this static status quo becomes trivial. This is the reason why particles move along geodesics. However, in another static status quo the field configuration will be different. This requires a separate coordinate transformation for every static status quo. The alternative is that we accept a curved coordinate system.

The presented picture supposes that nowhere the field excitations are so violently that it becomes impossible to define a local curvature.

Coherent state

A **coherent state** is a specific kind of state of the quantum harmonic oscillator whose dynamics most closely resemble the oscillating behavior of a classical harmonic oscillator system.

The coherent state $|\alpha>$ is defined to be the 'right' eigenstate of the annihilation operator $\mathcal{A}$. Formally, this reads:

$$|\mathcal{A}\,\alpha>=\ \alpha|\alpha> \tag{1}$$

Since $\mathcal{A}$ is not Hermitian, α is a hyper complex number that is not necessarily real, and can be represented as

$$\alpha = |\alpha|\exp(k\,\theta) \tag{2}$$

where
$\qquad \theta$ is a real number.
$\qquad |\alpha|$ is the amplitude and
$\qquad \theta$ is the phase of state $|\alpha>$.

This formula means that a coherent state is left unchanged by the annihilation or the creation of a particle. The eigenstate of the annihilation

operator has a [Poissonian](http://en.wikipedia.org/wiki/Poissonian)[13] number distribution. A Poisson distribution is a necessary and sufficient condition that all annihilations are statistically independent. (Shot noise is characterized by a Poisson distribution. See [information detection](.).)

The coherent state's location in the complex plane ([phase space](.)[14]) is centered at the position and momentum of a classical oscillator of the same phase θ and amplitude. As the phase increases the coherent state circles the origin and the corresponding disk neither distorts nor spreads. The disc represents Heisenberg's uncertainty. This is the most similar a quantum state can be to a single point in phase space.

Distributions and fields

The concepts that have been introduced so far invite the introduction of Hilbert distributions and Hilbert fields.

Hilbert distributions

Hilbert distributions are sets of Hilbert vectors, in which each vector corresponds to the current granule of a member of a set of chains. Thus, these vectors are eigenvectors of the strand operator in the current Hilbert space. All past and future granules in a chain correspond with a Hilbert vector in their corresponding Hilbert spaces, but the vectors of a Hilbert distribution correspond with the corresponding current granule, thus with a Hilbert vector in the current Hilbert space.

Also the granules that compose a horizon are eigenvectors of the strand operator. An **elementary Hilbert distribution** is a set of Hilbert vectors that belong to an elementary particle.

[13] http://en.wikipedia.org/wiki/Poissonian

[14] http://en.wikipedia.org/wiki/Phase_space

Hilbert field

A Hilbert field is a superposition of the QPAD's that are attached to the Hilbert vectors in a Hilbert distribution. In principle all Hilbert distributions are Hilbert fields.

A **private Hilbert field** is a Hilbert field that belongs to an elementary Hilbert distribution. However, if a complicated particle consists of a set of elementary particles, then we consider the superposition of the private fields of the elementary particles as the private field of the complicated particle. The Hilbert field is a skew field. The Hilbert book model only considers Hilbert fields whose values are taken from a division ring.

The covering field is the superposition of all private fields. It is a Hilbert field

Optics and quantum physics

If all QPAD's would be similar, then the Hilbert field can be considered as the convolution of this QPAD and a distribution of Dirac delta functions that correspond to the Hilbert distribution. This picture resembles (ideal) ray optics and if we take the Fourier transform then it resembles (ideal) Fourier optics. This is the reason that wave mechanics has so much similarity with optics. The characterization "ideal" indicates the restriction that all blurs are equal. In practical optics the blurs are not equal and change with position in the image surface. In quantum physics the same happens[15], but the blur may also change with the type and properties of the particle.

The Optical Transfer Function characterizes the information transfer capability of an imaging system. In the image plane this OTF has only a local validity and it changes with the angular and chromatic characteristics of the light beam. Also the phase homogeneity of the light plays an important role. In similar way the Fourier transform of the

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[15] See: What is curvature

QPAD characterizes the information transfer capability of a physical system.

Nothing is said yet about detecting the information that is carried by the particles. That will be treated [later](#)[16].

Dynamics

The extended separable Hilbert space model can only represent a **static status quo**. By using this ingredient, dynamics can be implemented by a model that consists of an ordered sequence of such extended Hilbert spaces. It corresponds to an equivalent sequence of extended quantum logics.

In order to give this model a name, we can call it the **Hilbert book model**. Passing through the sequence is like glancing through a book, where each page describes a static status quo.

The chains of the strand operator pass through a range of Hilbert book pages. A loop must be interpreted as a pair of chains that split at the start and merge at the end. The split and the merge occur between pages.

What is important is that each static status quo holds both the current state and ALL preconditions for the next static status quo. Thus in principle the duration of the progression between subsequent static status quos is unimportant. The Hilbert book model takes all progression steps to be of equal length.

Spacetime

This procedure introduces a new parameter that acts as a global progression step counter. This parameter must not be confused with our common notion of time, which only has local validity.

The dynamic model implies that space is not the only granular quantity. It also means that progression occurs in discrete steps. Further, it indicates that against general acceptance, fundamentally, space and progression

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[16] Information detection

have little to do with each other. With other words, no support exists for a fundamental physical spacetime quantity.

That does not say that no relation between the fundamental space step and the fundamental time step can exist. The Minkowski signature is a clear prove of such relation. It can already be understood from the ratio between the Planck-length and the Planck-time. A further more complex relation is set by the properties of space and the properties of the displacement group.

When the smallest possible space step

$$l_{Pl} = \sqrt{\hbar G/c^3} \qquad (1)$$

and the smallest possible coordinate time step

$$t_{Pl} = \sqrt{\hbar G/c^5} \qquad (2)$$

are put into the Minkowski signature,

$$\Delta\tau^2 = \Delta t^2 - \Delta q^2/c^2 \qquad (3)$$

then the corresponding proper time step $\Delta\tau$ is zero.

The number of Planck-time steps equals the number of global progression steps. The number of Planck-length steps must always be equal to or lower than the number of Planck-time steps. A free photon never takes a non-zero Δt step. The number of its space steps always equals the number of its time steps.

Any particle that does not travel with light speed skips some of its space steps. Any particle can take a space step in a direction that differs from the direction of a previous step.

Relativity

Wiki states: "One Planck-time is the time it would take a photon travelling at the speed of light to cross a distance equal to one Planck-length. Theoretically, this is the smallest time measurement that will ever

be possible, roughly 10^{-43} seconds. Within the framework of the laws of physics as we understand them today, for times less than one Planck-time apart, we can neither measure nor detect any change."

Nothing occurs in that period. It is as if universe does not exist in that period. Nature just steps over this period. The steps need not be exactly equal to the Planck units, but they have the same order of magnitude. In the model these steps are taken in synchrony. This follows from the fact that a separable Hilbert space can only represent a static status quo. It also holds for a Hilbert space that is extended with static fields. In the Hilbert book model dynamics is implemented via universe wide progression steps. A progression step occurs when an extended Hilbert space is followed by a subsequent extended Hilbert space.

The origin of the existence of the space step follows from the inaccuracy of the coupling between the strand operator and the GPS operator. It shares this origin with the existence of physical fields.

The Hilbert book model uses the concept that the state of the universe can be considered as a sequence of static status quos. With respect to Einstein's special relativity this might at first sight seem an odd idea. This holds especially with respect to the relativity of simultaneity. However, as will be shown[17], special relativity perfectly fits the Hilbert book model.

The unit sphere of the Hilbert space is an affine space. It houses all unit length eigenvectors. This also holds for the eigenvectors of the position operator. This means that between two realizations of the Hilbert space the eigenvector that corresponds to the origin of position can be freely selected. Or with other words the origin of position can be selected freely.

Differences between positions in subsequent members of the sequence of extended separable Hilbert spaces can be interpreted as displacements.

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[17] See "On the origin of physical dynamics; Dynamics; Relativity

The displacement is a coordinate transformation. For the properties of this transformation it does not matter where the displacement starts or in which direction it is taken.

The same holds for displacements that concern sequence members that are located further apart. The corresponding displacements form a group. The displacement is a function of both the position and the sequence number. The displacement $z, t \rightarrow z', t'$ can be interpreted as a coordinate transformation and can be described by a matrix.

$$\begin{bmatrix} t' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & \delta \\ \beta & \alpha \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} \qquad (1)$$

The matrix elements are interrelated. When the displacement concerns a uniform movement, the interrelations of the matrix elements become a function of the speed v. The group properties together with the isomorphism of space fix the interrelations.

$$\begin{bmatrix} t' \\ z' \end{bmatrix} = 1/\sqrt{1 + kv^2} \begin{bmatrix} 1 & kv \\ -v & 1 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} \qquad (2)$$

If k is positive, then there may be transformations with $kv^2 \gg 1$ which transform time into a spatial coordinate and vice versa. This is considered to be unphysical. The Hilbert book model also supports that vision.

The condition k = 0 corresponds to a Galilean transformation

$$\begin{bmatrix} t' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -v & 1 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} \qquad (3)$$

The condition $k < 0$ corresponds to a Lorentz transformation. We can set $kc^2 = -1$, where c is an invariant speed that corresponds to the maximum of v.

$$\qquad (4)$$

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$$\begin{bmatrix} t' \\ z' \end{bmatrix} = 1/\sqrt{1 - v^2/c^2} \begin{bmatrix} 1 & -v/c^2 \\ -v & 1 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix}$$

The Lorentz transformation corresponds with the situation in which a maximum speed occurs.

Since in each progression step photons step with a non-zero space step and both step sizes are fixed, the speed of the photon at microscopic scale is fixed. No other particle goes faster, so in the model a maximum speed occurs. With other words when sequence members at different sequence number are compared, then the corresponding displacements can be described by Lorentz transformations.

Lorentz transformations introduce the phenomena that go together with relativity, such as length contraction, time dilatation and relativity of simultaneity that occur when two inertial reference frames are considered.

$$\Delta t_c = \left(\Delta t_p - \Delta z_p \, v/c^2\right)/\sqrt{1 - v^2/c^2} \tag{5}$$

The term $\Delta z_p \, v/c^2$ introduces time dilatation. If $\Delta t_p = 0$ then depending on v and Δz_p the time difference Δt_c is non-zero.

These phenomena occur in the Hilbert book model when different members of the sequence of Hilbert spaces are compared. Usually the inertial frames are spread over a range of Hilbert book pages.
Since the members of the sequence represent static status quos, the relativity of simultaneity restricts the selection of the inertial frames. Only one of the inertial frames can be situated completely in a single member of the sequence. In that case the other must be taken from a range of sequence elements.

It means that when proper time is taken to be directly related with the progression parameter, thus when the corresponding inertial frame is

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fully located in a single sequence member, then coordinate time must differ from the progression parameter.

Continuity equations

All equations of motion are in fact continuity equations that treat the local information generation, annihilation and transfer.

Total change within V = flow into V + production inside V   (1)

$$\frac{d}{dt} \int_V \rho_0 \, dV = \oint_S \hat{\boldsymbol{n}} \rho_0 \frac{\boldsymbol{v}}{c} \, dS + \int_V s_0 \, dV \tag{2}$$

$$\int_V \nabla_0 \rho_0 \, dV = \int_V \langle \boldsymbol{\nabla}, \boldsymbol{\rho} \rangle \, dV + \int_V s_0 \, dV \tag{3}$$

Here $\hat{\boldsymbol{n}}$ is the normal vector pointing outward the surrounding surface S, $\boldsymbol{v}(q)$ is the velocity at which the charge density $\rho_0(q)$ enters volume V and s_0 is the source density inside V. $\boldsymbol{\rho}$ stands for $\rho_0 \boldsymbol{v}/c$.
The combination of c and $\boldsymbol{\rho}(q)$ is a quaternionic skew field $\rho(q)$ and can be seen as a probability amplitude distribution (QPAD).

$$\rho = \rho_0 + \boldsymbol{\rho} \tag{3}$$

$\rho(q)\rho^*(q)$ can be seen as a probability density distribution (PDD). Depending on their sign selection, quaternions come in four sign flavors. In a QPAD the quaternion sign flavors do not mix. So, there are four QPAD sign flavors. They differ in one or more signs of their imaginary base vectors.

$$\psi^{\textcircled{0}}, \psi^{\textcircled{1}}, \psi^{\textcircled{2}} \text{ and } \psi^{\textcircled{3}}; \;\; \psi = \psi^{\textcircled{0}}; \; \psi^* = \psi^{\textcircled{3}} \tag{4}$$

The field ψ is supposed to have the same sign flavor as the non-curved background coordinate system.

These sign flavors can combine in coupled pairs. Ordered coupled pairs characterize elementary particles. A continuity equation describes the distribution of the sign flavor switch.

The field $\rho(q)$ contains information on the distribution $\rho_0(q)$ of the considered charge density as well as on the current density $\boldsymbol{\rho}(q)$, which represents the transport of this charge density.

Where $\rho(q)\rho^*(q)$ can be seen as a probability density of finding the center of charge at position q, the probability density distribution $\tilde{\rho}(p)\tilde{\rho}^*(p)$ can be seen as the probability density of finding the center of the corresponding wave package at location p. $\tilde{\rho}(p)$ is the Fourier transform of $\rho(q)$.

The two independent sign selections of quaternions lead to four different field sign flavors. In the equations below the field sign flavors ψ^x and ψ^y can be any of $\psi^{\textcircled{0}}, \psi^{\textcircled{1}}, \psi^{\textcircled{2}}$ or $\psi^{\textcircled{3}}$. The numbers indicate the number of imaginary base factors that differ with respect to the local coordinate system. In its most basic form the continuity equation that describes the dynamics of the charges of elementary particles is given by:

$$\nabla\psi^x = m\,\psi^y \tag{5}$$

For the antiparticle:

$$\nabla^*\psi^{x*} = m\,\psi^{y*} \tag{6}$$

m is the coupling factor. For some particles m is zero. The sign flavors $\psi^{\textcircled{1}}$ and $\psi^{\textcircled{2}}$ occur in three different forms that are indicated by the colors r, g and b.
Elementary particles for which ψ^x equals $\psi^{\textcircled{0}}$ or $\psi^{\textcircled{3}}$ and m is non-zero are fermions. Other elementary particles are bosons.
The above equations do not yet show the effect of interactions. Thus they describe free moving particles.

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Lagrangian density

The Hilbert book model prefers to derive the equations of motion of elementary particles from continuity equations.

In physics the Lagrangian appears to be a very powerful instrument. With respect to the Hilbert book model (HBM) it appears to be not the proper entry point. A single Hilbert book page contains a complete description of the current static status quo. That means a complete description of the field configuration, which includes a description of the anchor points to the fields. These anchor points correspond to Hilbert vectors. When the fields are known, then also their Fourier transforms are known. This means that not only the probability distributions of positions are known, but also the probability distribution of momentums. Thus these data in fact comprise the complete description in terms of the Hamiltonian density rather than the description in terms of the Lagrangian density. Luckily enough the Hamiltonian density of the private field of each particle can be converted in a corresponding Lagrangian density, but curvature may hamper easy conversion. However, in general, locally the situation can be solved without much trouble. In this way the behaviour of a single private field in the environment constituted by all other private fields can be studied.

The consequence of this structure is that the Hamiltonian of a private field ψ of a free elementary particle does not explicitly contain the parameter t[18] and that this private field ψ becomes its time dependence by adding a phase:

$$\psi(q,t) = \chi(q) \exp(-iE\,t)$$

$$\tilde{\psi}(q,t) = \tilde{\chi}(q) \exp(-iE\,t)$$

$$H\psi = E\psi$$

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[18] See: Eliahu Comay:, "Physical Consequences of Mathematical Principles", (Progress in Physics, October 2009 Vol 4), http://www.tau.ac.il/~elicomay/MathPhys.pdf

$\tilde{\psi}(p)$ is the Fourier transform of $\psi(q)$. $\tilde{\chi}(p)$ is the Fourier transform of $\chi(q)$.

The Hamilton density $\mathcal{H}_{HBM}\big(\psi(q), \tilde{\psi}(p), t\big)$ in the Hilbert book model is then a function of $\psi(q)$, $\tilde{\psi}(p)$ and t, while its representation $\mathcal{H}_H\big(\chi(q), \tilde{\chi}(p)\big)$ in the Hilbert space $\mathbf{H}$ is a function of $\chi(q)$ and $\tilde{\chi}(p)$. This Hilbert space represent a single page of the book.

The coupling factor

The generalized equation of motion for elementary particles can be transformed to an equation that looks like the Lagrangian and that enables the computation of the coupling factor from the field ψ.

$$\nabla \psi^x = m\, \psi^? \tag{5}$$

$$\psi^{?*}\, \nabla \psi^x = \nabla(\psi^{?*}\, \psi^x) = m\, \psi^{?*}\, \psi^? = m\, \left|\psi^?\right|^2 \tag{6}$$

$$\mathcal{L} \overset{?}{=} \psi^{?*}\, \nabla \psi^x - m\, \psi^{?*}\, \psi^? \tag{7}$$

$$\int_V \nabla(\psi^{?*}\, \psi^x)\, dV = m \int_V \left|\psi^?\right|^2 dV = mg \tag{8}$$

g is a real constant.

Information detection

All information that is transmitted by nature is carried by clouds of information carrying quanta (see figure 1).

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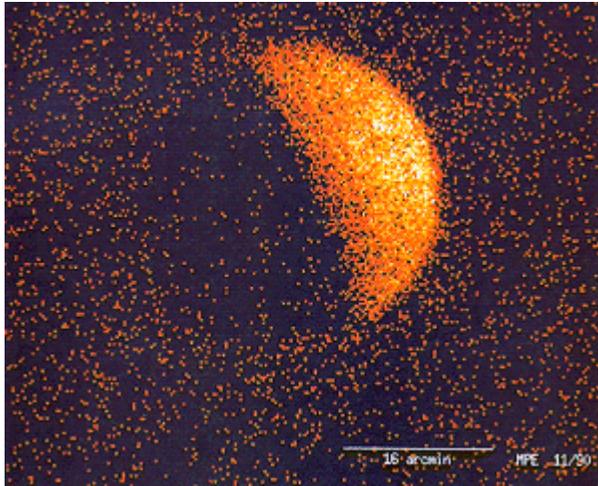
Figure 1: Intensified low dose image of the moon

The clouds themselves carry secondary information in their shape and their movement characteristics. It looks as if all quanta are generated by a series of Poisson processes. These facts become apparent when observations or measurements are done at very low dose rates [3]. The shape of the cloud is set by the corresponding QPAD's.

As indicated before, coherent states act as Poisson processes. The same holds for other QPAD's that support creation and annihilation of substates.

Rigged Hilbert space

The rigged partner $\mathbf{H}$ of a separable Hilbert space H is not a separable Hilbert space, but a Gelfand triplet. It is an ordered set $(\varPhi, \mathrm{H}, \varPhi^x)$, where H is the Hilbert space used to generate $\varPhi$ and $\varPhi^x$. The eigenspaces of normal operators in a Gelfand triplet need not be countable. They can be continuous spaces such as the full set of quaternions. The name of Hilbert is misused to identify the Gelfand triplet as a rigged Hilbert space. This paper uses the Gelfand triplet $\mathbf{H}$ in order to provide a background GPS system and to couple the equivalent of the separate Hilbert space strand operator to the corresponding GPS operator. Both the equivalent strand operator and the GPS operator reside in the rigged Hilbert space $\mathbf{H}$. In

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this way the granules of the chains that reside in separable Hilbert space get their position. Another use of the background GPS operator is the coupling of field values to a position value. For that purpose the field values must be attached to the corresponding eigenvectors in rigged Hilbert space **Ħ**.

Discussion

The Hilbert book model
This model of physical reality does not contain singularities. Nor does it contain infinities. The only infinity it uses is the infinity of the dimension of the separable Hilbert space.
The model is fundamentally granular. The only continuities that the extended Hilbert space uses are the continuity of the background coordinate system that it borrows from its rigged partner and the continuity of the shapes of the QPAD's.

Gravity and inertia
In the Hilbert book model, the gravitation field is treated as a derived field. It has long range effects due to the fact that its charges (the local metric tensors that describe the local curvature) do not get compensated by opposite charges as happens with electric charges[19]. Prove is given by the existence of inertia, which can only be explained by analyzing the influence of the universe of particles on a local particle [3], [4]. Locally this influence causes an enormous potential Φ, which according to Sciama can be related to the gravitational constant G. Uniform movement of a particle does not raise other field activity than a field reconfiguration, but any acceleration of the particle goes together with an extra vector field [4].

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[19] However Mendel Sachs describes a way to also include the curvature caused by EM fields into account.

Quantum clouds

The notion of quantum cloud needs clarification. The quantum cloud that corresponds to the private field of an elementary particle only contains the current granules of that particle as information carrying quanta. A field that consists of a superposition of the private fields of a set of elementary particles corresponds to a quantum cloud that contains quanta that correspond to the current granules of these particles. Not only the quanta carry information. Also the shape of the cloud that contains the quanta contains interpretable information.

When the cloud consists of emitted particles, then the process that controls the emission can be considered as a Poisson process. Upon detection an elementary particle is fully absorbed or it is converted into other particles from which at least one is absorbed. A detected particle was emitted by some body. During its travel it may have been reflected against or deflected by other bodies. The corresponding quantum clouds are affected correspondingly.

A quantum cloud can gain and lose quanta. An emission generates quanta and the corresponding private fields, which then form the shape of the cloud. The quantum cloud that corresponds to a private field disappears with its last quantum.

Testing theories

You can falsify a theory when its conclusions according to a selected logic are not consistent with its presumptions. If you take classical logic as a criterion then QM is a wrong theory. If you take quantum logic as a criterion, then more of quantum physics will pass, but you will have difficulty in checking quantum field theory. Only after extending quantum logic, such that it includes fields, you can handle quantum fields as well. Still this equipment does only reach to test static situations.

Strand model

The main difference between the Hilbert space approach that is taken here and Schiller's approach lays in the interpretation of the source of the observable data. The principle fundamental postulate of Schiller's strand model is that the crossing switches of strands deliver the observable data. In the Hilbert book model the switch of private field to another sign flavor of the field carries the observable data.

Further, Schiller's strand model derives fields from strand tangles. In the Hilbert approach the shape and the dynamics of the chains are controlled by fields.

In both pictures the described concepts may form the basis of a consistent model. Both models claim to deliver the proper equations of movement. [2], [3]. The reason of this conformance lays in the similarity of the basic field constituents.

Both Schiller's strand model and the Hilbert book model take their claims still further. Both models claim that the model fully explains the standard model and that no further particles than those specified by the standard model exist.

Apart from the difference with respect to the main postulate of strand model, an important difference exists between the approach presented in the Hilbert book model and Schiller's strand model. Schiller presents the gravitation field as a separate field that is mainly determined by distant fluctuations of tangle tails. The Hilbert book model treats the gravitation field as a field that is derived from the superposition of all fields that are private fields of particles.

Summary of scratches

The following scratches have been treated here.
1. Due to its link with traditional quantum logic quantum a model of physics must be based on separable Hilbert spaces, but quite often it is based on a non-separable Hilbert space.

2. Neither the separable Hilbert space nor the rigged Hilbert space can represent dynamics. They can only represent a static status quo.
3. The separable Hilbert space cannot represent physical fields. It must be extended in order to cope with fields. In models based on a non-separable Hilbert space fields are often represented by operators.
4. Nature is fundamentally granular. The usual GPS-like operators do not support granularity.
5. It is impossible to represent a continuum GPS-like operator in separable Hilbert space.
6. Gravitation field is usually seen as an independent field.
7. Identifying elementary particle types via quantum field theory is a burden and not transparent.

Summary of repairs

The following repairs have been suggested.
1. Base quantum physics on a book of Hilbert spaces, where each page is an extended infinite dimensional separable Hilbert space that represents the current static status quo. The extension is done by blurring a subset of the Hilbert vectors.
2. Introduce a strand operator whose eigenspace consists of one dimensional chains of granules, where each granule gets its position from a background GPS coordinate system that is generated by a GPS operator that houses in rigged Hilbert space.
   a. One of the granules of each chain is special. It is blurred. A corresponding eigenvector gives it its position.
   b. The blur is a QPAD. It anchors on the granule and on the corresponding Hilbert vector.
   c. The granule corresponds to a ground state of the QPAD.
3. During dynamic steps the QPAD keeps the chain smooth.

4. Elementary particles are anchored on the special granules of one or more chains. The corresponding QPAD's together form the particle's private field, which is also its wave function.
5. Together the private fields form an overall covering field.
6. The static covering field can be decomposed into a rotation free longitudinal part and a divergence free transverse part.
    a. This decomposition runs along curved lines.
    b. The curvature can be used to define a derived curvature field.
    c. The private curvature field of a particle enables the determination of the mass of the particle.
7. Glancing through the pages of the book of Hilbert spaces reveals the dynamics of the system. Dynamics couples the static parts of the fields.
8. The elementary particles that make up the standard model are identifiable by an ordered pair of quaternionic field sign flavors that together form the private field of the particle.

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Part two

On the Origin of Physical Dynamics

On the Origin of Physical Dynamics

Abstract

When physics must be based on an axiomatic foundation then the law set of traditional quantum logic is a valid candidate. However, at first sight, these axioms do not treat physical fields and they do not treat dynamics. It only prescribes the static relations that exist between quantum logical propositions that treat static subjects. These subjects are considered to be physical subjects or their properties. Amongst these propositions statements exist that describe everything that can be said about the static condition of a given physical item. Such propositions represent that item.

Traditional quantum logic is lattice isomorphic to the set of closed subspaces of an infinite dimensional separable Hilbert space Ħ. That is why quantum mechanics is usually done with the aid of Hilbert space features.

The representation of a physical field does not fit in a separable Hilbert subspace. Physical fields have a universe wide range and their presentation would cover all of a whole Hilbert space.

Piron has shown that a candidate Hilbert space can be defined by using one of three division rings for the specification of the inner products. The choice comprises the real numbers, the complex numbers and the quaternions. The choice for the quaternions means that manipulations of the Hilbert space, such as Fourier transforms, in general use such multi-dimensional numbers.

According to Helmholtz decomposition theorem, the quaternionic Fourier transform can be divided in a complex longitudinal Fourier transform and

a transverse Fourier transform. For quaternionic functions this means that they can be *locally* split into a one-dimensional rotation free part and a two-dimensional divergence free part. This also holds for the static versions of vector fields.

This e-paper indicates that traditional quantum logic can be expanded to extended quantum logic, which includes the influences of physical fields in the form of potential propositions that concern virtual items. Dynamic extended quantum logic is lattice isomorphic with the set of subspaces of *a set of* Hilbert spaces. The fields take care of the coherence between these Hilbert spaces.

In this complicated way the axioms of traditional quantum logic form the constraints of the dynamics of quantum physics. When the dynamics of the universe would be put to a hold, then the axioms of extended quantum logic would describe all static constraints and the preconditions that are put to that universe. In the developed model, dynamics means that universe steps from one static status quo to the next. After the step the conditions are changed and the static constraints are reestablished. If we find the laws that control the steps, then we have found a complete axiomatic foundation of physics. Classical physics forms another constraint of dynamical quantum physics. This e-paper studies what happens during the step.

Solutions are given for coping with the inherent countability of the eigenspaces of operators in the separable Hilbert space and for coping with the apparent graininess of some physical quantities. A classification of skew Hilbert fields will be generated that corresponds closely to the Maxwell fields. Further, this e-paper investigates what happens in the infinitesimal steps that nature takes in order to arrive at the next static status quo. In this way the origin of dynamics and the origin of special and general relativity may be revealed.
The e-paper unifies particle states with physical fields and treats the equivalent of the gravitation field as a derived curvature field.

Introduction

The aim of this paper is to build upon a fundament consisting of a minimal set of axioms and then derive as much as is possible from fundamental physics by using only purely mathematical methods.

Observations and measurements at low dose rates learn that ALL information that we receive comes to us in small packages that we call quanta. The distribution of the probability of these information carrying quanta determines the picture that we get from reality.

The development of quantum physics in its early days went violently. The consequence is that many of the fundaments of this theory are not constructed carefully. Fundamental repair is required.
This e-paper repairs the fundaments without disturbing the building. All equations of motion keep their validity.

The fundament

This e-paper builds on the following postulates:

1. The rock fundament of physics is an ordered sequence of instances of traditional quantum logic.
2. All physical information is transferred in the form of clouds of information carrying quanta.
3. The shape of this cloud is determined by a QPAD that generates a tendency to keep these quanta together.

The first point suggests the name **Hilbert book model** for the model that is described in this paper. Each next page of the book describes a subsequent static status quo.

Equations of motion

All equations of motion are in fact continuity equations that treat the local information generation, annihilation and transfer.

Total change within V = flow into V + production inside V

The logic of the model

This e-paper is not about reality. It is about a model that could be a reflection of part of reality.

When reasoning about physical reality, it is safe to follow the rules of classical logic. If one starts with a true statement and these rules are followed, then the path of reasoning stays with truth. Classical logic is based on about [25 axioms](#)[20]. A significant part of these axioms defines the structure of the logic as a half-ordered set and some other axioms expand this to define the set as a mathematical lattice. The other axioms have more to do with the rules that must be followed in order to reason logically. May be it is a good starting point to use logic itself as a fundament of physics.

In the first decades of the last century it was discovered that nature itself cheats with classical logic. Numerous observations of the behavior of small particles revealed that some of the interrelations between these observations are in conflict with classical logic. Birkhoff and von Neumann interpreted this conflict and came to the conclusion that nature obeys its own kind of logic. They named this logic **quantum logic**.

The model that is discussed here builds its foundation on traditional quantum logic. This e-paper is not about quantum logic. It uses quantum logic because [traditional quantum logic](#)[21] defines the **static** framework in which quantum dynamics takes place. Traditional quantum logic prescribes the potential relations that may exist between quantum logical propositions. Amongst these propositions statements exist that describe everything that can be said about the static condition of a given physical

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[20] Appendix: Quantum logic
[21] Appendix: History of quantum logic

item. Such propositions represent that item. These propositions form the top of a hierarchy of propositions that treat the current values of the properties of the considered item. It means that traditional quantum logic can represent physical items.

Traditional quantum logic is lattice isomorphic to the set of closed subspaces of an infinite dimensional separable Hilbert space H. That is why quantum mechanics is usually done with the aid of Hilbert space features. The representation of a physical field does not fit in a Hilbert subspace. Physical fields have a universe wide territory and their presentation would cover all of a complete Hilbert space.

Piron has shown that a candidate Hilbert space can be defined over one of three division rings. The choice comprises the real numbers, the complex numbers and the quaternions. The choice for the quaternions means that manipulations of the Hilbert space, such as the Fourier transforms, in general operate on these multi-dimensional numbers. In the model the representations of physical fields are Hilbert fields. Hilbert fields are blurred Hilbert distributions. Hilbert distributions are sets of Hilbert vectors that are eigenvectors of a special position operator that we will call strand operator. The blur is a local field excitation that is attached to the corresponding Hilbert vector. The blur is characterized by a continuous spread function. This spread function represents a QPAD. The territory of this function may reach all Hilbert vectors. In this way these fields not only cover the whole separable Hilbert space, but because these functions are smooth the Hilbert fields also become differentiable.

The eigenspace of a quaternionic normal operator may consist of a number set that is everywhere dense in the quaternionic number space. For example it may consist out of all rational quaternions. Apart from Hilbert fields the much simpler Hilbert functions exist. Hilbert functions can be defined with the help of a normal operator. Using the eigenvalues and the inner products of the eigenvectors with a selected Hilbert vector that vector can be converted in a hyper complex function. Hilbert

functions can be interpreted as sampled versions of continuous functions. They are not differentiable.

When the eigenspace of a normal operator is granular it can still cover the whole quaternionic number space, but it will have a lattice structure and this lattice will show preferred directions. These preferred directions are incompatible with the isotropy that characters the spaces that are found in nature.

Another example is the eigenspace of the strand operator. It does not cover the whole quaternionic number space. It possesses inner horizons and outer horizons. Within the outer horizon and outside the inner horizons its eigenvalues can be distributed freely through the imaginary part of the quaternionic number space. However, these eigenvalues have a stochastic inaccuracy.

All these operators can be used to construct Hilbert functions. The strand operator can be used to construct Hilbert fields. In order to represent the anchors of physical particles Hilbert fields seem to be better suited than Hilbert functions. By using suitable blurs the Hilbert fields are differentiable. The Hilbert functions are not differentiable. At the utmost they are quasi differentiable.

As a consequence, the theory that is derived here is largely based on the properties of these multidimensional transforms and on the properties of Hilbert fields. Any Hilbert field can be split in a rotation free longitudinal part and a divergence free transverse part. The direction in which a field is rotation free may change with the values of the local coordinates. As long as the direction stays stationary, the corresponding coordinates can be considered as belonging to a complex plane that is embedded in a quaternionic space. Selection of another coordinate system gives a different topology of the field decomposition.

The blurs that constitute the Hilbert fields do not fit inside the realm of an infinite dimensional separable Hilbert space, but their values can be temporary attached to Hilbert vectors. The separable Hilbert space **Ħ** can be embedded in a rigged Hilbert space **Ħ**. This Gelfand triple can be extended to a **Hilbert sandwich** that apart from the Gelfand triple consists of a GPS coordinate system and a covering field. The combination of GPS coordinate system and the covering field decomposes the **static** covering field into dubble cover. The decomposition defines a curvature. That curvature defines a derived field which is also part of the sandwich[22]. The sandwich consists of six layers and represents a static status quo.

Fourier transforms can be defined in a separable Hilbert space, but there they expose sampling characteristics that do not occur in a corresponding rigged Hilbert space **Ħ**. Similarly the notions of differentiation and integration are easily implemented in a rigged Hilbert space **Ħ**, but do not fit in the corresponding separable Hilbert space **Ħ**. Without the blurring trick, differentiation is impossible in the realm of a separable Hilbert space **Ħ**.

The set of closed subspaces of a rigged Hilbert space **Ħ** is no longer lattice isomorphic with the set of propositions in a traditional quantum logic system. We do not want to offer the isomorphism with quantum logic in order to achieve differentiability of functions. This differentiability is already introduced by the blurs that are attached to the Hilbert vectors. This approach delivers a cleaner model that becomes even better comprehensible when we interpret the blur as a QPAD. Further, nature has a fundamental granular character, which fits naturally to a separable Hilbert space.

In a three dimensional vector space a Fourier transform of a vector field can locally be divided in a one-dimensional longitudinal, (locally)

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[22] In this way the sandwich starts to resemble a club sandwich.

complex Fourier transform and a two-dimensional transverse Fourier transform. The longitudinal transform works only on the longitudinal part of the field that is being transformed. The transverse transform works only on the transverse part of the field that is being transformed. This also applies to the case where this vector space is formed by the imaginary quaternions and the fields have quaternionic values.

The division in a longitudinal part and a transverse part of a function or a field *has only a local validity*. It holds as long as the longitudinal direction is sufficiently (= within accepted inaccuracy) stationary. The split is the subject of the Helmholtz decomposition theorem. Multi-dimensional Dirac delta functions show the same decomposition as the multi-dimensional Fourier transform. The splits lose their significance when the field gets too wild.

The fact that this field categorization has only local validity and that it is related to an imaginary direction causes that the quaternionic Fourier transform must be considered to operate in a curved coordinate space. The differentiability of quaternionic functions and Hilbert fields also offers this categorization. Interestingly, Fourier transformation converts differentiation into multiplication with the canonical coordinate.

For a given field this situation can be solved by using two coordinate systems. One in which the coordinates show a curvature of the field that is set by the longitudinal direction and one in which the field has no curvature. The field values stay the same, but the coordinates that act as parameters change. This concept can be extended to a covering field, which is the superposition of all Hilbert fields that exist in the Hilbert space. Using the coordinate system for which the covering field has no curvature the **universe wide Fourier transform** can be taken. However, if the field configuration changes, then the coordinate system that delivers the universe wide Fourier transform changes as well.

For a given field and a given coordinate system it is possible to define a decomposition related local curvature. That curvature can be used to define a derived field. We will call this partner field the **curvature field** of that combination.

In a given coordinate system the fields can be categorized according to their symmetry properties. These categorizations must also cope with the curvature that is related to that coordinate system.

In the described way, traditional quantum logic and the Helmholtz decomposition theorem together form a set of laws that define the relational static status quo that would exist in nature when dynamics could be put to a hold. This paper points out that traditional quantum logic can be expanded such that it includes the representation of *static* physical fields.

The indifference of the properties of physical items for which picture is used, the Schrödinger picture or the Heisenberg picture[23] indicates that time is not a property of physical items but instead it is a parameter that characterizes the progress of dynamics. This is the reason why dynamics can be included into the model by representing nature by a sequence of such extended quantum logics. The fields regulate the coherence between subsequent quantum logics. This also means that the model can include dynamics by representing nature by a sequence of Hilbert spaces. The blurs in the Hilbert fields regulate the coherence between subsequent Hilbert spaces. It means that the blurs are smooth functions of the progression step counter. The progression step counter is a global parameter! It differs from our common notion of time. The blur acts as a probability density distribution. When the parameter is a position coordinate, then the probability density specifies the chance that during the next change the current position changes to this new coordinate. The form of the probability density distribution is such, that this change is has

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[23] Dynamics: Schrödinger or Heisenberg picture

a tendency to be minimal. The probability density is the squared modulus of the hyper complex probability amplitude. This last value contributes to the local field value.

As a consequence of these blur properties, the subsequent Hilbert spaces do not differ much. The same holds for the configuration of the fields in sub-sequent stages of the static status quo. In fact the fields can be seen as a storage place for the conditions that determine the relation between the past, the current and the future static status quo.

In this view the fields represent relations between sets of potential propositions. These propositions are statements that say everything that can be said about the static condition of virtual items. The extension of the logic with blurs means that some of the propositions are not precise. These propositions possess a stochastic inaccuracy.

The set of propositions in dynamic extended quantum logic is no longer isomorph with the set of closed subspaces of a single extended separable Hilbert space. It is isomorphic with the closed subspaces of a series of extended separable Hilbert spaces. One member of this set of Hilbert spaces is the currently actual Hilbert space. It contains the representatives of actual physical items. The other members are *past or future* Hilbert spaces. They contain the representations of non-actual physical items.

Each *past or future* extended separable Hilbert space corresponds to an instance of a *past or future* (extended) quantum logic. These non-actual quantum logics represent potential replacements of the actual traditional quantum logic. A non-actual quantum logic differs from other non-actual quantum logics in the fact that their propositions have a different configuration in terms of their atomic predicates or in terms of their sub-ordered propositions. In a similar sense they will differ from the actual quantum logic.

The replacement will be made between logics that have a great resemblance with each other and the values of the predicates within equivalent propositions will be the same or close to each other. In fact, each extended quantum logic contains the preconditions for the next extended quantum logic in the sequence. A replacement can be seen as a combined annihilation and creation. Annihilation must not be followed by creation and creation must not be preceded by annihilation. With other words annihilation and creation is done during progression steps.

A redefiner, which steps from the actual Hilbert space to a future one, implements dynamics. The redefinition step exchanges the actual Hilbert space against a future Hilbert space whose selection is determined by the current extended Hilbert space. The previous actual Hilbert space becomes the nearest past Hilbert space.

In order to be able to control the coherence between subsequent Hilbert spaces, the blurs that constitute the Hilbert fields act as probability density distributions (PDD's). In fact, they are QPAD's whose squared modulus is a PDD. These distributions have a form that minimizes change during the step from the current Hilbert space to its successor. As a consequence physical quantities do not become observable as continuous objects. Observables become available in the form of information carrying quanta that form the outcome of stochastic processes. The form of the clouds of information quanta is described by the QPAD's that together form the Hilbert fields.

Dynamics can be interpreted as a sequence of steps in which each step leads nature from the conditions of one static status quo to the conditions of the next static status quo. The laws that define the static status quo are fairly clear. During the steps several things happen. The laws that govern the dynamics are rather obscure. The steps couple the static ingredients into a dynamic mixture. For example, the step couples the longitudinal part of the field with its transverse part. The steps are taken universe wide. A redefiner with a universe wide domain controls these steps. The

step counter presents a universe-wide progression parameter. This parameter must not be confused with our common notion of time, but it cannot be denied that it has some relation with it.

During the step particles move forced by their own momentum and by the surrounding covering field to their new position. As a consequence their private fields get redistributed in space. Thus the covering field and its derived partner the curvature field will change. This delivers the preconditions for the next step. These activities are all infinitesimal.

Inertia represents the influence of the whole universe on the condition of a local physical item. In fact it is a bilateral relation. The distant particles together deliver the largest contribution. Only the curvature field takes part in inertia. The primary fields have charges that compensate each other's universe wide contributions. Inertia shows that distant field sources do not interfere with uniform movement. However, due to inertia, acceleration *goes together with* an extra local field contribution. The words "*goes together with*" mean that no causal relation exists.

Thus, acceleration of particles goes together with changes of the local curvature field. This causes a small change in the local metric. It is already indicated that uniform movement of particles causes a reconfiguration of the covering field. The local field influences the steps that are taken by the particles. This is all that happens during the infinitesimal progression step.

We must now analyze what change of curvature, acceleration and field reconfiguration does during an infinitesimal progression step.

[Inertia](#)[24] can guide part of the way. Roughly, the driving force comes from the difference $\Delta E(r, t)$ between the current curvature field and the

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[24] Influence: Inertia

previous curvature field. It is contained in an equivalent of the stripped version of one of [Maxwell's equations](#)[25].

(1)

$$\Delta \boldsymbol{E}(\boldsymbol{r}, t) = G \cdot \frac{\partial \mathbf{v}}{\partial t}$$

G is the gravitational constant.

During the progression step, the described infinitesimal adventure happens to all particles.

Please notice the switch from covering field to curvature field. It is essential!

Recapitulation and extrapolation

In summary: Traditional quantum logic is usually defined via its structure as an orthomodular lattice. This logic only defines part of the *static* skeleton of the frame in which quantum physics operates. It does not state anything about physical fields. The primary fields are the consequence of the stochastic inaccurate coupling between the position operator in separable Hilbert space and the GPS-like operator in rigged Hilbert space. The Helmholtz/Hodge decomposition theorem defines the structure of static physical fields. In that way this theorem plays a similar role as traditional quantum logic. However, the decomposition has only local validity, where quantum logic has global validity. Extended quantum logic encompasses both law sets. These law sets do not specify or even touch the source of *dynamics*. Dynamics couples the static fields. The coupling not only applies to parts of the same field. It also concerns different fields. For example dynamics couples electrostatic fields with magnetostatic fields into dynamical electromagnetic fields. The gravitation field administrates the curvature of observable space that is caused by the decomposition properties of the primary fields. Thus, instead of a separate field the gravitation field can be considered as the

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[25] Dynamics: Unitary transform: Inertia and the progression step

result of the properties of the other fields. [Inertia][26] reveals the importance of the gravitation field.

Both the propositions about a quantum physical system and physical fields are closely related. However, this relation only gets relevant when dynamics comes into play. Dynamics causes a continuing redefinition of the propositions. This converts the current static status quo into the next one. When one proposition is changed it interchanges its constituting atomic predicates with other predicates. The change can even involve the exchange of atomic predicates against atomic predicates that are of another type. It is also possible that the configuration of a complex system that consists of simpler components is altered.

The static physical fields can be interpreted as storage of the preconditions for the next step. The physical fields are the representatives of the influences that go together with the sticky resistance of the set of propositions against the changes that occur due to the redefinitions of the propositions that describe physical items. This sticky resistance also occurs in propositions that are sub-ordered to other propositions. Inertia is a feature that shows this resistance explicitly.

The propositions about quantum physical items can be represented by closed subspaces of a Hilbert space. The presence of dynamics means that the relations between these subspaces are not stationary. They change between subsequent Hilbert spaces. It is also possible to give the physical fields a "representation" in Hilbert space by attaching their anchor points to Hilbert vectors. However, it must be clear that quantum physical items and physical fields are not the same stuff. Physical fields cannot be represented by closed Hilbert subspaces. They cover the whole universe and as a consequence they cover the whole Hilbert space. However, the strength of individual fields may be concentrated around separate excited places that are represented by single Hilbert vectors or a small set of

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[26] Influence: Inertia

Hilbert vectors. Such private fields diminish with distance. Together the private fields form a covering field. For a given coordinate system that covering field has a partner curvature field that can be interpreted as gravitational field.

The actual physical items are distributed in space and are surrounded by potentials that act as a kind of blur. This is why quantum physics has much in common with optics[27]. The superposition of the separate blurs characterizes the information transfer quality of the corresponding field. For each particle a separate blur characterizes the quantum generation properties of that particle. At not too short distances the electromagnetic fields have the same shape as gravitational field. Locally, the EM fields and the gravitation field are based on the same Hilbert distributions. As is indicated above, the gravitation field is a derived field. The main difference lays in the fact that the charges of electromagnetic fields have the same size but may have different sign such that they may partly compensate each other's influence. The charge (mass) of the gravitational field is always positive, but it may differ in size. Another difference is that the gravitation field is the consequence of the decomposition properties of the other fields. Mass appears to be an expression of space curvature and on its turn this curvature is an expression of the rotation properties of the non-gravitational fields. The curvature fields that correspond with private fields do not compensate each other's influence. The masses of all physical items work together in order to create the immense potential that causes inertia.

GPS coordinates

One of the most intriguing facts is that a GPS operator[28] that resides in the separable Hilbert space **H**, cannot be used to define the position of particles. Due to the granularity of its eigenspace it would immediately introduce unnatural preferred directions. In contrast its equivalent, the

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[27] Optics
[28] The Hilbert GPS

GPS operator that resides in rigged Hilbert space **H** is very useful as a coordinate system for determining the location of field values. In this way it can be used to locate the field excitations that go together with particles. This continuous GPS operator cannot be used directly in order to locate the Hilbert vectors that represent particles. Thus there exists no GPS like operator that can be used to locate particles in Hilbert space. An alternative is formed by the [strand operator](#)[29]. The strand operator uses the continuous GPS operator as a background coordinate system. Its eigenspace depends on the configuration of the covering field.

Test proposition

It is an elucidating experience to try to implement a complicated quantum logical proposition in the representation of quantum logic in Hilbert space. In that way we may discover how dynamics emerges in this static skeleton. For that reason, we choose as an example a predicate with quantifiers rather than a clean proposition.

The selected example proposition (♠) is

"All items in universe influence each other's position".

We will already give the final conclusion of this experiment here: A well-ordered replacement of atomic predicates in an enveloping proposition appears to occur without extra field activity, but any deviation of a well ordered replacement causes an extra field activity in the form of a local influence of the complete set of all propositions.

This explains the interaction between fields and physical items. A local deviation of the uniformity of the distribution of physical items can still cause a slight influence of neighboring items. At small distances the influences can be large. The influence of fields can be implemented in the

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[29] Hilbert spaces: Strand operator

separable Hilbert space. Via an action = reaction game the interaction between fields and Hilbert subspaces form the source of dynamics.

What further happens during the implementation of our example proposition (♠) is completely governed by mathematics. Thus, *for our example* no further extension of quantum logic is needed to transform it into a useful version of dynamic quantum logic. However, nothing is said yet of what occurs during the infinitesimal progression steps. During this step one static status quo is converted to the next static status quo. This will be the main subject of this e-paper.

Numbers

As number spaces we use the 2^n-ons of Warren Smith rather than the hyper complex numbers based on the Cayley-Dickson construction. Up to the octonions the corresponding number spaces are similar. (See http://www.math.temple.edu/~wds/homepage/nce2.pdf[30]). For higher n the 2^n-ons behave in a nicer way. They keep more of their number characteristics. We use the quaternions (n=2) as the number space that is used to define the inner product of the Hilbert space. However, we tolerate operators to have eigenvalues that are higher dimensional 2^n-ons. We also use 2^n-ons in order to set the values of physical fields.

When we use these numbers as eigenvalues or as field values, then we apply their number characteristics as well as their storage capacity. A 2^n-on contains 2^n real numbers. We also tolerate that eigenvalues of operators and values of fields support multiple sign selections, such as the inversion of the real axis and the handedness (chirality) of external vector products for their eigenvalues. 2^n-ons contain n independent imaginary base numbers. Each new independent base number introduces a new sign selection. The sign selections translate into 2^n different skew fields.

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[30] Appendix: 2^n-on construction

With $n > m$, the 2^n-ons act like 2^m-ons in their lower m dimensions. Further, the 2^n-ons contain several subspaces of 2^m-ons. We may use smoothly curved manifolds that are crossed by curves which form trails of 2^n-on numbers and that are locally touched by tangent spaces that can be interpreted as 2^n-on number spaces.

When the members of a set 2^n-ons approach zero, then in their mutual arithmetic actions they are getting more and more the characters of lower dimensional 2^m-ons. In the same sense, when two 2^n-ons approach each other, their mutual arithmetic actions are getting more and more the characteristics the arithmetic of lower dimensional 2^m-ons.

The implementation of the proposition (♠) leads to a story of manipulators and manipulated observables. The number waltz feature ($c=ab/a$) of the 2^n-ons that becomes a noticeable effect for $n>1$ seems to play a significant role in our model. If this model applies to quantum physics, then it may reveal why special relativity exists and brings clearness in the different notions of time that exist in quantum physics. The curvature introduced by the spatial variance of what the longitudinal direction is reveals how the mentioned influences can be implemented as component fields which are defined on a curved coordinate system. This holds for gravitational fields as well as for the other fields such as electromagnetic fields.

Implementing quantum physics in a complex Hilbert space hides these interesting features and diminishes the insight that higher dimensional 2^n-ons can reveal.

Prospect

The article shows that there is a need to extend traditional quantum logic such that it not only includes the representations of fields but also includes axioms, which specify the dynamic underpinning of quantum physics.

In the course of this project several fundamental aspects of physics get uncovered.

Comments

Version 5

Version 5 builds on the content of previous versions. This new version stands on itself, but it reorganizes and extends the contents of previous versions. One reason is that the text in some paragraphs relies on the text of a series of other paragraphs, so that it is impossible to configure the paragraphs in a streaming order without repeating much of the content. Here we may take the solution to refer to future paragraphs and add a "back to XXX" reference after the target text.

Project

This project is far from finished. Most parts I have rewritten several times. Some ingredients are already included before they are finalized and before they are put at the proper position in the context. I try to make the whole paper consistent with its parts and I try to keep my goal to include nothing that did not follow directly or indirectly from the axioms of traditional quantum logic. I only tolerate mathematics as a valid tool and ingredient. I will not use or accept intuition as a reason to include a subject in the text. An exception might be the treatise of horizons. However, I will use indications retrieved from previous experiences. I will also not tolerate the usefulness of a concept or its acceptance in the physical community as a valid argument to include that concept. If you encounter places where I did not succeed in that goal, then you may conclude that I still have to work on that section. When the paper gets its final version, then no deviations of my goal should result. However, partly due to my progressed age (~70), I might never reach that condition. Then, *you* reader might take over and finish the job. But first think of the possibility that we will succeed. *What does it mean that all of fundamental physics is based on mathematics and on the ~25 axioms of traditional quantum logic?! Well, another input is the observation that ALL information about nature*

arrives in the form of clouds of information carrying quanta. The cloud as a whole carries secondary information.

References
This e-paper contains no lengthy reference list. References to other documents are usually presented inline and are mostly put in the form of hyperlinks. The appendix and a sometimes referenced [toolkit](#)[31] contains a collection of stuff that otherwise must be grasped from internet. As is done in this article, much of the contents of the toolkit are directly or indirectly obtained from Wikipedia or from publicly accessible publications. In that case the text is adapted to the requirements of the papers that use this information. Most texts on internet are based on complex Hilbert spaces, so where necessary I have converted these texts into quaternionic versions.

Equation editor
This paper is prepared with MS Word 2010. This word processor version contains a rather capable equation editor and a large series of fonts including Cambria Math. However, the equation editor does not cooperate with the paragraph indexing in order to automatically enumerate the out of line equations. For that reason equations are enumerated manually and relative to the current paragraph header. References inside that paragraph just use the equation number. References from outside of the paragraph are hyperlinks that refer to the paragraph header. The hyperlink text may then include the equation number. In that case, you must move manually to the referred equation inside the target paragraph.

EM fields
This paper draws significantly from the book on electromagnetic field theory of Bo Thidé. That book has a different goal and uses different premises. The book does not use the quaternionic field approach as is done here, but its contents easily translate to quaternions. Further its

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[31] http://www.crypts-of-physics.eu/Toolkit.pdf

formulation is very precise, it links formulas to physical concepts and most of all it is online:
http://www.plasma.uu.se/CED/Book/EMFT_Book.pdf[32].

Strands

I took some ideas from the research of Christoph Schiller as it is presented in his online book http://www.motionmountain.net/research.html .
If the strand model is a valid approach to a model of physical reality and if the Hilbert book model that is presented here is also a valid approach, then strands have essential correspondences with the chains in the eigenspace of the strand operator of the Hilbert book model. At least the basic constituents of fields correspond in both models.

Notation note

This paper uses {} in order to indicate a set or a function.
Depending on the context $\{|f_s>\}_s$ means an ordered set of vectors $|f_s>$ where s is the ordering index. In other contexts $\{|f_s>\}_s$ means a vector function $|f(s)>$ where s is the (discrete or quaternionic) parameter.
Continuous functions are presented in the normal way.
$f(\{q_j\}_j)$ is a function $f(q_1, q_2, q_3, q_4,\dots q_n,)$ of the set of parameters $(\{q_j\}_j$, where $j = 1, 2, \dots, n$. The index constraint n might be infinity. Nature itself is finite, however it lives in a model that has an infinite scope.

The appendix and the toolkit contain information about other notation and naming conventions that are used in this paper.

We use the symbols $\breve{Q}$ and Q for operators whose eigenspace is a coordinate system that is curved with respect to the eigenspace of the respective idealized operators $\breve{Q}$ and Q.

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[32] Hilbert field equations

Acquired indications

Several indications stimulated the development of the theory that is presented in this paper. They are listed in this section.

Studying physics

When I was studying physics I was triggered by two facts that have significant influence on my current insights.

Classical versus quantum physics

After finishing the semesters that treated classical physics I started taking lessons in quantum mechanics and I was immediately amazed by the large difference in the way that classical mechanics was handled and the way that quantum mechanics was done. Questioning the teachers did not bring much relief. Their explanation was that the difference is due to the superposition principle. Investigating this reply reveals quickly that this is an alternative description of the different way of working, but no explanation. So, I dived into the library and into scientific bookshops until I finally found a booklet from P. Mittelstaedt: (*Philosophische Probleme der modernen Physik*, BI Hochschultaschenbücher, Band 50, 1963) that contained a chapter on quantum logic. I concluded that this produced the answer that I was looking for. Small particles obey a kind of logic that differs from classical logic. As a result their dynamic behavior differs from the behavior of larger objects. I searched further and encountered papers from Garret Birkhoff and John von Neumann that explained the correspondence between quantum logic and Hilbert spaces. In those years C. Piron wrote his papers that finalized my insight in this subject, but first I must explain the other fact that triggered me.

The rediscovery of quaternions

Quantum physics appeared to be done in the realm of Hilbert spaces. Operators in those spaces delivered the eigenvalues that played the role of values of observable quantities.

I had problems with the fact that according to the in those days commonly accepted theory the operators, which deliver observable values

as their eigenvalues or as their expectation values, had to be self-adjoint and as a consequence these operators could only deliver real valued eigenvalues. Nature has a clear 3+1 dimensional structure and there was no logical indication in the quantum theory that was lectured in those days that explained why four observable values must cling together. I started searching for a number system that could deliver this extra connectivity and I quickly discovered a number system with 3+1 dimensions that supported addition, multiplication and division. It took me more time to discover that this number system was already discovered more than a century before by [William Rowan Hamilton](http://nl.wikipedia.org/wiki/William_Rowan_Hamilton)[33] when he was walking with his wife over a bridge in Dublin. He was so glad about his discovery that he carved the corresponding formula into the sidewall of the bridge. The inscription has faded away, but it is now molded in bronze and fixed to the same wall. When an assistant professor told me the story I started to read papers on quaternions and discovered the work of Constantin Piron.

Birkhoff and von Neumann already discovered that the set of propositions in a traditional quantum logic system is lattice isomorphic with the set of closed subspaces of an infinite dimensional separable Hilbert space Ħ. Piron proved that the inner product of this Hilbert space must be specified with members of a division ring. There are only three suitable division rings: the real numbers, the complex numbers and the quaternions. I went for the widest choice and started studying quaternionic Hilbert spaces.

Representation restriction

After discovering that traditional quantum logic can be represented inside an infinite dimensional separable Hilbert space, it is a disappointment to discover that this presentation does not cover physical fields and does not cover dynamics.

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[33] http://nl.wikipedia.org/wiki/William_Rowan_Hamilton

Extended quantum logic

However, it appears that it is possible to expand traditional quantum logic in a way that corresponds to adding blurs to a subset of the Hilbert vectors. The blurred subsets may represent the anchor points of particles. In this way the superposition of the blurs may represent the physical fields. This leads directly to the existence of Maxwell-like fields in the realm of such an extended Hilbert space. This also leads to an extended quantum logic that covers physical fields.

Dynamic quantum logic

Dynamics can be implemented by representing dynamic quantum logic as a sequence of extended traditional quantum logics that each represents a static status quo. This dynamic stepping can be detailed further. See <u>Progression step details</u>[34]. With respect to the Hilbert space the dynamic model uses a sequence of extended Hilbert spaces. It can be compared to a book, where each page represents a static status quo.

Curved space

When quaternions are taken as the division ring, then Fourier transforms become quaternionic Fourier transforms. The ideal Euclidean formulation of the multi-dimensional Fourier transform cannot cope with a variable direction in which the Fourier analyzed function or field is rotation free. This can be circumvented by converting the parameter domain of the field by a coordinate transformation such that the resulting field has a stationary direction in which it is rotation free. This corresponds with accepting the existence of a curved coordinate space. This curved space is subject of general relativity. With other words, extended quantum logic supports general relativity.

Intensified imaging

After finishing my study I started my career in the development laboratory for high-tech electronic appliances of a big electronics company. My task consisted of the analysis and measurement of the

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[34] Acquired indications: Progression step details

visual trajectory, starting from the radiation source and ending after interpretation of the image in the brain of the observer. At those times (~1975) this was fundamental research, because both the measuring methods and the modeling methodology in this area were still in their childhood. The target products for the laboratory were night vision devices and X-ray image intensifiers.

Intensified imaging is required at low radiation dose rates and in situations where the radiation detection capability of the human eye is unsuitable. This occurs with starlight scene imaging and with X-ray shadow imaging of patients. The low dose rate is necessary due to the fact that no active scene lightning can be supplied or due to the fact that hazardous gamma ray effects must be avoided.

When the snowing image produced by image intensifying equipment is observed, then it becomes immediately clear that this image is built up from a large number of separate spots that together form a rather noisy picture of the object. The impression is that clouds of quanta are detected rather than waves of radiation.

The research not only concerned perception experiments and measurement. We also had to devise the standards for the measurements as well. So we took part in the establishment of develop worldwide standards for the specification and measurement of the Optical Transfer Function (OTF) and its modulus the MTF. We also took part in the committees that created the standards for the Detective Quantum Efficiency (DQE).

The fact that these standards were not only required but were also successful is in itself very astonishing. We needed these standards because we could model the visual trajectory as a chain of which the first elements consisted of a set of Poisson generators. The generators are characterized with their efficiency and a spatial, angular and chromatic distribution.

Next in the chain are attenuating binomial processes. Statistically a blur can also be considered as a binomial process. The information is spread over a larger area. A Poisson process can be combined with a subsequent binomial process into a generalized Poisson process that has a lower efficiency.

The chain also contains light lenses and particle lenses. Further, the equipment aided chain contains detection surfaces that convert radiation quanta into electrons or electrons into radiation. This chain might also contain scintillation layers that convert high energy X-ray quanta in large series of low energy light quanta. It might contain fiber optic plates that just transport images, usually from a curved to a flat surface. It might contain channel plates that convert single electrons into clouds that contain about hundred thousand electrons. It might contain image receivers that convert the image into an electric signal or into a photographic plate.

The equipment aided chain may and the unaided chain will also contain the eye of a human observer. Intensified images are detected by the cones in the fovea. At very low light levels the adapted eye detects the images via the rods in the fovea. Rods have a much lower acuity than cones. Therefore they have a much longer integration time. In general, measures that reduce noise have both positive and negative effects on the information content of the signal. There is an optimum condition. My task was to find that condition.

After the detection in the fovea the received signal is handled by a large series of preprocessors that act in parallel as well as in sequence. The preprocessors associate the signals that are received by receptors that lay in each other's neighborhood. The association tests a detail pattern that is typical for the considered preprocessor. The associated signal is only passed further when its signal to noise ratio surpasses a given boundary level. In this way the higher regions of the information processing are not disturbed by unnecessary noise.

All preprocessors work in this way as noise filtering decision centers. The association results in a categorization of the encoded image. The signal that reaches the folded fourth layer of the visual cortex represents the completely coded version of the received image. In the human brain, a folded surface of about four square millimeters is devoted to each image receptor in the fovea. This code is interpreted further in the brain. As early as possible the filtering process stops noise and details of the image that do not carry useful information from proceeding further in the chain.

Due to this design, already the unaided brain-eye combination is well suited to perceive and interpret images in a very large dynamic range of circumstances. Apart from the fact that the visual channel can adapt from somewhat above starlight conditions until bright daylight conditions, the visual trajectory appears to be optimized for handling signals that enter the eye in the form of clouds of quanta that are generated by Poisson processes.

All vertebrate visual trajectories work according to the sketched principles. Over billions of years evolution has exploited the fact that information that comes to living species is generated by Poisson processes. The visual trajectory of vertebrates is optimized for handling this information for the benefit of the survival of the owner of this channel. See: http://www.crypts-of-physics.eu/Howthebrainworks.pdf[35] .

This fact is a strong indication that all visual information comes to us in the form of clouds of quanta. When looking at low dose rates through an intensified viewer, it becomes clear that this assumption is valid. The perceived noisy image is built from separate dots that represent the detected quanta. No radiation wave is visible. What you see is just a streaming cloud of quanta.

The fact that visual information is generated by Poisson processes indicates a more general feature of physics. ALL information that is

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[35] Part three: How the brain works

transferred by electromagnetic radiation can be considered to be generated by Poisson processes. This becomes apparent when measurements are done at very low dose rates. In the static model the Poisson processes represent a lateral distribution. In addition, taken over a sequence of Hilbert spaces the Poisson processes represent a temporal distribution.

Granular GPS

In separable Hilbert space a normal operator has a countable eigenspace. This allows to the usage of the rational quaternions as the eigenspace of a normal operator. However, nature appears to support a minimum for the distance between two positions. This renders a position related operator granular. In separable Hilbert space Ħ, the granularity of the eigenspace of a GPS-like coordinate operator presents problems with the fact that a dense packaging of the granules generates unrealistic preferred directions. Its non-granular equivalent in the corresponding rigged Hilbert space Ħ does not suffer this restriction. It can be used as coordinate system for fields, but it cannot be used to locate particles inside the separable Hilbert space Ħ. Inside the separable Hilbert space the fields are attached via anchor points to a subset of the Hilbert vectors and all Hilbert vectors touch their values.

Progression step details

After the former indications the theory reaches the stage that it becomes sensible that the model of nature, which takes its foundation on quantum logic, steps from one static status quo to the next. It dawns that this is the way that dynamics is implemented. What happens during these steps is still mysterious. The Hilbert space itself only suggests a Euclidean signature of observable space time. However, Einstein and others proved that observable spacetime has a Minkowski signature. This discrepancy has its origin in the group properties of displacements. For uniform movements this leads to the Lorentz displacements group. An early conclusion is that **coordinate time** does not play the role of the fourth

dimension in the quaternionic eigenspace of a spacetime-like operator. It also differs from the role of the counter of the progression steps.

Both inertia and Feynman's approach of the path integral may guide what happens during an [infinitesimal dynamical step](#)[36].

Release and removal of quanta
During the step interactions take place and particles are emitted or absorbed. The information is carried by clouds of quanta. The quanta carry the information that they collect during the dynamical step from GPS and GMS related data.
An indication for this fact houses in the structure of the creation and annihilation operators. These operators consist of a part that relates directly to the GPS operator and a part that directly relates to the GMS operator.

Fields and QPAD's
Some subsets of Hilbert vectors represent elementary particles. It means that these vectors are blurred. The blur is a QPAD whose form is typical for the elementary particle type. Elementary particles combine to form more complex particles.
The superposition of all QPAD's that correspond to the separate particles forms the covering field.
A repositioning of a particle means a reconfiguration of the covering field and vice versa.

A detailed list of indications and considerations
1. All information comes to us in the form of clouds of quanta.
2. These clouds get their shape via a combination of QPAD's.
3. Each type of elementary particle is characterized by a set of Hilbert vectors and a particular kind of QPAD.

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[36] Dynamics: Unitary transform: Infinitesimal dynamical step

4. The information contained in the quanta and in the cloud is the only information that becomes observable.
5. This information consists of the information that is carried by the separate quanta and by the probability distribution that describes the cloud.
6. Each quantum in the cloud carries a set of information data.
7. This set contains a 3D position, a 3D momentum and chirality qualifiers.
8. The information that is carried by the quanta becomes available via an interaction process.
9. The information is measured in Planck units, eventually related via physical constants, such as the speed of light.
10. The QPAD that characterizes a particle becomes part of the field that exists in the surroundings of the particle.
11. Physical fields consist from the superposition of the QPAD's of the separate particles.
12. Curvature and torsion of the path of the particle are secondary characteristics, which are introduced via the probability distributions that make up the field that exists in the direct environment of the particle.
13. In contrast to torsion, curvature appears to be linked with gravity. The photon path has a helix structure. The photon has no mass.
14. Curvature in the path of a particle is caused by the local rotation that exists in the surrounding field(s).
15. The rotation properties of the field determine the local decomposition of the static field.
16. This local decomposition determines a curvature of observable space.

17. On its turn this local curvature specifies a metric and the local metric specifies a curvature field.
18. The curvature field has all the characteristics of the gravitation field.
19. The generation of a given kind of quantum has a typical probability.
20. There exist anti-quanta. The generation of an anti-quantum is equivalent to the annihilation of the corresponding quantum.
21. Creation and annihilation operators have QPAD's as their eigenfunctions.
22. In their simplest form these probability distributions are Poisson distributions.
23. The generation of shot noise is characterized by Poisson distributions.
24. At high dose rates the Poisson distributions become Gaussian (normal) distributions.
25. For more complicated configurations the QPAD must be considered rather than its squared modulus: the probability density distribution (PDD).
26. Bosons are characterized by QPAD's that remain invariant under a rotation of 2π.
27. The QPAD of a two boson system is invariant under perturbation of the bosons.
28. The creation and annihilation operators of bosons are characterized by a non-zero commutator.
29. Photons form the simplest boson type. Their paths have a helix form.
30. The probability distribution of the corresponding quanta resembles a Poisson distribution.

31. Fermions are characterized by QPAD's that change sign under a rotation of 2π.
32. The QPAD of a two or more fermion system changes sign under perturbation of the fermions.
33. With each fermion type an anti-type exists.
34. A quaternionic QPAD can also contain chirality information.
35. When chirality is taken into account then a QPAD must be used rather than a PDD.
36. Electric charge is related to the chirality properties of the corresponding particle.
37. The creation and annihilation operators of fermions are characterized by a non-zero anti-commutator.
38. Creation and annihilation operators can be split in a part that resides in configuration space and a part that resides in Fourier space.
39. A quant can be emitted (created), absorbed (annihilated) and it can be virtual, which means that it is annihilated shortly after its creation.
40. Non-actual quanta belong to previous or future events.
41. Only actual quanta deliver observable information.
42. Emitted and absorbed actual quanta belong to the current version of events.
43. During each dynamical step information is collected both from configuration space related sources and from momentum space related sources.
44. The part of the collected information that resides in configuration space delivers the 3D position information to the emitted/absorbed/virtual quant.

45. The part of the collected information that resides in Fourier space delivers the 3D momentum information to the emitted/absorbed/virtual quant.
46. The sum of an even function and its Fourier transform is invariant under Fourier transformation.
47. The difference between an odd function and its Fourier transform is invariant under Fourier transformation.
48. Apart from a scale factor, the functions that characterize linear and spherical harmonics are invariant under Fourier transformation.
    a. The scale factor is 1, i, -1 or –i.
49. The harmonic functions are also related to creation and annihilation operators.
50. The harmonic functions contain a factor that equals a Gaussian probability distribution.
51. Strand model
    a. Any knot can be represented topologically by equations in Cartesian coordinates x, y, z of the form: $x = f(t), y = g(t), z = h(t)$, where $f(t)$, $g(t)$ and $h(t)$ are Fourier series with finitely many terms.
    b. Only in 3D space knots cannot all be unknotted.
    c. There exist three basic types of elementary particles that can be distinguished via the number of strands[37] / Hilbert vectors involved. These basic types are the bosons, the quarks and the leptons.
    d. The bosons can be distinguished in four categories:
        i. The photons have a helix form and no chirality. They have no mass and are involved in EM interaction.

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[37] Strands

ii. the $W^\pm$ bosons have a knotted form (overhand knot) and possess chirality. They have mass and are involved in weak interaction.

iii. the Z bosons have a knotted form (figure eight knot) and no chirality. They have mass and are involved in weak interaction.

iv. The gluons have rectangle loop shape ($\propto$) and no electric, but color charge. They have no mass and are involved in strong interaction.

e. The quarks and the leptons can be distinguished in three generations.

f. The particles are distinguished via the QPAD of the corresponding quanta and the kind of information that is carried by these quanta.

g. There exist three basic forms of interaction that are distinguished via the number of strands/Hilbert vectors that are involved in the interaction event.

h. These basic forms of interaction can be related to Reidemeister moves.

52. Particles become observable via their interactions, thus via the quanta that are generated due to these interactions.

53. All motion observed in nature minimizes action.

54. Uniform motion preferably occurs via a geodesic and obeys the geodesic equation.

55. The visual trajectory of vertebrates is devised in order to cope with a huge dynamical range of light conditions ranging from starlight conditions up to bright daylight conditions

56. Over billions of years, evolution has exploited the fact that information that comes to living species is generated by Poisson processes. The visual trajectory of vertebrates is optimized for

handling this information for the survival of the owner of this channel.

57. The Banach–Tarski theorem states that a spherical surface can be split in five pieces that can form two spheres of the same volume. The statement does not hold in the eigenspace of a coordinate operator that resides in separable Hilbert space.

58. In separable Hilbert space, at least one coordinate operator has lattice sampling properties. Its eigenspace shows preferred directions.

59. At the lowest scale it is not clear how the granules of an eigenspace of a Hilbert position operator are geometrically arranged. On a larger scale they appear to be influenced by fields.

60. The geometric sampling of normal operators between subsequent Hilbert spaces may differ.

61. Particles can be considered as sources and drains of information carrying quanta.

62. These sources and drains play their role in a continuity equation that treats information carried by quanta.

63. The concept of measurement has no significance at Planck scales.

The indications and considerations that are treated in this chapter will steer the development of the theory that is subject of this e-paper.

First conclusion

The standard model can be retrieved via categorization of the particle types and their interactions. This comes down to categorizing QPAD's and categorization of information packages that are carried by generated quanta.

Logic

Logics

Quantum logic differs from classical logic in one of its axioms. The set of propositions in classical logic is isomorphic with the set of [Venn diagrams](#)[38]. The set of propositions of traditional [quantum logic](#)[39] is far more complex. This significant difference is due to the weakening of just a single one of the set of more than 25 axioms. It is lattice isomorphic with the set of closed subspaces of an infinite dimensional separable [Hilbert space](#)[40] **Ħ**. The isomorphism means that quantum logical propositions can be represented by closed subspaces of a Hilbert space. The inner products of that Hilbert space can be defined by using numbers of a 2^n-on number space. Taking $n>2$ for that purpose raises numeric problems with the closure of the subspaces. Traditional quantum logic does not include any axioms that treat dynamics and it does not treat the influences of physical fields. It only specifies stationary relations that are possible between physical items and their properties.

Example proposition

In order to discover the emergence of dynamics we will implement a quantum logical proposition in Hilbert space and test its truthfulness. We will introduce in this example proposition physical fields as well as dynamics.

The example proposition(♠) is:

> *All items in universe influence each other's position*. (♠)

It can be answered with either yes or no. And, if we succeed, it can be implemented in Hilbert space. So, in that case it is a valid quantum logical proposition.

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[38] http://en.wikipedia.org/wiki/Venn_Diagram

[39] Appendix: Quantum logic

[40] Appendix: The separable Hilbert space

Proving 'yes' is cumbersome, but the 'no' is hardly less difficult. It requires finding an item of which the position is not influenced by at least one of the other items. For this purpose it is necessary to implement notions of *items*, the *universe*, *influences* and *position* in Hilbert space.

The statement includes quantifiers (*position*) and dynamic operational elements (*influence*). The set of axioms of traditional quantum logic does not treat dynamic operational elements. At least it does not do that in a realistic way. As we will see, the *influence* of the *universe* of propositions (*items*) will put particular restrictions to the extension of quantum logic into the realm of an extended dynamic logic. This restriction is manifested in the occurrence of [physical fields](#)[41] and [inertia](#)[42].

Translated in physical terms inertia means that in contrast to a *uniform movement*, the *acceleration* of an *item* will go together with the *action* of a *physical field*. Notice that we use the words "goes together with" instead of "generates" or "causes".

Translated in logical terms a conclusion of the analysis of inertia runs: "During a redefinition of a proposition the exchange of atomic predicates in that proposition must be done in well-ordered and controlled steps. Otherwise the *community of propositions* will *influence* the considered proposition."

Again it must be noticed that there is no causal relation between the event of being well-ordered and the event of influencing. With other words, the inertial interaction is instantaneous.

When nature's logic is put in axioms, then influences that correspond to physical fields must follow from the axioms. Together with the

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[41] Functions and fields

[42] Influence: Inertia

specification of the origin of dynamics this will then result in a dynamic version of quantum logic.

I assume that this category of logic does not yet exist in mathematics. There exists a version of <u>dynamic operational quantum logic</u>[43], but it does not cover or mention the effects of the representation of physical fields in logic and it does not specify the *origin of dynamics*.

Atomic predicates

Atomic propositions are statements that are either true or false and which cannot be broken down into other simpler propositions. When an atomic proposition concerns a property, then it may contain the value of that property. We will call that kind of atomic propositions atomic predicates. For example "The speed is 5." The identity or the category of the property is "speed". The value of the property is 5 with a certain inaccuracy. The inaccuracy is typical for the category of the atomic statement. Only discrete properties can be observed without inaccuracy. The dimension of the value is "meter per second", but that is another atomic statement and it is a fixed statement. Both the dimension and the inaccuracy form extra information that is part of the type definition of the atomic predicate category "speed".

In fact there exist no continuous properties that relate to Hilbert vectors. The smallest inaccuracy is set by Planck units. On the other hand the granularity of the properties must not cause a regular lattice structure of the property space. This need not lead to contradictions, but it leads to <u>special solutions</u>[44] for the operators that deliver the value of the observable properties.

The atomic predicates form a set with a particular lattice structure. In this set we only consider atomic predicates that are independent of all other

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[43] Discussion: Dynamic logic
[44] Hilbert space: Limitedness: Investigating a special operator

atomic predicates. Several choices of such sets exist. A subset consisting of members of a chosen set may be [canonical conjugates](45) of members of another set. However, canonical conjugates are always dependent. So they cannot be member of the same selected set.

In Hilbert space the type definitions of atomic predicates that concern numeric variables are represented by **operators**. The values of the properties in the atomic predicates correspond to the eigenvalues of the operators or they are expectation values. Expectation values are statistically determined via a probability characteristic that characterizes both the operator and a physical item. See [Wave function](46).

In separable Hilbert space **Ħ** the eigenspaces of all normal operators are granular. The granularity is a result of the stochastic inaccurate coupling between its eigenvalues and corresponding eigenvalues of a corresponding operator in rigged Hilbert space that has a continuum as eigenspace.
This stochastic inaccuracy also afflicts the corresponding atomic predicate.

Type definitions

Type definitions are propositions that describe and categorize subjects without specifying their variable values. An **atomic predicate type** is the type definition of a category of atomic predicates and specifies the type of property that these propositions treat. The definition also contains the *physical dimension* (unit) of the property, the inaccuracy and the allowed *range* of the potential values of this property. For example, if that category is "speed", then the definition contains the *physical dimension* meters per second. The minimum of the absolute value is zero and the maximum of the absolute value is c. Speed is an imaginary quaternion.

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[45] Functions and fields: Canonical conjugate
[46] Functions and fields: Characteristic function

When the type definition concerns a more complex object that can act as an individual the definition will be called an **item type** definition. Item type definitions use **atomic predicate types**.

When that item cannot be broken into simpler objects that still can act as an individual, then the type definition is an **elementary type** definition. Elementary type definitions are constructed of type definitions of atomic predicates.
The elementary types form (a rather small) subset of the whole set of type definitions. Elementary types appear to divide into two categories**: bosons** and **fermions**. The fermions can be divided in **leptons** and **quarks.** The bosons can be divided in **photons**, **W-bosons**, **Z-bosons** and **gluons**.

The private field determines the elementary particle type and the basic properties of the particle. These include spin, rest mass and charge Several types of charge exist. Electric charge, isospin, color charge and hyper charge are types of charges. In fact rest mass is one of them, but since the gravitation field is an administrator its value must follow from the spin and the other charges. Variable properties are position, momentum and angular momentum,

If the item is not an elementary type, then its type specification is a **system or sub-system type** definition. A (sub)-system type definition is constructed of elementary item type definitions and atomic predicate types.

The type definitions form a set with a different lattice structure. Its structure is isomorph with the structure of classical logic.

In Hilbert space no representation for item type definitions exists. However, in Hilbert space atomic predicate types are represented by operators.

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Strands as type definitions

The notion of [strand](#)[47] is introduced by Christoph Schiller. Apart from its crossing switch events, a strand is not observable. In the strand model the boson types are all represented by a single strand. Reversely a strand is nearly equivalent to the type definition of the simplest boson, which is the photon. A photon has a helix shape. A strand does not need to have that shape.

The Hilbert book model defines a [strand operator](#)[48] that has an eigenspace in which chains of granules reside. These chains come close to Schiller's strands. In each chain one granule is special and is called the current granule. Only the current granule and its immediate neighborhood can deliver observable values. If operators are type definitions, then the notion of a strand comes close to that type definition.

The shape of a chain has a direct relation with the configuration of the current covering field. Taken over an ordered sequence of Hilbert spaces the strand fluctuates under the influence of the changing field configuration. The current granule separates the chain in a "virtual past sub-chain" and a "virtual future sub-chain". The words "past" and "future" are misleading while these parts do not really correspond to the actual past or future of the chain. They depend on the current field configuration, rather than on the past or future configuration.

Items

The first problem that is raised by constructing the representation of proposition (♠) is to determine what in this representation stands for an item. The simplest solution is to attach a subspace of the Hilbert space to the item. The corresponding proposition can be phrased as: "*This is the item*". Something either belongs to the subspace or it is outside that subspace. Everything that can be attributed to the item can also be attributed to this subspace. Each of these propositions belongs to a

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[47] Strands

[48] Hilbert space: Limitedness: Strand operator

hierarchy for which the mentioned proposition forms the top. All sub-ordered propositions correspond to subspaces of the item's subspace. In this way the universe of items can be represented by a set of mutual orthogonal subspaces of the Hilbert space. Rays that are spanned by a single Hilbert vector and that are connected with a numeric value can be considered as atomic predicates. Subspaces spanned by such rays that are related to the same type of value can be considered as statements with a wider scope. The rays can be subspace of an items subspace. The subspace that corresponds to a conglomerate of elementary items also represents that conglomerate as an item. The configuration of the subspace that represents an item will change as a function of the parameter that measures the progression of the dynamic behavior of the item. It is possible that not only the values of the atomic predicates change. The types of these atomic predicates may change as well. This happens for example with atomic types that are each other's canonical conjugate. It is also possible that the configuration of the subspace changes more drastically.

In a set of subsequent Hilbert spaces the subspace that represents the item can be moved around with respect to a selected base consisting of eigenvectors of a normal operator. In this way it may be possible to implement the dynamics of items. This moving around does not mean that the vectors are moved around. It means that at each step of the move the set of vectors that span the considered subspace is **redefined**. The redefinition corresponds to a redefinition of the corresponding proposition. Alternatively, it is also possible to redefine the selected normal operator. Thus, redefinition and the laws that govern redefinition convert the static quantum logic into a dynamic version of quantum logic. It will be shown that physical fields play a significant role in this redefinition.

With his bra-ket notation Dirac has provided us with a marvelous symbolism for vectors and even for operators. He did not provide us with symbols for subspaces. However, it is easy to extend his symbolism and

indicate a subspace with a set of vectors that spans that subspace. For example $\{|f_s>\}_s$ indicates a set of element vectors $|f_s>$ with enumerator s that span a closed subspace. This set identifies the subspace. Different sets may identify the same closed subspace.

It is sensible to have one vector inside the item's subspace that is considered as characteristic for the location of the representation of the item in Hilbert space. We reserve the name **locator** for this vector. When the item is redefined, that vector may be redefined as well. This characteristic vector can be used to obtain a precise location of the subspace in Hilbert space. The process via which the locator is determined depends on the requirements. The requirements may be set in relation to an operator. For example the vector that corresponds with the expectation value of the operator for that subspace can be chosen as the locator. In that case the state vector[49] that corresponds with that operator may play the role of the locator. Two or more bosons can share the same locator. Fermions that possess the same property values cannot share the same vector as a locator.

Atomic predicates are not considered to be statements that fully describe a physical item. The statement "This is the item" forms the top of a hierarchy of statements that all say something about the item. The hierarchy contains statements that define the item's type. Other members of the hierarchy specify the items constituents. Still other statements concern the item's atomic variables that together with the type definition specify the item's identity. For atoms the variables of the subsystems are hidden from the outside of the atom. This means that atoms can be considered as modules[50].

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[49] States

[50] Part four or http://www.cryps-of-physics.eu/ThereExistsATendencyInNatureToReduceComplexity.pdf

Representation of items

A subspace in a single separable Hilbert space Ħ cannot represent all properties of a physical item. The fact that the position of the item is known means that an eigenvector of the position operator resides within the subspace that represents the item. Say that this subspace covers position values in a certain region. Heisenberg's uncertainty principle now states that the value of the momentum of the item is uncertain. Any values of this property must correspond with eigenvectors of the momentum operator that also reside in this subspace. For elementary particles the subspace will be too small in order to guarantee sufficiently sure property values. Sufficient information could be collected when the Hilbert space also contains past and future data, such that the momentum can be derived/estimated from those data. The physical fields contain such preconditions. For a free elementary particle the momentum can be derived from the Fourier transform of the QPAD that controls the position of the particle. This QPAD is the wave function of the particle. Together with the subspace that represents the particle, the wave function represents all information that can be retrieved from the particle. Since all particles have such QPAD's these private fields get intermixed. Thus in the neighborhood of other particles the superposition of the private fields must be reckoned rather than a single private field.

Via its wave function a particle is identified with its private field. The notion of private field transfers quantum theory into quantum field theory. The dynamics of the particle are represented by the dynamics of their private field.

Vacuum

Multidimensional subspaces exist that do not represent a dynamical item. They can be considered as **vacuum**. It is still possible that the subspace represents a [ground state](#)[51]. We will assume that on the average the 'filled'

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[51] Functions and fields: Quaternionic Fourier transform split: Ladder operator: Ground state

and the vacuum subspaces are evenly distributed over a connected part of the Hilbert space. The phrase "evenly distributed" means that the distance between the representations of items makes sense. Here we do not mean the distance related to the norm of Hilbert vectors, but the coordinate related distance.

"Vacuum" does not say that these subspaces are empty. It is rather an indication that the subspace does not represent a dynamical object. Instead the subspace may represent a ground state.

Vacuum does not generate observable information quanta. In vacuum the clouds of quanta are empty. (However the combined vacuum states can cause an observable effect. The Casimir effect is an observable phenomenon.)

Hilbert spaces

Dual views of a Hilbert space

We only consider infinite dimensional separable [Hilbert spaces](#)[52] and their [Gelfand triple](#)[53], the rigged Hilbert space, which is not a Hilbert space, but just got its name.

In Hilbert space normal operators exist whose eigenvectors form an orthonormal base of the Hilbert space. The canonical conjugate of that normal operator has a set of eigenvectors that is completely disjoint of the former orthonormal base. This fact defines pairs of views of the same Hilbert space that are related via canonical conjugation.

The corresponding orthogonal bases do not touch. Every base vector is a linear combination with non-zero coefficients of all members of "the other base". All coefficients have the same modulus.

Position

The original proposition (♠) speaks about the position of the item. The position must be related to something that is available in the separable Hilbert space. This Hilbert space is defined over a number space. Thus we might attach a number of this number space (or a higher 2n-on) to the Hilbert subspace that represents the item. That number must represent position. The natural way of attaching numbers to subspaces of a Hilbert space is via the concept of eigenvalues of normal operators. Any symmetry transform of a selected normal GPS coordinate operator might meet the requirements.

However, there exists a significant drawback. The eigenspaces of all normal operators that reside in a separable Hilbert space $Ħ$ are countable. In addition the eigenspace of the position operator in $Ħ$ must be granular. The granularity means that the difference between two different positions

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[52] Appendix; The separable Hilbert space
[53] Appendix; Gelfand triple

must be equal to or larger than the Planck-length. A GPS coordinate system that is constructed from a dense package of fixed size granules generates preferred directions. If we want to avoid this, then we must use a position operator whose eigenspace does not contain multidimensional sets of granular eigenvalues. Still each of the eigenvectors must have eigenvalues that deliver position values as eigenvalue. Thus there must be a relation with a background coordinate system. That background coordinate system must form a continuum. This background coordinate system can be delivered by a GPS-like operator that resides in the corresponding rigged Hilbert space $\mathbb{H}$. That operator has the continuum of the quaternionic number system as its eigenspace. Apart from the real axis of this hyper complex number system it shows no preferred directions. So for **position values** we must take rescue in rigged Hilbert space $\mathbb{H}$. The eigenspace of the GPS-like operator does not show a natural granularity. The continuum GPS operator is not a part of the separable Hilbert space $\mathbb{H}$, but an equivalent operator with a countable eigenspace exists in $\mathbb{H}$. It cannot be used to locate the vectors of the separable Hilbert space. However, we can use it to give field values an approximate location. What we have obtained are two GPS-like operators. One resides in separable Hilbert space and has a countable eigenspace. The other resides in rigged Hilbert space and has an continuum eigenspace. This continuum is at least usable as a background coordinate system. The eigenspace of the first GPS-like operator forms a dense coverage of the second GPS-like operator. Both GPS-like operators do not support granularity.

For a given field we may choose a GPS-like operator Q, which resides in separable Hilbert space and has an equivalent $\breve{Q}$ in rigged Hilbert space, such that for that field we can work with the ideal form of the quaternionic Fourier transform. That means that by using these coordinates as parameters, the field that will be analyzed has decompositions that run along straight lines in the eigenspaces of Q and $\breve{Q}$. $\breve{Q}$ introduces a new coordinate system that is curved with respect to the original GPS-like coordinate system that is eigenspace of operator $\breve{Q}$.

The new coordinates are characterized by the fact that the considered field when formulated using these coordinates shows a decomposition into static parts that runs along straight coordinate lines. A Fourier transform taken in these coordinates has universe wide validity. The canonical conjugate P of operator Q also shows a similar behavior for the Fourier transform of the analyzed field that was first stated in Q coordinates and after transformation is specified in P coordinates. The same relation holds for operator $\breve{Q}$ and the canonical conjugate $\breve{P}$.

Physical coordinates

Coordinates are not necessarily physical quantities in the way that they can be considered as properties of physical items. The physical coordinates of identifiable physical items are granular. The granularity means that at a given progression step they can only change with a step that either is zero or is equal to a Planck-length. Coordinates that are eigenvalues of normal operators in separable Hilbert space $Ħ$ are countable. The set of rational quaternions is countable, but this set is not granular. In rigged Hilbert space $Ħ$ the eigenspace of a normal operator may be uncountable. It means that this space forms a continuum. The set of all imaginary quaternions forms a continuum. In a given static status quo, only a countable and granular subset of these eigenvalues can be physical quantities.

We took the Planck-length here as THE minimum distance between positions. The Planck-length is derived via dimensional analysis. The important thing is that a minimum exists. Planck-length is a proper name for it and the exact size is less important.

Generating a Hilbert space GPS

The first step is the introduction of a suitable GPS system in Hilbert space. This can be done by taking an orthonormal base of Hilbert vectors and add quaternion values to them. Due to the separability of the Hilbert space this number set must be countable. Let us take the rational

quaternions as an example. This construction defines a normal operator Q with countable infinite number of eigenvectors $|\varrho\rangle$ and corresponding eigenvalues ϱ. We will use the name **coordinate space** for the eigenspace of the **coordinate operator** Q.

The quaternions clearly have an origin. In contrast, the unit sphere of the Hilbert space, which contains all eigenvectors of Q is an affine space. The eigenvectors of Q form an orthonormal base. This singles out the eigenvector that belongs to the origin of the eigenspace. It indicates that Q must only be used for relative locations. Also the real axis has no equivalent in the isomorphism between the unit sphere of the Hilbert space and the eigenspace of Q. So, we will neglect this part of the eigenspace of Q during the specification of a GPS-like operator. (It appears that nature does the same). We will only look at the imaginary part of the eigenspace of Q.

When we speak about the (Q) **coordinate distance** between two vectors $|f\rangle$ and $|g\rangle$ in Hilbert space, then we mean the numerical distance between the values of $\langle f|Q f\rangle/\langle f|f\rangle$ and $\langle g|Q g\rangle/\langle g|g\rangle$.

Q has an infinite but countable number of eigenvalues. A location in coordinate space represents a location on the unit sphere of Hilbert space.

The fact that Q must be bounded means that Q has a **boundary** $\Im$ at a finite distance from its origin.

Take the polar decomposition of the normal coordinate operator Q in a unitary part $Ų$ and a positive operator $Ņ$. The eigenspace of $Ų$ is the **uni-coordinate space**. Like the unit sphere of the Hilbert space, the uni-coordinate space *is an affine space*. Besides of that also no preferred direction should exist in this unit sphere. But that is not the case!

The eigenspace of Q consists of all eigenvalues of Q. The eigenspace is not a closed set and it does not include infinity. If the eigenspace of Q was granular, then in order to be able to act as a kind of GPS the granules must have a fixed size. A dense packing of the granules would create preferred directions. It means

that in that case Q is not isotropic. In contrast, the unit sphere of the Hilbert space is isotropic. This sphere contains all eigenvectors of Q. With granularity spread in a regulated order, the granularity raises preferred imaginary directions. As a consequence the size of an infinitesimal step will depend on direction. This does not generally correspond with physical reality. Only in condensed matter such conditions may occur. We can conclude that regulated spread granularity of the eigenspace of Q leads to unphysical eigenvalues. Thus, let us restrict to countability. However, this restriction prohibits the use of sets of eigenvalues as parameters in differentiation operations.

With artificial means the eigenspace of the coordinate operator may be closed by adding all limits of converging rows of eigenvalues. In this away a closed set of quaternions results. However, most members of this closed set are not eigenvalues of the coordinate operator Q. The set is eigenspace of a corresponding coordinate operator $\breve{Q}$ in a rigged Hilbert space $\math**{H}$. Still, the use of the separable Hilbert space $\mathbf{H}$ coordinate operator Q will always prevent differentiation. Thus, for realistic physical conditions an alternative for this coordinate operator Q must be sought. The coordinate operator $\breve{Q}$ that has its residence in the rigged Hilbert space $\mathbf{H}$ does not suffer from preferred imaginary directions and has an eigenspace that is a continuum. For that reason we can use it as a **background coordinate operator**. In the future we will indicate the background operator $\breve{Q}$ as **the (background) GPS operator**. We will use the name **GPS like operator** for any operator that has an eigenspace that can be obtained via an invertible continuous transform or a reflection from the eigenspace of operator $\breve{Q}$.

Canonical conjugate

The four dimensions of the quaternions enable the split of Q into one Hermitian and three anti-Hermitian components. Via the inner product of the Hilbert space, each of these components gets a canonical conjugate. This creates a GMS-like operator.

$$< \varrho_\mu | \wp_\mu > = \tilde{f}_\mu(\wp_\mu) = < \wp_\mu | \varrho_\mu >^* = f_\mu^{*}(\varrho_\mu) \tag{1}$$
$$= exp\,(\boldsymbol{n_\mu} \cdot \wp_\mu \cdot \varrho_\mu / \hbar)$$

$\mu = 0, 1, 2, 3$ is the index of the dimension.

$\boldsymbol{n_\mu} = i,\ \boldsymbol{i},\ \boldsymbol{j},\ \boldsymbol{k}$ are imaginary base numbers.

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$$n_\mu^2 = -1 \tag{2}$$

The Hilbert vector $|\varphi_\mu >$ is eigenvector of operator Q_μ and corresponds with eigenvalue φ_μ.
The Hilbert vector $|\wp_\mu >$ is eigenvector of operator $\mathcal{P}_\mu$ and corresponds with eigenvalue $\wp_\mu$.

The constant $\hbar$ relates to the size of the granules.
For each dimension index μ holds:

$$< \varphi_\mu | \mathcal{P}_\mu \, f > \cdot \Delta\varphi_\mu \; = \; \mathbf{n}_\mu \cdot \hbar \cdot \Delta < \varphi_\mu | f > = \; \mathbf{n}_\mu \cdot \hbar \cdot \Delta f(\varphi_\mu) \tag{3}$$

$$[\mathcal{P}_\mu, Q_\mu] \; = \; \mathcal{P}_\mu Q - Q_\mu \mathcal{P}_\mu \; = \mathbf{n}_\mu \cdot \hbar \tag{4}$$

The definitions of the four canonical conjugates also define four (decoupled) complex Fourier transforms. The granularity decouples the Fourier transforms.

The Hilbert space GMS

The GMS operator $\breve{\mathcal{P}}$ of the rigged Hilbert space $\mathbf{H}$ is the canonical conjugate of the rigged Hilbert GPS operator $\breve{Q}$. Both operators reside in the Gelfand triple that corresponds to the separable Hilbert space $\mathcal{H}$. The canonical conjugate $\mathcal{P}$ of Q is formed from the combination of the four $\mathcal{P}_\mu$ operators. The same reasoning that is used for the Q operator also holds for the $\mathcal{P}$ operator. It means that also the $\mathcal{P}$ operator has a countable eigenspace and it has a boundary $\mathcal{C}$. Both boundaries have a one to one correspondence with the unit sphere Θ of the Hilbert space, but none of the eigenvectors of the Q operator coincides with an eigenvector of the $\mathcal{P}$ operator.

GPS stands for Global Positioning System.
GMS stands for Global Momentum System.
For positioning purposes only the imaginary part of the eigenspaces are used. The real part is ignored.

Like with positions, in a given static status quo not all momentum eigenvalues of the GMS operator are physical quantities. Only a countable subset deserves that qualification.

The fourth dimension

Often time is perceived as the fourth dimension. However, Piron and Einstein prove that when space is occupying the imaginary part, our common notion of time is unfit to act as the fourth dimension of the hyper complex number space. Einstein's special relativity indicates that a Minkowski signature characterizes the common spacetime concept. It means that a rectangular triangle relation exists between the spacetime step, the space step and the coordinate time step, where the coordinate time steps acts as the hypotenuse. Thus the coordinate time step is not perpendicular to the space step as the fourth dimension in quaternion space would be.

In combination with Garret Birkhoff and John von Neumann, Constantin Piron proved that the values of inner products of Hilbert vectors must belong to a division ring. When observables must stay expressible in such numbers, then they can maximally be quaternions. In that case, the real part of the quaternion of which the imaginary part represents a space coordinate, cannot be coordinate time.

Another argument is the following. Neither traditional quantum logic nor the corresponding separable Hilbert space $Ħ$ can represent dynamics. Thus, time is not an observable that fits in this separable Hilbert space $Ħ$. However, both traditional quantum logic and the corresponding Hilbert space may contain items that represent the precondition of change.

Due to the fact that the Hilbert space is separable, the observable quantities must be countable. In fact many physical quantities are granular. For example the granularity of space is characterized by the Planck-length l_{Pl}. The fourth dimension is supposed to be granular as well.

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Two quantities, other than space and time that are known to be granular are action and entropy. The granularity of action is characterized by the Planck constant. The granularity of entropy is characterized by the Boltzmann constant. Both are valid candidates for the fourth dimension. Energy is not a valid candidate, because it represents action per unit of time. Thus, it would introduce a notion of time via this backdoor. Action represents change. Entropy represents potential change. Field values represent preconditions of change.

Another possibility is to use the spacetime step as the fourth dimension. This step is perpendicular to the space step. This interpretation immediately poses the question what then the physical significance is of this spacetime step.

Until we encounter the requirement to fill it, the gap of the fourth dimension can be left open. One thing is for sure; coordinate time does not fit in that gap.

Time and dynamics

Dynamics and its progression parameter time do not fit in a Hilbert space that can only represent a static status quo. That also means that this Hilbert space does not support the corresponding operator. However, the static representation of the preconditions of change is represented in this Hilbert space. Its interpretation is then as the precondition for the change that will be applied in the next dynamical step.

For example potential displacement is characterized by momentum, which is the canonical conjugate of space. A progression step is required in order to determine the actual displacement. The progression step occurs between the instants of validity of subsequent Hilbert spaces. As a consequence the displacement gets its significance by comparing subsequent Hilbert spaces.

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This means that the duration of the progression step is unimportant.

When the smallest possible space step $l_{Pl} = \sqrt{\hbar G/c^3}$ and the smallest possible coordinate time step $t_{Pl} = \sqrt{\hbar G/c^5}$ are put into the Minkowski signature, $\Delta t^2 = \Delta \tau^2 - \Delta q^2/c^2$ then the corresponding spacetime step $\Delta \tau$ is zero.

The number of Planck-time steps equals the number of global progression steps. The number of Planck-length steps must always be lower than the number of Planck-time steps. The photon never takes a non-zero spacetime step. The number of its space steps always equals the number of its time steps.

Any particle that does not travel with light speed skips some of its space steps. Any particle can take a space step in a direction that differs from the direction of a previous step.

According to the Minkowski signature of spacetime the proper time step of information transfer is zero. The Hilbert book model takes the duration of the progression step equal to the proper time step of information transfer.

Displacement goes together with a reconfiguration of the fields. An acceleration of an item goes together with an extra field component.

Action is change. In this respect its role is similar to the role of displacement. Also the action step gets its significance by comparing subsequent Hilbert spaces. Fields represent the preconditions for the next action step.

Hilbert functions

Coordinate operators enable the definition of a special type of functions. Take a coordinate operator Q. Next take an arbitrary Hilbert vector $|f>$. Construct the inner products of this vector with all eigenvectors $\{|\varphi>\}$. of

Q. Use the eigenvalues {ǫ} of Q as variable. Now we have defined **Hilbert function** $f(ǫ)$ as

$$f(ǫ) = < f|ǫ >; for\ all\ ǫ\ in\ \{ǫ\} \tag{1}$$

Now the Fourier transform $\tilde{f}(\wp)$ of $f(ǫ)$ is defined using the canonical conjugate $\mathcal{P}$ of Q via:

$$\tilde{f}(\wp) = < \tilde{f}|\wp >; for\ all\ \wp\ in\ \{\wp\} \tag{2}$$

This is again a Hilbert function, but it uses a different coordinate operator ($\mathcal{P}$).

Hilbert functions are sampled functions. They are not differentiable. They can be approximated by a corresponding continuous function, which may be differentiable.

The continuous approximation of $f(ǫ)$ is indicated as $\overset{\rightharpoonup}{f}(\tilde{ǫ})$. Both the function and its parameters are smooth.

The sampled version of continuous function $g(q)$ is indicated as $\ddot{g}(\ddot{q})$.

The components of Hilbert functions are always decoupled. The same holds for their Fourier transforms. *For Hilbert functions no divergence and no curl exists.*

The components of (quaternionic) continuous functions are always coupled. The same holds for their Fourier transforms. For continuous functions divergence and curl may exist. However, inside a separable Hilbert space **Ħ** continuous functions only can act as Hilbert vectors. This is the case in ℓ^2 space[54].

Limitedness

Countability

The separable Hilbert space **Ħ** has a countable dimension. It means that the eigenvalues of normal operators may offer a dense coverage of a connected part of the number space, but it is not a closed coverage. The number space is a continuum. The eigenspace does not include all limits

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[54] http://en.wikipedia.org/wiki/Lp_space#Hilbert_spaces

of all convergent rows. At least a closed subset of the whole number space is densely covered by the set of eigenvectors. An eigenvector represents an atomic predicate that represents the corresponding attribute of the considered item. The eigenvector lies inside the subspace that represents the item. The corresponding atomic predicate states that the corresponding attribute of the item lies inside the environment that is represented by the eigenvector.

Granularity

The fact that the separable Hilbert space H has a countable number of orthonormal base vectors does not on itself render the eigenspace of every normal operator granular. We could cover a closed subset of the whole quaternionic number space with a countable number of rational quaternions. However, the Planck-length sets a minimum difference for positions and this renders the corresponding position operator granular. The way this granularity is distributed may cause particular features. For example dense packing causes preferred directions. Preferred directions do not commonly occur in nature. Such directions occur in condensed matter. Thus, dense packing or any other kind of organized packing does not generally occur in nature. It may occur in horizons. (It happens in the horizons of black holes). This means that the physical use of a granular coordinate operator is restricted to specific situations. However, from the Q operator a corresponding background GPS operator $\breve{Q}$ can be derived that resides in the corresponding rigged Hilbert space $ℍ$. The set of closed subspaces of this rigged Hilbert space $ℍ$ is not lattice isomorph with traditional quantum logic. Thus, it is not a proper model of that logic. This conflicts with our primary goal.

Both Q and $\breve{Q}$ are not suitable as granular position operator. We must find a possible realization of a granular position operator that resides in the separable Hilbert space.

Investigating an alternative operator

In order not to generate preferred directions the alternative operator must not support an eigenspace that contains multidimensional subsets that are not horizons. Still it must deliver positions as eigenvalues. Part of the

solution is that this new operator relates to a background GPS coordinate system.

A corresponding continuous GPS coordinate operator that can deliver such a GPS background coordinate system can only reside in the rigged Hilbert space Ħ that corresponds to the considered separable Hilbert space.

The eigenspace of the target operator may consist of
- a set of separate points (granules)
- a set of curves (chains of granules)
- a set of horizons (surfaces consisting of granules)

These elements are located with respect to the mentioned background coordinate system.

It must be possible to locate the current position of ALL physical particles with the eigenvalue set of the new operator.

The operator must exhibit the granularity of the position attribute. At the same time, the position must not be related to a fixed lattice. As a consequence: Any position difference must be equal or larger than the Planck-length.

Further, a sensible reason must be found for the existence of the granules. They must not just fall from heaven.

Between subsequent Hilbert spaces the position may stay stationary. However, when a difference occurs, it must again be equal or larger than the Planck-length. On its turn this means that between subsequent Hilbert spaces the eigenspaces of the target operator must be related.

Apart from the horizons the solution may be given by a set of chains of granules. Each chain has a sub-chain of past granules, a current granule and a sub-chain of future granules. The space step may be zero. Otherwise, during the step to the next Hilbert space, the first granule in

the future chain of the current Hilbert space becomes the current granule in the subsequent Hilbert space. At the same instance the current granule turns into the last granule in the past chain.

In each chain only the current granules will deliver observable values. Fields take care that in each chain sufficient smoothness exists around the current granules. For that reason the field in the surrounds of the current granules acts like a QPAD that regulates the position of that granule. The result of this investigation is a strand operator.

The reason behind granules

The fact that the QPAD's anchor on the granules can be solved quickly when we can find a reason for the granules to be part of the corresponding quaternionic probability amplitude distribution (QPAD). This reason is fully supplied when the granule is the ground state of the QPAD.

Now the quest changes to the reason why the QPAD exists. This reason can be found in the coupling of the eigenspace of the position operator that houses in separatable Hilbert space and the eigenspace of the GPS-like operator in the rigged Hilbert space that delivers the background coordinate system. This coupling is inaccurate in a stochastic sense. The QPAD reflects this stochastic relation.

Several types of QPAD's exist. Every type of QPAD corresponds with a type of elementary particle. The types can be grouped in categories. This diversity and partitioning is the secret behind the standard model.

However, no great diversity exists with respect to the ground states. All localizable types feature nearly the same ground state.

The scale of the extent of the ground state is of the order of the Planck-length. This sets the size of the granules that represent the eigenvalues of the new position operator.

When the Fourier transformation of the full QPAD is taken, then the result gives information on the displacement that will be performed in the next progression step.

Apart from a constant, the ground state of the QPAD is invariant under Fourier transformation. This constant is one of (1, i, -1 or -i). This means that the momentum operator that corresponds to the new position operator has also a granular eigenspace.

Strand operator

With exception of its horizons, the eigenspace of the strand operator does not cause preferred directions. Thus, its eigenvalues do not suffer the anomalies of the eigenspace of the Q operator. The elements in its eigenspace have a direction, but that direction is related to local physical conditions.

A strand operator $\mathbb{Q}$ can be defined along the following steps:
- Take a chain of granules.
- All granules have the same size.
- Each granule in this chain can be given an integer ordering number.
- The background coordinate GPS operator can be used to give each granule in a chain a unique position.
- The coupling of the granule with the position in the background coordinates is not precise. The inaccuracy is stochastic and is of the order of the Planck-length. This effect determines the size of the granules.
- Each chain consists of a past sub-chain, a current granule and a future sub-chain. The ordering number of the current granule is zero.
- If the set of Hilbert spaces steps to the subsequent Hilbert space, then the position of the current granule stays stationary or it becomes the position of the last granule in the past sub-chain. In that case the current granule becomes the place of what was the first granule in the future sub-chain.
- A QPAD that extends beyond the size of the granule takes care that in each chain sufficient smoothness exists around the current granules.

- The granule is the ground state of this QPAD.
- Define a set of such chains.
- These measures leave a freedom that corresponds to a fluctuation of the chains over subsequent Hilbert spaces.
- Each separate QPAD in the chain contains information on the displacement in the next progression step.
- Taken over a small set of subsequent Hilbert spaces, the movement of the current granule reflects the influence of the QPAD that controls the smoothness of the chain in the surround of the current granule.
- This distribution describes the properties of a moving, rotating and diffusing cloud of virtual information carrying quanta.
- Depending on how the distribution is viewed, the QPAD describes the probability density of the information carried by these quanta.
- In any case the squared modulus of the QPAD describes the probable position of the current granule.
- Taking the Fourier transform of the distribution reveals similar information about the canonical conjugated coordinate.

Further:
1. The eigenspace of the strand operator can only house a finite number of chains.
2. The eigenspace of the strand operator does not house volumes.
3. The eigenspace of the strand operator houses horizons.
4. These horizons have the shape of bubbles.
5. The bubbles consist of densely packed granules.
6. In this configuration the granules take their minimal (ground state) shape.

Chains can split and they can merge. The corresponding creation and annihilation occurs during a progression step and is controlled by the combined effect of Poisson distributions and spatial QPAD'sthat are attached to the current granules.

Horizons

Because the normal strand operator is bounded, a boundary surface exists at a finite distance from the origin of the background GPS coordinate system. We will call this boundary the **outer horizon**.

The covered space has an outer horizon, but it may also contain closed **inner horizons**. Outside the outer horizon and inside the inner horizons no strands exist.

Black holes

Most inner horizons are borders of black holes. These horizons are bubbles that consist of densely packed granules. The QPAD's that are attached to these granules have taken their minimal possible size. Each granule is connected to a Hilbert vector which is eigenvector of the strand operator. That Hilbert vector represents a quantum logical proposition. It carries a single bit of information that indicates its membership of the set of eigenvectors of the strand operator. The inner horizons form an exception to the rule that the granules must not form a multidimensional subset.

When a large piece of matter collapses into a black hole, then the QPAD's that are attached to the anchor points of particles collapse into their smallest possible shape. They take the shape of the granule and all granules group at the horizon such that they form the horizon.

When matter falls onto the horizon of the black hole then the QPAD's of these particles are forced into their smallest possible extent, which is their ground state. Next these granules are added to the horizon of the black hole

Start horizon

Inner horizons and outer horizons exist. In the Hilbert space that describes the first element in the sequence of static status quos, the eigenspace of the strand operator may consist of a **start horizon**. It is at the same time an inner and an outer horizon.

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The idea that at the start of the universe an inner horizon coincides with the outer horizon is speculation. Still the concept of a start horizon is an intriguing possibility and it fits well into the Hilbert book model..

The start horizon can be interpreted as a bubble that existed in empty space and that converted into matter[55]. It is at same time an inner horizon and an outer horizon. Its inner side is empty. Outside its outer side nothing exists. The start horizon is a bubble that is densely covered with granules. In the start horizon the "granules" were huge. As a consequence that bubble was instable. The huge "granules" granules collapsed into their ground state. Despite the fact that the former state offered the capability to form bubbles, the ground state that also offers this capability is much more stable. After the implosion the new more stable granules spread over the space that came available and their QPAD folded out, such that it took more space than just the size of the granule.

After the implosion, the preconditions for forming the start horizon are gone. There is no indication that during the lifetime of the universe a similar implosion happened more often.

Information horizons
Information horizons exist in different types.

A black hole has its own particular type of information horizon. Information cannot pass through that horizon. Due to the strong curvature, in the neighborhood of the horizon of a black hole the speed of information carrying light particles goes to zero. This information horizon is an event horizon. Not only information cannot pass. In fact any particle cannot pass the horizon. Instead the particle or debris of that particle are transferred into their ground state and added to the horizon. Some of the debris may escape.

Any physical item has its own **private information horizon**. Since light transports all information and has a limited speed, the private information horizon is in fact the image of the **start horizon**. This differs

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[55] See Birth of the universe

from the outer horizon. That **private information horizon** is set by the most distant items from which light can reach the observing item.

The private information horizon depends on the position of the observer. In a universe, a multitude of such private information horizons may exist. They might even be disjoint. This type of information horizon is determined by the reach of light since the start of the universe.

Configuration space

Since the unit sphere of the Hilbert space is an affine space and all eigenvectors of the GPS operator are represented in that space, the GPS can be considered to have no origin.

The chains may be closed or they start and end at a horizon. Further they may split and merge. This corresponds with creation and annihilation of particles that anchor on these chains.

Vacuum is not empty. It is the space in between horizons where chains may exist. Very short closed chains are spread all over vacuum. The granules in very short closed chains may represent the anchors of virtual particles.

Only the direct environment of the current granule of the chain is relevant. The QPAD that guides the current granule becomes part of the surrounding fields. It forms the basic constituent of the field. Its introduction extends the concept of separable Hilbert space. In a similar way it extends the concept of quantum logic.

Statistics

The QPAD is a constituent of the field that surrounds the granule. The creation and annihilation operators of fields have eigenfunctions that are Poisson distributions. Such distributions are produced by Poisson processes. A Poisson process can be combined with a subsequent binomial process in order to form a generalized Poisson process that has a lower efficiency than the original Poisson process. The efficiency is

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weakened by the weakening that is introduced by the binomial process. The spatial spread introduced by the QPAD can be interpreted as a binomial process with a spatial varying weakening factor. The spread function is equal to the squared modulus of the QPAD.

Canonical conjugate

The ground states of the QPAD's that determine the spatial spread functions are all (nearly) equal. Apart from a factor (1, i, -1 or $-i$) they are invariant under Fourier transformation. This means that the canonical conjugate of the strand operator has the same basic format. It is also constituted of granules.

Chain interpretation

In a single Hilbert space a chain may represent a piece of a **potential** past, present and future path of a particle. The present part of the path is formed by the direct surround of a single granule that acts as the current granule. In this single Hilbert space the granule corresponds to a Hilbert vector which is an eigenvector of the strand operator. The local path is determined by the current configuration of the field(s) that influence(s) the path. As a consequence, when taken over a sequence of Hilbert spaces, the chains fluctuate. This gives chains a place in the Hilbert book model. It must be noticed that the chains do not reflect the actual path. That only holds for the direct neighborhood of the current granule.

Taken over a sequence of Hilbert spaces the granules that represent the actual state of the chain represent the actual path of the corresponding particle.

Vacuum

The inaccuracy in the coupling between the background coordinate system and the eigenvalues of the position operator also plays a role in the space where little or no actual current granules exist. In this space virtual granules may exist during a very short period, such as single progression step. In fact the granules are part of a chain that forms a very short loop. These virtual granules form the main content of vacuum.

Virtual granules only occur inside the outer horizon and outside the inner horizons of the strand operator. A virtual granule can be interpreted as the ground state of the corresponding QPAD and may come close to the ground state of a linear or spherical harmonic oscillator. This ground state corresponds with the minimal extend that the QPAD can take.

In the Hilbert book model, the vacuum has a constant density ρ_{vac} of virtual granules.

In the Hilbert book model the space between horizons is stochastically, but on the average uniformly covered with virtual granules. At every progression step these virtual granules are redistributed. The actual granules exist in between these virtual granules, but they possess a wider spread of the corresponding QPAD's. These wider QPAD's tend to last longer at a (nearly) stationary location.

Fundamental measures and units

Events are instants of creation or annihilation of quanta. After creation the quantum becomes observable. After annihilation the quantum is no longer observable.

A *change* is the stepwise variation of the information carried by a quantum.

The *information* carried by a quantum is its position, its momentum its chirality and other characteristics of the corresponding particle.

The *distance* between two items equals the number of granules that fit between them.

The *progression time* between two events equals the number of progression steps between them.

The *action* in a progression interval equals the number of progression steps in that interval during which a change took place.

The *entropy* of a system equals the number of steps during which a change can take place in that system. It equals the number of granules in that system.

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In relation to the [covering field][56], a QPAD provides secondary information.

The basic measures of physics are:

$$\text{The Planck-length, } l_{Pl} = \sqrt{\hbar G/c^3} = 1.6 \cdot 10^{-35} \text{ m} \tag{1}$$

$$\text{The Planck (coordinate) time, } t_{Pl} = \sqrt{\hbar G/c^5} = 5.4 \cdot 10^{-44} \text{ s} \tag{2}$$

$$\text{The unit of action is the constant of Planck, } \hbar = 1.055 \cdot 10^{-34} \tag{3}$$

$$\text{The unit of entropy is the constant of Boltzmann, } k = 1.38 \times 10^{-23} \tag{4}$$

Numbers

Sign selections

Four possibilities exist due to the sign selections of the quaternions. One sign selection is covered by the conjugation $a \rightarrow a^*$. This selection switches the sign of all three imaginary base vectors. The other is caused by switching the sign of a single binary base vector $a \rightarrow a^\otimes$. For this sign selection one of the three available base vectors is selected. When relevant, then these choices are indicated by colors (r, g or b). Both methods switch the handedness (chirality). When both sign selections combine then the superscript $a \rightarrow a^\oplus$ is used. This combination does not switch handedness. Also this selection is colored.

It is also possible to use the extended **quaternionic conjugation concept**:

$$a^* = a^{\circled{3}} \tag{1}$$

$$a^\otimes = a^{\circled{1}} \tag{2}$$

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[56] Functions and fields; Hilbert field; Covering field

$$a^{\oplus} = a^{*\otimes} = a^{\textcircled{2}}$$ (3)

The encircled number stands for the number of switched base vectors. For the single sign switch $a^{\textcircled{1}}$, three independent direction selections are possible. We indicate these choices with r, g and b.
Similarly for the double sign switch $a^{\textcircled{2}}$, three independent direction selections are possible. We indicate these choices also with r, g and b. This direction belongs to the non-switched direction.
Without closer description the value of $a^{\textcircled{1}\textcircled{2}}$ is $a^{\textcircled{3}}$. It means that the colors are suspected to be the same.
The change from a to $a^{\textcircled{1}}$ or $a^{\textcircled{3}}$ cause a switch of the handedness of a.

$$a^{**} = a^{\textcircled{3}\textcircled{3}} = \left(a^{\textcircled{3}}\right)^{\textcircled{3}} = a$$ (4)

$$a^{\textcircled{1}\textcircled{1}} = a^{\textcircled{2}\textcircled{2}} = a$$ (5)

$$a^{\textcircled{1}\textcircled{2}} = a^{\textcircled{2}\textcircled{1}} = a^{\textcircled{3}}$$

The effects of the quaternionic conjugation are visible in the base numbers 1, $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$:

$$1^* = 1$$ (6)

$$\mathbf{i}^{\textcircled{3}} = -\mathbf{i}; \mathbf{j}^{\textcircled{3}} = -\mathbf{j}; \mathbf{k}^{\textcircled{3}} = -\mathbf{k};$$ (7)

The blue colored sign selection is given by

$$\mathbf{i}^{\textcircled{1}} = +\mathbf{i}; \mathbf{j}^{\textcircled{1}} = +\mathbf{j}; \mathbf{k}^{\textcircled{1}} = -\mathbf{k};$$ (8)

$$\mathbf{i}^{\textcircled{2}} = -\mathbf{i}; \mathbf{j}^{\textcircled{2}} = -\mathbf{j}; \mathbf{k}^{\textcircled{2}} = +\mathbf{k};$$ (9)

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In the blue colored sign selection, **k** follows the rules of complex conjugation. This renders its direction to a special direction.

The selected color direction is called the **longitudinal** direction. The the perpendicular directions are the transverse directions. Apart from that they are mutual perpendicular and perpendicular to the longitudinal direction, they have no preferred direction.

Sign selections and quaternionic distributions

Quaternionic distributions are supposed to obey a distribution wide sign selection. Thus, the distribution is characterized by one of the eight quaternionic sign flavors.

$$\psi^{⓪}, \psi^{①}, \psi^{①}, \psi^{①}, \psi^{②}, \psi^{②}, \psi^{②}, \text{or } \psi^{①}$$

Many of the elementary particles are characterized by an ordered pair of two field sign flavors. These fields are coupled with a coupling strength that is typical for the particle type. These particles obey a characteristic continuity equation[57].

Product rule

We use the quaternionic product rule.

$$ab = a_0 b_0 - \langle \boldsymbol{a}, \boldsymbol{b} \rangle + a_0 \boldsymbol{b} + \boldsymbol{a} b_0 + \boldsymbol{a} \times \boldsymbol{b} \tag{1}$$

$$\langle \boldsymbol{a}, \boldsymbol{b} \rangle = a_1 b_1 + a_2 b_2 + a_3 b_3 \tag{2}$$

$$\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{i}(a_2 b_3 - a_3 b_2) + \boldsymbol{j}(a_3 b_1 - a_1 b_3) + \boldsymbol{k}(a_1 b_2 - a_2 b_1) \tag{3}$$

Operators

The sign selections of operator $\nabla = (\nabla_0, \boldsymbol{\nabla})$ depend on the sign selections of position operator Q, which determines the sign selections for its eigenvalues $q = (q_0, \boldsymbol{q})$.

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[57] Hilbert field equations; Continuity equation for charges

Normally we consider the sign selection for operators Q and ∇ fixed to operators $Q^{\circledcirc}$ and $\nabla^{\circledcirc}$. Sometimes we chose ∇^* instead of operator ∇.

Quaternionic sign selection are directly connected with the concepts of **parity** and **spin**.

For quaternionic functions symmetry reduces the differences that are produced by conjugation and anti-symmetry stresses the differences. The same holds for operators.

Matrices
Another possibility is to present sign selections by [matrices](58).

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \qquad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

The σ_1 matrix switches the complex fields that together form the quaternion field.

$$\begin{bmatrix} \varphi_b \\ \varphi_a \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \varphi_a \\ \varphi_b \end{bmatrix}$$

The σ_2 matrix switches the real parts and the imaginary parts of the complex fields that together form the quaternion field and it switches both fields.

$$i \begin{bmatrix} -\varphi_b \\ \varphi_a \end{bmatrix} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} \varphi_a \\ \varphi_b \end{bmatrix}$$

The σ_3 matrix switches the sign of the first complex field.

$$\begin{bmatrix} -\varphi_a \\ \varphi_b \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi_a \\ \varphi_b \end{bmatrix}$$

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[58] http://www.vttoth.com/qt.htm

$$\sigma_k^2 = -i\sigma_1\sigma_1\sigma_1 = I$$

The Pauli matrices are involutory.
The [determinants](http://en.wikipedia.org/wiki/Determinant)[59] and [traces](http://en.wikipedia.org/wiki/Trace_of_a_matrix)[60] of the Pauli matrices are:

$$\det(\sigma_k) = -1$$

$$\mathrm{Tr}(\sigma_k) = 0$$

$$\alpha_k = \begin{bmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{bmatrix}$$

$$\alpha_1 = \begin{bmatrix} 0 & \boldsymbol{i} \\ -\boldsymbol{i} & 0 \end{bmatrix} \tag{1}$$

$$\alpha_2 = \begin{bmatrix} 0 & \boldsymbol{j} \\ -\boldsymbol{j} & 0 \end{bmatrix} \tag{2}$$

$$\alpha_3 = \begin{bmatrix} 0 & \boldsymbol{k} \\ -\boldsymbol{k} & 0 \end{bmatrix} \tag{3}$$

$$\beta = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{4}$$

The α_k matrices together select the imaginary base vectors. The β matrix exchanges the sign of all imaginary base vectors. Thus the β matrix implements the quaternionic conjugate. The conjugation also exchanges right handedness against left handedness.

Another way of exchanging right handedness against left handedness is the exchange of the sign of one of the imaginary base vectors.

$$\begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \psi_R \\ \psi_L \end{bmatrix} \tag{5}$$

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[59] http://en.wikipedia.org/wiki/Determinant
[60] http://en.wikipedia.org/wiki/Trace_of_a_matrix

$$\psi^* = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \psi \tag{6}$$

The gamma matrices[61] translate directly from complex fields to fully featured quaternionic fields. In this way four sign flavors of quaternionic fields are constructed from four complex fields. This is represented by four dimensional matrices and four dimensional spinors. The equivalent of the β matrix is the γ_β matrix.

$$\begin{bmatrix} \varphi_{La} \\ \varphi_{Lb} \\ \varphi_{Ra} \\ \varphi_{Rb} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varphi_{Ra} \\ \varphi_{Rb} \\ \varphi_{La} \\ \varphi_{Lb} \end{bmatrix} \tag{7}$$

It is false to interpret the matrices as vectors. They form a shorthand for handling spinors.

The Pauli matrix σ_1 represents the sign selection $a \rightarrow a^\otimes$, while the β matrix represents the sign selection $a \rightarrow a^*$. The other Pauli matrices and the α matrices implement the resulting part of the quaternion behavior for spinors.

Construction
The Cayley-Dickson construction formula enables the generation of a quaternion from two complex numbers:

$$p = a_0 + a_1\mathbf{k} + \mathbf{i}(b_0 + b_1\mathbf{k}) \tag{1}$$

$$q = c_0 + c_1\mathbf{k} + \mathbf{i}(d_0 + d_1\mathbf{k}) \tag{2}$$

$$(a, b)(c, d) = (ac - db^*; a^*d + cb) \tag{3}$$

$$\tag{4}$$

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[61] Appendix; Gamma matrices

$$r = pq$$

$$r_0 = a_0c_0 - a_1c_1 - b_0d_0 - b_1d_1 \qquad (5)$$

$$r_k = a_0c_1 - a_1c_0 - b_0d_1 + b_1d_0 \qquad (6)$$

$$r_i = a_0d_0 + a_1d_1 + b_0c_0 - b_1c_1 \qquad (7)$$

$$r_j = -a_1d_0 + a_0d_1 + b_0c_1 + b_1c_0 \qquad (8)$$

Apart from the Cayley-Dickson construction the 2^n-on construction exists.[62]

Colored signs

In the following text, the consequences for the product of the sign choices of the conjugate ③ is indicated by blue color $\pm$. The extra consequence ① for the product of the choice of the handedness of the cross product is indicated by red color $\pm$. The mixed sign selection ② acts on both sign colors.

The handedness can be switched by changing the sign of all imaginary base vectors.

$$\boldsymbol{ij = k \rightarrow (-i)(-j) = ij = -k} \qquad (1)$$

The sign selections split the ring of quaternions in four different realizations.

Path characteristics

The Frenet-Serret frame is devised for describing curved paths of particles

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[62] Appendix; 2^n-on construction.

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Let $\{\alpha_{qt}\}_t = \alpha(q,t)$ describe a curved path consisting of infinitesimal steps through a landscape $\{\alpha_q\}_q = \alpha(q)$ of imaginary quaternions α_{qt}, such that $||\dot{\alpha}(q(t))|| = 1$ for all t.

The 3D Frenet-Serret frame for the above path is given by:

$$\boldsymbol{T}(q(t)) := \frac{\partial \boldsymbol{\alpha}(q(t))}{\partial t} = \boldsymbol{T}(t) = \dot{\boldsymbol{\alpha}}(t) \tag{1}$$

$$\kappa(t) := ||\dot{\boldsymbol{T}}(t)|| \tag{2}$$

$$\kappa(t) \cdot \boldsymbol{N}(t) := \dot{\boldsymbol{T}}(t) \tag{3}$$

$$\boldsymbol{B}(t) := \boldsymbol{T}(t) \times \boldsymbol{N}(t) \tag{4}$$

$$||\boldsymbol{T}(t)|| = ||\boldsymbol{N}(t)|| = ||\boldsymbol{B}(t)|| = 1 \tag{5}$$

$\boldsymbol{T}(t)$ is the **tantrix** of curve $\boldsymbol{\alpha}(q(t))$ at instance t.
$\boldsymbol{N}(t)$ is the **principal normal** of curve $\boldsymbol{\alpha}(q(t))$ at instance t. It is only defined when $\kappa(t) \neq 0$.
$\boldsymbol{B}(t)$ is the **binormal** of curve $\boldsymbol{\alpha}(q(t))$ at instance t.
$\boldsymbol{T}(t)$, $\boldsymbol{N}(t)$ and $\boldsymbol{B}(t)$ are imaginary quaternions.
$\kappa(t)$ is the curvature of curve at $\boldsymbol{\alpha}(q(t))$ at instance t.
$r(t) = 1/\kappa(t)$ is the radius of curvature at instance t.
$\tau(t)$ is the torsion of curve $\boldsymbol{\alpha}(q(t))$ at instance t.

$$\begin{bmatrix} \dot{\boldsymbol{T}}(t) \\ \dot{\boldsymbol{N}}(t) \\ \dot{\boldsymbol{B}}(t) \end{bmatrix} = \begin{bmatrix} 0 & \kappa(t) & 0 \\ -\kappa(t) & 0 & \tau(t) \\ 0 & -\tau(t) & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{T}(t) \\ \boldsymbol{N}(t) \\ \boldsymbol{B}(t) \end{bmatrix} \tag{6}$$

The Frenet-Serret curves have particular characteristics. The path may be curved and curled. The path is completely determined by its tantrix, curvature and torsion given by functions of t. Each coordinate of the quaternionic function $\boldsymbol{\alpha}(q(t))$ has its own set of characteristics. This means

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that for a given quaternionic function these characteristics are quaternions rather than real numbers and they are all functions of parameter t.

Path equations

The path equations are given by

$$\dot{\boldsymbol{T}}(t) = \kappa(t) \cdot \boldsymbol{N}(t) \tag{1}$$

$$\dot{\boldsymbol{N}}(t) = -\kappa(t) \cdot \boldsymbol{T}(t) + \tau(t) \cdot \boldsymbol{B}(t) = -\kappa(t) \cdot \boldsymbol{T}(t) + \tau(t) \cdot \boldsymbol{T}(t) \times \boldsymbol{N}(t) \tag{2}$$

$$\dot{\boldsymbol{B}}(t) = -\tau(t) \cdot \boldsymbol{N}(t) = \boldsymbol{T}(t) \times \dot{\boldsymbol{N}}(t) + \dot{\boldsymbol{T}}(t) \times \boldsymbol{N}(t) \tag{3}$$

$$= \tau(t) \cdot \boldsymbol{T}(t) \times \boldsymbol{B}(t)$$

Curve length

The curve length $l(a, b)$ is defined by:

$$l(a, b) = \int_{x=a}^{x=b} |\dot{\boldsymbol{\alpha}}(q(x))| \, dx \tag{1}$$

The integration over the square of the modulus delivers the **action** S of the curve.

$$S(a, b) = \int_{x=a}^{x=b} |\dot{\boldsymbol{\alpha}}(q(x))|^2 \, dx \tag{2}$$

Reparameterization

The path characteristics $\kappa(t)$ and $\tau(t)$ together with the curve length and the curve action are independent of any reparameterization $s(t)$ of the progression parameter t.
A natural reparameterization is given by $s(t) = l(t_0, t)$.
This turns the curve $\boldsymbol{\alpha}(q(t))$ into a **natural** curve $\boldsymbol{\gamma}(q(s))$:

$$\boldsymbol{\gamma}(q(s)) = \boldsymbol{\alpha}(q(t)) \tag{1}$$

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Curves on a surface which minimize length between the endpoints are called geodesics.

The natural curve corresponds to a [geodesic](http://en.wikipedia.org/wiki/Geodesic)[63].

The consequence is that in three-dimensional space the corresponding movement obeys the [geodesic equation](http://en.wikipedia.org/wiki/Geodesic)[64]. The Lagrangian is an equivalent of this equation.

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[63] http://en.wikipedia.org/wiki/Geodesic

[64] Equations of motion; Lagrangian

Functions and fields

Distributions in quaternionic Hilbert space

Using a compact normal operator $\mathcal{Q}$ and a second distribution operator ρ with the same eigenvectors $\{|q_i\rangle\}_q$ but with eigenvalues $\{\rho_q\}$ we can generate a Hilbert [distribution](http://en.wikipedia.org/wiki/Distribution_(mathematics))[65].

$$\rho(q_i) = \langle q_i | \rho \, q_i \rangle$$

(1)

Operator ρ need not be a compact normal operator. Its spectrum of eigenvalues may be confined to a discrete set of points. Its eigenvectors are used. Its eigenvalues need not be used. . If they are used, the eigenvalues may consist of any kind of hyper complex number.

A Hilbert distribution is not differentiable. It can be seen as a combination of a set of Dirac delta functions that are multiplied with hyper complex numbers. If all numbers are quaternions, then it is a linear combination of Dirac delta functions that each represents a Hilbert vector.

The Hilbert space is separable. This means that the set of eigenvalues of an operator is countable. Thus a Hilbert distribution $\rho(q_i)$ is always discrete:

$$\rho(q_i) = \sum_{i=1}^{N} q_{E\,i} \cdot \delta(q_i - q_i)$$

(2)

The factors $q_{E\,i}$ are hyper complex 2^n-ons.

A Hilbert function is also a Hilbert distribution. (The reverse is not true). A special form of Hilbert distribution is the representation of a QPAD as a Hilbert function.

As stated before, every (quaternionic) Hilbert function can be split into four decoupled components. And every Hilbert function has a Fourier transform that consists of four decoupled Fourier transforms.

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[65] http://en.wikipedia.org/wiki/Distribution_(mathematics)

As in the case of a Hilbert function, a Hilbert distribution can represent a very dense coverage. In that case the distribution may become quasi differentiable.

Convolution of a Hilbert distribution with a blurring spread function can render the result (mostly) differentiable. In fact in the convolution the distribution is represented by a set of Dirac delta functions. Depending on the blur, the result may still be singular for example on the definition points of the Hilbert distribution. The blur may represent a probability distribution. Those blurs are well-formed.

A convolution means that every member of the Hilbert distribution is blurred with the same blur. It may also be done with a different blur, but then the result is no longer a convolution.

A special kind of Hilbert distributions is formed by the **elementary Hilbert distributions**. These distributions contain a single or only a few Hilbert vectors. They form the anchor points under the private fields, which represent elementary particles. Private fields are special kinds of Hilbert fields.

The Hilbert book model uses Hilbert distributions that consist of eigenvectors of the strand operator. The eigenvalues of the strand operator are taken from a continuum background coordinate system. In the Hilbert book model every elementary particle anchors on one ore more eigenvectors of the strand operator. Each elementary particle type has its own type of blur.

Hilbert field

By blurring the Hilbert distribution with a suitable spread function, the distribution can be transformed into a mostly continuous function. When the blur is the same for every element of the Hilbert distribution, then this converts the Hilbert distribution $\rho(q)$ into a [skew Hilbert field](http://en.wikipedia.org/wiki/Skew_field)[66] $\phi(q)$ via the convolution:

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[66] http://en.wikipedia.org/wiki/Skew_field

$$\phi(q) = f(q) \circ \rho(q) \qquad (1)$$

With a given Hilbert distribution $\rho(q)$, each blurring function $f(q)$ causes in this way a corresponding Hilbert field that is characterized by the blurring function $f(q)$.

Mathematically this convolution is impossible in a separable Hilbert space, but the corresponding rigged Hilbert space $\mathbf{H}$ is a suitable alternative. The vectors in an orthonormal base consisting of eigenvectors of the normal operator $\mathcal{Q}$ that resides in the separable Hilbert space $\mathcal{H}$ are represented in the rigged Hilbert space $\mathbf{H}$ by corresponding Dirac delta functions. We only use the vectors that belong to the Hilbert distribution $\rho(q)$. The values of the result of the convolution can be attached to the same orthonormal base vectors of the separable Hilbert space $\mathcal{H}$. This procedure attaches the field onto the separable Hilbert space $\mathcal{H}$. In principle the field covers all vectors of the separable Hilbert space.

When the blurs differ per element of the Hilbert distribution, then the Hilbert field can still be interpreted as the superposition of the contributing blurs, but it can no longer be considered as a convolution. Like with the convolution, the location of the blur must be reckoned in this superposition.

When there are only a few types of blurs, then each type constitutes via convolution with a corresponding Hilbert distribution a type specific Hilbert field. The covering Hilbert field can then be interpreted as the superposition of the (blur) type specific Hilbert fields.

The blurs are not hanging as a lose substance in the separable Hilbert space $\mathcal{H}$. The blurs are spread over the Hilbert vectors. Each Hilbert vector in the domain of a blur touches this blur and carries the local value of that blur.

Via superposition the Hilbert fields that correspond to the same Hilbert distribution form a covering Hilbert field.

A Hilbert field or type specific subfield can be categorized according to its:
- Symmetries
- Conjugation
- Corresponding blur function

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- Corresponding Hilbert distribution

Hilbert fields are differentiable. The dimension related components of a Hilbert field are coupled. The differential of a symmetric field or field part is anti-symmetric.
The differential of an anti- symmetric field or field part is symmetric.

Sampled Hilbert field
In separable Hilbert space, a **sampled Hilbert field** consists of its values attached to the eigenvectors of a normal operator, whose eigenspace acts as a coordinate system. A sampled Hilbert field is NOT differentiable. Its dimension related components are decoupled. It closely approximates a corresponding Hilbert field. The countable eigenspace of the normal operator closely matches the corresponding continuous eigenspace of a coordinate operator that resides in rigged Hilbert space **H**.

It can be interpreted as the distributed superposition of a number of Hilbert functions.

Blur function
The blur is a spread function. It is the reason of the significant similarity between optics and quantum physics. On the other hand, the blur is a probability distribution. This is the source of quantum noise. The probability distribution can be a probability amplitude distribution or its squared modulus, which is a probability density distribution. A quaternionic QPAD has the advantage that its squared modulus can specify the probability and the parameter can specify the full location, while the resulting factor represents related data in the form of a unitary quaternion. This quaternion can also carry its sign selection data, which includes its chirality, its spin and its parity. Compared to a complex amplitude distribution, this is a wealth of extra information. The shape of the blur contains secondary information. For example the Fourier transform of the blur offers momentum related data and the rotation of the blur represents angular momentum related data.

The simplest kind of blur that belongs to a particle relates to its [ground state[67]](#).

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[67] Functions and fields; Quaternionic transform split; Ground state

Hermite functions, which are eigenfunctions of the Fourier transformation, have a Gaussian blur. Coherent states[68], which are eigenfunctions of creation or annihilation operators have a Poissonian blur.

Quantum shot noise[69] produces a Poisson distribution. When large numbers of quanta are produced the distribution approaches a Gaussian distribution. A binomial process that follow a noise generating Poisson process can be combined with that binomial process into a generalized Poisson process with a lower efficiency. The binomial process represents a weakening effect. Spatial blur can be interpreted as a binomial process. This is because it represents a spatial diffusion effect. In the static model the Poisson processes only represent a lateral distribution. Taken over a sequence of Hilbert spaces the Poisson processes represent an additional temporal distribution. The efficiency of the detection of quanta is characterized by the detective quantum efficiency[70] (DQE) of the detector. Together with the Fourier transform of the spatial spread function this determines the signal to noise ratio in the information stream. The spread has an integrating (smoothing) effect. A sharper spread improves the signal, but also increases the noise. Any temporal integration reduces the noise. The effect of the lateral spread can be characterized by the Optical Transfer Function (OTF).

When the quanta are given a direction, then the blur becomes the equivalent of a QPAD. In the strand model the observable values of crossing switches of strands form QPAD's. See: http://www.motionmountain.net/research.html .

The blur plays a role when canonical conjugate operators occur together or in sequence. An extra blur is caused by the inaccuracy of the combination of these operators.
The blur has many functions and interpretations:

- Convolution with a smooth spread function makes a Hilbert distribution differentiable.
- The spread ensures the compactness of corresponding operators. It also reduces the frequency range that is covered by its Fourier transform.

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[68] Functions and fields; Quaternionic transform split; Coherent state

[69] http://en.wikipedia.org/wiki/Quantum_noise

[70] http://en.wikipedia.org/wiki/Detective_quantum_efficiency

- The spread function represents a probability density distribution or more in detail a quaternionic QPAD.
- Each elementary blur has a Fourier transform
- The probability distribution is characteristic for the inaccuracy of the expectation value of a category of operators, such as the GPS operator, the GMS operator, the Fourier transform, the creation/annihilation operators, the ladder operators and the number and ladder operators.
- The ground state is characterized by a typical spread function.
- The spread represents the probability that non-actual items exchange roles with actual items.
- The non-actual items represent subspaces of non-actualHilbert spaces that are ready to exchange roles with the currently valid Hilbert space.
- The non-actual items represent non-actual quantum logical propositions that may exchange roles with currently actual propositions.
- The non-actual quantum logical propositions are elements of a non-actual traditional quantum logic that is ready to exchange roles with the currently actual traditional quantum logic.
- The blur can be interpreted as a spatial quantum noise distribution.
- The blur can be interpreted as a spatial distribution of crossing switches of strands.
- The blur can be interpreted as a spatial distribution of generations or annihilations of quanta.
- The annihilation of a quant is equivalent to the generation of the corresponding anti-quant.
- The blur works as storage of past, present and future conditions.
- The blur can be squeezed in order to reflect the importance of momentum versus position.
- A ground state blur has in each direction a symmetric cut.
- An odd-times differentiated ground state blur has in one direction an asymmetric cut.
- An even-times differentiated ground state blur has in each direction a symmetric cut.

- The blur represents the sticky resistance of the universe against unordered changes (= changes of uniform movement in a geodesic). This is proved by the existence of [inertia](71)[71].
- The blur represents the sticky resistance of the collection of all propositions against unordered redefinitions.
- Blurs can be categorized according to the corresponding particle type.
- The superposition of blurs forms a field.
- A particle can be interpreted as the local excitation of this field.
- During a progression step the blur may get distorted.

In short: Without blur (quantum) physics is impossible!

Bypassing granularity

The fact that the Hilbert space is separable means that normal operators have a countable number of eigenvalues. That may still be an infinite number, but it means that the eigenspace of these operators is a countable set. It is not a continuum. It is possible to define a procedure that attaches an increasing natural number to each eigenvector and to each eigenvalue. It means that functions that are defined using such eigenvalues as parameters or as function values cannot be differentiated.

This does not mean that differentiable functions cannot exist in Hilbert space. For example, ℓ^2 is isomorphic with a separable Hilbert space H, and consists of integrable and differentiable functions, but, as with any separable Hilbert space, the eigenvalues of operators in ℓ^2 do not form a continuum. The mentioned functions act as Hilbert vectors. They are NOT Hilbert functions.

It is possible to use a trick that enables differentiation of fields that are defined as functions with eigenvalues of a normal operator as their parameter values. The trick consists of blurring all or a subset of the corresponding eigenvectors. When the blur is differentiable, then the field becomes differentiable as well. Still, if the blur extends wide enough, all

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[71] Influence; Inertia

members of an orthonormal base of Hilbert vectors touch a value of this field.

The blur does not fit IN the considered separable Hilbert space Ħ. It anchors onto a vector of this separable Hilbert space Ħ. In addition, all members of an orthonormal base of the Hilbert space touch a value of the blur.

The fact that differentiable quaternionic functions have an isotropic multi-dimensional parameter space (in the imaginary part of the quaternions) means that in contrast to the eigenspaces of coordinate operators in separable Hilbert space Ħ, this parameter space is continuous. All its dimension related components of the quaternionic functions are coupled. Instead in the canonical conjugated coordinate space a decoupling exists along not necessarily straight radial lines that decompose rotation free and divergence free parts of the quaternionic functions.

In our model the real part of quaternions that are applied as parameters appears to play a rather minor or at least a quite different role. For that reason, in most cases the results of differential geometry are more applicable than the theory of regular quaternionic functions.

Differential geometry also decomposes local space into three independent coordinate directions. These dimensions are selected according to the divergence and rotation properties of the analyzed functionality. This is similar to the approach in the Helmholtz or Hodge decomposition theorem. For example, the Frenet-Serret frame[72] features three mutually perpendicular directions.

The basic constituent and private field

There is only one basic constituent to Hilbert fields. That constituent is a QPAD. A small subset of Hilbert vectors forms an elementary Hilbert

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[72] Path characteristics

[distribution](#)[73]. Basic constituents exist in different forms. A corresponding **private field** is formed from the superpositions of the basic constituents that are attached to this elementary Hilbert distribution. In the rigged Hilbert space the corresponding Hilbert vectors in this distribution are represented by Dirac delta functions. These Dirac delta functions are convoluted with the corresponding basic constituent. In this superposition the Hilbert vectors form the anchor points of the basic constituents. The ground state of the basic constituent corresponds to a granule.

The basic constituent covers the whole separable Hilbert space. In separable Hilbert space a normal operator has a set of eigenvectors that forms an orthonormal base of the separable Hilbert space. The corresponding eigenvalues can be taken as parameters of functions that have values of the basic constituent as their function values. This means that every member of an orthonormal base of the separable Hilbert space touches a value of the constituent. Via linear combination of the eigenvectors any Hilbert vector can be reached and the corresponding value of the basic constituent can be closely approximated.

Via the anchor points and via the touching values the private fields are embedded in separable Hilbert space **Ħ**. The private field represents an elementary particle and the physical fields that belong to that particle. The anchor points are eigenvectors of a strand operator. The corresponding eigenvalues are taken from a background coordinate system, which is in fact the eigenspace of a GPS-like operator that resides in the rigged Hilbert space that belongs to the separable Hilbert space. The strand operator resides in separable Hilbert space and has an equivalent in rigged Hilbert space.

In this rigged Hilbert space the eigenvectors of the strand operator get their GPS-value. However, this coupling is inaccurate in a stochastic

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[73] Functions and fields; Elementary Hilbert distribution

sense. The blur that is attached to the eigenvector reflects this inaccuracy. Its ground state defines a granule.

Via its wave function a particle is identified with its private field. The notion of private field transfers quantum theory into quantum field theory. The dynamics of the particle are represented by the dynamics of their private field. The fact that the private field anchors on a Hilbert vector also identifies the particle with that Hilbert vector.

In some cases the private field is wide spread. That happens when the private field has the shape of a wave. In that case the anchor Hilbert vector is an eigenvector of the momentum operator rather than an eigenvector of the position operator. Also in that case the coupling with the background coordinate system is inaccurate in a stochastic sense. The background coordinate system in this case is the continuum canonical conjugate GSM system of the continuum GPS coordinate system. (Without further notice we will always assume that the anchor Hilbert vector is an eigenvector of the position operator).

The eigenspace of the strand operator contains a set of chains of granules. In each chain one granule is singled out and represents the current granule. It forms the anchor point of the chain's basic constituent. In fact the granule represents the ground state of the blur. It represents the central part of the QPAD and it equals zero over a region of the size of the Planck length.

Depending on its type each elementary particle owns one or more of these anchor points. Also depending on the type of the elementary particle the QPAD might have typical characteristics, but the ground states of these different QPAD's must all have nearly the same format and the same characteristics.

The strand operator possesses an outer horizon. This guarantees its compactness.

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The private field is a continuous function with parameters that are taken from a **selected** coordinate system. *The field itself is independent of the selection of this coordinate system*. Thus, only the when the field is taken as a function of the coordinates it depends on the coordinate selection. The selected coordinate system is related to a corresponding orthonormal base of the Hilbert space. That base consists of eigenvectors of a normal operator that resides in separable Hilbert space. Its eigenvalues are spread dense in the background coordinate system. That background coordinate system corresponds to the eigenspace of a GPS-like operator, which resides in rigged Hilbert space. This last eigenspace is a continuum.

We **assume** that in the context of the Hilbert book model all basic constituents are **differentiable** with respect to a selected coordinate system. This means that the basic constituents have a local divergence and a local curl. This corresponds to two vector fields. These vector fields are formed by imaginary quaternions. One is divergence free and the other is rotation free. This divides the imaginary part of the differential of the basic constituent locally in two components, a divergence free part and a rotation free part. Fourier transformation converts differentiation into a product of the original Fourier transform with the argument.

By redistributing the eigenvalues of the coordinate system a new coordinate system can be established for which the decomposition runs along straight coordinate lines. An appropriate reorientation of this coordinate system puts the decomposition in the canonical conjugated coordinate system along straight radial lines. In this coordinate system the ideal form of the Fourier transform can be applied to the considered configuration of the field. In this idealized condition the Fourier transform can be considered as three independent complex Fourier transforms. This trick can only be done for a static status quo, thus for a single separable Hilbert space and the static fields that are attached to it. Each static status quo has its own field configuration and asks for a adapted coordinate system in order to reach the idealized condition.

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When subsequent Hilbert spaces are considered, the private fields [move](74) together with the corresponding elementary Hilbert distribution. Apart from a linear movement the private fields may rotate. When a given Hilbert distribution contains just one Hilbert vector, then the constituent can rotate free around that point. If the Hilbert distribution contains multiple Hilbert vectors, then apart from these anchor points a center of movement exists. If it contains two vectors, then one axis is fixed with respect to the anchor points. If it contains three independent vectors, then the private field can only rotate together with these anchor points.

The movements are stochastic and have average characteristics such as position, speed, rotation axis, rotation phase and chirality. At each position within the private field these data may differ. Also the relative position of the carrying Hilbert vectors with respect to each other may change.

The basic constituent can be interpreted as the QPAD whose squared modulus describes the probable location of the carrying Hilbert vector. The private field does that for all its anchor points.

The granularity of the eigenspace of the position operator determines the minimal distance that can exist between the carrying vectors. It also describes the maximal change in average position that can occur during a single progression step. Apart from zero it also describes the minimal change.

It is sensible to select the coordinate system such that the members of the elementary Hilbert distribution are eigenvectors of the corresponding position operator. When a Fourier transform is taken, then this can no longer be valid. In that case the members of the elementary Hilbert

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[74] Dynamics

distribution must be linear combinations of the eigenvectors of the canonical conjugate of the original coordinate operator.

The Hilbert vectors that are member of a given elementary Hilbert distribution can be interpreted as eigenvectors of a strand operator. The corresponding eigenvalue is the position value of a granule that is the current granule of the chain.

The private fields overlap and because they are all QPAD's their superposition causes an interaction between the particles that anchor on these fields.

Covering field

Physical fields are not identifiable physical items. In the Hilbert book model, physical fields are represented by <u>Hilbert fields</u>[75]. For each Hilbert field, every member of an orthonormal base of the Hilbert space corresponds to a value of the field. If for this base the set of eigenvectors of a normal operator is selected, then in this way this field can be coupled to a parameter system that is formed by the corresponding eigenvalues of the normal operator.

These parameters are not necessarily physical quantities. The physical coordinates of identifiable physical items are granular. They can only change with steps that are equal to a Planck-length. Coordinates that are eigenvalues of normal operators in separable Hilbert space are countable. The set of rational quaternions is countable, but this set is not granular in the sense that a difference has a minimal step size. In rigged Hilbert space the eigenspace of a normal operator may be uncountable. It means that this space forms a continuum. The set of all imaginary quaternions forms a continuum. In a given static status quo, only a countable and granular subset of these eigenvalues can represent physical quantities.

Each elementary particle corresponds to a **private field**. A **covering field** is formed by the superposition of these private fields. Each private field that belongs to an elementary particle is characterized by a complicated

[75]Distributions and fields; Hilbert fields

blur function whose characteristics are typical for the particle type. That blur function can be interpreted as a complicated *QPAD*. This blur function anchors on **a small number of Hilbert vectors,** which together span a Hilbert subspace. These vectors form an elementary Hilbert distribution. The number of Hilbert vectors on which this elementary Hilbert distribution is based depends on the type of the elementary particle. The anchor points correspond to current granules of chains that reside in the eigenspace of the strand operator. The granules are ground states of basic field constituents.

The blurs of elementary particles are smooth and fade out at long distances. As a consequence the covering field is smooth as well and its squared modulus can be considered as a single - very wide spread - QPAD.

Taken over a series of static status quos the simplest form of blur is a Poisson distribution. Thus, dynamically, the covering field can also be seen as a series of parallel Poisson processes.

Depending on the type of the constituting particles the covering field can be divided in subfields. Each type has its own subfield.

Depending on the coordinate operator that is selected for the background coordinate system, the parameters of the probability distributions are GPS related or GMS related.

Decomposition

The imaginary part of a Hilbert field can be decomposed in a rotation free part and a divergence free part.

The Helmholtz decomposition splits the static vector field $\boldsymbol{F}$ in a (transversal) divergence free part $\boldsymbol{F_t}$ and a (one dimensional longitudinal) rotation free part $\boldsymbol{F_l}$.

$$F = F_t + F_l = \nabla \times A - \nabla \varphi \tag{1}$$

Here φ is a scalar field and A is a vector field. In quaternionic terms φ and A are the real and the imaginary part of a quaternionic field. F is an imaginary quaternion.[76]

The significance of the terms "longitudinal" and "transversal" can be understood by computing the local three-dimensional Fourier transform of the vector field F, which we call $\widetilde{F}$. Next decompose this field, at each point k, into two components, one of which points longitudinally, i.e. parallel to k, the other of which points in the transverse direction, i.e. perpendicular to k.

$$\tag{2}$$
$$\widetilde{F}(k) = \widetilde{F}_l(k) + \widetilde{F}_t(k)$$

$$\langle k, \widetilde{F}_t(k) \rangle = 0 \tag{3}$$

$$k \times \widetilde{F}_l(k) = 0 \tag{4}$$

The Fourier transform converts gradient into multiplication and vice versa. Due to these properties the inverse Fourier transform gives:

$$F = F_l + F_t \tag{5}$$

$$\langle \nabla, F_t \rangle = 0 \tag{6}$$

$$\nabla \times F_l = 0 \tag{7}$$

so this split indeed conforms to the Helmholtz decomposition.

This interpretation relies on idealized circumstance in which the decomposition runs along straight lines. This idealized condition is in general not provided. In normal conditions the decomposition and the

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[76] See next paragraph

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interpretation via Fourier transformation only work locally and with reduced accuracy.

If we take the covering field as the subject, then the above idealized picture no longer fits. The covering field is a superposition of a very large number of constituents that each bear on their own anchor point. These anchor points disturb the ideal picture. As a result the k lines are no longer straight lines but they get curved in the neighborhood of anchor points. The curvature of these lines can be used to define a local curvature value.

The decomposition depends on the choice of the selected coordinate system. In general such decomposition runs along curved lines. However, for a fixed field configuration it is possible to select a coordinate system for which the decomposition runs along straight lines. For this coordinate system it is possible to define a globally valid multidimensional Fourier transform that consists of a set of complex Fourier transforms. With respect to this coordinate system, other coordinate systems possess a locally defined curvature. In the idealized coordinate system all typical shapes are spatially invariant.

Decomposition and quaternionic Fourier transform

The above relations are the consequence of the properties of the quaternionic Fourier transform with respect to differentiation in an idealized coordinate system. The quaternionic differentiation of a quaternionic field runs;

$$
\begin{aligned}
g(q) &= \nabla f(q) \\
&= \nabla_0 f_0(q) \mp \langle \boldsymbol{\nabla}, \boldsymbol{f}(q) \rangle \pm \nabla_0 \boldsymbol{f}(q) + \boldsymbol{\nabla} f_0(q) \\
&\pm \left(\pm \boldsymbol{\nabla} \times \boldsymbol{f}(q) \right)
\end{aligned}
\tag{1}
$$

The colored $\mp$ and $\pm$ signs refer to the sign selections of quaternionic multiplication.

In Fourier space differentiation becomes multiplication with the canonical conjugate coordinate and therefore the equivalent equation becomes:

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$$\tilde{g}(k) = k\tilde{f}(k) \tag{2}$$
$$= k_0 \tilde{f}_0(k) \mp \langle \mathbf{k}, \tilde{\boldsymbol{f}}(k) \rangle \pm k_0 \tilde{\boldsymbol{f}}(k) + \mathbf{k}\tilde{f}_0(k)$$
$$\pm \left(\pm \mathbf{k} \times \tilde{\boldsymbol{f}}(k) \right)$$

For the imaginary parts holds:

$$\mathbf{g}(q) = \pm\nabla_0\boldsymbol{f}(q) + \nabla f_0(q) \pm \left(\pm\nabla \times \boldsymbol{f}(q) \right) \tag{3}$$

$$\tilde{\mathbf{g}}(k) = \pm k_0\tilde{\boldsymbol{f}}(k) + \mathbf{k}\tilde{f}_0(k) \pm \left(\pm\mathbf{k} \times \tilde{\boldsymbol{f}}(k) \right) \tag{4}$$

For the static part ($\nabla_0 f(q) = 0$) holds:

$$\mathbf{g}(q) = \nabla f_0(q) \pm \left(\pm\nabla \times \boldsymbol{f}(q) \right) \tag{5}$$

$$\tilde{\mathbf{g}}(k) = \mathbf{k}\tilde{f}_0(k) \pm \left(\pm\mathbf{k} \times \tilde{\boldsymbol{f}}(k) \right) \tag{6}$$

Since

$$\nabla \times \nabla f_0(q) = \mathbf{0} \tag{7}$$

and

$$\langle \nabla, \nabla \times \boldsymbol{f}(q) \rangle = 0 \tag{8}$$

this conforms to the previous paragraph[77].

Curvature field

The decomposition properties of the covering field determine the curvature of a secondary coordinate system with respect to the original

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[77] http://www.plasma.uu.se/CED/Book/EMFT_Book.pdf ;Formulas:F.104, F.105

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GPS coordinate system. That curvature on its turn defines a local metric and this metric defines a secondary tensor field which we will call **curvature field**. In this view the curvature field is derived from the covering field, which is built via superposition from the private fields of the separate particles.

The curvature that is caused by the blur of an elementary distribution represents a **private curvature field** and is independent of any electric charge (or other charges) of the elementary distribution. The curvature of the private curvature field is non-negative. It can be thought of being distributed over the domain of the private curvature field or its equivalent "charge" value being located at a center point. This "charge" is called mass and the center point is the center of mass.

At each location the local curvature can also act as a guide for the local direction of chains in that environment.

About the field concept

It is common practice to treat the EM fields and the gravitation field as different and independent subjects. In this interpretation, the gravitation field generates the curvature of the coordinate system in which the other fields must operate.

This paper takes a different approach. It puts the reason for the curvature of the coordinates in the properties and configuration of the covering field. The curvature that exists in this way is used to derive the curvature field. The wave function is interpreted as a private field that is part of the covering field. In this way it also contributes to the curvature field. This picture unifies all fields.

Functions in quaternionic Hilbert space

Due to their definition the Hilbert functions are only defined for an infinite but countable number of parameter values that lay dense in quaternion space. The Hilbert functions are close to a corresponding

differentiable function that resides in the corresponding rigged Hilbert space **Ħ**. In contrast to what holds for the Hilbert function, the dimension related components of the differentiable function are coupled.

A locatable probability distribution can be described by the convolution of a Dirac delta function that corresponds to the Hilbert vector, which represents the location of the weighted center of that distribution and a function $f(q)$ that describes the distribution relative to that location. In this way a blurred Hilbert vector is defined. This means that a blurred Hilbert vector can be closely approximated by a Hilbert function that is defined by the combination of a sharp *locator* Hilbert vector and a sharp *shape* Hilbert vector. We will use the addition "Hilbert" to the name of a continuous function for the Hilbert function that closely approximates that continuous function.

Thus, in Hilbert space the representative of the blurred locator Hilbert vector by a Hilbert function is a **Hilbert blur** or more specifically a **Hilbert QPAD.**

It is also possible to use an <u>elementary Hilbert distribution</u>[78] as the anchor of the continuous QPAD. This construct may represent an elementary particle. It is closely approximated by a private Hilbert field that is formed by the superposition of the Hilbert functions that are formed by a small set of locator Hilbert vectors and a single shape Hilbert vector.

<u>Pure states</u>[79] are characterized by blurred elementary Hilbert distributions.

Elementary Hilbert distribution

An elementary Hilbert distribution is a discrete distribution in which a single or a small number of Hilbert vectors participate. Together these

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[78] Functions and fields; Elementary Hilbert distribution

[79] States; State definition; Pure states

Hilbert vectors span a closed subspace that represents an un-blurred elementary item.

A unary distribution uses only one Hilbert vector that is eigenvector of the position operator.
A binary distribution uses two Hilbert vectors that are different eigenvectors of the position operator.
A ternary distribution uses three Hilbert vectors that are different eigenvectors of the position operator.

If the eigenvectors are selected such that they belong to mutually perpendicular imaginary (base) eigenvalues, then the elementary distributions are restricted to the mentioned three classes.

Characteristic functions

Now the position is connected to eigenvectors of the strand operator. The physical item is connected to a subspace rather than to a single vector. This subspace is spanned by the eigenvectors. So we can use a localizer that represents the (weighted) average position as a more precise indicator of the position of the physical item. On the other hand physical items are characterized by a state.

A state is either a [wave function](#)[80] or a probability density operator. Both use background coordinate position as their parameter. The wave function is a QPAD. Each wave function can be approximated by a Hilbert function. The squared modulus of the wave function indicates the probability of finding the position of the localizer.

The probability density operator is a weighted projection operator that is related both to the subspace that represents the item and to the position operator. It represents the probability that after measuring the position the parameter of the density distribution is found as the result.

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[80] States

Differentiation

Let $\breve{Q}$ be the selected coordinate operator.

The nabla operator ∇ is directly related to operator $\breve{Q}$. Thus, the sign selections for $\breve{Q}$ transfer to the sign selections for operator ∇. Due to sign selection four nabla operations exist. With a fixed nabla operator there may exist four results. Normally we reduce the use of the nabla operator to ∇ and ∇^*.

$$g(q) = \nabla f(q) \tag{1}$$

$$= \nabla_0 f_0(q) \mp \langle \boldsymbol{\nabla}, \boldsymbol{f}(q) \rangle \pm \nabla_0 \boldsymbol{f}(q) + \boldsymbol{\nabla} f_0(q) \pm \left(\pm \boldsymbol{\nabla} \times \boldsymbol{f}(q) \right)$$

$$\bar{g}(q) = \nabla^{①} f(q) \tag{2}$$

$$= \nabla_0 f_0(q) \pm \langle \boldsymbol{\nabla}, \boldsymbol{f}(q) \rangle \pm \nabla_0 \boldsymbol{f}(q) - \boldsymbol{\nabla} f_0(q) \pm \left(\mp \boldsymbol{\nabla} \times \boldsymbol{f}(q) \right)$$

∇ turns a symmetric field $f(q)$ into an anti-symmetric field $\nabla f(q)$ and an anti-symmetric field into a symmetric field.

The fact that $\nabla f(q) = 0$ means that $f(q)$ is constant or that at location q function $f(q)$ is in a maximum, a minimum, a saddle point or an asymmetric plateau. The consequence of this restriction is:

$$\nabla_0 f_0(q) = \mp \langle \boldsymbol{\nabla}, \boldsymbol{f}(q) \rangle \pm \nabla_0 \boldsymbol{f}(q) + \boldsymbol{\nabla} f_0(q) = \mp \left(\pm \boldsymbol{\nabla} \times \boldsymbol{f}(q) \right) \tag{3}$$

The fact that $\nabla^{①} f(q) = 0$ leads to different equations.

$$\nabla_0 f_0(q) = \pm \langle \boldsymbol{\nabla}, \boldsymbol{f}(q) \rangle \pm \nabla_0 \boldsymbol{f}(q) - \boldsymbol{\nabla} f_0(q) = \pm \left(\pm \boldsymbol{\nabla} \times \boldsymbol{f}(q) \right) \tag{4}$$

The quaternionic Laplace operator Δ is defined by

$$h(q) = \Delta f(q) = \nabla^{①} \nabla f(q) = \nabla \nabla^{①} f(q) = \nabla_0{}^2 f(q) + \boldsymbol{\nabla}^2 f(q) \tag{5}$$

A quaternionic function that fulfills $\Delta f(q) = 0$ is a harmonic function.

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A quaternionic function that fulfills $\boldsymbol{\nabla}^2 f(q) = 0$ is a spatial harmonic function.

For quaternionic functions in general:

$$\nabla\big(f(q)g(q)\big) \neq \big(\nabla f(q)\big)g(q) + f(q)\nabla g(q) \qquad (6)$$

Covariant derivative

The covariant derivative plays a role in the Lagrangian and in the equation of motion.

The covariant derivative D is defined as

$$Df(q) = \nabla f(q) - \boldsymbol{A}(q)\, f(q) \tag{1}$$

This is interesting with respect to a gauge transformation of the form

$$f'(q) = G(q)\, f(q) \tag{2}$$

$$G^*(q)\, G(q) = 1 \tag{3}$$

$$\nabla G(q) = \boldsymbol{H}(q)G(q) \tag{4}$$

where with a corresponding vector potential transformation

$$\boldsymbol{A}'(q) = \boldsymbol{A}(q) + \boldsymbol{H}(q) \tag{5}$$

$$D' = \nabla - \boldsymbol{A}(q) - \boldsymbol{H}(q) \tag{6}$$

The following step is in general not valid for quaternionic functions. However, we assume that it is valid for $G(q)$ and $f(q)$.

$$\nabla\big(G(q)f(q)\big) \neq \big(\nabla G(q)\big)f(q) + G(q)\nabla f(q) \tag{7}$$

$$D'f'(q) = \boldsymbol{H}(q)\, G(q)\, f(q) + G(q)\, \nabla f(q) \tag{8}$$

$$-\boldsymbol{A}(q)G(q)f(q) - \boldsymbol{H}(q)G(q)f(q)$$

$$= G(q)\big(\nabla f(q) - \boldsymbol{A}(q)\, f(q)\big)$$

$$D'f'(q) = G(q)Df(q) \tag{9}$$

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Thus with that transformation pair not only the modulus of the function stays invariant but also the modulus of the covariant derivative stays invariant. Further

$$f'^*(q)D'f'(q) = f^*(q)\,G^*(q)\,\,G(q)\,Df(q) \tag{10}$$

$$= f^*(q)\,Df(q)$$

Above the right sided covariant derivative D is defined

$$\vec{D}f(q) = \vec{\nabla}f(q) - \boldsymbol{A}(q)\,f(q) \tag{11}$$

The left sided covariant derivative is defined as:

$$f(q)\overleftarrow{D} = f(q)\overleftarrow{\nabla} - f(q)\,\boldsymbol{B}(q) \tag{12}$$

We will use $\overleftrightarrow{D}$ for both left sided and right sided covariant derivative:

$$\overleftrightarrow{D}f(q) = \frac{\vec{\nabla}f(q) + f(q)\overleftarrow{\nabla}}{2} - \boldsymbol{A}(q)\,f(q) - f(q)\,\boldsymbol{B}(q) \tag{13}$$

Canonical conjugate

Remember that the operator $\check{Q}$ that resides in rigged Hilbert space $\boldsymbol{H}$ is defined such that the decomposition of the covering field runs along straight lines. At least we suppose that for the environment that we investigate an operator exists that does this with sufficient accuracy. First we restrict to a selected longitudinal direction. This restricts to a complex subspace of the full quaternionic number space.

The canonical conjugate of the operator $\check{Q}$ is the operator $\check{P}$. It is defined by using a complex subspace of a quaternionic number space that is used to specify inner products. It is defined by specifying the function that defines the inner products of the eigenvectors $|q>$ of $\check{Q}$ and $|p>$ of $\check{P}$ with real eigenvalues q and p.

$$< q|p > = \tilde{f}(p) = < p|q >^* = f^*(q) = exp\ (\boldsymbol{k} \cdot p \cdot q/\hbar) \tag{1}$$

The imaginary base number $\boldsymbol{k}$ belongs to a complex subspace of the quaternionic number space. The constant $\hbar$ is Planck's constant and relates to the granularity of the eigenspaces. If the Fourier transform of the ground state of a chain's QPAD is taken, then apart from a factor $(1, \boldsymbol{k}, -1\ or - \boldsymbol{k})$ the same function results. The average spread of the granule in phase space is characterized by $\hbar$.

Due to its specification, the canonical conjugate operator $\check{P}$ can be interpreted as a generator of displacement of the eigenvalues of $\check{Q}$. For this purpose the considered function $f(q)$ must be differentiable.

$$\check{P} = \boldsymbol{k} \cdot \hbar \cdot \frac{\partial}{\partial q} \tag{2}$$

$$< q|\check{P} f > = \boldsymbol{k} \cdot \hbar \cdot \frac{\partial}{\partial q} f(q) \tag{3}$$

This interpretation of the operator $\check{P}$ shows that the complex canonical conjugate shown here corresponds with the imaginary direction in which the differentiated function $f(q)$ is rotation free.

The definition leads to the commutator:

$$[\check{P}, \check{Q}] = \check{P}\check{Q} - \check{Q}\check{P} = \boldsymbol{k} \cdot \hbar \tag{4}$$

The sign selections of $\check{P}$ depend on the sign selections of $\check{Q}$.

Complex Fourier transform
The specification of the complex canonical conjugate also defines a **complex** Fourier transform.

Also here the imaginary direction must correspond to the direction in which the analyzed function is rotation free.

Let $|f>$ be the generator of a quaternionic function that is generated with the help of the eigenvectors and eigenfunctions of operator $\check{Q}$ with canonical conjugate $\check{P}$.

The Fourier transform $\mathcal{F}_i = U_{qp}$ converts the base $\{|q>\}_q$ into the base $\{|p>\}_p$. The inverse Fourier transform U_{pq} does the reverse. These transforms reside both in separable Hilbert space H, as well as in rigged Hilbert space $\mathcal{H}$.

$$< q|f > = < f|q >^* = f^*(q) = \sum_p (< q|p > \cdot < p|f >) \tag{1}$$

$$= \sum_p < p \, (< p|q >)|f >$$

$$= \sum_p < p \, U_{pq}|f >$$

$$= \sum_p < p|U_{qp} \, f >$$

$$< p|f > = \sum_q (< p|q > \cdot < q|f >) \tag{2}$$

$$= \sum_q < q|U_{pq} \, f >$$

When summation is replaced by integration the Fourier transformation is confined to the rigged Hilbert space. There it can be applied to continuous functions.

The complex Fourier transform of a symmetric (complex) function is a cosine transform. It is a real function.

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The complex Fourier transform of an anti-symmetric (complex) function is a sine transform. It is an imaginary function.

Through complex Fourier transformation the operators $\breve{P}$ and $\breve{Q}$ exchange roles.

The Hilbert function $\tilde{f}(p) = <f|p>$ denotes the Fourier transform of the Hilbert function $f(q) = <f|q>$

Heisenberg's uncertainty

The Heisenberg's uncertainty principle is a consequence of the definition of the combination of the canonical conjugate and the definition of the Hilbert field. It means that a small spread of q values goes together with a large spread of p values and vice versa.

$$\Delta q \cdot \Delta p \geq \hbar/2 \tag{1}$$

A **squeezed** coherent state[81] is any state such that the uncertainty principle is saturated. That is:

$$\Delta q \cdot \Delta p = \hbar/2 \tag{2}$$

See: http://en.wikipedia.org/wiki/Squeezed_coherent_state.
For animations: http://gerdbreitenbach.de/gallery/.

The ground states of the basic field constituents are squeezed coherent states.

The quaternionic displacement generator

The formula that defines $\breve{P}$ as a complex displacement generator:

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[81] Functions and fields: Quaternionic Fourier transform split: Functions invariant under Fourier transform: Coherent states

$$< q | \breve{P} f > = \ \boldsymbol{k} \cdot \hbar \cdot \frac{\partial}{\partial q} f(q) \tag{1}$$

can more generally be written as a quaternionic displacement generator for eigenvalues of operator $\breve{Q}$.

$$< q | \breve{P} f > = \ \hbar \cdot \nabla_q < q | f > = \ \hbar \cdot \nabla_q f^*(q) \tag{2}$$

$$= \ \hbar \int_p \nabla_q < q | p > \cdot < p | f > = \int_p < q | p > p < p | f >$$

This means that for all q and its canonical conjugated p holds:

$$\nabla_q < q | p > = \ < q | p > p \tag{3}$$

Here $|q >$ is the eigenvector belonging to eigenvalue q and $|p >$ is the eigenvector belonging to eigenvalue p. ∇_q represents quaternionic differentiation with respect to eigenvalues of operator $\breve{Q}$.

It is <u>shown</u>[82] that locally the operator ∇_q splits field $f^*(q)$ in a longitudinal rotation free part and a transverse divergence free part.

Idealized field conditions
Only in a complex subspace of the quaternionic number space the relation (3) between the canonical conjugates p and q can be simplified to:

$$< q | p > = \ \exp\left(\boldsymbol{k} \cdot p \cdot \frac{q}{\hbar} \right)) \tag{1}$$

The longitudinal direction runs in p space. The above simplification can only be valid when the longitudinal direction runs along straight radial lines. This simplification also enables the specification of a complex Fourier transform that is based on this formula (1).

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[82] Decomposition: Decomposition and quaternionic Fourier transform

It behaves as if the analyzed function is constant in dimensions that belong to other quaternionic imaginary directions. It is well-known that the Fourier transform of a constant delivers a Dirac delta function. Thus, the complex longitudinal Fourier transform equals a cut through the quaternionic Fourier transform of the full 3D imaginary quaternionic function or field.

This is similar to the cut through the 2D optical transfer function that is obtained when the Fourier analysis of the imaging device is confined to the image of a thin slit.

The configuration of the analyzed field determines whether the conditions are sufficiently ideal. Otherwise the field configuration induces at every location a local curvature of the actual background coordinate system that is defined using operator $\breve{Q}$ The eigenspace of the actual operator $\breve{Q}$ is curved with respect to the eigenspace of the idealized operator $\check{Q}$.

The position operator $\check{Q}$ is defined such that when the analyzed function or field is specified with $\check{Q}$ coordinates the longitudinal direction is stationary. It runs along straight radial lines. The construction of such an idealized position operator is possible for a given configuration of the analyzed field. When the analyzed field is the static covering field, then it holds for that field and not for a part of this field or the covering field that belongs to another static status quo.

In this paper, when nothing else is indicated, we confine Fourier analysis to the ideal quaternionic Fourier transform. When nothing is indicated we presume $\check{Q}$ coordinates and analysis of the covering field.

The formula below specifies the local relation between canonical conjugated coordinates when field conditions are not idealized.

$$\nabla_q < q|p > = < q|p > p \tag{2}$$

Affine space

The eigenvectors of a normal operator form an orthonormal base of the separable Hilbert space Ħ. This orthonormal base defines an affine unit sphere. Apparently, the correspondence with a $\breve{Q}$ type GPS operator that is equipped with an origin in its eigenspace is not a natural mapping for this affine Hilbert unit sphere. On the other hand, like this Hilbert unit sphere, the imaginary eigenspace of the $\breve{Q}$ type GPS operator has no preferred direction. When viewed from a particular Hilbert vector the mapping becomes more natural.

If a field covers all vectors of an orthonormal base, then it covers all of Hilbert space. The orientation along the longitudinal direction of the (covering) field is not natural for the Hilbert space, but it is natural for the combination of the field and a position operator that keeps the longitudinal lines straight. Thus apart from a shift of the origin, the position operator $\breve{Q}$ is fully determined by the properties of the field.

The origin of the eigenspace of the $\breve{Q}$ operator may be interpreted as the position of the observer. That selection would consume the last freedom for this operator.

There exists a point to point relation between an arbitrary $\breve{Q}$ type GPS operator and the $\breve{Q}$ operator. This point to point relation defines the curvature field.

Quaternionic Fourier transform split

The longitudinal Fourier transform represents only part of the full quaternionic Fourier transform. It depends on the selection of a radial line $\boldsymbol{k}(q)$ in p space that under ideal conditions runs along a straight line.

$$\mathcal{F}_{\mathbf{k}}\big(g(q)\big) = \mathcal{F}(g(q), \boldsymbol{k}(q)) \tag{1}$$

Or

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$$\mathcal{F}_{\parallel}\big(g(q)\big) \overset{\text{def}}{=} \mathcal{F}\Big(g_{\parallel}(q)\Big) \tag{2}$$

It relates to the full quaternionic Fourier transform $\mathcal{F}$

$$\mathcal{F}\big(g(q)\big) = \tilde{g}(p) \tag{3}$$

The inverse Fourier transform runs:

$$\mathcal{F}^{-1}\big(\tilde{g}(p)\big) = g(q) \tag{4}$$

The split in longitudinal and transverse Fourier transforms corresponds to a corresponding split in the multi-dimensional Dirac delta function.

We consider a field $g(q)$ that equals the quaternionic differentiation of another field f with respect to a selected (ideal) coordinate system Q.

$$g(q) = \nabla_q f \tag{5}$$

We use the results of the paragraph on decomposition. We only use the static and imaginary version of field $g(q)$.

For the static imaginary part $\mathbf{g}(q)$ holds:

$$\mathbf{g}(q) = \nabla f_0(q) \pm \big(\pm \nabla \times \mathbf{f}(q)\big) = \mathbf{g}_l(q) + \mathbf{g}_t(q) \tag{6}$$

In Fourier space differentiation becomes multiplication with the canonical conjugate coordinate $\mathbf{p}$ and therefore the equivalent equation becomes:

$$\tilde{\mathbf{g}}(p) = \mathbf{p}\tilde{f}_0(p) \pm \big(\pm \mathbf{p} \times \tilde{\mathbf{f}}(p)\big) = \tilde{\mathbf{g}}_l(p) + \tilde{\mathbf{g}}_t(p) \tag{7}$$

Since

$$\nabla \times \nabla f_0(q) = \mathbf{0} \rightarrow \nabla \times \mathbf{g}_l(q) = \mathbf{0} \tag{8}$$

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and

$$\langle \nabla, \nabla \times \boldsymbol{f}(q) \rangle = 0 \rightarrow \langle \nabla, \boldsymbol{g}_t(q) \rangle = 0 \tag{9}$$

Now we take

$$< q|\check{P} f > = \hbar \cdot \nabla_q < q|f > = \hbar \cdot \nabla_q f^*(q) = \mathbf{g}(q) \tag{10}$$

$$= \int_p < q|p > \cdot < p|g >$$

The static imaginary part is

$$< q|\check{P} f > = \hbar \cdot \nabla_q < q|f > = \hbar \cdot \nabla_q \boldsymbol{f}^*(q) = \mathbf{g}(q) \tag{11}$$

$$= Im\left(\int_p < q|p > \cdot < p|\boldsymbol{g} > \right) = \int_p Im(< q|p > \cdot < p|\boldsymbol{g} >)$$

$$= \int_p Im(< q|p > \cdot < p|\boldsymbol{g}_l >) + \int_p Im(< q|p > \cdot < p|\boldsymbol{g}_t >)$$

$$= \int_p Im(< q|p > \cdot \tilde{\boldsymbol{g}}_l(p)) + \int_p Im(< q|p > \cdot \tilde{\boldsymbol{g}}_t(p))$$

The left part is the longitudinal inverse Fourier transform of field $\tilde{\boldsymbol{g}}(p)$.
The right part is the transverse inverse Fourier transform of field $\tilde{\boldsymbol{g}}(p)$.
For the Fourier transform of $\mathbf{g}(q)$ holds the split:

$$\tilde{\boldsymbol{g}}(p) = \int_q Im(< p|q > \cdot \boldsymbol{g}_l(q)) + \int_p Im(< p|q > \cdot \boldsymbol{g}_t(q)) \tag{12}$$

$$= \int_q Im(< p|q > \cdot \boldsymbol{g}(q))$$

The longitudinal direction is a one dimensional (radial) space. The corresponding transverse direction is tangent to a sphere in 3D. Its direction depends on the field $\boldsymbol{g}(q)$ or alternatively on the combination of field f and the selected (ideal) coordinate system $\breve{Q}$.

For a weakly curved coordinate system $\breve{Q}$ the formulas hold with a restricted accuracy and within a restricted region.

Alternative transverse plane

The Cayley-Dickson construction, as well as Warren Smith's construction formula shows that the transverse part can be considered as a complex number space multiplied with a fixed imaginary quaternionic base number. The selection of the imaginary base number i is arbitrary as long as it is perpendicular to k. The resulting plane is spanned by axes i and ik. When base number i is divided away, then a normal complex number space results.

Also here a complex Fourier transform can be defined in a way that is similar to the longitudinal Fourier transform. It must be reckoned that the sign selections for these directions differ.

Alternative approach to Fourier transform

The following draws from the work of S. Thangavelu[83].

Let us take the non-abelian group $\mathbb{H}_1$ which is $\mathbb{R} \otimes \mathbb{R} \otimes \mathbb{R}$ with the group law

$$(x, y, t)(x_0, y_0, t_0) = (x + x_0; y + y_0; t + t_0 + x\, y_0) \tag{1}$$

Then it is clear that $\mathbb{H}_1$ is non-abelian and the Lebesgue measure $\mathrm{d}x\, \mathrm{d}y\, \mathrm{d}t$ is both left and right invariant Haar measure on $\mathbb{H}_1$. With this measure we can form the Hilbert space $L^2(\mathbb{H}_1)$. Let $\Gamma = \mathbb{Z} \otimes \mathbb{Z} \otimes \mathbb{Z}$. Then it is easy to

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[83] http://www.math.iitb.ac.in/atm/faha1/veluma.pdf

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check that Γ is a subgroup of $\mathbb{H}_1$ so that we can form the quotient $M = \Gamma/\mathbb{H}_1$ consisting of all right cosets of Γ. Functions on M are naturally identified with left Γ-invariant functions on $\mathbb{H}_1$. As the Lebesgue measure $dx\,dy\,dt$ is left Γ-invariant we can form $L_2(M)$ using the Lebesgue measure restricted to M. As a set we can identify M with $[0, 1)^3$ and we just think of $L^2(M)$ as $L^2([0, 1)^3)$.

Fourier expansion in the last variable allows us to decompose $L^2(M)$ into a direct sum of orthogonal subspaces. Simply define $\mathcal{H}_k$ to be the set of all $f \in L^2(M)$ which satisfy the condition

$$f(x, y, t + s) = \exp(2\pi i k s) f(x, y, t) \tag{2}$$

Then $\mathcal{H}_k$ is orthogonal to $\mathcal{H}_j$ whenever $k \neq j$ and any $f \in L^2(M)$ has the unique expansion

$$f = \sum_{k=-\infty}^{\infty} f_k \; ; \; f_k \in \mathcal{H}_k \tag{3}$$

In quaternionic terms, the split sees ik as imaginary quaternion $\boldsymbol{k}$ and the quaternionic Hilbert space is split in components according to the imaginary direction of $\boldsymbol{k}$, where the choice is between three mutually perpendicular directions.

For the moment, we are mainly interested in $\mathcal{H}_1$ which is a Hilbert space in its own right. It is interesting to note that functions in $\mathcal{H}_1$ are also invariant under the left action of Γ.

Our next example of a unitary operator is the following. Consider the map $J : \mathcal{H}_1 \to \mathcal{H}_1$ given by

$$J(x, y, t) = (-x, y, t - xy) \tag{4}$$

$$J^{\dagger}(x, y, t) = (x, -y, t - xy) \tag{5}$$

$$J^{\dagger} = J^{-1} \tag{6}$$

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$$J^2(x, y, t) = J(-x, \cdot\, y; t - xy) = (-x, -y; t) \tag{7}$$

$$J^4 = I \tag{8}$$

$$J(0, 0, t) = (0, 0, t) \tag{9}$$

$$J f(x, y, t) = f(J(x, y; t)) = f(-x, \cdot\, y, t - xy) \tag{10}$$

Weil-Brezin transform

Next consider the Weil-Brezin transform V:

$$V f(x, y, t) = exp(2\pi \boldsymbol{k} t) \sum_n f(x + n) \exp(2\pi \boldsymbol{k} n y) \tag{11}$$

$$\int_{y=0}^{1} |V f(x, y, t)|^2 dy = \int_{x=0}^{1} \sum_{n=-\infty}^{n=\infty} |f(x + n)|^2 dx \tag{12}$$

$$\iiint_0^1 |V f(x, y, t)|^2 dx \, dy \, dt = \int_0^1 |f(x)|^2 dx \tag{13}$$

V is unitary.

See also <u>Zak transform</u>

Fourier transform

We define the Fourier transform $\mathcal{F}$ by:

$$\mathcal{F} = V^\dagger J V \tag{14}$$

- $\mathcal{F}^4 f = f$; for every $f \in L^2(\mathbb{R})$
- $\mathcal{F}^2 f(x) = f(-x)$; for almost every $x \in \mathbb{R}$
- $\|\mathcal{F} f\|^2 = \|f\|^2$

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For $f \in L^1(\mathbb{R}) \setminus L^2(\mathbb{R})$ the Fourier transform is given by

$$\mathcal{F}f(\xi) = \int_{x \in \mathbb{R}} f(x) \cdot \exp(2\pi \boldsymbol{k}\, x) dx \tag{15}$$

If we further assume that $\mathcal{F}f \in L^1(\mathbb{R})$ then for almost every x we have

$$f(\xi) = \int_{\xi \in \mathbb{R}} \mathcal{F}f(\xi) \exp(2\pi \boldsymbol{k}\, x\, \xi) dx \tag{16}$$

Functions invariant under Fourier transform

In this section we confine to a complex part of the Hilbert space.
See http://en.wikipedia.org/wiki/Hermite_polynomials.
There exist two types of Hermite polynomials: (1, 2)

1. The probalist's Hermite polynomials:

$$H_n^{prob}(z) = (-1)^n \exp(\tfrac{1}{2}z^2) \frac{d^n}{dz^n} \exp(-\tfrac{1}{2}z^2).$$

2. The physicist's Hermite polynomials

$$H_n^{phys}(z) = (-1)^n \exp(z^2) \frac{d^n}{dx^n} \exp(-z^2)$$
$$= \exp(\tfrac{1}{2}z^2) \left(z - \frac{d}{dz}\right) \exp(-\tfrac{1}{2}z^2)$$

These two definitions are *not* exactly equivalent; either is a rescaling of the other:

$$H_n^{phys}(z) = 2^{n/2} H_n^{prob}(z\sqrt{2}) \tag{3}$$

In the following we focus on the physicist's Hermite polynomials.

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The Gaussian function $\phi(z)$ defined by

$$\varphi(x) \;=\; exp(-\pi \, z^2) \tag{4}$$

is an eigenfunction of $\mathcal{F}$. It means that its Fourier transform has the same form.

As $\mathcal{F}^4 = I$ any λ in its spectrum $\sigma\,(\mathcal{F})$ satisfies $\lambda^4 = 1$: Hence,

$$\sigma\,(\mathcal{F}) \;=\; \{1; -1;\; i; -i\}. \tag{5}$$

We take the Fourier transform of the expansion:

$$exp(-\tfrac{1}{2}\, z^2 + 2\, z\, c - c^2) \;=\; \sum_{n=0}^{\infty} exp(-\tfrac{1}{2}\, z^2)\, H_n(z)\, c^n/n! \tag{6}$$

First we take the Fourier transform of the left hand side:

$$\frac{1}{\sqrt{2\pi}} \int_{z=-\infty}^{\infty} exp(-\boldsymbol{k}\, z\, p_z)\, exp(-\tfrac{1}{2}\, z^2 + 2\, z\, c - c^2)\; dz \tag{7}$$

$$= \; exp(-\tfrac{1}{2}\, p_z^2 - 2\,\boldsymbol{k}\, p_z\, c + c^2)$$

$$= \; \sum_{n=0}^{\infty} exp(-\tfrac{1}{2}\, p_z^2)\, H_n(p_z)\, (-\boldsymbol{k}\, c)^n/n!$$

The Fourier transform of the right hand side is given by

$$\frac{1}{\sqrt{2\pi}} \sum_{n=0}^{\infty} \int_{z=-\infty}^{\infty} exp(-\boldsymbol{k}\, z\, p_z) \cdot exp(-\tfrac{1}{2}\, z^2)\, H_n(z)\, c^n/n!\; dz \tag{8}$$

Equating like powers of c in the transformed versions of the left- and right-hand sides gives

$$\tag{9}$$

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$$\frac{1}{\sqrt{2\pi}} \int_{z=-\infty}^{\infty} \exp(-\boldsymbol{k}\, z\, p_z) \cdot exp(-\tfrac{1}{2}\, z^2)\, H_n(z)\, c^n/n!\; dz$$

$$= (-\boldsymbol{k})^n \cdot \exp(-\tfrac{1}{2}\, p_z^2)\, H_n(p_z)\, \frac{c^n}{n!}$$

Let us define the Hermite functions $\psi_n(z)$

$$\psi_n(z) \overset{\text{def}}{=} \ <z|\psi_n> = c_n \exp(-\tfrac{1}{2}\, z^2)\, H_n(z) \tag{10}$$

$$|\mathcal{F}\,\psi_n> = |\psi_n> (-\boldsymbol{k})^n \tag{11}$$

with suitably chosen c_n so as to make

$$\|\psi_n\|^2 \ = \ 1 \tag{12}$$

$$c_n = \frac{1}{\sqrt{2^n n!\, \sqrt{\pi}}} \tag{13}$$

The importance of the Hermite functions lie in the following theorem.

"The Hermite functions ψ_n; $n \in \mathbb{N}$ form an orthonormal basis for $L^2(\mathbb{R})$"

Consider the operator

$$H \ = \ -\tfrac{1}{2}\frac{d^2}{dz^2} + \tfrac{1}{2}\, z^2 \tag{14}$$

Apply this to $\psi_n(z)$:

$$H \cdot \psi_n(z) = (\tfrac{1}{2} + n)\, \psi_n(z) \tag{15}$$

Thus, ψ_n is an eigenfunction of H.

Let $f = \psi_{4k+j}$ be any of the Hermite functions. Then we have

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$$\sum_{n=-\infty}^{\infty} f(y + n) \cdot \exp\left(-2 \pi \, k \, x \, (y + n)\right) \tag{16}$$

$$= (-k)^j \sum_{n=-\infty}^{\infty} f(x + n) \, exp(2 \pi \, k \, n \, y)$$

Proof: As

$$\mathcal{F} = V^\dagger J V \tag{17}$$

the equation

$$\mathcal{F} f = (-k)^j f \tag{18}$$

translates into

$$J V f(x; \, y; \, t) = (-k)^j \, V f(x; \, y; \, t) \tag{19}$$

With the definition of V and $t = xy$:

$$V f(x, y, t) = exp(2 \pi \, k \, t) \sum_n f(x + n) \, exp(2 \pi \, k \, n \, y) \tag{20}$$

QED.

The vectors $|\psi_n\rangle$ are eigenvectors of the Fourier transform operator with eigenvalues $(-k)^n$. The eigenfunctions $\psi_n(x)$ represent eigenvectors $|\psi_n\rangle$ that span the complex Hilbert space $\mathbf{H}_k$.
For higher n the central parts of $\psi_n(x)$ and $|\psi_n(x)|^2$ become a sinusoidal form.

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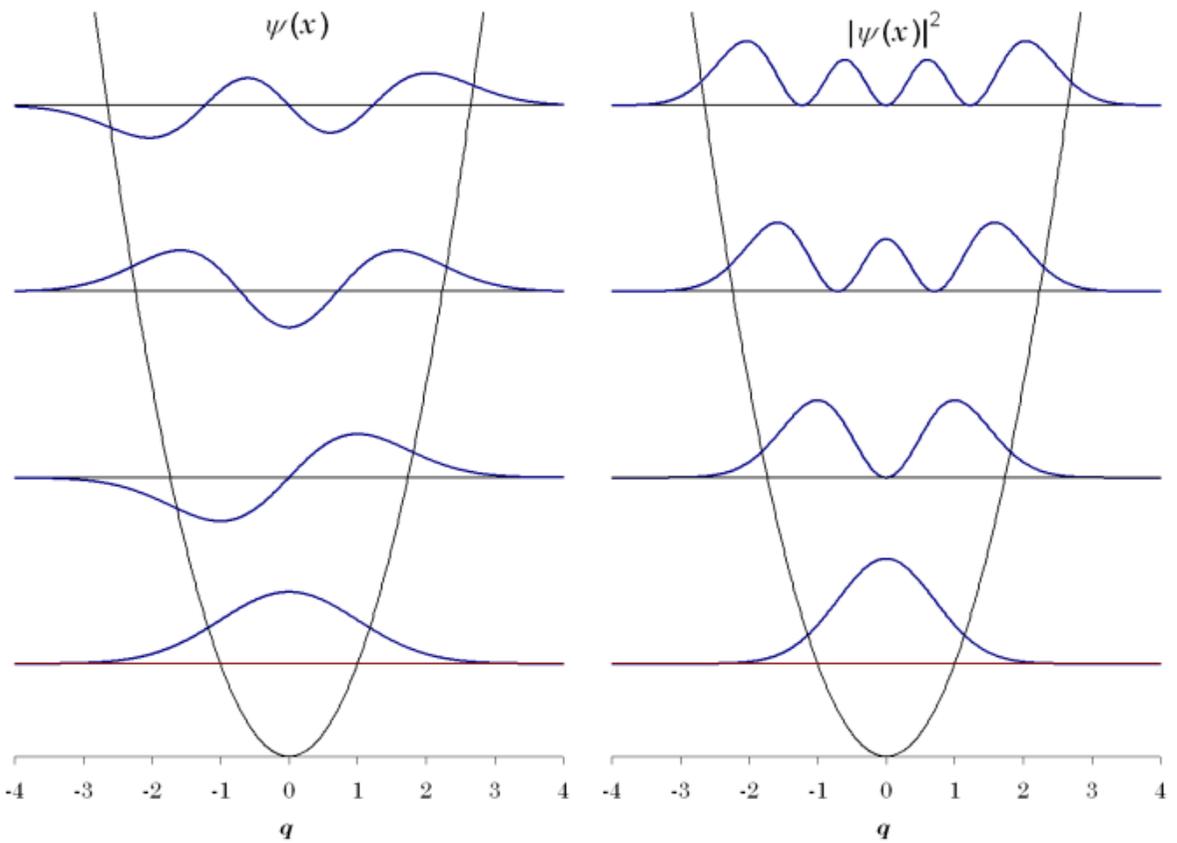
A **coherent state**[84] is a specific kind of [state][85] of the quantum harmonic oscillator whose dynamics most closely resemble the oscillating behavior of a classical harmonic oscillator system. The ground state is a [squeezed coherent state][86].

The ground state here differs from the ground state of the QPAD. That ground state equals zero in the close neighborhood of the center. The size of that neighborhood is of the order of the Planck length. Thus in this region the QPAD has the form of a stretched turban mold. It has a form similar to the second state in the picture of $|\psi(x)|^2$, thus the lowest state

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[84] http://en.wikipedia.org/wiki/Coherent_state
[85] States
[86] Canonical conjugate: Heisenberg's uncertainty

where $\psi(x)$ is asymmetric. Asymmetric states are better localizable than symmetric states.

Ladder operator

The Hermite functions ψ_n represent [Fock states](http://en.wikipedia.org/wiki/Fock_state)[87].

Boson ladder operators are characterized by

$$\mathcal{A}|\psi_n> = \sqrt{n}\,|\psi_{n-1}> \tag{1}$$

$$\mathcal{A}^\dagger|\psi_n> = \sqrt{n+1}\,|\psi_{n+1}> \tag{2}$$

$$\mathcal{A} = \frac{1}{\sqrt{2}}\left(c_1\frac{d}{dq} + c_2 q\right) = -\boldsymbol{k}\,\check{P}\sqrt{\frac{1}{2\,\hbar\,m\,\omega}} + \check{Q}\sqrt{\frac{m\,\omega}{2\,\hbar}}$$

$$\mathcal{A}^\dagger = \frac{1}{\sqrt{2}}\left(-c_1\frac{d}{dq} + c_2 q\right) = \boldsymbol{k}\,\check{P}\sqrt{\frac{1}{2\,\hbar\,m\,\omega}} + \check{Q}\sqrt{\frac{m\,\omega}{2\,\hbar}}$$

In the Heisenberg picture, the operators have the following time dependence:

$$\mathcal{A}(t) = \mathcal{A}(t_0)\exp\left(-\boldsymbol{k}\,\omega\,(t-t_0)\right) \tag{3}$$

$$\mathcal{A}^\dagger(t) = \mathcal{A}^\dagger(t_0)\exp\left(\boldsymbol{k}\,\omega\,(t-t_0)\right) \tag{4}$$

We can also define an enumeration operator N which has the following property:

$$N = \mathcal{A}^\dagger\,\mathcal{A} \tag{5}$$

$$N|\psi_n> = |\psi_n> n \tag{6}$$

In deriving the form of $\mathcal{A}^\dagger$, we have used the fact that the operators X and P_x, which represent observables, are Hermitian. These observable

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[87] http://en.wikipedia.org/wiki/Fock_state

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operators can be expressed as a linear combination of the ladder operators as

$$\check{Q}(t) = \sqrt{\frac{\hbar}{2\,m\,\omega}}\ (\mathcal{A}^\dagger(t) + \mathcal{A}(t)) \tag{7}$$

$$\check{P}(t) = \boldsymbol{k}\sqrt{\tfrac{1}{2}\,\hbar\,m\,\omega}\ (\mathcal{A}^\dagger(t) - \mathcal{A}(t)) \tag{8}$$

The $\check{\boldsymbol{Q}}$ and $\check{\boldsymbol{P}}$ operators obey the following identity, known as the canonical commutation relation:

$$[\check{Q},\check{P}] = \boldsymbol{k}\,\hbar \tag{9}$$

Using the above, we can prove the identities

$$H = \hbar\,\omega\,(\mathcal{A}^\dagger\,\mathcal{A} + \tfrac{1}{2}) = \hbar\,\omega\,(N + \tfrac{1}{2}) \tag{10}$$

$$[\mathcal{A}^\dagger, \mathcal{A}] = 1 \tag{11}$$

Now, let $|f_E\rangle$ denote an energy eigenstate with energy E. The inner product of any ket with itself must be non-negative, so

$$< \psi_E\,\mathcal{A}|\mathcal{A}\,\psi_E > = < \psi_E E|\,\mathcal{A}^\dagger\,\mathcal{A}\,\psi_E > \geq 0 \tag{12}$$

Expressing $\mathcal{A}^\dagger\mathcal{A}$ in terms of the Hamiltonian H:

$$< \psi_E|(H/(\hbar\,\omega) - \tfrac{1}{2})\,\psi_E > = (E/(\hbar\,\omega) - \tfrac{1}{2}) \geq 0 \tag{13}$$

so that

$$E \geq \tfrac{1}{2}\,\hbar\,\omega. \tag{14}$$

Note that when $|\mathcal{A}\,\psi_E > = |0>$ (is the zero ket i.e. a ket with length zero), the inequality is saturated, so that

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$$E = ½\, ℏ\, \omega \tag{15}$$

It is straightforward to check that there exists a state satisfying this condition; it is the ground state

$$|\psi_{ground}> = |\psi_{E_n}>; \; (n = 0) \tag{16}$$

Using the above identities, we can now show that the commutation relations of $\mathcal{A}$ and $\mathcal{A}^\dagger$ with H are:

$$[H, \mathcal{A}] = -ℏ\, \omega\, \mathcal{A} \tag{17}$$

$$[H, \mathcal{A}^\dagger] = ℏ\, \omega\, \mathcal{A}^\dagger \tag{18}$$

Thus, provided $|\mathcal{A}\ \psi_E>$ is not the zero ket,

$$|H\, \mathcal{A}\, \psi_E> = |[H, \mathcal{A}] + \mathcal{A}\, H\, \psi_E> \tag{19}$$

$$= |-ℏ\, \omega\, \mathcal{A} + \mathcal{A}\, E\, \psi_E>$$

$$= |-ℏ\, \omega\, \mathcal{A} + \mathcal{A}\, E\, \psi_E>$$

$$= (E - ℏ\, \omega)\, |\mathcal{A}\, \psi_E>$$

Similarly, we can show that

$$|H\, \mathcal{A}^\dagger\, \psi_E> = (E + ℏ\, \omega)\, |\mathcal{A}^\dagger\, \psi_E> \tag{20}$$

In other words, $\mathcal{A}$ acts on an eigenstate of energy E to produce, up to a multiplicative constant, another eigenstate of energy $E - ℏ\, \omega$, and $\mathcal{A}^\dagger$ acts on an eigenstate of energy E to produce an eigenstate of energy $E + ℏ\, \omega$. For this reason, a is called a "lowering operator", and $\mathcal{A}^\dagger \mathcal{A}$ "raising operator". The two operators together are called ladder operators. In quantum field theory, $\mathcal{A}$ and $\mathcal{A}^\dagger$ are alternatively called "annihilation"

and "creation" operators because they destroy and create particles, which correspond to our quanta of energy.

Given any energy eigenstate, we can act on it with the lowering operator $\mathcal{A}$, to produce another eigenstate with $\hbar\omega$-less energy. By repeated application of the lowering operator, it seems that we can produce energy eigenstates down to $E = -\infty$. However, this would contradict our earlier requirement that $E \geq \hbar\omega/2$.

Ground state

Therefore, there must be a ground-state energy eigenstate, which we label $|f_{ground}\rangle$, such that

$$|\mathcal{A}\,\psi_{ground}\rangle = |0\rangle;\ \text{(zero ket)}. \tag{1}$$

In this case, subsequent applications of the lowering operator will just produce zero kets, instead of additional energy eigenstates. Furthermore, we have shown above that

$$|H\,\psi_{ground}\rangle = (\tfrac{1}{2}\hbar\omega)\,|\psi_{ground}\rangle \tag{2}$$

Finally, by acting on $|\psi_{ground}\rangle$ with the raising operator and multiplying by suitable normalization factors, we can produce an infinite set of energy eigenstates

$$\{|\psi_{ground}\rangle, |\psi_{E_1}\rangle, |\psi_{E_2}\rangle, \ldots, |\psi_{E_n}\rangle\}, \tag{3}$$

such that

$$|H\,\psi_{E_n}\rangle = \hbar\omega\,(n + \tfrac{1}{2})\,|\psi_{E_n}\rangle \tag{4}$$

which matches the energy spectrum.

This method can also be used to quickly find the ground state wave function of the quantum harmonic oscillator.

Indeed

$$|\mathcal{A}\,\psi_{ground} >= \, |0 > \tag{5}$$

becomes

$$\psi_{ground}(x) = \psi_0(x) \overset{\text{def}}{=} < x|\,\psi_{ground} >= \, - \frac{\hbar}{m\,\omega} \frac{d}{dx}\,\psi_0(x) \tag{6}$$

so that

$$d\,\psi_0(x) = \psi_0(x)\,\frac{\hbar}{m\,\omega}\,x\,d\,x \,\Rightarrow \ln\!\big(\psi_0(x)\big) \,=\, \frac{m\,\omega}{2\hbar}\,x^2 \,+\, const \tag{7}$$

After normalization this leads to the following position space representation of the ground state wave function.

$$\psi_0(x) \,=\, \sqrt[4]{\frac{m\,\omega}{\pi\hbar}}\; e^{-\frac{m\,\omega}{2\hbar}x^2} \tag{8}$$

Coherent state

A **coherent state** is a specific kind of state[88] of the quantum harmonic oscillator[89] whose dynamics most closely resemble the oscillating behavior of a classical harmonic oscillator system.

The coherent state |α> is defined to be the 'right' eigenstate of the annihilation operator $\mathcal{A}$. Formally, this reads:

$$|\mathcal{A}\,\alpha >= \, \alpha|\alpha > \tag{1}$$

Since $\mathcal{A}$ is not Hermitian, α is a hyper complex number that is not necessarily real, and can be represented as

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[88]States

[89] Functions invariant under Fourier transform

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$$\alpha = |\alpha| \exp(\boldsymbol{k}\,\theta) \tag{2}$$

where θ is a real number. $|\alpha|$ is the amplitude and θ is the phase of state $|\alpha>$.

This formula means that a coherent state is left unchanged by the annihilation or the creation of a particle. The eigenstate of the annihilation operator has a [Poissonian](http://en.wikipedia.org/wiki/Poissonian)[90] number distribution A Poisson distribution is a necessary and sufficient condition that all annihilations are statistically independent.

The coherent state's location in the complex plane ([phase space](http://en.wikipedia.org/wiki/Phase_space)[91]) is centered at the position and momentum of a classical oscillator of the same phase θ and amplitude. As the phase increases the coherent state circles the origin and the corresponding disk neither distorts nor spreads. The disc represents Heisenberg's uncertainty. This is the most similar a quantum state can be to a single point in phase space.

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[90] http://en.wikipedia.org/wiki/Poissonian
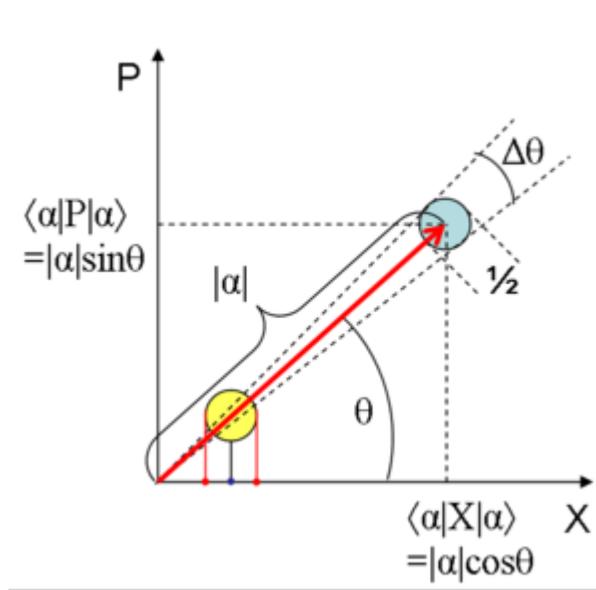
[91] http://en.wikipedia.org/wiki/Phase_space

Phase space plot of a coherent state. This shows that the uncertainty (blur) in a coherent state is equally distributed in all directions. The horizontal and vertical axes are the X and P quadratures of the field, respectively. Oscillations that are said to be *in quadrature*, if they are separated in phase by $\pi/2$ radians. The red dots on the x-axis trace out the boundaries of the quantum noise. Further from the origin the relative contribution of the quantum noise becomes less important.

The representation of the coherent state in the basis of Fock states is:

$$|\alpha> = \exp(-\tfrac{1}{2}|\alpha|^2) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n> = \exp(-\tfrac{1}{2}|\alpha|^2)\exp(\alpha \mathcal{A}^\dagger)|0> \tag{3}$$

where |n> are Hermite functions (eigenvectors of the Hamiltonian). This is a Poissonian distribution. The probability of detecting *n* photons is:

$$\mathcal{P}(n) = \exp(-\langle n \rangle)\frac{\langle n \rangle^n}{n!} \tag{4}$$

Similarly, the average photon number in a coherent state is

$$\langle n \rangle = \langle \mathcal{A}^\dagger \mathcal{A} \rangle = |\alpha|^2 \tag{5}$$

and the variance is

$$(\Delta n)^2 = Var(\mathcal{A}^\dagger \mathcal{A}) = |\alpha|^2 \tag{6}$$

Squeezing
The squeezing operator can squeeze a state more or less in the direction of either P or Q. The operator is defined as:

$$Sq(z) = \exp\left(\tfrac{1}{2}(z^*\mathcal{A} + z\mathcal{A}^\dagger)\right) \tag{1}$$

$$z = r\exp(\boldsymbol{k}\,\theta) \tag{2}$$

The ground state is a saturated squeezed coherent state where

$$\Delta p = \Delta q \text{ and } \Delta q \cdot \Delta p = \hbar/2 \tag{3}$$

Base transforms

Now we have discovered the following base transforms:
Position⟺momentum:

$$<q|p> = \frac{1}{\sqrt{2\pi\hbar}} \exp(\frac{\boldsymbol{k} \, q \, p}{\hbar}) \tag{1}$$

Position⟺Fock state:

$$<q|n> = \sqrt[4]{\frac{m\omega}{\pi\hbar}} \frac{1}{\sqrt{2^n \, n!}} \exp(-\frac{m\omega}{2\hbar} \, q^2) H_n(q\sqrt{\frac{m\omega}{\hbar}}) \tag{2}$$

Fock state⟺coherent state:

$$<n|z> = \frac{1}{\sqrt{n!}} \, z^n \exp(-\tfrac{1}{2}|z|^2) \tag{3}$$

Harmonic oscillating Hilbert field

Take the ingredients of the complex harmonic oscillator and interpret these as similar ingredients of a harmonic oscillating Hilbert field that is based on a Gaussian blur. The blur delivers the conditions of the ground state.

$$\psi_0(r) = \sqrt[4]{\frac{m \, \omega}{\pi\hbar}} \, e^{-\frac{m \, \omega}{2\hbar} r^2} \tag{1}$$

This means that the ground state corresponds with a Gaussian charge distribution. Higher states correspond to a blurred current. We indicate this current as vector potential $\boldsymbol{\varphi}$. Its time derivative $\dot{\boldsymbol{\varphi}}$ is perpendicular to $\boldsymbol{\varphi}$. The other ingredients are P, Q, $\mathcal{A}$ and $\mathcal{A}^\dagger$.

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$$Q \Leftrightarrow \boldsymbol{\varphi} = \varphi_x = \sqrt{\frac{\hbar}{2m\omega}} (\mathcal{A}_x + \mathcal{A}_x^\dagger)$$ (2)

$$\mathcal{A}_x - \mathcal{A}_x^\dagger = 0$$ (3)

$$P \Leftrightarrow m\dot{\boldsymbol{\varphi}} = m\dot{\varphi}_y = \sqrt{\frac{m\omega\hbar}{2}} (-\mathcal{A}_y + \mathcal{A}_y^\dagger)$$ (4)

$$\mathcal{A}_y + \mathcal{A}_y^\dagger = 0$$ (5)

$$\mathcal{A} \Leftrightarrow \boldsymbol{A} = i\mathcal{A}_x - ik\mathcal{A}_y = \sqrt{\frac{m\omega}{2\hbar}} \left(\boldsymbol{\varphi} + \frac{\dot{\boldsymbol{\varphi}}}{\omega} \right) = \sqrt{\frac{m\omega}{2\hbar}} \left(i\varphi_x + ik\frac{\dot{\varphi}_y}{\omega} \right)$$ (6)

$$\mathcal{A}^\dagger \Leftrightarrow \boldsymbol{A}^\dagger = i\mathcal{A}_x^\dagger + ik\mathcal{A}_y^\dagger = \sqrt{\frac{m\omega}{2\hbar}} \left(\boldsymbol{\varphi} - \frac{\dot{\boldsymbol{\varphi}}}{\omega} \right) = \sqrt{\frac{m\omega}{2\hbar}} \left(i\varphi_x - ik\frac{\dot{\varphi}_y}{\omega} \right)$$ (7)

The $\boldsymbol{\varphi}$ field and the $\dot{\boldsymbol{\varphi}}$ field are mutually perpendicular. If both fields are subjected to a synchronized quantum harmonic oscillation, then an oscillating wave results. We take the same ground state for each of the fields. These ground states correspond to a spherical symmetric Gaussian blur.

When bounds of the cavity are removed or relaxed, then the higher order modes may differ in a phase shift. The sign selections set the eigenvalues of the spin operator. The result is an elliptically polarized wave that moves in directions along $\boldsymbol{\varphi} \times \dot{\boldsymbol{\varphi}}$.

$\boldsymbol{\varphi}$ no longer stands for a single position, but instead for a Gaussian distribution of positions. Similarly $\dot{\boldsymbol{\varphi}}$ does not stand for a single moving particle, but for a moving Gaussian cloud of virtual particles.
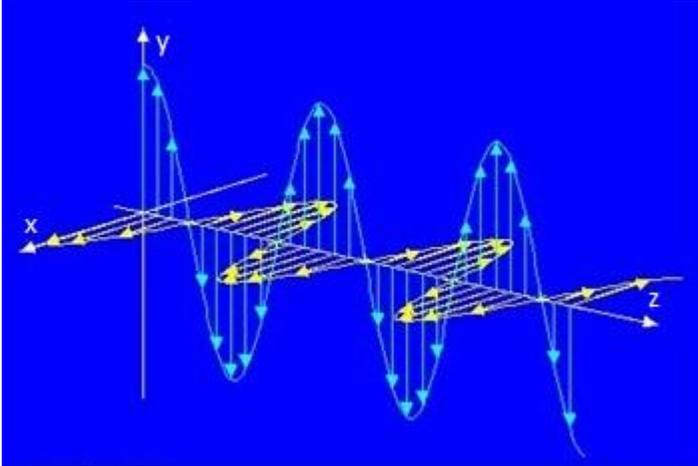
Annihilator and creator

The annihilator $\mathcal{A}$ and the creator $\mathcal{A}^\dagger$ are examples of boson operators. This is a consequence of their commutation relations.

$$\mathcal{A} + \mathcal{A}^\dagger = \alpha\,\boldsymbol{\varphi} \tag{1}$$

$$\mathcal{A} - \mathcal{A}^\dagger = \beta\,\dot{\boldsymbol{\varphi}} \tag{2}$$

$$\mathcal{A} = \tfrac{1}{2}\alpha\,\boldsymbol{\varphi} + \tfrac{1}{2}\beta\,\dot{\boldsymbol{\varphi}} \tag{3}$$

$$\mathcal{A}^\dagger = \tfrac{1}{2}\alpha\,\boldsymbol{\varphi} - \tfrac{1}{2}\beta\,\dot{\boldsymbol{\varphi}} \tag{4}$$

$$\left[\mathcal{A}(f), \mathcal{A}^\dagger(g)\right] = <f|g> \tag{5}$$

$$\left[\mathcal{A}(f), \mathcal{A}(g)\right] = 0 \tag{6}$$

$$\left[\mathcal{A}^\dagger(f), \mathcal{A}^\dagger(g)\right] = 0 \tag{7}$$

The corresponding fermion operators are:

$$\left\{\mathcal{B}(f), \mathcal{B}^\dagger(g)\right\} = <f|g> \tag{8}$$

$$\{\boldsymbol{\mathcal{B}}(f), \boldsymbol{\mathcal{B}}(g)\} = 0 \tag{9}$$

$$\{\boldsymbol{\mathcal{B}}^\dagger(f), \boldsymbol{\mathcal{B}}^\dagger(g)\} = 0 \tag{10}$$

The fermion operators can be represented by imaginary quaternionic base numbers:

$$\boldsymbol{\mathcal{B}} + \boldsymbol{\mathcal{B}}^\dagger = \boldsymbol{i} \tag{11}$$

$$\boldsymbol{\mathcal{B}} - \boldsymbol{\mathcal{B}}^\dagger = \boldsymbol{j} \tag{12}$$

$$\boldsymbol{\mathcal{B}} = \tfrac{1}{2}(\boldsymbol{i} + \boldsymbol{j}) \tag{13}$$

$$\boldsymbol{\mathcal{B}}^\dagger = \tfrac{1}{2}(\boldsymbol{i} - \boldsymbol{j}) \tag{14}$$

$$(\boldsymbol{\mathcal{B}} + \boldsymbol{\mathcal{B}}^\dagger)(\boldsymbol{\mathcal{B}} - \boldsymbol{\mathcal{B}}^\dagger) = \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{B}} - \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{B}}^\dagger + \boldsymbol{\mathcal{B}}^\dagger\boldsymbol{\mathcal{B}} - \boldsymbol{\mathcal{B}}^\dagger\boldsymbol{\mathcal{B}}^\dagger \tag{15}$$

$$= \boldsymbol{\mathcal{B}}^\dagger\boldsymbol{\mathcal{B}} - \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{B}}^\dagger = \boldsymbol{i}\,\boldsymbol{j}$$

Rotational symmetry
In case of rotational symmetry in the imaginary part of quaternion space, the exponential function must be replaced by a Bessel function. The corresponding Fourier transform then becomes a [Hankel transform](http://en.wikipedia.org/wiki/Hankel_transform)[92].

The spherical harmonics are eigenfunctions of the square of the [orbital angular momentum](#) operator $-i\hbar \boldsymbol{r} \times \boldsymbol{\nabla}$ and therefore they represent the different [quantized](#) configurations of [atomic orbitals](#).

Spherical harmonics
The following draws from the work of [S. Thangavelu](#)[93].

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[92] http://en.wikipedia.org/wiki/Hankel_transform

[93] http://www.math.iitb.ac.in/atm/faha1/veluma.pdf

In this subsection we look for eigenfunctions of the Fourier transform which have spherical symmetry. As in the one dimensional case we consider functions of the form

$$f(x) = p(x) \exp(-\pi|x|^2) \tag{1}$$

This will be an eigenfunction of $\mathcal{F}$ if and only p satisfies

$$\int_{\mathbb{R}^n} p(x - iy) \exp(-\pi \cdot |x|^2) \, dx = \lambda \, p(y) \tag{2}$$

Here in quaternion terms x and iy represent two mutually perpendicular imaginary numbers while x and y are parallel. Thangavelu uses complex numbers. We keep as close as is possible to his text.

If (2) is true for all $y \in \mathbb{R}^n$ then we should also have

$$\int_{\mathbb{R}^n} p(x + y) \exp(-\pi \cdot |x|^2) \, dx = \lambda \, p(iy) \tag{3}$$

Integrating in polar coordinates the integral on the left is

$$\int_{r=0}^{\infty} |\mathbb{S}^{n-1}| \left(\int_{\mathbb{S}^{n-1}} p(y + r\,\omega) \, d\sigma(\omega) \right) exp(-\pi \cdot r^2) \cdot r^{n-1} dr \tag{4}$$

where $d\sigma(\omega)$ is the normalised surface measure on the unit sphere $\mathbb{S}^{n-1}$.

If p is homogeneous of degree m then

$$p(i\,y) = i^m \, p(y) \tag{5}$$

and hence for such polynomials the equation

$$\int_{\mathbb{R}^n} p(x + y) \exp(-\pi \cdot |x|^2) \, dx = \lambda \, i^m \, p(iy) \tag{6}$$

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will be satisfied for

$$\lambda = (-i)^m \tag{7}$$

if p has the mean value property

$$\int_{\mathbb{S}^{n-1}} p(y + r\,\omega)d\sigma(\omega) = p(y) \tag{8}$$

Such functions are precisely the harmonic functions satisfying

$$\Delta u = 0 \tag{9}$$

Thus we have proved:

Let

$$f(x) = p(x)\,exp(-\pi\,|x|^2) \tag{10}$$

where p is homogeneous of degree m and harmonic. Then

$$\mathcal{F}f = (-i)^m\,f \tag{11}$$

Let $\mathcal{P}^m$ stand for the finite dimensional space of homogeneous harmonic polynomials of degree m:

The above theorem says that the finite dimensional subspace of $L^2(\mathbb{R}^n)$ consisting of functions of the form

$$p(x)\,\exp(-\pi\,|x|^2);\ p \in \mathcal{P}^m \tag{12}$$

is invariant under the Fourier transform.
We claim that the following extension is true.
Let

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$$f \in L^2(\mathbb{R}^n \tag{13}$$

be of the form

$$f(x) = p(x)g(|x|); \ p \in \mathcal{P}^m \tag{14}$$

Then

$$\mathcal{F} f(\xi) = p(\xi)g(|\xi|) \tag{15}$$

Thus the subspace of functions of the form

$$f(x) = p(x)g(|x|); \ p \in \mathcal{P}^m \tag{16}$$

is invariant under the Fourier transform.

Let
$$f \in L^2(\mathbb{R}^n) \tag{17}$$

be of the form

$$f(x) = p(x)g(|x|); \ p \in \mathcal{P}^m \tag{18}$$

Then

$$\mathcal{F}_n (f) = (-i)^m p \, \mathcal{F}_{n+2m} \, g \tag{19}$$

The above result is known as the Hecke-Bochner formula for the Fourier transform.

We conclude our discussion on invariant subspaces with the following result which shows that the Fourier transform of a radial function reduces to an integral transform whose kernel is a Bessel function. This relates to the Hankel transform.

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Let J_α stand for the Bessel function of type $\alpha > -1$
If

$$f(x) = g(|x|) \tag{20}$$

is radial and integrable then

$$\mathcal{F}_n(f)(\xi) = c_n \int_0^\infty g(r) \cdot \frac{J_{\frac{n}{2}-1}(2\pi r |\xi|)}{(2\pi r |\xi|)^{\frac{n}{2}-1}} \cdot r^{n-1} dr \tag{21}$$

Spherical harmonic transform

Next we like to decompose 2D and 3D functions into wave-like basic patterns that have simple radial and angular structures[94]. In that case, the base functions must take the separation-of-variable form:

$$R(r)\Phi(\varphi) = \frac{1}{\sqrt{2\pi}} R(r) \exp(im\varphi) \tag{1}$$

for 2D and

$$R(r)\Theta(v)\Phi(\varphi) = R(r)\Omega(v,\varphi) \tag{2}$$

$$\Omega(v,\varphi) = Y_{lm}(v,\varphi) = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_{lm}(v) \exp(im\varphi) \tag{3}$$

for 3D where (r,φ) and (r,v,φ) are the polar and spherical coordinates respectively. m and l are integers. $l \geq 0$ and $|m| \leq l$.

The base functions are eigenfunctions of the Laplacian. They represent wave-like patterns. The associated angular transform is closely related to the normal Fourier transform. For polar coordinates this reduces to a simple complex 1D Fourier transform.

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[94] http://lmb.informatik.uni-freiburg.de/papers/download/wa_report01_08.pdf

The radial base function is a Bessel function $J_m(kr)$ for polar coordinates and a spherical Bessel function $j_l(kr)$ for spherical coordinates. The parameter k can take either continuous or discrete values, depending on whether the region is infinite or finite. For functions defined on $(0, \infty)$, the transform with $J_m(kr)$ as integral kernel and r as weight is known as the **Hankel transform**. For functions defined on a finite interval, with zero-value boundary condition for the base functions, one gets the **Fourier-Bessel series**. For the 3D case the transform is called **Spherical Harmonic (SH) transform**.

Polar coordinates
The Laplacian in polar coordinates is:

$$\nabla^2 \psi = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \varphi^2} \tag{1}$$

The Helmholtz differential equation is

$$\nabla^2 \psi(r, \varphi) = -k^2 \psi(r, \varphi) \tag{2}$$

$$\psi(r, \varphi) = R(r) \Phi(\varphi) \tag{3}$$

$$\Phi(\varphi) = -m^2 \Phi(\varphi) \tag{4}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial R(r)}{\partial r} \right) = \left(\frac{m^2}{r^2} - k^2 \right) R(r) \tag{5}$$

The solution is:

$$\Phi_m(\varphi) = \exp(im\varphi) \tag{6}$$

$$R(r) = aJ_m(kr) + bY_m(kr) \tag{7}$$

J_m is the m-th order Bessel function. The Neumann function Y_m is singular at $r = 0$. Therefore $a = 1$ and $b = 0$.

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In finite solutions, the boundary conditions determine what set of functions can be used as base functions. The reference in the footnote shows which choices can be relevant.

Spherical coordinates

The Laplacian in polar coordinates is:

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2 \sin^2 v}\frac{\partial^2\psi}{\partial\varphi^2} + \frac{1}{r^2 \sin v}\frac{\partial}{\partial v}\left(\sin v\frac{\partial\psi}{\partial v}\right) \tag{1}$$

The Helmholtz differential equation is

$$\nabla^2\psi(r,\varphi,v) = -k^2\psi(r,\varphi,v) \tag{2}$$

$$\psi(r,\varphi,v) = R(r)\Omega(\varphi,v) \tag{3}$$

$$\Omega(\varphi,v) = Y_{lm}(\varphi,v) \tag{4}$$

$$Y_{lm}(v,\varphi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_{lm}(v)\exp(im\varphi) \tag{5}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial R(r)}{\partial r}\right) = \left(\frac{l(l-1)}{r^2} - k^2\right)R(r) \tag{6}$$

A non-singular solution for $R(r)$ is:

$$R(r) = j_l(kr) \tag{7}$$

j_l is the spherical Bessel function of order l.

$$j_l(r) = \sqrt{\frac{\pi}{2r}}\,J_{l+\frac{1}{2}}(r) \tag{8}$$

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The spherical harmonic transform

The equivalent of the Fourier transform in terms of spherical harmonics depends on the boundary conditions. For example when the analysis is done over a limited region, then the zero boundary condition will give different results than the zero derivative boundary condition[95]. An infinite range will always request a zero value of contributions when the radius goes to infinity.

$$S_{klm} = \int_{r=0}^{\infty} \int_{\varphi=0}^{2\pi} \int_{v=0}^{\pi} f(r,\varphi,v)\ \psi_{klm}^{*}(r,\varphi,v)\ r^2 \sin v\ dr\ d\varphi\ dv \tag{1}$$

$$f(r,\varphi,v) = \sum_{k=1}^{\infty}\sum_{l=0}^{\infty}\sum_{m=-l}^{l} S_{klm}\ \psi_{klm}(r,\varphi,v) \tag{2}$$

$$\psi_{klm}(r,\varphi,v) = j_l(kr)\ Y_{lm}(\varphi,v) \tag{3}$$

The Fourier transform of a black hole

In its simplest form a black hole is a bubble that is covered with a blanket of ground states.

The blanket is a comb function that is convoluted with a ground state. The Fourier transform of this blanket is the product of the Fourier transform of the comb function and the Fourier transform of the ground state. Apart from a factor, the ground state is invariant under Fourier transformation. Also the comb function is invariant. Thus the Fourier transform of the blanket is a modulated comb function. The modulation does not reach far.

The most complicated component is the bubble. In its simplest form this is a pulse on the radius. If we interpret this pulse as a Dirac delta function, then the Fourier coefficients have the form:

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95 http://lmb.informatik.uni-freiburg.de/papers/download/wa_report01_08.pdf

$$\psi_{k00}(r) = j_0(kr_0) = \sqrt{\frac{\pi}{2r}} J_{1/2}(kr_0)$$

If we sum these coefficients, then we get a sampled spherical Bessel function. These spheres are blurred with the transformed blanket.

Spherical harmonics eigenvalues

See: http://en.wikipedia.org/wiki/Spherical_harmonics for more details. Spherical harmonics are best presented in polar coordinates. There exists a corresponding polar Fourier transform. This Fourier transform also has invariant functions. Like in the rectangular case, they form the basis for spherical harmonics.

Laplace's equation in spherical coordinates is:

$$\nabla^2 f = r^{-2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin(\theta)} \frac{\partial}{\partial \theta} \left(\sin(\theta) \frac{df}{d\theta} \right) + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2 f}{\partial \varphi^2} = 0 \tag{1}$$

Try to find solutions in the form of the eigenfunctions of the Fourier transform.
By separation of variables, two differential equations result by imposing Laplace's equation:

$$f(r, \theta, \varphi) = R(r) \cdot Y(\theta, \varphi) \tag{2}$$

$$R^{-1} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) = \lambda \tag{3}$$

$$\frac{1}{Y \sin(\theta)} \frac{\partial}{\partial \theta} \left(\sin(\theta) \frac{\partial Y}{\partial \theta} \right) + \frac{1}{Y \sin^2(\theta)} \frac{\partial^2 Y}{\partial \varphi^2} = -\lambda \tag{4}$$

The second equation can be simplified under the assumption that Y has the form

$$Y(\theta, \varphi) = \Theta(\theta) \Phi(\varphi) \tag{5}$$

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Applying separation of variables again to the second equation gives way to the pair of differential equations

$$\frac{1}{\Phi(\varphi)}\frac{d^2\Phi(\varphi)}{d\varphi^2} = -m^2 \tag{6}$$

$$\lambda \sin^2(\theta) + \frac{\sin(\theta)}{\Theta(\theta)}\frac{d}{d\theta}\left[\sin(\theta)\frac{d\Theta}{d\theta}\right] = m^2 \tag{7}$$

for some number m. A priori, m is a complex constant, but because Φ must be a periodic function whose period evenly divides 2π, m is necessarily an integer and Φ is a linear combination of the complex exponentials $\exp(\pm i\, m\, \theta)$. The solution function $Y(\theta,\varphi)$ is regular at the poles of the sphere, where $\theta = 0, \pi$. Imposing this regularity in the solution Θ of the second equation at the boundary points of the domain is a [Sturm–Liouville problem](http://en.wikipedia.org/wiki/Sturm%E2%80%93Liouville_problem)[96] that forces the parameter λ to be of the form $\lambda = \ell(\ell + 1)$ for some non-negative integer with $\ell \geq |m|$; this is also explained below in terms of the orbital angular momentum. Furthermore, a change of variables $t = cos\theta$ transforms this equation into the Legendre equation, whose solution is a multiple of the [associated Legendre function](http://en.wikipedia.org/wiki/Associated_Legendre_function)[97]. $P_l^m(cos(\theta))$. Finally, the equation for R has solutions of the form $R(r) = A\, r^\ell + B\, r^{-\ell-1}$; requiring the solution to be regular throughout $\mathbb{R}^3$ forces $B = 0$.

Here the solution was assumed to have the special form

$$Y(\theta,\varphi) = \Theta(\theta)\,\Phi(\varphi) \tag{8}$$

For a given value of ℓ, there are $2\ell + 1$ independent solutions of this form, one for each integer m with $-\ell \leq m \leq \ell$. These angular solutions are a

[96] http://en.wikipedia.org/wiki/Sturm%E2%80%93Liouville_problem
[97] http://en.wikipedia.org/wiki/Associated_Legendre_function

product of trigonometric functions, here represented as a complex exponential, and associated Legendre functions:

$$Y_l^m(\theta, \varphi) = N \exp(i\,m\,\varphi)\, P_l^m(cos(\theta)) \tag{9}$$

which fulfill

$$r^2\,\nabla^2 Y_l^m(\theta, \varphi) = -l\,(l + 1)\,Y_l^m(\theta, \varphi) \tag{10}$$

Here Y_l^m is called a spherical harmonic function of degree ℓ and order m, P_l^m is an associated Legendre function, N is a normalization constant, θ represents the colatitude and ϕ represents the longitude. In particular, the colatitude[98] θ, or polar angle, ranges from 0 at the North Pole to π at the South Pole, assuming the value of $\pi/2$ at the Equator, and the longitude[99] φ, or azimuth[100], may assume all values with $0 \le \varphi < 2\pi$. For a fixed integer ℓ, every solution $Y(\theta, \varphi)$ of the eigenvalue problem

$$r^2\,\nabla^2 Y = -l\,(l + 1)\,Y \tag{11}$$

is a linear combination of Y_l^m. In fact, for any such solution, $r^\ell\,Y(\theta, \varphi)$ is the expression in spherical coordinates of a homogeneous polynomial that is harmonic, and so counting dimensions shows that there are $2\ell + 1$ linearly independent of such polynomials.

The general solution to Laplace's equation in a ball centered at the origin is a linear combination of the spherical harmonic functions multiplied by the appropriate scale factor r^l,

$$f(r, \theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_l^m r^l Y_l^m(\theta, \varphi) \tag{12}$$

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[98] http://en.wikipedia.org/wiki/Colatitude
[99] http://en.wikipedia.org/wiki/Longitude
[100] http://en.wikipedia.org/wiki/Azimuth

where the f_l^m are constants and the factors $\cdot r^l\, Y_l^m$ are known as [solid harmonics](#)[101]. Such an expansion is valid in the ball

$$r \;<\; R \;=\; 1/\lim_{l\Rightarrow\infty} sup\; |f_l^m|^{1/l} \tag{13}$$

Orbital angular momentum

In quantum mechanics, Laplace's spherical harmonics are understood in terms of the [orbital angular momentum](#)[102]

$$\boldsymbol{L} \;=\; -i\,\hbar\,\boldsymbol{x}\times\boldsymbol{\nabla} \;=\; L_x\,\boldsymbol{i} \;+\; L_y\boldsymbol{j} \;+\; L_z\,\boldsymbol{k} \tag{1}$$

The spherical harmonics are eigenfunctions of the square of the orbital angular momentum

$$\boldsymbol{L}^2 \;=\; -r^2\,\nabla^2 \;+\; \left(r\,\frac{\partial}{\partial r} + 1\right) r\,\frac{\partial}{\partial r} \tag{2}$$

$$=\; \frac{1}{\sin(\theta)}\,\frac{\partial}{\partial\theta}\,\sin(\theta)\frac{\partial}{\partial\theta} \;-\; \frac{1}{\sin^2(\theta)}\cdot\frac{\partial^2}{\partial\varphi^2}$$

Laplace's spherical harmonics are the joint eigenfunctions of the square of the orbital angular momentum and the generator of rotations about the azimuthal axis:

$$L_z \;=\; -i\cdot\hbar\cdot\left(x\frac{\partial}{\partial y} - y\,\frac{\partial}{\partial z}\right) = i\,\hbar\,\frac{\partial}{\partial\varphi} \tag{3}$$

These operators commute, and are densely defined self-adjoint operators on the Hilbert space of functions f square-integrable with respect to the normal distribution on $\mathbb{R}^3$:

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[101] http://en.wikipedia.org/wiki/Solid_harmonics

[102] http://en.wikipedia.org/wiki/Orbital_angular_momentum

$$(2\pi)^{-\frac{3}{2}} \int_{\mathbb{R}^3} |f(x)|^2 \exp(-|x|^2/2) < \infty \qquad (4)$$

Furthermore, $\boldsymbol{L}^2$ is a positive operator.

If Y is a joint eigenfunction of L^2 and L_z, then by definition

$$\boldsymbol{L}^2 Y = \lambda Y \qquad (5)$$

$$L_z Y = m Y \qquad (6)$$

for some real numbers m and λ. Here m must in fact be an integer, for Y must be periodic in the coordinate φ with period a number that evenly divides 2π. Furthermore, since

$$\boldsymbol{L}^2 = L_x^2 + L_y^2 + L_z^2 \qquad (7)$$

and each of L_x, L_y, L_z are self-adjoint, it follows that $\lambda \geq m^2$.

Denote this joint eigenspace by $E_{\lambda,m}$, and define the raising and lowering operators by

$$L_+ = L_x + i L_y \qquad (8)$$

$$L_- = L_x - i L_y \qquad (9)$$

Then L_+ and L_- commute with $\boldsymbol{L}^2$, and the Lie algebra generated by L_+, L_-, L_z is the special linear Lie algebra, with commutation relations

$$[L_z, L_+] = L_+ \qquad (10)$$

$$[L_z, L_-] = -L_- \qquad (11)$$

$$[L_+, L_-] = 2 L_z \qquad (12)$$

Thus $L_+ \colon E_{\lambda,m} \to E_{\lambda,m+1}$ (it is a "raising operator") and $L_- \colon E_{\lambda,m} \to E_{\lambda,m-1}$ (it is a "lowering operator"). In particular, $L_+^k \colon E_{\lambda,m} \to E_{\lambda,m+k}$ must be zero for k sufficiently large, because the inequality $\lambda \geq m^2$ must hold in each of the

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nontrivial joint eigenspaces. Let $Y \in E_{\lambda,m}$ be a nonzero joint eigenfunction, and let k be the least integer such that

$$L_+^k \, Y \; = \; 0 \tag{13}$$

Then, since

$$L_- \, L_+ \; = \; \mathbf{L}^2 - L_z^2 - L_z \tag{14}$$

it follows that

$$0 \; = \; L_- \, L_+^k \, Y \; = \; \left(\lambda - (m + k)^2 - (m + k)\right) Y \tag{15}$$

Thus $\lambda \; = \; \ell(\ell + 1)$ for the positive integer $\ell \; = \; m + k$. \hfill (16)

Spherical harmonics expansion

The Laplace spherical harmonics form a complete set of orthonormal functions and thus form an orthonormal basis of the Hilbert space of square-integrable functions. On the unit sphere, any square-integrable function can thus be expanded as a linear combination of these:

$$f(\theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_l^m Y_l^m(\theta, \varphi) \tag{1}$$

This expansion holds in the sense of mean-square convergence — convergence in $\underline{L^2}$ of the sphere — which is to say that

$$\lim_{N \to \infty} \int_0^{2\pi} \int_0^{\pi} \left| f(\theta, \varphi) - \sum_{l=0}^{N} \sum_{m=-l}^{l} f_l^m Y_l^m(\theta, \varphi) \right|^2 \sin(\theta) \, d\theta \, d\varphi = 0 \tag{2}$$

The expansion coefficients are the analogs of Fourier coefficients, and can be obtained by multiplying the above equation by the complex conjugate of a spherical harmonic, integrating over the solid angle Ω, and utilizing

the above orthogonality relationships. This is justified rigorously by basic Hilbert space theory. For the case of orthonormalized harmonics, this gives:

$$f_l^m = \int_\Omega f(\theta, \varphi) \, Y_l^{m*}(\theta, \varphi) d\Omega \tag{3}$$

$$= \int_0^{2\pi} d\varphi \int_0^\pi d\theta \, \sin(\theta) \, f(\theta, \varphi) \, Y_l^{m*}(\theta, \varphi)$$

If the coefficients decay in ℓ sufficiently rapidly — for instance, exponentially — then the series also converges uniformly to f.
A real square-integrable function f can be expanded in terms of the real harmonics $Y_{\ell m}$ above as a sum

$$f(\theta, \varphi) = \sum_{l=0}^\infty \sum_{m=-l}^l f_l^m Y_l^m(\theta, \varphi) \tag{4}$$

Convergence of the series holds again in the same sense.

Spin weighted spherical harmonics

Regard the sphere $\mathbb{S}^2$ as embedded into the three-dimensional imaginary part of the quaternionic number field. At a point $\mathbf{x}$ on the sphere, a positively oriented orthonormal basis of tangent vectors at $\mathbf{x}$ is a pair $\mathbf{a}$, $\mathbf{b}$ of vectors such that

$$(\mathbf{x}, \mathbf{a}) = (\mathbf{x}, \mathbf{b}) = (\mathbf{a}, \mathbf{b}) = 0 \tag{1}$$

$$(\mathbf{a}, \mathbf{a}) = (\mathbf{b}, \mathbf{b}) = 1 \tag{2}$$

$$\langle \mathbf{x}, \mathbf{a} \times \mathbf{b} \rangle > 0 \tag{3}$$

where the first pair of equations states that $\mathbf{a}$ and $\mathbf{b}$ are tangent at $\mathbf{x}$, the second pair states that $\mathbf{a}$ and $\mathbf{b}$ are unit vectors, $\mathbf{a}$ and $\mathbf{b}$ are orthogonal, and the $\{\mathbf{x}, \mathbf{a}, \mathbf{b}\}$ is a right-handed basis of $\mathbb{R}^3$.

$\underline{\textbf{a}}\cdot\sin(\theta)$

$\underline{\textbf{a}}\cdot\cos(\theta)$

$d_a = c_a\cdot\cos(\theta) - c_b\cdot\sin(\theta)$

$d_b = c_a\cdot\text{cin}(\theta) - c_b\cdot\cos(\theta)$

$\underline{\textbf{b}}\cdot\cos(\theta)$

$-\underline{\textbf{b}}\cdot\sin(\theta)$
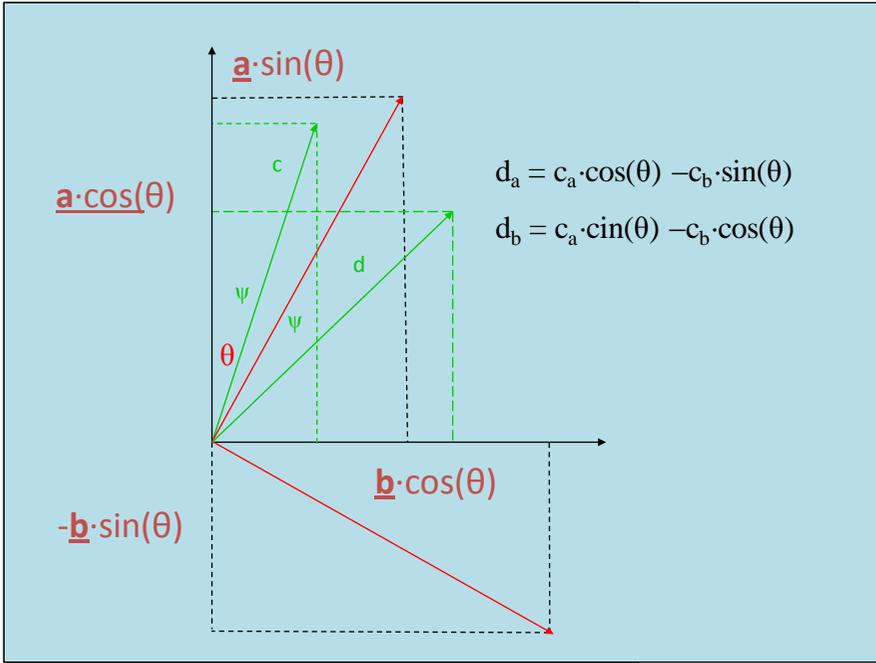
Figure 3: θ and the parameters a and b of the spin-weight function f.

A spin-weight s function f is a function accepting as input a point $\mathbf{x}$ of S^2 and a positively oriented orthonormal basis of tangent vectors at $\mathbf{x}$, such that

$$f\big(\mathbf{x}, \mathbf{a}\,\cos(\theta) - \mathbf{b}\,\sin(\theta), \mathbf{a}\,\sin(\theta) + \mathbf{b}\,\cos(\theta)\big) = \exp(i\,s\,\theta)\,f(\mathbf{x},\mathbf{a},\mathbf{b}) \tag{4}$$

for every rotation angle θ.

Following Eastwood & Tod (1982), denote the collection of all spin-weight s functions by $\mathbf{B}(s)$. Concretely, these are understood as functions f on $\mathbf{C^2}\backslash\{0\}$ satisfying the following homogeneity law under complex scaling

$$f(\lambda\,\mathbf{z}, \bar{\lambda}\,\bar{\mathbf{z}}) = \left(\frac{\bar{\lambda}}{\lambda}\right)^s f(\mathbf{z},\bar{\mathbf{z}}) \tag{5}$$

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This makes sense provided s is a half-integer.

Abstractly, $\mathbf{B}(s)$ is isomorphic to the smooth vector bundle underlying the antiholomorphic vector bundle $O^*(2{\cdot}s)$ of the Serre twist on the complex projective line $\boldsymbol{CP^1}$. A section of the latter bundle is a function g on $\boldsymbol{C^2}\backslash\{0\}$ satisfying

$$g\left(\lambda\,\boldsymbol{z},\bar{\lambda}\,\bar{\boldsymbol{z}}\right) = \left(\bar{\lambda}\right)^{2s} g(\boldsymbol{z},\bar{\boldsymbol{z}}) \tag{6}$$

Given such a g, we may produce a spin-weight s function by multiplying by a suitable power of the Hermitian form

$$P(\boldsymbol{z},\bar{\boldsymbol{z}}) = \boldsymbol{z}\,\bar{\boldsymbol{z}} \tag{7}$$

Specifically, $\mathfrak{f} = P^{-s}g$ is a spin-weight s function. The association of a spin-weighted function to an ordinary homogeneous function is an isomorphism.

Eth

The spin weight bundles $\mathbf{B}(s)$ are equipped with a differential operator $\eth$ (eth). This operator is essentially the [Dolbeault operator](#)[103],

$$\eth = \partial + \partial^* \tag{1}$$

Thus for $\mathfrak{f}\in\boldsymbol{B}(s)$,

$$\eth\,\mathfrak{f} \equiv P^{-s+1}\,\partial(P^s\,\mathfrak{f}) \tag{2}$$

defines a function of spin-weight $s + 1$.

Spin-weighted harmonic functions

See http://en.wikipedia.org/wiki/Spin-weighted_spherical_harmonics for more details.

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[103] http://en.wikipedia.org/wiki/Dolbeault_operator

Just as conventional spherical harmonics are the eigenfunctions of the Laplace-Beltrami operator on the sphere, the spin-weight s harmonics are the eigensections for the Laplace-Beltrami operator acting on the bundles $\mathcal{E}(s)$ of spin-weight s functions.

The spin-weighted harmonics can be represented as functions on a sphere once a point on the sphere has been selected to serve as the North Pole. By definition, a function η with *spin weight* s transforms under rotation about the pole via

$$\eta \;\rightarrow\; \exp(i\,s\,\psi)\,\eta \tag{1}$$

Working in standard spherical coordinates, we can define a particular operator $\eth$ acting on a function η as:

$$\eth\eta \;=\; -\sin^s(\theta)\,\{\frac{\partial}{\partial\theta} + \frac{i}{\sin(\theta)}\frac{\partial}{\partial\varphi}\}[\sin^{-s}(\theta)\,\eta] \tag{2}$$

This gives us another function of θ and φ. [The operator $\eth$ is effectively a covariant derivative operator in the sphere.]

An important property of the new function $\eth\eta$ is that if η had spin weight s, $\eth\eta$ has spin weight $s + 1$. Thus, the operator raises the spin weight of a function by 1. Similarly, we can define an operator which will lower the spin weight of a function by 1:

$$\bar{\eth}\eta \;=\; -\sin^{-s}(\theta)\left\{\frac{\partial}{\partial\theta} - \frac{i}{\sin(\theta)}\frac{\partial}{\partial\varphi}\right\}[(\sin^s(\theta)\,\eta] \tag{3}$$

We extend the function Y_l^m to ${}_sY_l^m$ according to

$${}_0Y_l^m\,(\theta,\varphi) = Y_l^m(\theta,\varphi) \tag{4}$$

$$l \;=\; 0,1,2,\dots;\; m \;=\; -l,\dots 0,\dots l \tag{5}$$

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The spin-weighted spherical harmonics are then defined in terms of the usual spherical harmonics as:

$$_sY_l^m = \sqrt{\frac{(l-s)!}{(l+s)!}}\, \eth^s\, Y_l^m; \; 0 \leq s \leq l \tag{6}$$

$$_sY_l^m = \sqrt{\frac{(l+s)!}{(l-s)!}}\, (-1)^s\, \bar\eth^s\, Y_l^m; \; -l \leq s \leq 0 \tag{7}$$

$$_sY_l^m = 0; \; l < |s|; \tag{8}$$

The functions $_sY_l^m$ then have the property of transforming with spin weight s.

Other important properties include the following:

$$\eth(_sY_l^m) = +\sqrt{(l-s)\,(l+s+1)}\; _{s+1}Y_l^m \tag{9}$$

$$\bar\eth(_sY_l^m) = -\sqrt{(l+s)\,(l-s+1)}\; _{s-1}Y_l^m \tag{10}$$

Special Fourier transform pairs

Functions that keep the same form through Fourier transformation are:

$$f(q) = \exp(-|q|^2) \tag{1}$$

$$f(q) = \frac{1}{|q|} \tag{2}$$

$$f(q) = comb(q) \tag{3}$$

The comb function consists of a set of equidistant Dirac delta functions.

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Other examples of functions that are invariant under Fourier transformation are the linear and spherical harmonic oscillators and the solutions of the Laplace equation.

Complex Fourier transform invariance properties

Each even function $f(q) \iff \tilde{f}(p)$ induces a Fourier invariant:

$$h(q) = \sqrt{2\pi}\, f(q) + \tilde{f}(q). \tag{1}$$

$$\tilde{h}(q) = \sqrt{2\pi}\, h(q) \tag{2}$$

Each odd function $f(q) \iff \tilde{f}(p)$ induces a Fourier invariant:

$$h(q) = \sqrt{2\pi}\, f(q) - \tilde{f}(q). \tag{3}$$

A function $f(q)$ is invariant under Fourier transformation *if and only if* the function f satisfies the differential equation

$$\frac{\partial^2 f(q)}{\partial q^2} - t^2 f(q) = \alpha\, f(q), \text{ for some scalar } \alpha \in C. \tag{4}$$

The Fourier transform invariant functions are fixed apart from a scale factor. That scale factor can be 1, k, -1 or $-k$. k is an imaginary base number in the longitudinal direction.

Fourier-invariant functions show iso-resolution, that is, $\Delta_p = \Delta_q$ in the Heisenberg's uncertainty relation.

For proves see: http://www2.ee.ufpe.br/codec/isoresolution_vf.pdf.

Fourier transform properties

Parseval's theorem
Parseval's theorem runs:

$$\int f^*(q) \cdot g(q) \cdot dV_q = \int \tilde{f}^*(p) \cdot \tilde{g}(p) \cdot dV_p \tag{1}$$

This leads to

$$\int |f(q)|^2 \cdot dV_q = \int \left|\tilde{f}(p)\right|^2 \cdot dV_p$$

(1)

Convolution
Through Fourier transformation a convolution changes into a simple product and vice versa.

$$\mathcal{F}(f(q) \circ g(q)) = \tilde{f}(p) \cdot \tilde{g}(p)$$

(1)

Differentiation
Fourier transformation converts differentiation into multiplication with the canonical conjugated coordinate.

$$g(q) = \nabla f(q)$$

(1)

$$\tilde{g}(p) = p\tilde{f}(p)$$

(2)

Vacuum expectation value
The **vacuum expectation value** (also called **condensate** or simply VEV) of an operator is its average, expected value in the [vacuum](http://en.wikipedia.org/wiki/Vacuum_state)[104]. The vacuum corresponds to a ground state. The vacuum expectation value of an operator O is usually denoted by $\langle O \rangle$.

Hilbert field equations
Despite the obvious similarity, Hilbert field equations are not Maxwell field equations. First of all, the Hilbert field is a skew field. It carries the properties of the quaternions and Hilbert field theory uses the properties of the quaternionic Fourier transform.

Next Hilbert fields are mathematical (skew) fields, while Maxwell fields are physical fields in a 3D geometry. Finally the Hilbert fields are

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[104] http://en.wikipedia.org/wiki/Vacuum_state

constituted from a collection of member fields that each have one or more anchor points.

Statics and dynamics

In this section we mix statics and dynamics, but we start with a static status quo. When we confine to the static status quo we get the two sets of equations.

We will consider differentiations with respect to an idealized coordinate system.

The field $\varphi = \boldsymbol{\varphi} + \varphi_0$ consists of a real part φ_0 and an imaginary part $\boldsymbol{\varphi}$. We consider the effect of the nabla operator $\boldsymbol{\nabla}$ on this field.

$$\boldsymbol{F} = \boldsymbol{\nabla}\varphi = \boldsymbol{\nabla}\boldsymbol{\varphi} + \boldsymbol{\nabla}\varphi_0 = \boldsymbol{F}_\perp + \boldsymbol{F}_\parallel$$

The real valued field φ_0 is a scalar field. Its divergence $\boldsymbol{F}_\parallel = \boldsymbol{\nabla}\varphi_0$ is imaginary and can be considered to be a vector field. This vector field is rotation free.

The imaginary valued field $\boldsymbol{\varphi}$ conforms to a vector field. It is possible to take the curl $\boldsymbol{F}_\perp = \boldsymbol{\nabla} \times \boldsymbol{\varphi}$ of that imaginary field. This new field is again imaginary and is divergence free.

The idealized coordinate system is formed by the eigenspace of operator $\check{Q}$. We will only use the imaginary part of the eigenspace as coordinate system. Now introduce a progression parameter. We will place it as the real part q_0 of the eigenvalues q of operator $\check{Q}$. Thus a full eigenvalue is $q = \boldsymbol{q} + q_0$. We use this in order to define a nabla operator $\nabla=\boldsymbol{\nabla}+\nabla_0$.

When dynamics is supported as well, then these parts $\boldsymbol{F}_\perp$ and $\boldsymbol{F}_\parallel$ get coupled. A single separable Hilbert space $\boldsymbol{H}$, only meets the static parts of the Hilbert field. This means that in this Hilbert space the two parts do not couple. Coupling only takes place during the step from one Hilbert

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space to the next member in the sequence. Continuity equations describe the coupling between the parts.

The parameter that controls dynamics in these differential equations is the progression parameter. This parameter stands for the counter of the progression steps. For mathematical convenience we consider the progression parameter as a smooth parameter. Thus we switch from a fundamentally granular progression step counter to a continuous progression parameter. This progression parameter is not our common notion of time. The derivative of the field f for this parameter is defined as $\nabla_0 f$.

In order to make the step from the integer progression step to the continuous progression parameter possible there must be a mechanism that reduces change, such that no violent steps are taken. On the other hand the mechanism must not be so strong that only a few steps are taken after which the universe is put to an eternal hold. How this in practice is regulated is shown by the phenomenon [inertia](#)[105]. Inertia is installed by the community of all particles. Locally this community generates an enormous potential. This potential works the same in all directions, so when nothing happens it has no influence on a local particle. A uniform movement of a local particle corresponds with the existence of a local vector potential. With other words, where the real part φ_0 of field φ corresponds to the "charge density" of the particle, the imaginary part $\boldsymbol{\varphi}$ of the field φ corresponds to a uniform moving particle. Also this vector potential does not apply any action. However, when the particle accelerates, then this goes together with the existence of an extra vector field that counteracts the acceleration. Thus, inertia does not counteract uniform movement. Uniform movements cause redistribution of the particles and with it a reconfiguration of the fields. This disturbance of the static status quo is the motor that keeps dynamics going. The tolerance of inertia with respect to uniform movement is the reason that the movement does not get killed.

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[105] Influence:Inertia

The quaternionic nabla

According to the definition of quaternionic differentiation the split of quaternionic nabla operator in a real part and an imaginary part is defined by:

$$g(q) = \nabla f(q) = g_0(q) + \mathbf{g}(q) \tag{1}$$

$$g_0(q) = \nabla_0 f_0(q) \mp \langle \nabla, \mathbf{f}(q) \rangle \tag{2}$$

$$\mathbf{g}(q) = \pm \nabla_0 \mathbf{f}(q) + \nabla f_0(q) \pm (\pm \nabla \times \mathbf{f}(q)) \tag{3}$$

$$= \pm \nabla_0 \mathbf{f}(q) + \mathbf{g}_l(q) + \mathbf{g}_t(q)$$

The second term on the right treats imaginary divergence. The last term treats the rotation. The first term is raised due to the dynamic coupling of the static fields.

In Fourier space the equivalent equation are:

$$\tilde{g}(p) = pf(q) = \tilde{g}_0(p) + \tilde{\mathbf{g}}(p) \tag{4}$$

$$\tilde{g}_0(p) = p_0 \tilde{f}_0(p) \mp \langle \mathbf{p}, \tilde{\mathbf{f}}(p) \rangle \tag{5}$$

$$\tilde{\mathbf{g}}(q) = \pm p_0 \tilde{\mathbf{f}}(p) + \mathbf{p} \tilde{f}_0(p) \pm \left(\pm \mathbf{p} \times \tilde{\mathbf{f}}(p) \right) \tag{6}$$

$$= \pm p_0 \tilde{\mathbf{f}}(p) + \tilde{\mathbf{g}}_l(\mathbf{p}) + \tilde{\mathbf{g}}_t(\mathbf{p})$$

Blurring the charges

We may represent the members of the Hilbert distribution with Dirac delta functions. These Dirac delta functions can be multiplied with a hyper complex number. Such a distribution raises problems with the nabla operator.

However, since these members represent anchor points and since each anchor point attaches to a QPAD, it has more sense to start directly with these blurred anchor points. We introduce the quaternionic function $\rho(q)$

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that represent the presence of one or more blurred anchor points in its real part and represent the flow of these blurred anchor points in its imaginary part. This leads to integral and differential continuity equations.

Not all anchor points must be equal. When this is true, it is better to categorize them and treat each category separately. Each member of such a category represents a charge that is typical for that category.

The QPAD $\rho(q)$ can be interpreted as the combination of a scalar potential $\rho_0(q)$ and a vector potential $\boldsymbol{\rho}(q)$.

Continuity equation for charges

Continuity equation

When $\rho_0(q)$ is interpreted as a charge density distribution, then the conservation of the corresponding charge is given by the continuity equation:

$$\text{Total change within } V = \text{flow into } V + \text{production inside } V \tag{1}$$

$$\frac{d}{dt}\int_V \rho_0 \, dV = \oint_S \hat{\boldsymbol{n}}\rho_0 \frac{\boldsymbol{v}}{c} \, dS + \int_V s_0 \, dV \tag{2}$$

$$\int_V \nabla_0\rho_0 \, dV = \int_V \langle \boldsymbol{\nabla}, \boldsymbol{\rho} \rangle \, dV + \int_V s_0 \, dV \tag{3}$$

Here $\hat{\boldsymbol{n}}$ is the normal vector pointing outward the surrounding surface S, $\boldsymbol{v}(t, \boldsymbol{q})$ is the velocity at which the charge density $\rho_0(t, \boldsymbol{q})$ enters volume V and s_0 is the source density inside V. In the above formula $\boldsymbol{\rho}$ stands for

$$\boldsymbol{\rho} = \rho_0 \boldsymbol{v}/c \tag{4}$$

It is the flux (flow per unit area and unit time) of ρ_0 .

The combination of $\rho_0(t, \boldsymbol{q})$ and $\boldsymbol{\rho}(t, \boldsymbol{q})$ is a quaternionic skew field $\rho(t, \boldsymbol{q})$ and can be seen as a probability amplitude distribution (QPAD).

$$\rho \stackrel{\text{def}}{=} \rho_0 + \boldsymbol{\rho} \tag{5}$$

$\rho(t, \boldsymbol{q})\rho^*(t, \boldsymbol{q})$ can be seen as an overall probability density distribution (PDD). $\rho_0(t, \boldsymbol{q})$ is a charge density distribution. $\boldsymbol{\rho}(t, \boldsymbol{q})$ is the current density distribution.

Depending on their sign selection, quaternions come in four sign flavors. In a QPAD the quaternion sign flavors do not mix. So, there are four QPAD sign flavors.

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Still these sign flavors can combine in pairs or in quadruples.

The quaternionic field $\rho(t, \boldsymbol{q})$ contains information on the distribution $\rho_0(t, \boldsymbol{q})$ of the considered charge density as well as on the current density $\boldsymbol{\rho}(t, \boldsymbol{q})$, which represents the transport of this charge density.

Where $\rho(t, \boldsymbol{q})\rho^*(t, \boldsymbol{q})$ can be seen as a probability density of finding the center of charge at position $\boldsymbol{q}$, the probability density distribution $\tilde{\rho}(t, \boldsymbol{p})\tilde{\rho}^*(t, \boldsymbol{p})$ can be seen as the probability density of finding the center of the corresponding wave package at location $\boldsymbol{p}$. $\tilde{\rho}(t, \boldsymbol{p})$ is the Fourier transform of $\rho(t, \boldsymbol{q})$.

The dimension of $\rho_0, \boldsymbol{\rho}$ and ρ is $[XTL^{-3}]$, the dimension of s_0 is $[XL^{-3}]$. The factor c has dimension $[T^{-1}L]$. $[X]$ is an arbitrary dimension. It attaches to the charge.

The conversion from formula (2) to formula (3) uses the Gauss theorem[106]. This results in the law of charge conservation

$$s_0(t, \boldsymbol{q}) = \nabla_0\rho_0(t, \boldsymbol{q}) \mp \langle \boldsymbol{\nabla}, \big(\rho_0(t, \boldsymbol{q})\boldsymbol{v}(t, \boldsymbol{q}) + \boldsymbol{\nabla} \times \boldsymbol{a}(t, \boldsymbol{q})\big)\rangle \tag{6}$$

$$= \nabla_0\rho_0(t, \boldsymbol{q}) \mp \langle \boldsymbol{\nabla}, \boldsymbol{\rho}(t, \boldsymbol{q}) + \boldsymbol{A}(t, \boldsymbol{q})\rangle$$

$$= \nabla_0\rho_0(t, \boldsymbol{q}) \mp \langle \boldsymbol{v}(t, \boldsymbol{q}), \boldsymbol{\nabla}\rho_0(t, \boldsymbol{q})\rangle \mp \langle \boldsymbol{\nabla}, \boldsymbol{v}(t, \boldsymbol{q})\rangle \rho_0(t, \boldsymbol{q})$$

$$\mp \langle \boldsymbol{\nabla}, \boldsymbol{A}(t, \boldsymbol{q})\rangle$$

The blue colored $\pm$ indicates quaternionic sign selection through conjugation of the field $\rho(t, \boldsymbol{q})$. The field $\boldsymbol{a}(t, \boldsymbol{q})$ is an arbitrary differentiable vector function.

$$\langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \times \boldsymbol{a}(t, \boldsymbol{q})\rangle = 0 \tag{7}$$

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[106] http://en.wikipedia.org/wiki/Divergence_theorem

$A(t, q) \stackrel{\text{def}}{=} \nabla \times a(t, q)$ is always divergence free. In the following we will neglect $A(t, q)$.

In Fourier space the continuity equation becomes:

$$\tilde{s}_0(t, p) = p_0 \tilde{\rho}_0(t, p) \mp \langle p, \tilde{\rho}(t, p) \rangle \tag{8}$$

This equation represents a balance equation for charge (or mass) density. Here $\rho_0(q)$ is the charge distribution, $\rho(q)$ is the current density. This only treats the real part of the full equation. The full equation runs:

$$s(t, q) = \nabla \rho(t, q) = s_0(t, q) + s(t, q) \tag{9}$$

$$= \nabla_0 \rho_0(t, q) \mp \langle \nabla, \rho(t, q) \rangle \pm \nabla_0 \rho(t, q) + \nabla \rho_0(t, q)$$
$$\pm \left(\pm \nabla \times \rho(t, q) \right)$$

$$= \nabla_0 \rho_0(t, q) \mp \langle v(t, q), \nabla \rho_0(t, q) \rangle \mp \langle \nabla, v(t, q) \rangle \rho_0(t, q)$$

$$\pm \nabla_0 v(t, q) + \nabla_0 \rho_0(t, q) + \nabla \rho_0(t, q)$$

$$\pm \left(\pm (\rho_0(t, q) \nabla \times v(t, q) - v(t, q) \times \nabla \rho_0(t, q) \right)$$

$$s_0(t, q) = 2 \nabla_0 \rho_0(t, q) \mp \langle v(q), \nabla \rho_0(t, q) \rangle \mp \langle \nabla, v(t, q) \rangle \rho_0(t, q) \tag{10}$$

$$s(t, q) = \pm \nabla_0 v(t, q) \pm \nabla \rho_0(t, q) \tag{11}$$

$$\pm \left(\pm (\rho_0(t, q) \nabla \times v(t, q) - v(t, q) \times \nabla \rho_0(t, q)) \right)$$

The red sign selection indicates a change of handedness by changing the sign of one of the imaginary base vectors. (Conjugation also causes a switch of handedness). If temporarily no creation and no annihilation occur, then these equations reduce to equations of motion.

$$\nabla_0 \rho_0(t, q) \pm \nabla_0 \rho(t, q) = \pm \langle \nabla, \rho(t, q) \rangle - \nabla \rho_0(t, q) \mp \left(\pm \nabla \times \rho(t, q) \right) \tag{12}$$

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$$\nabla_0 \rho(t, \boldsymbol{q}) = \langle \boldsymbol{\nabla}, \boldsymbol{\rho}(t, \boldsymbol{q}) \rangle - \boldsymbol{\nabla}\rho_0(t, \boldsymbol{q}) \mp \boldsymbol{\nabla} \times \boldsymbol{\rho}(t, \boldsymbol{q}) \tag{13}$$

$$\nabla_0 \rho_0(t, \boldsymbol{q}) = \mp \langle \boldsymbol{\nabla}, \boldsymbol{\rho}(t, \boldsymbol{q}) \rangle \tag{14}$$

$$\nabla_0 \boldsymbol{\rho}(t, \boldsymbol{q}) = \mp \boldsymbol{\nabla}\rho_0(t, \boldsymbol{q}) \mp \boldsymbol{\nabla} \times \boldsymbol{\rho}(t, \boldsymbol{q}) \tag{15}$$

The field $\rho(t, \boldsymbol{q})$ can be split in a (relative) stationary background $\rho_b(\boldsymbol{q})$ and the moving private field $\rho_p(t, \boldsymbol{q})$.
If $\boldsymbol{v}(t, \boldsymbol{q})$ is a constant then

$$s_0(t, \boldsymbol{q}) = 2\nabla_0 \rho_0(t, \boldsymbol{q}) \mp \langle \boldsymbol{v}, \boldsymbol{\nabla}\rho_0(t, \boldsymbol{q}) \rangle \tag{16}$$

$$\boldsymbol{s}(t, \boldsymbol{q}) = \pm \boldsymbol{\nabla}\rho_0(t, \boldsymbol{q}) \mp \left(\pm \boldsymbol{v} \times \boldsymbol{\nabla}\rho_0(t, \boldsymbol{q}) \right) \tag{17}$$

$$s(t, \boldsymbol{q}) = 2\nabla_0 \rho_0(t, \boldsymbol{q}) \mp \langle \boldsymbol{v}, \boldsymbol{\nabla}\rho_0(t, \boldsymbol{q}) \rangle \pm \boldsymbol{\nabla}\rho_0(t, \boldsymbol{q}) \mp \left(\pm \boldsymbol{v} \times \boldsymbol{\nabla}\rho_0(t, \boldsymbol{q}) \right) \tag{18}$$

The continuity equation has a direct relation to a corresponding conservation law[107]. The conserved quantity is $\rho_0(t, \boldsymbol{q})$ or its integral

$$Charge = \int_V \rho_0 \, dV$$

Noether's theorem[108] provides the relation between conserved quantities, differentiable symmetries and the Lagrangian[109].

Properties
The particles described below have properties such a coupling factor m, a half integer or full integer valued spin and an electric charge that can be $0, \pm\frac{1}{3}e, \pm\frac{2}{3}e,$ or $\pm\ e$.

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[107] http://en.wikipedia.org/wiki/Conservation_law
[108] http://en.wikipedia.org/wiki/Noether's_theorem
[109] http://en.wikipedia.org/wiki/Lagrangian#Lagrangians_in_quantum_field_theory

Apart from the coupling factor, this paper does not explain why the particles have these properties. However, it is clear that the values of the properties are related to the sign flavors of the ordered pair of quaternionic fields that identify the particle.

Quaternion sign flavors

Quaternions know two independent conjugations. One conjugation $q \Leftrightarrow q^*$ switches the sign of all three imaginary base vectors of the values of the quaternion. The other conjugation $q \Leftrightarrow q^1$ switches the sign of just one imaginary base vector. In a quaternionic field the conjugation works field wide. The two independent conjugations raise four sign flavors for quaternionic fields. A quaternionic field will stick with one and no more than one sign flavor. The sign flavors are determined by sign selections.

The background coordinate system has its own kind of sign flavor. The sign flavor of the background coordinate system can act as a reference for comparing quaternion field sign flavors. The background coordinate system can be curved. In that case we use the local tangent space that acts as a quaternionic number space.

Quaternion fields come in four sign flavors[110]: $\psi^{⓪}$, $\psi^{①}, \psi^{②}$ and $\psi^{③}$. We will use the symbol ψ or $\psi^{⓪}$ for the sign flavor of the quaternionic field that has the same sign flavor as the local background coordinate system. The superscripts indicate the number of base vectors that changed sign.

$$\psi^{③} = \psi^* \tag{1}$$

And with the same symbolic:

$$\psi^{①} = \psi^1 \tag{2}$$

$$\tag{3}$$

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[110] The notion of "sign flavor" is used because for elementary particles "flavor" already has a different meaning.

$$\psi^{\textcircled{0}} = \psi$$

Often the symbols ψ and ψ^* will be used instead of the symbols $\psi^{\textcircled{0}}$ and $\psi^{\textcircled{3}}$.

In the later investigated continuity equations, pairs of field sign flavors will be treated that belong to the same base field ψ. For example:

$$\nabla\psi^x = m\,\psi^y \tag{4}$$

The factor m is a coupling factor. We sometimes call ψ^y the coupled field and $\psi^x \Longrightarrow \psi^y$ stands for the flip.
The continuity equation will use one of the pair $\{\psi^x, \psi^y\}$ as the transported field and the other pair member (ψ^y) as the source field. Each choice of an ordered pair of field sign flavors $\{\psi^x, \psi^y\}$ will result in a different equation. However, if $\psi^x = \psi^y$, then the coupling factor m is zero.

In many equations ψ^y is interpreted as a background field.

The same equation may accept different basic fields (ψ). The standard model appears to use three different field configurations for ψ. This means that as many different background fields exist. Each of the configurations has its own set of coupling factors. This paper does not explain why these three field configurations exist.

Each ordered pair $\{\psi^x, \psi^y\}$ represents an elementary particle type category. Each such pair corresponds to a specific continuity equation, which is also an equation of motion.

Some categories appear in triplets. The members of the triplet are coupled to directions of imaginary base vectors.

Sign flavor	Flip	Imaginary	Handedness	Isotropy

	$\psi^x \Longrightarrow \psi^y$	base vectors		
conjugation:	$\psi \Leftrightarrow \psi^{③}$	3	switch	isotropic
double flip:	$\psi \Leftrightarrow \psi^{②}$	2	neutral	anisotropic
single flip:	$\psi \Leftrightarrow \psi^{①}$	1	switch	anisotropic
No flip	$\psi = \psi$	0	neutral	isotropic
②① flip	$\psi^{②} \Leftrightarrow \psi^{①}$	3	switch	anisotropic

The quaternionic nabla operator ∇ uses the sign flavor of the background coordinate system. Antiparticles use the conjugated nabla operator ∇^*.

The Dirac equation

The Dirac equation appears to be a special form of continuity equation[111].

$$\rho(t, \boldsymbol{q}) = \boldsymbol{v}(t, \boldsymbol{q})\rho_0(t, \boldsymbol{q}) \tag{1}$$

The QPAD ψ(t, q) can be used to define a charge probability density and probability current density. The conventional form of the Dirac equation runs

$$\nabla_0\{\psi\} + \boldsymbol{\nabla}\boldsymbol{\alpha}\{\psi\} = m\beta\{\psi\} \tag{2}$$

$\boldsymbol{\alpha}$ and β represent the matrices that implement the quaternion behavior including the sign selections of quaternions for complex fields.
We keep the sign selections of the background coordinate system $(t, \boldsymbol{q})$ fixed. Thus $\boldsymbol{\alpha}$ and β only influence the elements of **spinor** $\{\psi\}$.

$$\alpha_1 = \begin{bmatrix} 0 & \boldsymbol{i} \\ -\boldsymbol{i} & 0 \end{bmatrix} \tag{3}$$

$$\alpha_2 = \begin{bmatrix} 0 & \boldsymbol{j} \\ -\boldsymbol{j} & 0 \end{bmatrix} \tag{4}$$

$$\alpha_3 = \begin{bmatrix} 0 & \boldsymbol{k} \\ -\boldsymbol{k} & 0 \end{bmatrix} \tag{5}$$

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[111] See http://www.vttoth.com/qt.htm.

$$\beta = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{6}$$

There exist also a relation between $\alpha_1, \alpha_2, \alpha_3$ and the [Pauli][112] matrices $\sigma_1, \sigma_2, \sigma_3$:

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \qquad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{7}$$

$$1 \longmapsto I, \qquad \boldsymbol{i} \longmapsto \sigma_1, \qquad \boldsymbol{j} \longmapsto \sigma_2, \qquad \boldsymbol{k} \longmapsto \sigma_3 \tag{8}$$

This combination is usually represented in the form of gamma matrices[113]. These matrices are not used in this paper. They are used when a complex Hilbert space must handle quaternionic behavior.

Transferring the matrix form of the Dirac equation into quaternionic format delivers two quaternionic fields ψ_R and ψ_L that couple two equations of motion.

$$\nabla_0 \psi_R + \boldsymbol{\nabla}\psi_R = m\psi_L \tag{9}$$

$$\nabla_0 \psi_L - \boldsymbol{\nabla}\psi_L = m\psi_R \tag{10}$$

The mass term m couples ψ_L and ψ_R. The fact $m = 0$ decouples ψ_L and ψ_R.

$$\psi_R = \psi_L^* = \psi_0 + \boldsymbol{\psi} \tag{11}$$

Thus the fields are each other's quaternionic conjugate.
Reformulating the quaternionic equations gives

$$\nabla\psi^{\circledcirc} = m\,\psi^{\circled{3}} \tag{12}$$

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[112] http://en.wikipedia.org/wiki/Pauli_matrices
[113] http://en.wikipedia.org/wiki/Gamma_matrices

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$$\nabla_0(\psi_0 + \boldsymbol{\psi}) + \boldsymbol{\nabla}(\psi_0 + \boldsymbol{\psi}) = m(\psi_0 - \boldsymbol{\psi}) \tag{13}$$

For the conjugated field holds

$$\nabla^* \psi^{\textcircled{3}} = m\, \psi^{\textcircled{0}} \tag{14}$$

$$\nabla_0(\psi_0 - \boldsymbol{\psi}) - \boldsymbol{\nabla}(\psi_0 - \boldsymbol{\psi}) = m(\psi_0 + \boldsymbol{\psi}) \tag{15}$$

This implements the reverse flip. The corresponding particle is the antiparticle.

$$\left(\psi^{\textcircled{0}}, \psi^{\textcircled{3}}\right) \leftrightarrow \left(\psi^{\textcircled{3}}, \psi^{\textcircled{0}}\right) \tag{16}$$

Summing the equations gives via

$$\boldsymbol{\nabla}\boldsymbol{\psi} = \boldsymbol{\nabla} \times \boldsymbol{\psi} - \langle \boldsymbol{\nabla}, \boldsymbol{\psi}\rangle \tag{17}$$

the result

$$\nabla_0 \psi_0 - \langle \boldsymbol{\nabla}, \boldsymbol{\psi}\rangle = m\, \psi_0 \tag{18}$$

The difference gives

$$\nabla_0 \boldsymbol{\psi} + \boldsymbol{\nabla}\psi_0 + \boldsymbol{\nabla} \times \boldsymbol{\psi} = -m\, \boldsymbol{\psi} \tag{19}$$

Just reversing the sign flavors does not work. The corresponding equation contains extra terms:

$$\nabla\psi^{\textcircled{3}} = \nabla_0(\psi_0 - \boldsymbol{\psi}) + \boldsymbol{\nabla}(\psi_0 - \boldsymbol{\psi}) = \nabla_0\psi_0 - \nabla_0\boldsymbol{\psi} + \boldsymbol{\nabla}\psi_0 - \boldsymbol{\nabla}\boldsymbol{\psi} \tag{20}$$

$$= (m\,\psi_0 + \langle \boldsymbol{\nabla}, \boldsymbol{\psi}\rangle) - (-m\,\boldsymbol{\psi} - \boldsymbol{\nabla}\psi_0) + \boldsymbol{\nabla}\psi_0$$
$$- (\boldsymbol{\nabla} \times \boldsymbol{\psi} - \langle \boldsymbol{\nabla}, \boldsymbol{\psi}\rangle)$$

$$= m\,\boldsymbol{\psi} + 2\langle \boldsymbol{\nabla}, \boldsymbol{\psi}\rangle + 2\boldsymbol{\nabla}\psi_0$$

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Thus if the reverse equation fits, then it will concern another field configuration ψ' that will not fit the original equation.

$$\nabla^*\nabla\psi = \nabla_0^2\,\psi + \langle\mathbf{\nabla},\mathbf{\nabla}\rangle\,\psi = m\nabla^*\psi^{\textcircled{3}} = m^2\psi \tag{21}$$

Compare with the continuity equations

$$\nabla_0\rho_0(t,\mathbf{q}) - \langle\mathbf{\nabla},\boldsymbol{\rho}(t,\mathbf{q})\rangle = s_0(t,\mathbf{q}) \tag{22}$$

And

$$\mathbf{s}(t,\mathbf{q}) = \nabla_0\boldsymbol{\rho}(t,\mathbf{q}) + \mathbf{\nabla}\rho_0(t,\mathbf{q}) + \mathbf{\nabla}\times\boldsymbol{\rho}(t,\mathbf{q}) \tag{23}$$

This means that

$$s_0(t,\mathbf{q}) \mapsto m\,\psi_0(t,\mathbf{q}) \tag{24}$$

$$\mathbf{s}(t,\mathbf{q}) \mapsto -m\,\boldsymbol{\psi} \tag{25}$$

Thus in the Dirac equation the mass term is a source term that depends on the (conjugate) field.

The following definitions specify another continuity equation:

$$\rho_{Dirac} \overset{\text{def}}{=} \psi\psi = \psi_0\psi_0 - \langle\boldsymbol{\psi},\boldsymbol{\psi}\rangle + 2\psi_0\boldsymbol{\psi} \tag{26}$$

$$\rho_{0\,Dirac} = \psi_0\psi_0 - \langle\boldsymbol{\psi},\boldsymbol{\psi}\rangle \tag{27}$$

$$\boldsymbol{\rho}_{Dirac} = 2\psi_0\boldsymbol{\psi} \tag{28}$$

$$\nabla\rho_{Dirac} = 2\psi\nabla\psi = 2m\psi\psi^{\textcircled{3}} = 2m|\psi|^2 = 2m\varphi \tag{29}$$

$$\nabla\psi\psi^{\textcircled{3}} = \nabla\varphi = 2\psi\nabla\psi = 2m\rho_{0\,Dirac} \tag{30}$$

The field φ is real and non-negative and represents a probability density distribution. This result defines two new continuity equations. $\rho_{0\,Dirac}$ has a Minkowski signature.

The interpretation of φ as the probability density distribution of **presence** leads to:

$$\int_V \varphi \, dV = 1 \tag{31}$$

$$\int_V \nabla \rho_{Dirac} \, dV = 2m \tag{32}$$

The coupling factor m for the antiparticle is the same.

The field ψ has an intrinsic spin[114]:

$$spin = \int_V (\nabla \psi_0) \times \boldsymbol{\psi} \, dV = \int_V \nabla \times (\psi_0 \boldsymbol{\psi}) \, dV = \tfrac{1}{2} \int_V \nabla \times \boldsymbol{\rho}_{Dirac} \, dV \tag{33}$$

The sign flavor flip reverses the spin.

Properties
The particles that obey the Dirac equation appear to have electrical charge and half integer valued spin. They are fermions.
The particles that obey equation (12) have the opposite charge from the corresponding antiparticles that obey equation (14). Both particle types can have spin up or spin down.

Interactions
The interaction free equation can be extended with interactions with other fields.

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[114] http://www.plasma.uu.se/CED/Book/EMFT_Book.pdf Section: Conservation of angular momentum, formula 4.70a

$$\nabla \psi = m \, \psi^* \tag{1}$$

$$\overset{\leftrightarrow}{\mathbb{D}} \psi = m\psi^* - e(A\psi - \psi B) + C \tag{2}$$

The field A is right covariant with ψ. The field B is left covariant with ψ. The two can be combined in Q-covariance. e is a coupling constant. Thus here $\overset{\leftrightarrow}{\mathbb{D}}$ is the two sided covariant derivative[115]. The field C represents a source.

For the interaction field A holds

$$\square A = (\nabla_0^2 - \langle \nabla, \nabla \rangle)A = e(\psi\psi) = e\rho_{Dirac} \tag{3}$$

$\square$ is the d'Alembert operator

The wave equation for the electromagnetic field in vacuum is

$$\square A = 0 \tag{4}$$

Besides of the one sided covariance also a Q covariance is possible due to the application of a quaternion waltz[116].

The background field
It is possible to get an explanation/implementation for the coupled field.
- Let $\psi(r)$ be a spherically symmetric QPAD for which the modulus $|\psi(r)|$ decreases with the distance r of the center location of $\psi(r)$ according to the function $1/r$.
- This dependence need not start directly at $r = 0$, but it must start close to zero.
- Let the universe be filled with QPAD's $\{\psi_i\}$ that on average are equal to $\psi(r)$ and who's center locations are randomly distributed over an affine space.

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[115] See paragraphs on covariant and Q-covariant derivative
[116] Fermion and boson equations; Q covariant derivative

- Define $\varphi(r)$ as the superposition of all $\{\psi_i\}$, taken at the center location of $\psi(r)$.

The contribution to the local superposition decreases with the distance of a ψ_i, however the number of contributing ψ_i's increases with that distance. So, the largest contribution comes from the most distant ψ_i's.

Now the charge of $\varphi(r)$ is huge and it is distributed over space in a similar way as the charge of $\psi(r)$. However the current $\boldsymbol{\varphi}(r)$ has its direction reverse to $\boldsymbol{\psi}(r)$. With other words:

$$\varphi(r) = c_0\,\psi^*(r) = c_0\,\psi^{\textcircled{3}}(r) \tag{1}$$

c_0 is a constant. Thus, when ψ is spherically symmetric and is embedded in a sea of QPAD's that on average are similar to ψ, while all these QPAD's decrease with r as $1/r$, then the above construction works.

The long range averaging over an increasing number of contributions and the random distribution of the ψ_i's take care that equation (1) holds.

We will call the renormalized superposition *the background field*.

This interpretation enables to interpret the transported field as the wave function and the coupled field as the environment.

This view lifts a tip of the veil that hides why the gravitational constant enters into the Planck units. The gravitational constant has to do with the total number of particles that exist[117].

The background field places the particle in its environment. It means that the equation of motion is at the same time describing interactions.

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[117] See D. Sciama: On the origin of inertia,
(http://www.adsabs.harvard.edu/abs/1953MNRAS.113...34S

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Prospect

The original Dirac equation can be transformed into two quaternionic equations:

$$\nabla\psi = m\,\psi^*\tag{1}$$

$$\nabla^*\psi^* = m\,\psi\tag{2}$$

The reverse equations (for the same field configuration) are more complicated:

$$\nabla\psi^{③} = m\,\psi + 2\langle\boldsymbol{\nabla},\boldsymbol{\psi}\rangle + 2\boldsymbol{\nabla}\psi_0\tag{3}$$

$$\nabla^*\psi = m\,\psi^{③} - 2\langle\boldsymbol{\nabla},\boldsymbol{\psi}\rangle - 2\boldsymbol{\nabla}\psi_0\tag{4}$$

We will analyze whether this is more general principle. For example the Majorana equation is to a certain extend similar to the Dirac equation.

The Majorana equation

The [Majorana equation](http://en.wikipedia.org/wiki/Majorana_equation)[118] differs from the Dirac equation in the way that the sign flavor of the transported field $\psi^{⓪}$ differs.

In fact two possible versions of the Majorana equation exist. The first is:

$$\nabla\psi^{①} = m_n\psi^{③}\tag{1}$$

While the second version is:

$$\nabla\psi^{②} = m_n\psi^{⓪}\tag{2}$$

The first version is in agreement with the use of the background field $\psi^{③}$ as the coupled field. The second equation uses the conjugate $\psi^{⓪}$ of the background field.

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[118] http://en.wikipedia.org/wiki/Majorana_equation

The sign selection $\psi^{\text{①}}$ only switches a single imaginary base vector. Like the conjugation, it switches the handedness of ψ. The sign selection $\psi^{\text{②}}$ switches two imaginary base vectors. It does not switch the handedness. In both equations two imaginary base vectors change their sign. These sign selections do not switch handedness. Three independent directions are possible. (That fact may not become observable).

For the conjugated equations hold:

$$\nabla^*\psi^{\text{②}} = m_n\psi^{\text{⓪}} \tag{3}$$

$$\nabla^*\psi^{\text{①}} = m_n\psi^{\text{③}} \tag{4}$$

Thus the conjugated equation does not switch the handedness. Three independent directions are possible.

Neutrinos are supposed to obey the Majorana equation.

When the first version of the Majorana equation holds, then

$$\psi\nabla\psi^{\text{①}} = m_n\psi\,\psi^{\text{③}} = m_n|\psi|^2 \tag{5}$$

$$\int_V \left(\psi\,\nabla\psi^{\text{①}} \right) dV = m_n \int_V |\psi|^2\, dV = m_n \tag{6}$$

For the conjugated field holds:

$$\psi^{\text{③}}\,\nabla^*\psi^{\text{②}} = m_n\psi^{\text{③}}\psi = m_n|\psi|^2 \tag{7}$$

$$\int_V \left(\psi^{\text{③}}\nabla^*\psi^{\text{②}}\right) dV = m_n \int_V |\psi|^2\, dV = m_n \tag{8}$$

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Properties

The particles that belong to this category are neutral fermions. They have half integer valued spin that can be either up or down.

The third category sign flavor switch

Apart from the Dirac equation and the Majorana equation, a third category equation is possible. In these equations the mass term flips the sign of only one imaginary base vector. As a result the handedness flips as well. The sign flavor of the background coordinate system can act as a reference for comparing quaternion sign flavors. The quaternionic nabla operator uses that same sign flavor. With respect to the background sign flavor, three different possibilities for the choice of the flipped imaginary base vector exist. It will become clear that this category corresponds to quarks.

The corresponding equation is:

$$\nabla\psi_i^{②} = m_{di}\,\psi^{③} \tag{1}$$

The index i runs over three color versions r, g and b. These colors relate to the selected direction. This particle features charge -⅓e.

For the conjugated equation holds:

$$\nabla^*\psi_i^{①} = m_{di}\,\psi^{⓪} \tag{2}$$

The so called down versions obey equation (1) and (2). When equation (1) holds, then

$$\psi^{⓪}\nabla\psi_i^{②} = m_{di}\,\psi^{⓪}\,\psi^{③} = m_{di}|\psi|^2 \tag{3}$$

$$\int_V \left(\psi^{⓪}\nabla\psi_i^{②}\right)\,dV = m_{di} \tag{4}$$

For each color i a down version $\left\{\psi_i^{②}, \psi^{③}\right\}$ and an up version $\left\{\psi_i^{①}, \psi^{⓪}\right\}$ exists.

Properties

In contrast to the $\left(\psi^{⓪}, \psi^{③}\right)$ flip, the $\left(\psi^{⓪}, \psi_i^{②}\right)$ flip and the $\left(\psi^{③}, \psi_i^{①}\right)$ flip are strongly anisotropic. The three choices for the flipped imaginary base vector may be linked with color charges.

The antiparticles have anti-color. The particles and antiparticles may be linked with the color charges and the up and down versions of quarks. The fact that only one of the three, or with the second version two of the three imaginary base vectors are flipped may account for the respective electrical charges, which are $+\frac{2}{3}$ or $-\frac{1}{3}$.

All particles of this category appear to have half integer valued spin. They can have spin up or spin down. The particles are fermions.

The rules

The ordered pair $\{\psi^x, \psi^y\}$ represents a category of elementary particle types.

The above treated particles appear to be fermions.

For antiparticles all participating fields and the nabla operator conjugate. Photons and gluons have zero coupling factor.

The rules are:

- If the coupling takes place between two field sign flavors with different handedness, then the corresponding particle is charged.
- The charge depends on the number and direction of the base vectors that differ.
- The count for each difference is $\pm\frac{1}{3}$e.

No elementary particle exists that obeys the rules and features electric charge $\frac{2}{3}$e. Such a particle may exist as a composite. Thus, according to

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these rules the up-quarks are not elementary particlesFor that reason, they do not belong to the standard model.

Anisotropic coupling fields

We have explored all particles that make use of the isotropic background field or the conjugate of the background field, which is also isotropic. These particles appear to be fermions. Next we like to explore particles that couple to anisotropic fields. These particles appear to be bosons. They all have integer valued spin.

The cross-sign flavor equations

These equations describe the situation that a flip is made from a $\psi_i^{\text{①}}$ field to a $\psi_i^{\text{②}}$ field or vice versa. The direction i might play no role.

$$\nabla \psi_i^{\text{②}} = m_{W_+} \psi_i^{\text{①}} \tag{1}$$

The conjugated equation is:

$$\nabla^* \psi_i^{\text{①}} = m_{W_+} \psi_i^{\text{②}} \tag{2}$$

Another form is

$$\nabla \psi_i^{\text{①}} = m_{W_-} \psi_i^{\text{②}} \tag{3}$$

The conjugated equation is:

$$\nabla^* \psi_i^{\text{②}} = m_{W_-} \psi_i^{\text{①}} \tag{4}$$

The sign flavor switch affects three imaginary base vectors and flips the handedness. As a consequence the particles have a full electric charge. It concerns two particles, the W^- and the W^+ bosons. These bosons carry electrical charges.

$$\tag{5}$$

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$$\psi_i^{(2)} \nabla \psi_i^{(2)} = m_{W_+} \psi_i^{(2)} \psi_i^{(1)}$$

$$\int_V \left(\psi_i^{(2)} \nabla \psi_i^{(2)} \right) dV = m_{W_+} \int_V \left(\psi_i^{(2)} \psi_i^{(1)} \right) dV = m_{W_+} g \tag{6}$$

$$\psi_i^{(1)} \nabla \psi_i^{(1)} = m_{W_-} \psi_i^{(1)} \psi_i^{(2)} \tag{7}$$

$$\int_V \left(\psi_i^{(1)} \nabla \psi_i^{(1)} \right) dV = m_{W_-} \int_V \left(\psi_i^{(2)} \psi_i^{(1)} \right) dV = m_{W_-} g \tag{8}$$

The Z boson
The particle that obeys:

$$\nabla \psi^{(0)} = m_Z \psi^{(2)}$$

Is a neutral boson.

$$\int_V \left(\psi_i^{(1)} \nabla \psi^{(0)} \right) dV = m_Z \int_V \left(\psi_i^{(1)} \psi_i^{(2)} \right) dV = m_Z g$$

Another possibility is:

$$\nabla \psi^{(3)} = m_{Z?} \psi^{(1)}$$

The non-sign flavor flip category
In this category no switch is performed. The field couples with itself.
The corresponding equation is:

$$\nabla \psi^x = m \psi^x \tag{1}$$

For the antiparticle holds:

$$\nabla^* \psi^{x*} = m \psi^{x*} \tag{2}$$

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And for the mass m holds

$$\int_V (\psi^{x*} \nabla \psi^x)\, dV = m \int_V |\psi^x|^2\, dV = g_x\, m \tag{3}$$

The equation describes neutral particles.
For the probability density no integral source or leakage exists. Thus m **must be zero**.

Fermion and boson equations

Elementary particles are identified by a pair of quaternionic field sign flavors. The antiparticle corresponds to the conjugated pair. The type of sign flavor switch determines the charge of the particle. From this combination it is not clear what the maximum value of the spin of the particle will be. It certainly has something to do with the isotropy of the coupled field.

Elementary particles with zero mass are not coupled and appear to be bosons. With the $W_{\mp}$ bosons the coupled field is in condition $\psi^{①}$ or $\psi^{②}$. For all fermions the coupled field is in condition $\psi^{③}$ or $\psi^{⓪}$.

Elementary fermions are elementary particles that are based on a coupled pair of field sign flavors of which the coupled member has sign flavor $\psi^{③}$

Elementary bosons are elementary particles that are based on non-coupled field sign flavors or on a coupled pair of field sign flavors of which the coupled member has sign flavor $\psi^{①}$ or $\psi^{②}$

Three fermion equations exist. Their *interaction free forms* are:

$$\nabla \psi^{⓪} = m_{e_-}\, \psi^{③} \tag{1}$$

$$\nabla^* \psi^{③} = m_{e_+}\, \psi^{⓪}, \text{ this concerns the antiparticle} \tag{1a}$$

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$$\nabla \psi^{\text{\textcircled{1}}} = m_n \, \psi^{\text{\textcircled{3}}} \tag{2}$$

$$\nabla \psi_i^{\text{\textcircled{2}}} = m_{di} \psi^{\text{\textcircled{3}}}; i = r, g, b \tag{3}$$

Three massive boson equations exist:

$$\nabla \psi^{\text{\textcircled{2}}} = m_{W_+} \psi^{\text{\textcircled{1}}} \tag{4}$$

$$\nabla \psi^{\text{\textcircled{1}}} = m_{W_-} \psi^{\text{\textcircled{2}}} \tag{5}$$

$$\nabla \psi^{\text{\textcircled{0}}} = m_Z \psi^{\text{\textcircled{2}}} \tag{6}$$

The massless particles are of the form $\{\psi^x, \psi^x\}$:

$$\nabla \psi^x = 0 \tag{7}$$

Four of these massless bosons exist.
Other possibilities do not appear in the standard model.

$$\nabla \psi^{\text{\textcircled{3}}} = m_{Z?} \psi^{\text{\textcircled{1}}} \tag{8}$$

$$\nabla \psi_i^{\text{\textcircled{1}}} = m_{?i} \psi^{\text{\textcircled{0}}}; i = r, g, b \tag{9}$$

$$\nabla \psi^{\text{\textcircled{3}}} = m_{e?} \, \psi^{\text{\textcircled{0}}} \tag{10}$$

$$\nabla \psi^{\text{\textcircled{2}}} = m_{n?} \, \psi^{\text{\textcircled{0}}} \tag{11}$$

General form

The general form of the equation for particle $\{\psi^x, \psi^y\}$ is:

$$\nabla \psi^x = m \, \psi^y \tag{1}$$

For the antiparticle:

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$$\nabla^* \psi^{x*} = m\, \psi^{y*} \tag{2}$$

For all particles holds:

$$\nabla_0 \psi_0 - \langle \nabla, \boldsymbol{\psi} \rangle = m\, \psi_0 \tag{3}$$

$$\nabla \times \boldsymbol{\psi}^x + \nabla \psi_0^x + \nabla_0 \boldsymbol{\psi}^x = m\, \boldsymbol{\psi}^y \tag{4}$$

$$\int_V \psi \psi^{\circledcirc} \, dV = 1 \tag{5}$$

$$\int_V \psi^{\textcircled{1}} \psi^{\textcircled{2}} \, dV = g \tag{6}$$

The factor g is real and non-negative.
Further, the equation for coupling factor m

$$\int_V (\psi^{y*} \nabla \psi^x) \, dV = m \int_V (\psi^{y*} \psi^y) \, dV = m \int_V |\psi^y|^2 \, dV \tag{7}$$

An equivalent of the Lagrangian may look like

$$\mathcal{L} \stackrel{?}{=} \psi^{y*} \nabla \psi^x - m\, \psi^{y*}\, \psi^y \tag{8}$$

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Survey of couplings

In the following table the attribution of particle names is speculative.

RLrl	e	Diff	Coupling	m		Particle	Multiplet
RL	-1	3	$\psi^{⓪}\psi^{③}$	m	fermion	electron	1
LR	1	3	$\psi^{③}\psi^{⓪}$	m	fermion	positron	1
lR	-⅓	1	$\psi^{①}\psi^{⓪}$	m_d	fermion	down-quark	3 colors
Ll	0	2	$\psi^{③}\psi^{①}$	m_z	boson	Z	3?
lr	-1	1	$\psi^{①}\psi^{②}$	m_{w_-}	boson	W^-	3?
rl	1	1	$\psi^{②}\psi^{①}$	m_{w_+}	boson	W^+	3?
RR	0	0	$\psi^{⓪}\psi^{⓪}$	0	boson	photon	
LL	0	0	$\psi^{③}\psi^{③}$	0	boson	photon	
rr	0	0	$\psi^{②}\psi^{②}$	0	boson	gluon	3?
ll	0	0	$\psi^{①}\psi^{①}$	0	boson	gluon	3?
Rl	⅓	1	$\psi^{⓪}\psi^{①}$	$m_{d?}$	boson	?	3 colors
rL	⅔	1	$\psi^{②}\psi^{③}$	m_u	boson	?	3 colors
Lr	-⅔	1	$\psi^{③}\psi^{②}$	$m_{u?}$	boson	?	3 colors
rR	0	2	$\psi^{②}\psi^{⓪}$	$m_{n?}$	boson	neutrino?	3?
lL	0	2	$\psi^{①}\psi^{③}$	m_n	boson	neutrino	3?
Rr	0	2	$\psi^{⓪}\psi^{②}$	$m_{Z?}$	boson	Z?	3?

Colophon:
RLrl; switch by 3, 2 or 1 imaginary base vectors
e; electric charge of particle
Diff; number of imaginary base vectors difference
Coupling; the field sign flavors that are coupled
Fermion/boson;
Particle; elementary particle category
Multiplet; multiplet structure

The neutrinos, Z and W bosons might show multiplicity.

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In the standard model three versions of fermion mass factors m exist. These versions are not (yet) explained by this model.

Remarkably, in the table several places for particles are still open.

Coupling factors

The integral probability densities are:

$$\int_V \left(\psi^{\textcircled{0}} \psi^{\textcircled{3}} \right) dV = \int_V |\psi|^2 \, dV = 1$$

$$g = \int_V \left(\psi^{\textcircled{1}} \psi^{\textcircled{2}} \right) dV = \int_V \left(\psi^{\textcircled{2}} \psi^{\textcircled{1}} \right) dV = \int_V \left| \psi^{\textcircled{1}} \right|^2 dV = \int_V \left| \psi^{\textcircled{2}} \right|^2 dV$$

The coupling factors are:

Primary	Coupling factor	reverse	Coupling factor
$\psi^{\textcircled{3}}, \psi^{\textcircled{0}}$	$m_e = \int_V \left(\psi^{\textcircled{0}} \nabla \psi^{\textcircled{0}} \right) dV$	$\psi^{\textcircled{0}}, \psi^{\textcircled{3}}$	
$\psi^{\textcircled{1}}, \psi^{\textcircled{3}}$	$m_n = \dfrac{1}{g} \int_V \left(\psi^{\textcircled{0}} \nabla \psi^{\textcircled{1}} \right) dV$	$\psi^{\textcircled{3}}, \psi^{\textcircled{1}}$	
$\psi^{\textcircled{3}}, \psi^{\textcircled{1}}$	$m_z = \int_V \left(\psi^{\textcircled{1}} \nabla \psi^{\textcircled{3}} \right) dV$	$\psi^{\textcircled{1}}, \psi^{\textcircled{3}}$	
$\psi_i^{\textcircled{2}}, \psi^{\textcircled{3}}$	$m_{ui} = \dfrac{1}{g_i} \int_V \left(\psi^{\textcircled{0}} \nabla \psi_i^{\textcircled{2}} \right) dV$	$\psi^{\textcircled{3}}, \psi_i^{\textcircled{2}}$	
$\psi_i^{\textcircled{1}}, \psi^{\textcircled{0}}$	$m_{di} = \dfrac{1}{g_i} \int_V \left(\psi^{\textcircled{3}} \nabla \psi_i^{\textcircled{1}} \right) dV$	$\psi^{\textcircled{0}}, \psi_i^{\textcircled{1}}$	
$\psi^{\textcircled{2}}, \psi^{\textcircled{1}}$	$m_{W+} = \dfrac{1}{g} \int_V \left(\psi_i^{\textcircled{2}} \nabla \psi_i^{\textcircled{2}} \right) dV$	$\psi^{\textcircled{1}}, \psi^{\textcircled{2}}$	$m_{W-} = \dfrac{1}{g} \int_V \left(\psi_i^{\textcircled{1}} \nabla \psi_i^{\textcircled{1}} \right) dV$

Most particle categories of the SM appear with three different coupling factors. This corresponds with three different field configurations of ψ. This paper does not explain that extra diversity.

Interactions

In complex quantum field theory, interactions are derived from covariant derivatives. In quaternion field theory this is not that straight forward. The problem is caused by the fact that for quaternionic fields in general:

$$\nabla(fg) \neq f\nabla g + (\nabla f)g \tag{1}$$

On the other hand quaternionic fields are interesting because a field f can rotate inside another field g under the influence of a quaternion waltz:

$$h = gf/g \neq f \tag{2}$$

The result is Q-covariance.

Covariant derivative

The covariant derivative plays a role in the Lagrangian and in the equation of motion.

The covariant derivative D of field $\psi(q)$ is defined as

$$D\psi(q, t) = \nabla\psi(q, t) - \mathbf{A}(q, t)\,\psi(q, t) \tag{1}$$

This is interesting with respect to a gauge transformation of the form

$$\psi'(q) = \varphi(q)\,\psi(q) \tag{2}$$

The field $\varphi(q)$ has a modulus that is equal to one:

$$\varphi(q)\,\varphi^*(q) = 1 \tag{3}$$

We suppose that a field $\mathbf{H}(q)$ exists such that

$$\nabla\varphi(q) = \mathbf{H}(q)\,\varphi(q) \tag{4}$$

A new version of the derivative can be obtained by a corresponding vector potential transformation

$$\mathbf{A}'(q) = \mathbf{A}(q) + \mathbf{H}(q) \tag{5}$$

$$D' = \nabla - \mathbf{A}(q) - \mathbf{H}(q) \tag{6}$$

The following inequality holds in general for quaternionic functions.

$$\nabla\big(\varphi(q)\psi(q)\big) \neq \big(\nabla\varphi(q)\big)\psi(q) + \varphi(q)\nabla\psi(q) \tag{7}$$

However, we assume that it is *an equality* for $\varphi(q)$ and $\psi(q)$.

$$D'\psi'(q) = \mathbf{H}(q)\,\varphi(q)\,\psi(q) + \varphi(q)\,\nabla\psi(q) \tag{8}$$

$$-\mathbf{A}(q)\varphi(q)\psi(q) - \mathbf{H}(q)\varphi(q)\psi(q)$$

$$= \varphi(q)\big(\nabla\psi(q) - \mathbf{A}(q)\,\psi(q)\big)$$

$$D'\psi'(q) = \varphi(q)\,D\psi(q) \tag{9}$$

Thus, with that transformation pair not only the modulus of the function stays invariant but also the modulus of the covariant derivative stays invariant.

Further

$$\psi'^*(q)D'\psi'(q) = \psi^*(q)\,\varphi^*(q)\,\varphi(q)\,D\psi(q) \tag{10}$$

$$= \psi^*(q)\,D\psi(q)$$

Above the right sided covariant derivative D is defined

$$\vec{D}\psi(q) = \vec{\nabla}\psi(q) - \mathbf{A}(q)\psi(q) \tag{11}$$

The left sided covariant derivative is defined as:

$$\psi(q)\overleftarrow{D} = \psi(q)\overleftarrow{\nabla} - \psi(q)\,\boldsymbol{B}(q) \tag{12}$$

We will use $\overleftrightarrow{D}$ for both left sided and right sided covariant derivative:

$$\overleftrightarrow{D}\psi(q) = \frac{\overrightarrow{\nabla}\psi(q) + \psi(q)\overleftarrow{\nabla}}{2} - \boldsymbol{A}(q)\,\psi(q) - \psi(q)\,\boldsymbol{B}(q) \tag{13}$$

Multiplication with a unitary factor corresponds with a displacement in the canonical conjugate space, thus with a shift of the momentum of the field.

Q Covariant derivative

The Q covariant derivative[119] relates to quaternionic field transformations of the form

$$\psi' = \varphi\,\psi\,\varphi^* \tag{1}$$

$$\varphi\,\varphi^* = 1 \tag{2}$$

This is the **quaternion waltz**. Let the imaginary field A be defined by:

$$\overrightarrow{\nabla}\varphi \overset{\text{def}}{=} \boldsymbol{A}\,\varphi \tag{3}$$

$$\varphi^*\overleftarrow{\nabla} = -\varphi^*\boldsymbol{A} = \left(\overrightarrow{\nabla}\varphi\right)^* \tag{4}$$

The following step is questionable, because with quaternionic functions in general

$$\nabla(fg) \neq f\nabla g + (\nabla f)g \tag{5}$$

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[119] Principle of General Q Covariance; D. Finkelstein, J. M. Jauch, S. Schiminovich and D. Speiser; Journal of Mathematical Physics volume 4, number 6, June 1963, 788-796

However we consider the rule valid for this special case. In fact we apply the covariant case twice.

$$\nabla\psi' = \nabla(\varphi\,\psi\,\varphi^*) = (\vec{\nabla}\varphi)\psi\,\varphi^* + \varphi\,(\nabla\psi)\,\varphi^* + \varphi\,\psi\,(\varphi^*\overleftarrow{\nabla}) \tag{6}$$

$$= A\,\varphi\,\psi\,\varphi^* + \varphi\,(\nabla\psi)\,\varphi^* - \varphi\,\psi\,\varphi^*A$$

$$\nabla\psi' = [\mathbf{A}, \psi'] + \varphi\,(\nabla\psi)\,\varphi^* \tag{7}$$

The general equation of motion is:

$$\nabla\psi^x = m\,\psi^y \tag{8}$$

Applying the quaternion waltz gives:

$$\nabla\psi^{x'} = [\mathbf{A}, \psi^{x'}] + \varphi(\nabla\psi^x)\varphi^* \tag{9}$$

$$= [\mathbf{A}, \psi^{x'}] + m\,\varphi\,\psi^?\,\varphi^* = [\mathbf{A}, \psi^{x'}] + m\psi^{y'}$$

Where

$$\varphi\,\psi^y\,\varphi^* \tag{10}$$

Thus, the general equation of motion due to the waltz is

$$\nabla\psi^{x'} = [\mathbf{A}, \psi^{x'}] + m\psi^{y'} \tag{11}$$

This equation describes the equation of motion including interactions that are due to the effect of the quaternion waltz under the influence of another field (φ).

The interpretation of the Q-covariant derivative is that the particle to which ψ^x and ψ^y belong not only moves due to the nabla operator, but

also rotates with respect to an outside field φ, which takes the particle in a quaternion waltz[120].

English quaternion waltz

When the rotation is slow compared to the current $\boldsymbol{\psi}^x$, then it becomes interesting to analyze an infinitesimal rotation. The quaternionic value of φ is close to 1. Thus $\varphi = 1 + \boldsymbol{\Delta\theta}$. $\boldsymbol{\Delta\theta}$ is imaginary. Let us investigate the transform $\psi^{x'} = \varphi\,\psi^x\,\varphi^*$.

$$\psi^{x'} \approx (1 + \boldsymbol{\Delta\theta})\,\psi^x\,(1 - \boldsymbol{\Delta\theta}) \tag{12}$$

$$= \psi^x + \boldsymbol{\Delta\theta}\,\psi^x - \psi^x\,\boldsymbol{\Delta\theta} - \boldsymbol{\Delta\theta}\,\psi^x\,\boldsymbol{\Delta\theta}$$

$$\approx \psi^x + \boldsymbol{\Delta\theta}\,\psi^x - \psi^x\,\boldsymbol{\Delta\theta}$$

$$= \psi^x - 2\,\boldsymbol{\psi}^x \times \boldsymbol{\Delta\theta}$$

$$\Delta\psi^{x'} \approx -2\,\boldsymbol{\psi}^x \times \boldsymbol{\Delta\theta} \tag{13}$$

$$\nabla\psi^{x'} \approx \nabla\psi^x - 2\,\nabla_0\boldsymbol{\psi}^x \times \boldsymbol{\Delta\theta} - 2\,(\boldsymbol{\nabla} \times \boldsymbol{\psi}^x) \times \boldsymbol{\Delta\theta} \tag{14}$$

$$= \nabla\psi^x - 2(\nabla_0\boldsymbol{\psi}^x + \boldsymbol{\nabla} \times \boldsymbol{\psi}^x) \times \boldsymbol{\Delta\theta}$$

$$= \nabla\psi^x - 2\,(\nabla\psi^x) \times \boldsymbol{\Delta\theta}$$

$$= m\,(\psi^y - 2\,\boldsymbol{\psi}^y \times \boldsymbol{\Delta\theta})$$

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[120] For an explanation of the quaternion waltz, see the Hilbert book model: http://www.crypts-of-physics.eu/OntheoriginofdynamicsBoek2.pdf, part two

Interpreting the flip event

The equations of motion indicate that a flip of field sign flavor occurs. The charge density distribution specifies the probability where this flip occurs. The current density distribution represents the transport of the location where the flip may occur.

The flip event can be observed. This is then the event of observing the corresponding quantum. The observation represents the interaction with another particle. The flip event may represent an electric charge and it may represent a color charge.

Photons and gluons are flipping at every progression step. That is why their coupling constant delivers zero.

Interpreting coupling factors

The gravitation field, which is a tensor field rather than a quaternionic field, is an administrator of the local curvature rather than that it is the cause of local curvature. The value of the local metric tensor accurately registers all aspects of the local curvature. From the gravitation field it is possible to derive centers of gravitation on which the field can be thought to be anchored. Such a center need not be the location of an actual cause. It can be the center of the activity of a local geometric anomaly, such as a black hole. Such a center is a (virtual) position that can be at a location where space does not even exist. This is possible when two coordinate systems are considered. One flat and the other curved. The curved system features geometric anomalies.

In this way it becomes possible to consider a black hole as a geometric anomaly, such that within which nothing, not even space exists. Instead space at its border is curved such that no information can penetrate that border. Every particle, elementary or not, that approaches the border is ripped apart and part of the debris is attached to the border of the BH. The rest of the debris escapes from the process.

It can be imagined that elementary particles that possess mass will also have a geometric anomaly at their center. The curvature at the border of that anomaly forms a center of gravity. The fact that the particle is formed by anti-symmetric private fields will already explain the presence of such a local hole.

Private fields of elementary particles are formed by pairs of coupled sign flavors of the same quaternionic probability amplitude distribution. The coupling factor that characterizes the coupling of the two sign flavors might also determine the curvature of the local geometric anomaly.

Interpreting the equations of free movement
The equations of movement are best interpreted when an extra differentiation step is added:

$$\nabla \psi^x = m \, \psi^? \tag{1}$$

$$\nabla^* \nabla \psi^x = m \, \nabla^* \psi^? \tag{2}$$

It means that a coupled oscillation takes place when the quantum moves. In case of leptons this means that:

$$\nabla^* \nabla \psi = m \, \nabla^* \psi^* \tag{3}$$

The coupled oscillation takes place along the direction in which the electron moves.
The anisotropic coupled quanta oscillate free in one or two directions and oscillate coupled along the direction of movement.

Extending the Zoo
This paper treated single fields and ordered pairs of sign flavors of the same base field. The set of particles may be extended by ordered triples and ordered quadruples of the same base field.

Higher order couplings

Couplings that constitute composite particles from elementary particles or other composite particles are not treated here. It is assumed that during these couplings the constituting elementary particles keep their basic properties; coupling factor, electric charge and angular momentum. The properties that characterize the coupling event are sources of secondary fields. These fields are known as physical fields.
It is thought that these secondary fields play a major role in the higher order couplings. The reason for this fact is that the properties influence the curvature of the parameter space.

Forbidden region

Fermions have asymmetric permutation wave functions. This fact has only significance when two or more states are considered. Let us consider the situation that the *two states are completely identical*[121] and are *nearly at the same location*. In that case the superposition of the two states is given by:

$$|\psi> = |n_1> |n_2> \pm |n_2> |n_1> \tag{1}$$

The plus sign holds for bosons and the minus sign holds for fermions. The images of the two cases are:

Boson pair	Fermion pair
Symmetric distribution	Asymmetric distribution

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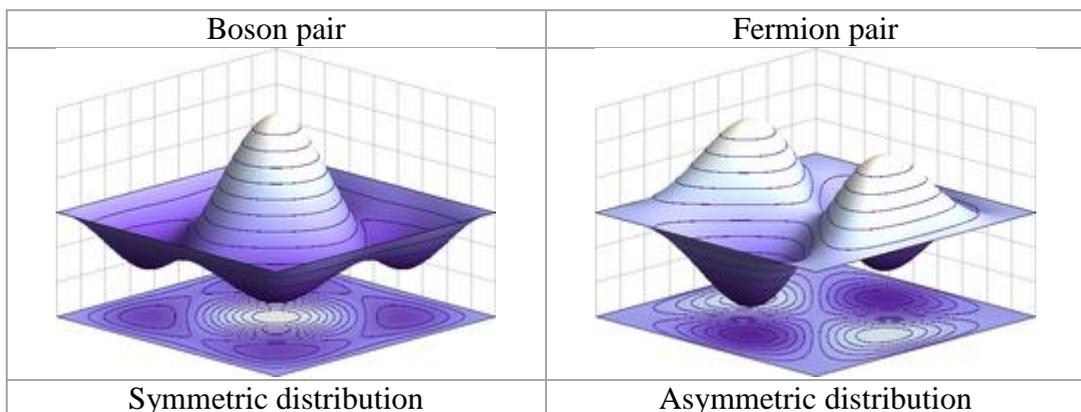
[121] http://en.wikipedia.org/wiki/Identical_particles

This is a two dimensional model, but it explains the general idea. Below the cut through the center of the asymmetric distribution is shown. When this is compared with the same cut of the squared modulus, then it reveals a forbidden region for the asymmetric distribution.
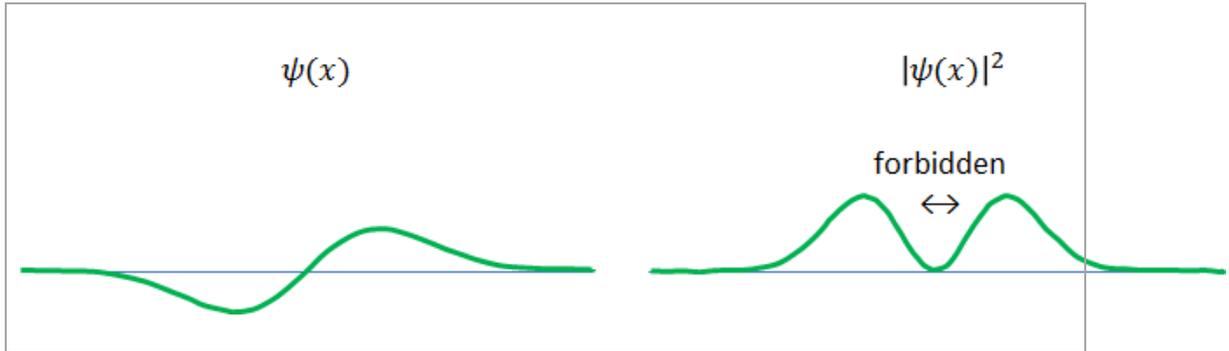
The particles were put at the closest possible position. Before the displacement occurs, the direction of the displacement is undefined. Thus the forbidden region has a spherical shape. When fermions go to their next position, they must step over the forbidden region. Bosons do not have that restriction.

Fourier transforms
The Fourier transform of the generalized equation

$$\nabla\psi = \nabla_0\psi_0 - \langle\nabla, \boldsymbol{\psi}\rangle + \nabla\times\boldsymbol{\psi} + \nabla\psi_0 + \nabla_0\boldsymbol{\psi} = m\,\psi^? \tag{1}$$

Gives

$$p\tilde{\psi} = p_0\tilde{\psi}_0 - \langle\boldsymbol{p}, \tilde{\boldsymbol{\psi}}\rangle + \boldsymbol{p}\times\tilde{\boldsymbol{\psi}} + \boldsymbol{p}\tilde{\psi}_0 + p_0\tilde{\boldsymbol{\psi}} = m\,\tilde{\psi}^? \tag{2}$$

For all field sign flavors hold:

$$p_0\tilde{\psi}_0 - \langle\boldsymbol{p}, \tilde{\boldsymbol{\psi}}\rangle = m\,\tilde{\psi}_0 \tag{3}$$

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$$\boldsymbol{p} \times \tilde{\boldsymbol{\psi}} + \boldsymbol{p}\tilde{\psi}_0 + p_0\tilde{\boldsymbol{\psi}} = m\,\tilde{\boldsymbol{\psi}}^? \tag{4}$$

$$\int_V \tilde{\psi}\tilde{\psi}^*\, dV = 1 \tag{5}$$

Let us consider the (anti)commutators

$$\left[\tilde{\psi}(p),\tilde{\psi}(p')\right] = \tilde{\psi}(p)\,\tilde{\psi}(p') - \tilde{\psi}(p)\,\tilde{\psi}(p') \tag{6}$$

$$\left\{\tilde{\psi}(p),\tilde{\psi}(p')\right\} = \tilde{\psi}(p)\,\tilde{\psi}(p') + \tilde{\psi}(p)\,\tilde{\psi}(p') \tag{7}$$

To be continued

Example potential

Spatial [Harmonic functions](http://en.wikipedia.org/wiki/Harmonic_function)[122] are suitable spread functions.
For a harmonic function $f(q)$ holds:

$$\Delta f(q) = \nabla\nabla^{①}f(q) = 0 \tag{1}$$

If there is a static spherically symmetric Gaussian charge density $\varrho\,(r)$:

$$\rho(q) = \frac{Q}{\sqrt{2\pi\sigma^2}^3}\exp(-|q|^2/(2\sigma^2)) \tag{2}$$

where Q is the total charge, then the solution $\phi\,(r)$ of [Poisson's equation](http://en.wikipedia.org/wiki/Poisson%27s_equation)[123],

$$\nabla^2\varphi(q) = -\frac{\rho(q)}{\varepsilon} \tag{3}$$

is given by

$$\varphi(q) = \frac{Q}{4\pi\varepsilon|q|}\operatorname{erf}\left(\frac{|q|}{\sqrt{2}\sigma}\right) \tag{4}$$

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[122] http://en.wikipedia.org/wiki/Harmonic_function
[123] http://en.wikipedia.org/wiki/Poisson%27s_equation

where erf(x) is the error function.

In fact the quaternionic Poisson's equation represents two separate equations:

$$(\nabla_0{}^2 - \mathbf{\nabla}^2)\varphi_0(q) = -\frac{\rho_0(q)}{\varepsilon} \tag{5}$$

$$(\nabla_0{}^2 - \mathbf{\nabla}^2)\boldsymbol{\varphi}(q) = -\frac{\boldsymbol{\rho}(q)}{\varepsilon} \tag{6}$$

Note that, for $|q|$ much greater than σ, the erf function approaches unity and the potential ϕ (r) approaches the point charge potential $\frac{Q}{4\pi\varepsilon|q|}$, as one would expect. Furthermore the erf function approaches 1 extremely quickly as its argument increases; in practice for $|q| > 3\sigma$ the relative error is smaller than one part in a thousand[124].

The definition of the quaternionic potential $\phi(q)$ is based on the convolution of a quaternionic distribution $\rho(q)$ with the real function $\varphi(q)$ See Newton potential and Bertrand's theorem in Wikipedia. The real part $\rho_0(q)$ of the distribution $\rho(q)$ can be interpreted as a charge distribution. The imaginary part $\boldsymbol{\rho}(q)$ can be interpreted as a current distribution. The convolution blurs the distribution such that the result becomes differentiable.

In configuration space holds:

$$\phi(q) = \rho(q) \circ \frac{1}{|q|}. \tag{7}$$

Reversely, according to Poisson's equation:

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[124] http://en.wikipedia.org/wiki/Poisson's_equation#Potential_of_a_Gaussian_charge_density

$$\rho(q) = = -\Delta\phi(q) \tag{8}$$

The real part of $\phi(q)$ presents a scalar potential. The imaginary part presents a vector potential.

$$\phi(q) = \phi_0(q) + \boldsymbol{\phi}(q) \tag{9}$$

In the above section:

The scalar potential is a blurred charge distribution.
The vector potential is a blurred current distribution.
Current is moving charge.
Mass is a form of charge.

(The selected blurring function has striking resemblance with the ground state of the quantum harmonic oscillator[125]).

In Fourier space holds:

$$\tilde{\phi}(p) = \tilde{\rho}(p) \cdot \frac{1}{|p|} . = \tilde{\phi}_0(p) + \tilde{\boldsymbol{\phi}}(p) \tag{10}$$

In Fourier space the frequency spectrum of the Hilbert distribution is multiplied with the Fourier transform of the blurring function. When this falls off when the frequencies go to infinity, then as a consequence the frequency spectrum of the potential is bounded. This is valid independent of the fact that the frequency spectrum of the Hilbert distribution is unbounded.

Equations of motion
The equation for the conservation of charge:

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[125] Functions and fields:Functions invariant under Fourier transformation:Ladder operator:Ground state

$$s_0(q) = \nabla_0 \rho_0(q) \mp \langle \nabla, \boldsymbol{\rho}(q) \rangle \tag{11}$$

We can define $\mathfrak{F}(q)$:

$$\mathfrak{F}(q) \overset{\text{def}}{=} \nabla \rho(q) \tag{12}$$

$$\mathfrak{F}_0(q) = \nabla_0 \rho_0(q) \mp \langle \nabla, \boldsymbol{\rho}(q) \rangle \tag{13}$$

$$\boldsymbol{\mathfrak{F}}(q) = \nabla \rho_0(q) \pm \nabla_0 \boldsymbol{\rho}(q) \pm \nabla \times \boldsymbol{\rho}(q) = \boldsymbol{\mathfrak{E}}(q) + \boldsymbol{\mathfrak{B}}(q) \tag{14}$$

$$\boldsymbol{\mathfrak{E}}(q) = -\nabla \rho_0(q) \mp \nabla_0 \boldsymbol{\rho}(q) \tag{15}$$

$$\boldsymbol{\mathfrak{B}}(q) = \pm \nabla \times \boldsymbol{\rho}(q) \tag{16}$$

The definition of $\boldsymbol{\mathfrak{B}}(q)$ and $\boldsymbol{\mathfrak{E}}(q)$ have the freedom of the [gauge transform](#)[126]

$$\boldsymbol{\rho}(q) \mapsto \boldsymbol{\rho}(q) + \nabla \phi_0 \tag{17}$$

$$\boldsymbol{\mathfrak{E}}(q) \mapsto \boldsymbol{\mathfrak{E}}(q) - \nabla \left(\nabla_0 \phi_0(q) \right) \tag{18}$$

$$\nabla^2 \phi_0 = \nabla_0^2 \phi_0 \tag{19}$$

This translates in the source free case $s_0(q) = 0$ into:

$$\nabla_0 \rho_0(q) = \pm \langle \nabla, \boldsymbol{\rho}(q) \rangle \tag{20}$$

$$\mathfrak{F}_0(q) = \nabla_0 \rho_0(q) \mp \langle \nabla, \boldsymbol{\rho}(q) \rangle = 0 \tag{21}$$

In the source divergence free case $\nabla s_0(q) = 0$ this means:

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[126] http://en.wikipedia.org/wiki/Gauge_fixing

$$\nabla_0 \boldsymbol{\nabla} \rho_0(q) = \pm \boldsymbol{\nabla} \langle \boldsymbol{\nabla}, \boldsymbol{\rho}(q) \rangle \tag{22}$$

$$\nabla_0 \boldsymbol{\nabla} \phi_0(q) = \pm \boldsymbol{\nabla} \langle \boldsymbol{\nabla}, \boldsymbol{\phi}(q) \rangle \tag{23}$$

$$\boldsymbol{\nabla} \langle \boldsymbol{\nabla}, \boldsymbol{\phi}(q) \rangle = \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{\phi}(q) + \nabla^2 \boldsymbol{\phi}(q) \tag{24}$$

Due to the fact that there are other charges present, the divergence of the scalar potential need be in the direction of the current $\rho(q)$, which for a spherical symmetric blur is also in the direction of the vector potential $\boldsymbol{\phi}(q)$. However, a tendency exists to minimize that difference. Thus $\nabla_0 \boldsymbol{\nabla} \phi_0(q)$ is parallel to $\boldsymbol{\phi}(q)$. With other words:

$$\boldsymbol{\phi}(q) \times \boldsymbol{\nabla} \langle \boldsymbol{\nabla}, \boldsymbol{\phi}(q) \rangle = 0 \tag{25}$$

Reckoning the sign selections for the sign $\pm$ of the conjugation and the handedness $\pm$ of the cross product will provide four different sets of equations. This will provide four different Hilbert fields.

Discrete distribution
If $\rho(q)$ is discrete, such that

$$\rho(q) = \Sigma_i \, q_{E_i} \cdot \delta(q - q_i) \tag{1}$$

where q'_E is a point charge at location q', then the contribution to the field $E(q)$ that is generated by a point charge at location q_i is given by:

$$d\boldsymbol{E}(q) = q_{E_i} \cdot \frac{q_i - q}{|q_i - q|^3} = -q_{E_i} \cdot \boldsymbol{\nabla} \cdot \frac{1}{|q_i - q|} \tag{2}$$

Differential potential equations
The gradient and curl of $\phi(q)$ are related. In configuration space holds:

$$\nabla \phi(q) = \nabla_0 \phi_0(q) \mp \langle \boldsymbol{\nabla}, \boldsymbol{\phi}(q) \rangle \pm \nabla_0 \boldsymbol{\phi}(q) \pm \boldsymbol{\nabla} \phi_0(q) \pm \left(\pm \boldsymbol{\nabla} \times \boldsymbol{\phi}(q) \right) \tag{1}$$

$$\mathfrak{E}(q) \stackrel{\text{def}}{=} -\boldsymbol{\nabla} \phi_0(q) \tag{2}$$

$$\mathcal{B}(q) \overset{\text{def}}{=} \nabla \times \phi(q) \tag{3}$$

$$\mathfrak{F}(q) \overset{\text{def}}{=} \nabla \phi(q) = \mathfrak{F}_0(q) \mp \mathfrak{E}(q) \pm \mathcal{B}(q) \pm \nabla_0 \boldsymbol{\phi}(q) \tag{4}$$

$$\mathfrak{F}_0(q) = \nabla_0 \phi_0(q) \mp \langle \nabla, \boldsymbol{\phi}(q) \rangle \tag{5}$$

$$\mathfrak{F}(q) = \mp \mathfrak{E}(q) \pm \mathcal{B}(q) \pm \nabla_0 \boldsymbol{\phi}(q) \tag{6}$$

When the field $\phi(q)$ is split into a private field $\phi_p(q)$ and a background field $\phi_b(q)$, then $\boldsymbol{\phi_p}(q)$ corresponds to the private field of the uniform moving item. When this item accelerates, then it goes together with an extra term $\nabla_0 \boldsymbol{\phi_p}(q)$. This is the reason of existence of inertia[127].

$$\langle \nabla, \mathfrak{E}(q) \rangle = -\nabla^2 \phi_0(q) = \rho_0(q) \tag{7}$$

$$\nabla \times \mathfrak{E}(q) = 0; \text{ Rotation free field} \tag{8}$$

$$\langle \nabla, \mathcal{B}(q) \rangle = 0; \text{ Divergence free } B \text{ field} \tag{9}$$

$$\nabla \times \mathcal{B}(q) = \nabla \langle \nabla, \boldsymbol{\phi}(q) \rangle - \nabla^2 \boldsymbol{\phi}(q) = \nabla \langle \nabla, \boldsymbol{\phi}(q) \rangle + \rho(q) + \nabla_0{}^2 \boldsymbol{\phi}(q) \tag{10}$$

$$\nabla \times \mathcal{B}(q) = \pm \nabla_0 \nabla \phi_0(q) + \rho(q) + \nabla_0{}^2 \boldsymbol{\phi}(q) \tag{11}$$

$$= \pm \nabla_0 \mathfrak{E}(q) + \rho(q) + \nabla_0{}^2 \boldsymbol{\phi}(q)$$

Since $\nabla_0 \boldsymbol{\phi}(q)$ is supposed to be parallel to $\nabla \phi_0(q)$, it is sensible to define $\boldsymbol{E}(q)$ as the total field in longitudinal direction:

$$\boldsymbol{E}(q) = -\nabla \phi_0(q) - \nabla_0 \boldsymbol{\phi}(q) = \mathfrak{E}(q) - \nabla_0 \boldsymbol{\phi}(q) \tag{12}$$

And

$$\boldsymbol{B}(q) = \mathcal{B}(q) \tag{13}$$

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[127] Influence; Inertia

With this definition:

$$\nabla \times \boldsymbol{E}(q) = -\nabla_0 \boldsymbol{B}(q) \tag{14}$$

$$\langle \nabla, \boldsymbol{B}(q) \rangle = 0 \tag{15}$$

$$\nabla \times \boldsymbol{B}(q) = \boldsymbol{\rho}(q) + \nabla_0 \boldsymbol{E}(q) \tag{16}$$

In Fourier space

In Fourier space holds:

$$\widetilde{\mathfrak{F}}(p) = p_0 \tilde{\phi}_0(p) - \langle \boldsymbol{p}, \tilde{\boldsymbol{\phi}}(p) \rangle \pm p_0 \tilde{\boldsymbol{\phi}}(p) \pm \boldsymbol{p}\tilde{\phi}_0(p) \pm \boldsymbol{p} \times \tilde{\boldsymbol{\phi}}(p) \tag{1}$$

$$\widetilde{\mathfrak{F}}(p) = p\tilde{\phi}(p) = \mathfrak{F}_0(p) \mp \widetilde{\mathfrak{E}}(p) \pm \widetilde{\mathfrak{B}}(p) \pm p_0\tilde{\boldsymbol{\phi}}(p) \tag{2}$$

$$\widetilde{\mathfrak{F}}_0(p) = p_0 \tilde{\phi}_0(p) - \langle \boldsymbol{p}, \tilde{\boldsymbol{\phi}}(p) \rangle \tag{3}$$

$$\widetilde{\mathfrak{E}}(p) = -\boldsymbol{p}\tilde{\phi}_0(p) \tag{4}$$

$$\widetilde{\boldsymbol{E}}(p) = -\boldsymbol{p}\tilde{\phi}_0(p) \pm p_0\tilde{\boldsymbol{\phi}}(p)$$

$$\widetilde{\mathfrak{B}}(p) = \boldsymbol{p} \times \tilde{\boldsymbol{\phi}}(p) \tag{5}$$

$$\widetilde{\mathfrak{F}}(p) = \mp\widetilde{\mathfrak{E}}(p) \pm \widetilde{\mathfrak{B}}(p) \pm p_0\tilde{\boldsymbol{\phi}}(p) \tag{6}$$

$$\langle \boldsymbol{p}, \widetilde{\mathfrak{E}}(p) \rangle = -\boldsymbol{p}^2 \tilde{\phi}_0(p) = \tilde{\rho}_0(p) \tag{7}$$

$$\boldsymbol{p} \times \widetilde{\mathfrak{E}}(p) = 0; \text{Rotation free field} \tag{8}$$

$$\langle \boldsymbol{p}, \widetilde{\mathfrak{B}}(p) \rangle = 0; \text{Divergence free } B \text{ field} \tag{9}$$

$$\boldsymbol{p} \times \widetilde{\mathfrak{B}}(p) = \boldsymbol{p}\langle \boldsymbol{p}, \tilde{\boldsymbol{\phi}}(q) \rangle - \boldsymbol{p}^2\tilde{\boldsymbol{\phi}}(q) = \boldsymbol{p}\langle \boldsymbol{p}, \tilde{\boldsymbol{\phi}}(p) \rangle + \tilde{\boldsymbol{\rho}}(p) \tag{10}$$

$$p \times \tilde{\mathfrak{B}}(p) = \pm p_0 p \tilde{\phi}_0(p) + \tilde{\rho}(p) = \pm p_0 \tilde{\mathfrak{E}}(p) + \tilde{\rho}(p) \tag{11}$$

If the distribution $\rho(q)$ is differentiable, then the same equations that hold for fields $\phi(q)$ and $\tilde{\phi}(p)$ hold for the non-blurred distributions $\rho(q)$ and $\tilde{\rho}(q)$.

Maxwell equations

First it must be noted that the above derived field equations hold for general quaternionic fields.
The resemblance with physical fields holds for electromagnetic fields as well as for gravitational fields and for any fields whose blurring function approximates

$$f(q) \approx \frac{1}{|q|}.$$

In Maxwell equations, $E(\mathbf{r})$ is defined as:

$$E(\mathbf{r}, t) \equiv -\nabla \phi_0(\mathbf{r}, t) - \frac{\partial A(\mathbf{r}, t)}{\partial t} = \mathfrak{E}(\mathbf{r}, t) - \frac{\partial A(\mathbf{r}, t)}{\partial t}$$

Further:

$$\langle \nabla, E(\mathbf{r}, t) \rangle = -\nabla^2 \phi_0(\mathbf{r}, t) - \frac{\partial \langle \nabla, A(\mathbf{r}, t) \rangle}{\partial t}$$

$$= \frac{\rho_0(\mathbf{r}, t)}{\varepsilon_0} - \frac{\partial \langle \nabla, A(\mathbf{r}, t) \rangle}{\partial t}$$

In Maxwell equations, $B(\mathbf{r})$ is defined as:

$$B(\mathbf{r}, t) \equiv \nabla \times A(\mathbf{r}, t) = \mathfrak{B}(\mathbf{r}, t)$$

Further:

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$$\nabla \times \boldsymbol{E}(\boldsymbol{r},t) = -\frac{\partial \boldsymbol{B}(\boldsymbol{r},t)}{\partial t}$$

$$\langle \nabla, \boldsymbol{B}(\boldsymbol{r},t) \rangle = 0$$

$$\nabla \times \boldsymbol{B}(\boldsymbol{r},t) = \mu_0 (\boldsymbol{j} + \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t})$$

Differentiable distribution

If the distribution $\rho(q)$ is differentiable, then the same equations that hold for fields $\phi(q)$ and $\tilde{\phi}(p)$ hold for the non-blurred distributions $\rho(q)$ and $\tilde{\rho}(q)$.

Using:

$$\boldsymbol{B} = \nabla \times \boldsymbol{\phi} = \boldsymbol{i}(\nabla_2 \phi_{\parallel} - \nabla_{\parallel}\phi_2) + \boldsymbol{j}(\nabla_{\parallel}\phi_1 - \nabla_1\phi_{\parallel}) + \boldsymbol{k}(\nabla_1\phi_2 - \nabla_2\phi_1) \tag{1}$$

gives

$$\nabla_0\phi_{\parallel}(q) = \mp\nabla_{\parallel}\phi_0(q) \tag{2}$$

$$\nabla_0\phi_1(q) = \mp\left(\nabla_2\phi_{\parallel}(q) - \nabla_{\parallel}\phi_2(q)\right) \tag{3}$$

$$\nabla_0\phi_2(q) = \pm\left(\nabla_1\phi_{\parallel}(q) - \nabla_{\parallel}\phi_1(q)\right) \tag{4}$$

$$\nabla_0\phi_0(q) = \langle\nabla, \boldsymbol{\phi}(q)\rangle = \nabla_{\parallel}\phi_{\parallel}(q) + \nabla_1\phi_1(q) + \nabla_2\phi_2(q) \tag{5}$$

And correspondingly in Fourier space

$$p_0\tilde{\phi}_{\parallel}(p) = \mp p_{\parallel}\tilde{\phi}_0(p) \tag{6}$$

$$p_0\tilde{\phi}_1(p) = \pm\left(p_{\parallel}\tilde{\phi}_2(p) - p_2\tilde{\phi}_{\parallel}(p)\right) \tag{7}$$

$$p_0\tilde{\phi}_2(p) = \mp\left(p_{\parallel}\tilde{\phi}_1(p) - p_1\tilde{\phi}_{\parallel}(p)\right) \tag{8}$$

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$$p_0 \tilde{\phi}_0(p) = \langle \boldsymbol{p}, \tilde{\boldsymbol{\phi}}(p) \rangle = p_{\parallel} \tilde{\phi}_{\parallel}(p) + p_1 \tilde{\phi}_1(p) + p_2 \tilde{\phi}_2(p) \tag{9}$$

The origin of mass

Conservation laws

Flux vector

The longitudinal direction $\mathbf{k}$ of $\boldsymbol{E}(q)$ and the direction $\mathbf{i}$ of $\boldsymbol{B}(q)$ fix two mutual perpendicular directions. This generates curiosity to the significance of the direction $\mathbf{k} \times \mathbf{i}$. With other words what happens with $\boldsymbol{E}(q) \times \boldsymbol{B}(q)$.

The **flux vector** $\mathfrak{S}(q)$ is defined as:

$$\mathfrak{S}(q) \overset{\text{def}}{=} \boldsymbol{E}(q) \times \boldsymbol{B}(q) \tag{1}$$

Conservation of energy

Field energy density

$$\langle \boldsymbol{\nabla}, \mathfrak{S}(q) \rangle = \langle \boldsymbol{B}(q), \boldsymbol{\nabla} \times \boldsymbol{E}(q) \rangle - \langle \boldsymbol{E}(q), \boldsymbol{\nabla} \times \boldsymbol{B}(q) \rangle \tag{1}$$

$$= -\langle \boldsymbol{B}(q), \nabla_0 \boldsymbol{B}(q) \rangle - \langle \boldsymbol{E}(q), \boldsymbol{\phi}(q) \rangle - \langle \boldsymbol{E}(q), \nabla_0 \boldsymbol{E}(q) \rangle$$

$$= -\tfrac{1}{2} \nabla_0 (\langle \boldsymbol{B}(q), \boldsymbol{B}(q) \rangle + \langle \boldsymbol{E}(q), \boldsymbol{E}(q) \rangle) - \langle \boldsymbol{E}(q), \boldsymbol{\phi}(q) \rangle$$

The **field energy density** is defined as:

$$u_{field}(q) = \tfrac{1}{2}(\langle \boldsymbol{B}(q), \boldsymbol{B}(q) \rangle + \langle \boldsymbol{E}(q), \boldsymbol{E}(q) \rangle) = u_B(q) + u_E(q) \tag{2}$$

$\mathfrak{S}(q)$ can be interpreted as the **field energy current density**.
The continuity equation for field energy density is given by:

$$\nabla_0 u_{field}(q) + \langle \boldsymbol{\nabla}, \mathfrak{S}(q) \rangle = -\langle \boldsymbol{E}(q), \boldsymbol{\phi}(q) \rangle = -\phi_0(q) \langle \boldsymbol{E}(q), \boldsymbol{v}(q) \rangle \tag{3}$$

This means that $\langle \boldsymbol{E}(q), \boldsymbol{\phi}(q) \rangle$ can be interpreted as a source term.
$\phi_0(q)\boldsymbol{E}(q)$ represents **force** per unit volume.
$\phi_0(q)\langle \boldsymbol{E}(q), \boldsymbol{v}(q) \rangle$ represents **work** per unit volume, or, in other words, the power density. It is known as the Lorentz power density and is equivalent to the time rate of change of the mechanical energy density of the charged particles that form the current $\boldsymbol{\phi}(q)$.

$$\nabla_0 u_{field}(q) + \langle \nabla, \mathfrak{S}(q) \rangle = -\nabla_0 u_{mechanical}(q) \tag{4}$$

$$\nabla_0 u_{mechanical} = \langle \boldsymbol{E}(q), \boldsymbol{\phi}(q) \rangle = \phi_0(q)\langle \boldsymbol{E}(q), \boldsymbol{v}(q) \rangle \tag{5}$$

$$\nabla_0 \left(u_{field}(q) + u_{mechanical}(q) \right) = -\langle \nabla, \mathfrak{S}(q) \rangle \tag{6}$$

$$\text{Total change within } V = \text{flow into } V + \text{production inside } V \tag{7}$$

$$u(q) = u_{field}(q) + u_{mechanical}(q) = u_B(q) + u_E(q) + u_{mechanical}(q) \tag{8}$$

$$U = U_{field} + U_{mechanical} = U_B + U_E + U_{mechanical} = \int_V u \, dV \tag{9}$$

$$\frac{d}{dt} \int_V u \, dV = \oint_S \langle \hat{\boldsymbol{n}}, \mathfrak{S} \rangle dS + \int_V s_0 \, dV \tag{10}$$

Here the source s_0 is zero.

How to interprete $U_{mechanical}$
$U_{mechanical}$ is the energy of the private field (wave function) of the involved particle(s).

Conservation of linear momentum

Field linear momentum
$\mathfrak{S}(q)$ can also be interpreted as the **field linear momentum density**. The time rate change of the field linear momentum density is:

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$$\nabla_0 \mathfrak{S}(q) = \boldsymbol{g}_{field}(q) = \nabla_0 \, \boldsymbol{E}(q) \times \boldsymbol{B}(q) + \boldsymbol{E}(q) \times \nabla_0 \boldsymbol{B}(q) \tag{1}$$

$$= \big(\nabla \times \boldsymbol{B}(q) - \boldsymbol{\rho}(q)\big) \times \boldsymbol{B}(q) - \boldsymbol{E}(q) \times \nabla \times \boldsymbol{E}(q) \tag{2}$$

$$\boldsymbol{G}(\boldsymbol{E}) = \boldsymbol{E} \times (\nabla \times \boldsymbol{E}) = \langle \nabla \boldsymbol{E}, \boldsymbol{E} \rangle - \langle \boldsymbol{E}, \boldsymbol{E} \rangle = \tfrac{1}{2}\nabla\langle \boldsymbol{E}, \boldsymbol{E} \rangle - \langle \boldsymbol{E}, \boldsymbol{E} \rangle \tag{3}$$

$$= -\nabla(\mathbf{EE}) + \tfrac{1}{2}\nabla\langle \boldsymbol{E}, \boldsymbol{E} \rangle + \langle \nabla, \boldsymbol{E} \rangle \boldsymbol{E}$$

$$= -\nabla(\mathbf{EE} + \tfrac{1}{2}\mathbf{1_3}\langle \boldsymbol{E}, \boldsymbol{E} \rangle) + \langle \nabla, \boldsymbol{E} \rangle \boldsymbol{E}$$

$$\boldsymbol{G}(\boldsymbol{B}) = \boldsymbol{B} \times (\nabla \times \boldsymbol{B}) = -\nabla(\mathbf{BB} + \tfrac{1}{2}\mathbf{1_3}\langle \boldsymbol{B}, \boldsymbol{B} \rangle) + \langle \nabla, \boldsymbol{B} \rangle \boldsymbol{B} \tag{4}$$

$$\boldsymbol{H}(\boldsymbol{B}) = -\nabla(\mathbf{BB} + \tfrac{1}{2}\mathbf{1_3}\langle \boldsymbol{B}, \boldsymbol{B} \rangle) \tag{5}$$

$$\nabla_0 \mathfrak{S}(q) = \boldsymbol{G}(\boldsymbol{B}) + \boldsymbol{G}(\boldsymbol{E}) - \boldsymbol{\rho}(q) \times \boldsymbol{B}(q) \tag{6}$$

$$= \boldsymbol{H}(\boldsymbol{E}) + \boldsymbol{H}(\boldsymbol{B}) - \boldsymbol{\rho}(q) \times \boldsymbol{B}(q) + \langle \nabla, \boldsymbol{B} \rangle \boldsymbol{B} + \langle \nabla, \boldsymbol{E} \rangle \boldsymbol{E}$$

$$= \boldsymbol{H}(\boldsymbol{E}) + \boldsymbol{H}(\boldsymbol{B}) - \boldsymbol{\rho}(q) \times \boldsymbol{B}(q) - \rho_0(q) \, \boldsymbol{E}(q)$$

$$= \boldsymbol{H}(\boldsymbol{E}) + \boldsymbol{H}(\boldsymbol{B}) - \boldsymbol{f}(q) = \mathcal{T}(q) - \boldsymbol{f}(q)$$

$\mathcal{T}(q)$ is the linear momentum flux tensor.
The linear momentum of the field contained in volume V surrounded by surface S is:

$$\boldsymbol{P}_{field} = \int_V \boldsymbol{g}_{field} \, dV = \int_V \rho_0 \boldsymbol{\phi} \, dV + \int_V \langle \nabla\boldsymbol{\phi}, \boldsymbol{E} \rangle \, dV + \oint_S \langle \hat{\boldsymbol{n}}, \boldsymbol{EA} \rangle dS \tag{7}$$

$$\boldsymbol{f}(q) = \boldsymbol{\rho}(q) \times \boldsymbol{B}(q) + \rho_0(q) \, \boldsymbol{E}(q) \tag{8}$$

Physically, $\boldsymbol{f}(q)$ is the Lorentz force density. It equals the time rate change of the mechanical linear momentum density $\boldsymbol{g}_{mechanical}$.

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$$\boldsymbol{g}_{mechanical}(q) = \rho_{0m}(q)\boldsymbol{v}(q) \tag{9}$$

The force acted upon a single particle that is contained in a volume V is:

$$\boldsymbol{F} = \int_V \boldsymbol{f} \, dV = \int_V (\boldsymbol{\rho} \times \boldsymbol{B} + \rho_0 \, \boldsymbol{E}) \, dV \tag{10}$$

Brought together this gives:

$$\nabla_0 \left(\boldsymbol{g}_{field}(q) + \boldsymbol{g}_{mechanical}(q) \right) = -\langle \nabla, \boldsymbol{\mathcal{T}}(q) \rangle \tag{11}$$

This is the continuity equation for linear momentum.
The component $\mathcal{T}_{ij}$ is the linear momentum in the i-th direction that passes a surface element in the j-th direction per unit time, per unit area.

$$\text{Total change within } V = \text{flow into } V + \text{production inside } V \tag{12}$$

$$\boldsymbol{g}(q) = \boldsymbol{g}_{field}(q) + \boldsymbol{g}_{mechanical}(q) \tag{13}$$

$$\boldsymbol{P} = \boldsymbol{P}_{field} + \boldsymbol{P}_{mechanical} = \int_V \boldsymbol{g} \, dV \tag{14}$$

$$\frac{d}{dt} \int_V \boldsymbol{g} \, dV = \oint_S \langle \hat{\boldsymbol{n}}, \boldsymbol{\mathcal{T}} \rangle dS + \int_V \boldsymbol{s}_g \, dV \tag{15}$$

Here the source $s_g = 0$.

Conservation of angular momentum

Field angular momentum
The angular momentum relates to the linear momentum.

$$\boldsymbol{h}(\boldsymbol{q}_c) = (\boldsymbol{q} - \boldsymbol{q}_c) \times \boldsymbol{g}(q) \tag{1}$$

$$\boldsymbol{h}_{field}(\boldsymbol{q}_c) = (\boldsymbol{q} - \boldsymbol{q}_c) \times \boldsymbol{g}_{field}(q) \tag{2}$$

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$$h_{mechanical}(q) = (q - q_c) \times g_{mechanical}(q) \tag{3}$$

$$\mathcal{K}(q_c) = (q - q_c) \times \mathcal{T}(q) \tag{4}$$

This enables the balance equation for angular momentum:

$$\nabla_0 \left(h_{field}(q_c) + h_{mechanical}(q_c) \right) = -\langle \nabla, \mathcal{K}(q_c) \rangle \tag{5}$$

Total change within V = flow into V + production inside V

$$J = J_{field} + J_{mechanical} = \int_V h \, dV \tag{6}$$

$$\frac{d}{dt} \int_V h \, dV = \oint_S \langle \hat{n}, \mathcal{K} \rangle dS + \int_V s_h \, dV \tag{7}$$

Here the source $s_h = 0$.

For a localized charge density contained within a volume V holds for the mechanical torsion:

$$\tau(q_c) = \int_V (q' - q_c) \times f(q') dV \tag{8}$$

$$= \int_V (q' - q_c) \times \left(\rho_0(q')E(q') + j(q') \times B(q') \right) dV$$

$$= Q(q - q_c) \times (E(q) + v(q) \times B(q))$$

$$J_{field}(q_c) = J_{field}(0) + q_c \times P(q) \tag{9}$$

Using

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$$\langle \boldsymbol{\nabla a}, \boldsymbol{b} \rangle = n_\nu \frac{\partial a_\mu}{\partial q_\nu} b_\mu \tag{10}$$

$$\langle \boldsymbol{b}, \boldsymbol{\nabla a} \rangle = n_\mu \frac{\partial a_\mu}{\partial q_\nu} b_\mu \tag{11}$$

holds

$$\boldsymbol{J}_{field}(\boldsymbol{0}) = \int_V \boldsymbol{q}' \times \mathfrak{S}(q') dV = \int_V \boldsymbol{q}' \times \boldsymbol{E}(q') \times \boldsymbol{\nabla} \times \boldsymbol{\phi}(q') \, dV \tag{12}$$

$$= \int_V (\boldsymbol{q}' \times \langle (\boldsymbol{\nabla\phi}), \boldsymbol{E} \rangle - \langle \boldsymbol{q}' \times \boldsymbol{E}, (\boldsymbol{\nabla\phi}) \rangle) \, dV$$

$$= \int_V \boldsymbol{q}' \times \langle (\boldsymbol{\nabla\phi}), \boldsymbol{E} \rangle dV$$

$$+ \int_V \boldsymbol{E} \times \boldsymbol{\phi} \, dV - \int_V \langle \boldsymbol{\nabla}, \boldsymbol{E} \boldsymbol{q}' \times \boldsymbol{\phi} \rangle dV$$

$$+ \int_V (\boldsymbol{q}' \times \boldsymbol{\phi}) \langle \boldsymbol{\nabla}, \boldsymbol{E} \rangle dV$$

Spin

Define the non-local spin term, *which does not depend on $\boldsymbol{q}'$* as:

$$\boldsymbol{\Sigma}_{field} = \int_V \boldsymbol{E}(q) \times \boldsymbol{\phi}(q) dV \tag{13}$$

Notice

$$\boldsymbol{\phi}(q) \times \boldsymbol{\nabla} \phi_0(q) = \phi_0 \boldsymbol{\nabla} \times \boldsymbol{\phi}(q) + \boldsymbol{\nabla} \times (\phi_0(q) \boldsymbol{\phi}(q))$$

And

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$$L_{field}(\mathbf{0}) = \int_V \mathbf{q}' \times \langle (\nabla \boldsymbol{\phi}), \mathbf{E} \rangle dV + \int_V \mathbf{q}' \times \rho_0 \boldsymbol{\phi} dV \tag{14}$$

Using Gauss:

$$\int_V \langle \nabla, \mathbf{a} \rangle dV = \oint_S \langle \hat{\mathbf{n}}, \mathbf{a} \rangle dS \tag{15}$$

And

$$\rho_0 = \langle \nabla, \mathbf{E} \rangle \tag{16}$$

Leads to:

$$\mathbf{J}_{field}(\mathbf{0}) = \boldsymbol{\Sigma}_{field} + \mathbf{L}_{field}(\mathbf{0}) + \oint_S \langle \hat{\mathbf{n}}, \mathbf{E}\mathbf{q}' \times \boldsymbol{\phi} \rangle dS \tag{17}$$

Spin discussion

The spin term is defined by:

$$\boldsymbol{\Sigma}_{field} = \int_V \mathbf{E}(q) \times \boldsymbol{\phi}(q) dV \tag{1}$$

In free space the charge density ρ_0 vanishes and the scalar potential ϕ_0 shows no variance. Only the vector potential $\boldsymbol{\phi}$ may vary with q_0. Thus:

$$\mathbf{E} = \nabla \phi_0 - \nabla_0 \boldsymbol{\phi} \approx -\nabla_0 \boldsymbol{\phi} \tag{2}$$

$$\boldsymbol{\Sigma}_{field} \approx \int_V (\nabla_0 \boldsymbol{\phi}(q)) \times \boldsymbol{\phi}(q) dV \tag{3}$$

Depending on the selected field Σ_{field} has two versions that differ in their sign. These versions can be combined in a single operator:

$$\boldsymbol{\Sigma}_{field} = \begin{bmatrix} \boldsymbol{\Sigma}^+{}_{field} \\ \boldsymbol{\Sigma}^-{}_{field} \end{bmatrix} \tag{4}$$

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If $\frac{\phi(q)}{|\phi(q)|}$ can be interpreted as tantrix (q_0)) and $\frac{\nabla_0\phi(q)}{|\nabla_0\phi(q)|}$ can be interpreted as the principle normal $\boldsymbol{N}(q_0)$, then $\frac{(\nabla_0\phi(q))\times\phi(q)}{|(\nabla_0\phi(q))\times\phi(q)|}$ can be interpreted as the binormal $\boldsymbol{B}(q_0)$.

From these quantities the <u>curvature and the torsion</u>[128] can be derived.

$$\begin{bmatrix} \dot{\boldsymbol{T}}(t) \\ \dot{\boldsymbol{N}}(t) \\ \dot{\boldsymbol{B}}(t) \end{bmatrix} = \begin{bmatrix} 0 & \kappa(t) & 0 \\ -\kappa(t) & 0 & \tau(t) \\ 0 & -\tau(t) & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{T}(t) \\ \boldsymbol{N}(t) \\ \boldsymbol{B}(t) \end{bmatrix} \tag{5}$$

States

Where a unique closed Hilbert subspace *represents* a given physical item, its **state** *characterizes* the probabilistic properties of that item. In quantum physics, a quantum state is a set of mathematical variables that as far as is possible describes the corresponding physical item. For example, the set of 4 numbers $\{n, l, m_l, m_s\}$ define part of the state of an electron within a hydrogen atom and are known as the electron's quantum numbers. The observables that determine the state are mutually compatible. The position of the electron within the atom is a hidden property. If two operators are each other's canonical conjugate, then only one of them can participate in the state, or the state must contain an account of the combination of both values. An example of such a combination is the ladder operator.

Quantum states can be either pure or mixed. **Pure states** cannot be described as a mixture of others. **Mixed states** correspond to a random process that blends pure states together. Realizations of elementary types are characterized by pure states.

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[128]Path characteristics

This indicates that the notion of state is closely related to the basic constituents of Hilbert fields. Stated in other words:

> The QPAD that represents the private field of an elementary particle also represents the state of that particle.
> The properties of the field are also the properties of the state.

In relation to the concepts defined earlier, a pure state corresponds to the blur of an elementary Hilbert distribution, while the blur itself can be interpreted as a QPAD. The state then also corresponds to the squared modulus of this QPAD, which is a probability density function. The blur is defined with respect to a background coordinate system. This coordinate system is delivered by a [GPS operator](#)[129] that resides in rigged Hilbert space. For example the operators $\breve{Q}$ and $\breve{Q}$ suit that purpose, or the coordinate system is represented by the canonical conjugate, the GMS operator that corresponds to $\breve{\mathcal{P}}$ or $\breve{P}$. With other words, several different states correspond with the same physical item. The position of the considered item is determined by the position of the anchor points of the QPAD. These anchor points correspond to eigenvectors of the strand operator. The eigenvalues of this strand operator are coupled to the background coordinate system. This coupling is inaccurate. The QPAD reflects the inaccuracy.

When performing an observation on a quantum state, the result is generally described by a probability distribution, and the form that this distribution takes is completely determined by the quantum state and the operators that are related to the observation of the quantum state. The result of an observation is only determined probabilistically. Even when the observed quantity is quantized it still can take a range of values, each with a corresponding probability. In relation to the observables that determine the state, a pure state is characterized by the blur of [a single elementary Hilbert distribution](#) and that blurred distribution corresponds

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[129]Hilbert spaces; Generating a Hilbert space GPS

in relation to these observables to a mathematical object known as a [wave function](). If another observable concerns the canonical conjugate, then the corresponding Fourier transform of the wave function must be considered.

The result of a "sharp" observation equals one of the eigenvalues that corresponds to the set of eigenvectors over which the probability distribution is non-zero. It means that the observable must have the same eigenvectors as the operator that is used to define the wave function. The probability of getting this vector and the corresponding value is given by the probability density that corresponds to the wave function value. The probalistic nature of observations reflects a core difference between classical and quantum physics. The granularity of observables that are afflicted with Planck limits forbids that differences are measured with precise accuracy.

Linear combinations (superpositions) of states can describe interference phenomena. A mixed state cannot be characterized by a single blurred elementary Hilbert distribution. Instead, it is described by the associated [density operator]() of that mixed state. It is still represented by a (blurred) closed Hilbert subspace, but that is no longer the subspace that is spanned by a single elementary Hilbert distribution.

Pure states can be represented by a single blurred elementary Hilbert distribution.

State definition

A measure μ on the closed subspaces of a Hilbert space obeys the rule:

$$\mu(B) = \sum_{i=1}^{i=n} A_i, \text{ for each set } \{A_i\} \text{ of closed subspaces} \tag{1}$$

Each Hilbert vector $|v>$ generates a measure $\mu_v(A)$ via the projection P_A of $|v>$ on A

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$$\mu_v(A) = \|P_A(v)\|^2 \tag{2}$$

[Gleason's theorem](#) states:

Let $\mu(A)$ be a measure on the closed subspaces A of a separable Hilbert space **H**, with dimension ≥ 3, then there exists a positive definite self adjoint operator T of the trace class, such that

$$\mu(A) = trace(TP_A) \tag{3}$$

Given a state P on a space of dimension ≥ 3, there is an Hermitian, non-negative operator ρ on **H**, whose trace is unity, such that

$$\forall_{|x>\in H}\{P(x) = <x|\,\rho\,x>\}, \text{ where } x \text{ is a ray spanned by } |x> \tag{4}$$

With each compact normal operator Q corresponds an orthonormal base of eigenvectors $\{|q>\}_q$ with eigenvalues q. As a consequence a notion of state is attached to each physical item combined with one or more mutually compatible compact normal operators $\{Q_j\}_j$.

In Hilbert space a state, or probability function, is a real function P on the Hilbert subspaces, with the following properties:

1. $P(0) = 0$

2. $\forall_{y\in H}\{P(y) \geq 0\}$, y is a Hilbert subspace

3. $\sum_j P(x_j) = 1$, where $|x_j>$ form an orthonormal base of **H**, and x_j is the ray spanned by $|x_j>$

4. $P(y) = \sum_{j=1}^{r} P(x_j)$ where x_j are mutually orthogonal rays spanning subspace y

Pure state

In particular, if some ray x_0 satisfies $P(x_0) = 1$, then according to Born's rule:

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$$\forall_{|x>\in H}\{P(x) = |< x_0, x >|^2\} \tag{1}$$

This happens when $|x_0>$ represents an unary pure state.

Item state

The state $P(\{q_j\}_j)$ is connected to a **wave function** $\psi(\{q_j\}_j)$, where

$$P(\{q_j\}_j) = \left|\psi(\{q_j\}_j)\right|^2 \tag{1}$$

and $\{q_j\}_j$ are the eigenvalues of eigenvector $\{|q_j >\}$ of the operators $\{Q_j\}_j$. Two operators A and B are compatible when their commutator is zero:

$$[A, B] \overset{\text{def}}{=} AB - BA = 0 \tag{2}$$

If the state is characterized by a *set* of independent properties, then each of these properties corresponds with a corresponding operator. These operators must be normal, but they need not be compact. It must be possible to construct a spectral decomposition for each of the operators. Further, the operators that together determine the state must be mutually compatible. The wave function is then the product of the probability amplitudes that correspond to the separate operators. Thus the resulting wave function is a characteristic that represents the probability amplitudes of a set of mutually compatible observables that correspond to the normal operators that determine the state.

The squared modulus of the probability amplitude is the probability density. The wave function will also be a function of a progression parameter. Position can be a state characterizing observable. However, like the progression parameter, spacetime does not occur as an eigenvalue of a Hilbert space operator. The operators may vary. For example an operator may be replaced by its canonical conjugate. In that case, care must be taken that the operators that form the changed state are still

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compatible. Thus, even with the same physical item, the wave function is not unique.

For the operator Q with eigenfunctions |q> and eigenvalues q the probability amplitude function $\psi(q)$ is given by the smoothed version of $< \psi|q >$

$$\psi(q) \cong < \psi|q > \tag{3}$$

When Q is a [compact normal operator](#) then the smoothed version of $\psi(q)$ is a continuous function. In that case the eigenvalues of the corresponding operator $\breve{Q}$ that resides in rigged Hilbert space **H** are used. Then $\psi(q)$ has a Fourier transform $\phi(p)$, where the operator $\breve{P}$ with eigenvectors |p> and eigenvalues p is the canonical conjugate of $\breve{Q}$. Like $\psi(q)$, the function $\phi(p)$ is also a function that characterizes the corresponding item and |φ> is a characterizing vector. The parameters q and p may be quaternionic.

$$\varphi(p) = \tilde{\psi}(q) = < \varphi|p > \tag{4}$$

With respect to the correspondence with traditional quantum logic, it is wrong to take any characteristic vector including the locator or any function including the wave function as the *representative* for the item. It is ridiculous to expect that a single Hilbert vector carries all properties of a complex physical item, such as a DNA molecule or an elephant.

In usual quantum mechanics the wave function can be interpreted as the combination of a stationary vector and a progression operator. The progression operator has the form $A \cdot \exp(S/\hbar)$. This was introduced by Dirac. A is Hermitian and positive. S is anti-Hermitian. Both operators are a function of parameter t. This is reflected in the Hamilton-Jacobi equation.

In contemporary quantum field theory the fields replace the wave function. Thus a field may be interpreted as the amplitude of the

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probability to find something at the location of the field value. For bosons that something may be interpreted as a virtual particle. For fermions that something may be interpreted as a pair of virtual particles. Each type of virtual particle has its own type of field.

There are some questions left with wave functions:
- Can it have non-zero values outside the subspace that represents the physical item?
    - o Answer: Yes. The private field covers the whole Hilbert space.
- Is the wave function a regular function?
    - o Answer: When universe is restricted by an outer horizon, then the wave function is regular.
- What happens to the representing subspace and to the wave function when a measurement on a particle is performed?
    - o Answer:
        - When the coordinate space stays the same, then both the subspace and the wave function will not be affected. However, the measurement may affect the state of the particle.
        - When the coordinate space changes into the canonical conjugate, then the subspace changes to other base vectors and the wave function is Fourier transformed into a new form.
- Has a system a wave function?
    - o Answer: In general a system must not have a wave function, but it has a density operator.

Probability density
[Gleason's theorem](http://en.wikipedia.org/wiki/Quantum_logic#Statistical_structure)[130] states that a probability measure $\mu(P)$ on the lattice $L(\mathbf{H})$ of projections P on closed subspaces of a Hilbert space $\mathbf{H}$ corresponds to a non-negative Hermitian operator ρ with trace 1, such

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[130] http://en.wikipedia.org/wiki/Quantum_logic#Statistical_structure

that $\mu(P) = \text{tr}(\rho P)$. When the projections P_q correspond to the rays formed by the eigenvectors $|q\rangle$ of operator Q and $\mu_i(P_q)$ corresponds to the considered physical item, then $\mu_i(P_q) = \langle q, \rho_i q\rangle$ corresponds to the square of the modulus of the wave function $\psi_i(q)$. ρ_i is the [probability density operator](#)[131] corresponding to μ_i. The probability measure μ is a regular function.

The [probability density function](#)[132] $P(q) = |\psi(q)|^2$ of an absolutely continuous random variable q is a function that describes the relative chance for this random variable to occur at a given point in the Q observation space. The probability for a random variable to fall within a given set is given by the integral of its density over the set.

The probability [density operator](#)[133] ρ is positive-semi-definite ($\forall_{|f\rangle\in H}\{\langle f|\rho f\rangle \geq 0\}$), self-adjoint ($\rho = \rho^\dagger$), and has trace one ($\text{tr}(\rho) = 1$). For the operator Q with eigenfunctions $|q\rangle$ and eigenvalues q with probability amplitude $\psi(q)$, the density operator ρ is given by

$$\rho = \sum_q\{|\langle \psi|q\rangle|^2 \cdot |q\rangle\langle q|\} \tag{1}$$

[Von Neumann entropy](#)[134] is defined using the density operator of physical items.

The entropy $S(\varrho)$ describes the departure of the system from a pure state. In other words, it measures the degree of mixture ([entanglement](#)[135]) of the state $|\psi\rangle$.

The operator A can be decomposed

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[131] Functions and fields:characteristic functions

[132] http://en.wikipedia.org/wiki/Probability_density_function

[133] http://en.wikipedia.org/wiki/Density_operator

[134] http://en.wikipedia.org/wiki/Von_Neumann_entropy

[135] http://en.wikipedia.org/wiki/Quantum_entanglement

$$A = \sum_a |a> a < a| \qquad (2)$$

For the state $|\psi>$ the **expectation value** $\langle A \rangle$ for the observable A is

$$\langle A \rangle \stackrel{\text{def}}{=} < \psi | A\, \psi > = \sum_q \{|< \psi | q >|^2 \cdot < q | A q >\} = tr(\rho A) \qquad (3)$$

A Hilbert field is a blurred Hilbert distribution. The blur represents a QPAD.

The squared modulus of the private field that belongs to a Hilbert distribution can act as a probability density function. The projection operator whose target domain is spanned by the Hilbert distribution can act as the probability operator.

States and blurs

Apparently a state is the same stuff as the basic constituent of a Hilbert field. Both can be characterized as QPAD's. The squared modulus of a probability amplitude distribution[136] is a probability density distribution (PDD). The state corresponds with a wave function or with a probability density operator.

- The state of a physical item can be interpreted as the probability of finding the parameter value when an observation is done that corresponds to the corresponding coordinate operator.
- The squared modulus of the blur can be interpreted as the probability of detecting a quantum at the location specified by the parameter value that corresponds to the corresponding coordinate operator.

Blurs are the building stones of Hilbert fields. In a similar way wave functions must be interpretable as the building stones of fields.

Blurs are private fields of elementary Hilbert distributions. Thus, wave functions must also be related to elementary Hilbert distributions.

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[136] http://en.wikipedia.org/wiki/Probability_amplitude; the quaternionic version is used.

Observables and field values

In separable Hilbert space observables are represented by [operators](). The observed value is represented by an eigenvalue or by the expectation value of the operator that represents the observable. The expectation value can be computed using the QPAD of the observed item. In order to do this the QPAD must be written as a function of the eigenvalues of the operator.

Scalar physical fields have numeric values. Vector and tensor fields consist of vectors and tensors that are constructed using numbers. Both the eigenvalues of operators and the values of fields may be hyper complex 2^n-ons[137].

Numbers

The Hilbert space can be specified by using a number space that allows the mutual orthogonalization and the closure of subspaces. The real numbers, the complex numbers and the quaternions can perform that job. Horwitz showed that even the octonions with some trouble can achieve this [138]. The real numbers, the complex numbers, the quaternions and the octonions are the only normed division algebras and they are the only alternative division algebras. In general the octonions are not associative, but the product of two octonions that belong to the same quaternionic subfield is associative. The alternative property of the octonions admits the closure of the subspace generated by (successively associated) products of the vector with octonion elements to order seven, i.e., after multiplication seven times by octonions, the subspace no longer grows.

Neither all quaternions nor all octonions commute. However, within complex subspaces the numbers commute. In general holds for 2^n-ons that they behave like 2^m-ons in their lower m dimensions.

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[137] see http://www.math.temple.edu/~wds/homepage/nce2.pdf or the [appendix]()
[138] see: http://arxiv.org/abs/quant-ph/9602001

We might take the following freedom. The fact that a given number space is used for specifying linear combinations of Hilbert space vectors does not mean that eigenvalues of operators must also be restricted to that same number space. In this sense a Hilbert space specified over the quaternions may allow eigenvalues of operators that are taken from the octonions or even higher 2^n-ons. The problem with higher dimension 2^n-ons is that their number characteristics deteriorate with n. However, as long as the (full) eigenvalues are not used to construct linear combinations of vectors, or to specify the inner products of the Hilbert space, there is no problem. This means that operators for which expectation values are used as parameters of functions must also have quaternionic or lower dimensional eigenvalues.

All higher dimensional 2^n-ons contain several subspaces that are lower dimensional 2^m-on number spaces. Further, 2^n-ons behave like 2^m-ons in their lower 2^m dimensions.

In general the elements of curves or curved manifolds are themselves not numbers. So, in general they cannot be used as eigenvalues. However, locally the elements of a curved manifold may resemble numbers of a 2^n-on number space. Also tangent spaces may resemble number spaces.

Number spaces can be **attached** as tangent spaces to smoothly curved manifolds. Smoothly curved trails of objects that locally resemble 2^n-ons can be treated with the Frenet-Serret frame toolkit. In that way the elements of the curves and the manifolds obtain number characteristics in a small enough environment.

Sequences or sets of operators can locally have eigenvalues that are numbers which can be considered as member of smooth curves or of the tangent space of a curved manifold at that location. In that way the elements of smooth curves or of curved manifolds can be related to the corresponding eigenvalues. 2^n-ons are ideally suited for this purpose. This means that the eigenspaces of the subsequent operators in a trail need not

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overlap. These eigenspaces are only used locally. When curvature and bending of the operator trail diminish, the dimension of the local number space can be lower. When the curvature and the bending increase, the dimension must be higher. This will be reflected in the dimensionality of the local eigenvalues. Apart from the application as eigenvalues of operators the 2^n-ons are suited as values of physical fields.

We will restrict to the 2^n-ons as extensions of the quaternions. As we stated, the higher dimension numbers created with the Cayley-Dickson construction are not so well behaved. Alternatives are the use of Clifford algebras, Jordan algebras or Grassmann algebras. We will show that in the Hilbert space the 2^n-ons for $n > 1$ automatically introduce these latter algebras for example through their number waltz.

The **niners** are the most extensive 2^n-on numbers that still keep a reasonable set of number characteristics. More precisely said the 2^n-ons, even those that have a higher dimension than the octonions, keep reasonable number characteristics in the space spanned by their coordinates that have an index lower than nine. The real numbers, the complex numbers, the quaternions and the octonions completely fall within these boundaries. The above hyperlink describes exactly what characteristics the niners retain.

The subspace of the 2^n-on field that is spanned by the first 2^m dimensions acts as a 2^m-on number space. Thus in a dynamic situation, an octionic operator acts locally as a quaternionic operator. In a smaller or more flat region it acts as a complex operator and at "nano"-locality as a real (or as an imaginary) operator.

2ⁿ-on construction
The 2^n-ons use the following doubling formula

$$(a, b)\ (c, d) = (a{\cdot}c - (b{\cdot}d^*)^*, (b^*{\cdot}c^*)^* + (b^*{\cdot}(a^*{\cdot}((b^{-1})^*{\cdot}d^*)^*)^*)^*) \tag{1}$$

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Up until the 16-ons the formula can be simplified to

$$(a, b)\,(c, d) = (a{\cdot}c - b{\cdot}d^*,\ c{\cdot}b + (a^*{\cdot}b^{-1}){\cdot}(b{\cdot}d)) \tag{2}$$

Up to the octonions the Cayley Dickson construction delivers the same as the 2^n-on construction. From n>3 the 2^n-ons are 'nicer' than the Cayley Dickson numbers. They keep more useful number characteristics. The 2^{n+1}-ons contain the 2^n-ons as the sub-algebra of elements of the form (a, 0)

Waltz details

The 16-ons lose the continuity of the map x => xy. Also, in general holds $xy{\cdot}x \neq x{\cdot}yx$ for 16-ons. However, for all 2^n-ons the base numbers fulfill $e_i e_j {\cdot} e_i = e_i {\cdot} e_j e_i$. All 2^n-ons feature a conjugate and an inverse. The inverse only exists for non-zero numbers. The 2^n-ons support the **number waltz**

$$c = a{\cdot}ba^{-1}. \tag{1}$$

Often the number waltz appears as a unitary number waltz

$$c = u^*{\cdot}bu \tag{2}$$

where u is a unit size number and u^* is its conjugate $u{\cdot}u^* = 1$.

In quaternion space the **quaternion waltz** $a{\cdot}b{\cdot}a^{-1}$ can be written as

$$a{\cdot}b{\cdot}a^{-1} = \exp(2{\cdot}\pi{\cdot}\tilde{\mathbf{i}}{\cdot}\phi){\cdot}b{\cdot}\exp(-2{\cdot}\pi{\cdot}\tilde{\mathbf{i}}{\cdot}\phi) \tag{3}$$

$$= b - \underline{\mathbf{b}}\bot + \exp(2{\cdot}\pi{\cdot}\tilde{\mathbf{i}}{\cdot}\phi){\cdot}\underline{\mathbf{b}}\bot{\cdot}\exp(-2{\cdot}\pi{\cdot}\tilde{\mathbf{i}}{\cdot}\phi)$$

$$= b - \underline{\mathbf{b}}\bot + \exp(4{\cdot}\pi{\cdot}\tilde{\mathbf{i}}{\cdot}\phi){\cdot}\underline{\mathbf{b}}\bot$$

$$\Delta b = (\exp(4{\cdot}\pi{\cdot}\tilde{\mathbf{i}}{\cdot}\phi) - 1){\cdot}\underline{\mathbf{b}}\bot \tag{4}$$

$$= (\cos(4{\cdot}\pi{\cdot}\phi) + \tilde{\underline{i}}{\cdot}\sin(4{\cdot}\pi{\cdot}\phi) - 1){\cdot}\underline{b}{\perp}$$

$$= \exp(2{\cdot}\pi{\cdot}\tilde{\underline{i}}{\cdot}\phi){\cdot}2{\cdot}\tilde{\underline{i}}{\cdot}\sin(2{\cdot}\pi{\cdot}\phi){\cdot}\underline{b}{\perp}$$

$$||\Delta b|| = ||2{\cdot}\sin(2{\cdot}\pi{\cdot}\phi){\cdot}\underline{b}{\perp}|| \tag{5}$$

Another way of specifying the difference is:

$$\Delta b = (a{\cdot}b - b{\cdot}a)/a = 2{\cdot}(\underline{a}{\times}\underline{b})/a \tag{6}$$

$$||\Delta b|| = 2{\cdot}||\underline{a}{\times}\underline{b}||/||a|| \tag{7}$$
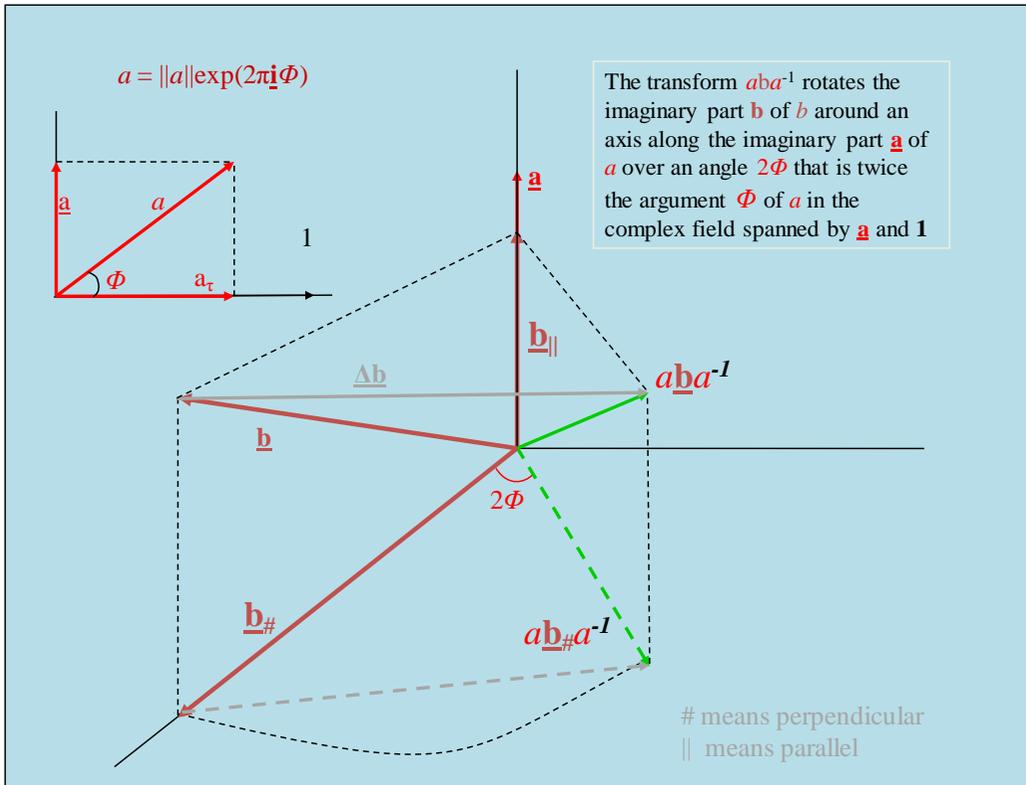
Figure 1. The rotation of a quaternion by a second quaternion.

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Infinitesimal number transformation

The number v is close to 1. Thus $v = 1 + \Delta s$. Let us investigate the transform $c = v^* \cdot b \cdot v$.

$$c = (1 + \Delta s^*) \cdot b \cdot (1 + \Delta s) \tag{1}$$
$$= b + \Delta s^* \cdot b + b \cdot \Delta s + \Delta s^* \cdot b \cdot \Delta s$$
$$\approx b + \Delta s^* \cdot b + b \cdot \Delta s$$
$$= b + \Delta s_0 \cdot b + 2 \cdot \mathbf{b} \times \Delta \mathbf{s}$$

$$\Delta b = \Delta s_0 \cdot b + 2 \cdot \mathbf{b} \times \Delta \mathbf{s} \tag{2}$$

This comes close to the effect of an infinitesimal number waltz, especially when $\Delta s_0 = 0$ In that case $\Delta b_0 = 0$ and $\Delta \mathbf{b}$ is perpendicular to $\Delta \mathbf{s}$.

For 2^n-ons with $n > 1$, $a \cdot b a^{-1}$ in general does not equal b. This effect stays unnoticed when quantum mechanics sticks to a complex Hilbert space.

$ab_{\#}a^{-1}$

$\underline{b}_{\#}2\sin^2(2\pi\Phi))$

$\underline{\Delta b}$

$\underline{b}_{\#}\underline{i}\sin(4\pi\Phi)$

2Φ

$\underline{b}_{\#}$

$\underline{\Delta b} = (-2\sin^2(2\pi\Phi) + \underline{i}\sin(4\pi\Phi))\underline{b}_{\#}$
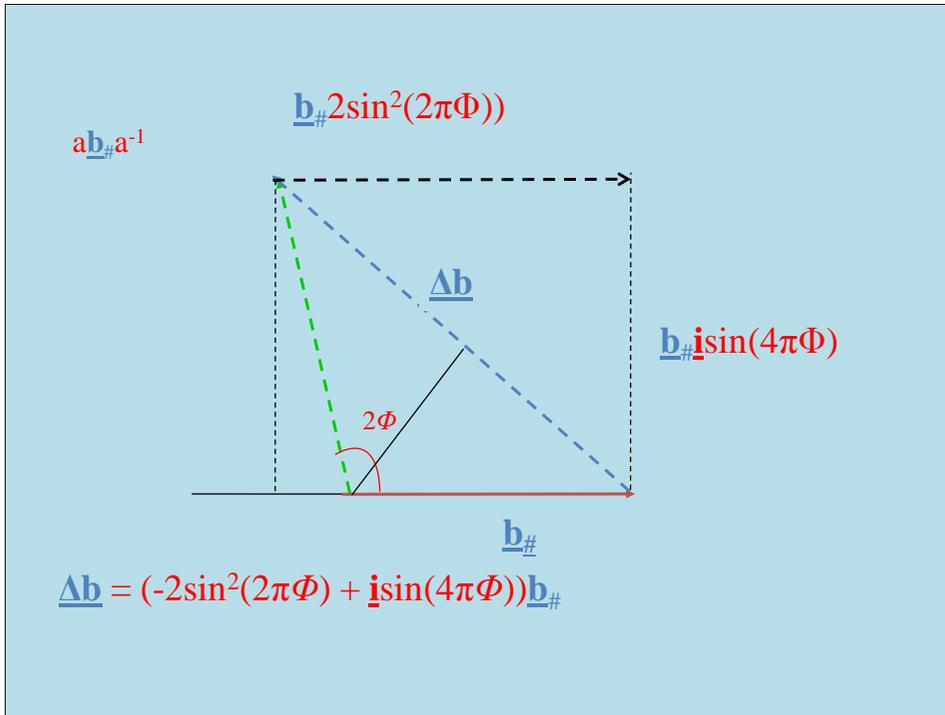
Figure 2: The difference after rotation

Sign selections

The paper that describes 2^n-ons does not describe the choice for right or left handedness of the external vector product. So, we do it here. The generally accepted convention is to let the handedness depend on the orientation of the underlying $\mathbb{R}^n$ space. However, when numbers are constructed via the Cayley-Dickson construction or the 2^n-on construction then the handedness follows from the applied construction formula. We want to get rid of these restrictions, because we want to give operators and fields the freedom to select the handedness and other sign selections of their (eigen)values.

The 2^n-ons have n independent binary base numbers and n sign selections. The real numbers do not offer a sign selection. The complex numbers offer the selection of the sign of the real or the imaginary axis. This is inherited by all higher 2^n-ons in the form of the conjugation. The quaternions have two independent imaginary base numbers and two

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independent sign selections that both affect the handedness of its external product. The octonions have three independent imaginary base numbers. These correspond to three independent sign selections for the handedness in external products that involve this new base number.

Need for spinors

In the number waltz the current manipulator only needs an argument α in order to turn the subject over 2α. This is typical behavior for spinors. Spinors also have a storage place for the handedness of rotations. By using the number waltz and the sign selections the 2^n-ons can perform the same act as the spinors. Spinors are only required when quantum mechanics is restricted to complex Hilbert spaces. Spinors are the carriers of the spin phenomenon. Thus, in our model the sign selections in combination with the number waltz form the carriers of spin.

The approach taken in this paper might cause a revival of the importance of the hyper complex numbers that turned in oblivion when Gibbs introduced his vector analysis.

Influence

The original example proposition (♠) talks about influencing the position of an item. This implies that the position of the item changes due to the mentioned influence. Thus when the influence occurs, the eigenvector that represents the position of the item is exchanged against another eigenvector. That other eigenvector corresponds to another environment inside the eigenspace of the position operator. The eigenvectors of the position operator move with respect to the subspace that characterizes the item. Another possibility is that the eigenvectors stay, but the corresponding eigenvalues change while the Hilbert subspace moves. In both cases the movement is relative. See Heisenberg picture versus Schrödinger picture[139].

Thus, there is a way to implement influence in Hilbert space. The influence causes a move of the item's subspace relative to one or more eigenvectors of the position operator. The original proposition (♠) claims that this movement is caused by other items. We must check whether this is true.

If this is true then influences are the motor behind the dynamics of the items.

The universe of items

The original proposition (♠) states that all items influence each other's position. This includes that all items influence the considered item. Part of the items compensates each other's influences on the currently considered item. It will be shown that this holds for the largest part.

Inertia

The influence may decrease with distance according to some function $f(r)$ of the distance r. However the number of contributing items increases with the distance. Depending on function $f(r)$ the most probable result is

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[139] Dynamics: Schrödinger or Heisenberg picture

that the strongest influence comes from the cooperative activity of the most distant items. Due to the enormous number of items in the universe, any variation of the influences of the distant items averages away. This also holds for the density distribution of the items. So there exists a fairly uniform background influence caused by the universe of items. What will happen, can be deduced from an equivalent of <u>Denis Sciama's analysis</u>[140]. We will take his analysis as a guide. Sciama's analysis uses a different setting: the (observed) 3D space and coordinate time. This setting raised critique because the approach involves instantaneous action on large distances. In Sciama's setting this is in conflict with special relativity. In our setting we do not (yet) encounter special relativity. We use the coordinate space defined by an appropriate coordinate operator and the progression parameter t that relates to the progression step counter as our setting. A location in coordinate space represents a location on the unit sphere of Hilbert space. This last location is taken by the eigenvector that corresponds to the first location. As stated before, the unit sphere of Hilbert space is an affine space. This means that we must treat position as relative data. With other words, the eigenspace of the coordinate operator has no absolute origin.

The most important aspects of the analysis are:

The total potential Φ at the location of the influenced subject is[141]

$$\Phi = -\int_V \frac{\rho}{r} dV = -\rho \int_V \frac{dV}{r} = 2\pi\, R^2 \rho \tag{1}$$

This conforms to a <u>Gaussian blur</u>[142] as a representative of the average blur function. The integral is taken over the coordinate space volume V. Indirectly, the integral is taken over the unit sphere of Hilbert space. This

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[140] See: <u>http://arxiv.org/abs/physics/0609026v4.pdf</u> and "On the origin of inertia", by Denis Sciama (<u>http://www.adsabs.harvard.edu/abs/1953MNRAS.113...34S</u>

[141] See: <u>http://en.wikipedia.org/wiki/Newtonian_potential</u>

[142] Hilbert field equations: example potential

is an affine space. The parameter r is the length of the vector from the actor to the location of the subject. The considered subject is located somewhere in the affine coordinate space. All other subjects have positions relative to that considered subject. At large distances, the density ϱ of the contributing items can be considered to be uniformly distributed. Also any variance in strength other than the dependence on r becomes negligible because the differences are blurred and averaged away. We already assumed that the average blur of the distributed matter in universe is a Gaussian blur. We take the average of the strength as the significant parameter. We combine it with ρ. Therefore the average of ρ can be taken out of the integral. Thus, apart from its dependence on the average value of ρ, Φ is a huge constant. Sciama relates Φ to the gravitational constant G.

$$G = {-c^2}/{\Phi}$$

As a consequence we can consider the universe as a very large rigid body. If nothing else happens then all influences compensate each other.

In the following equations we use imaginary quaternions rather than 3D vectors. In this way we can avoid the distracting factor i.

If the considered subject moves relative to the universe with a uniform speed $\mathbf{v}$, then a vector potential $\mathbf{A}$ is generated.

$$\mathbf{A} = -\int_V \frac{\mathbf{v} \cdot \rho}{c \cdot r} dV \tag{2}$$

Both ρ and $\mathbf{v}$ are independent of r. Together with the constant c they can be taken out of the integral. Thus

$$\mathbf{A} = \Phi \cdot \mathbf{v}/c \tag{3}$$

What we have here is the reverse of the definition of the potential that goes together with a charge distribution. When we defined a [Hilbert field](#) we started in fact from a charge distribution and a current distribution and we considered the influence of these distributions on the universe. Here we consider the influence of the universe on a local charge or current. For this purpose we use the same volume integrals!

The field that we treat in studying inertia is a curvature field rather than a covering field. The curvature field derives from the covering field by taking the curvature that is caused by the decomposition of the covering field as its charge.

The notions of charge and current correspond to equivalent notions in [Noether's theorem](#)[143]. Here we talk about inertia and curvature fields. Thus charge may symbolize mass.

Here the progression parameter t plays the role of "time". Be aware, this is not our usual notion of time.

According to Helmholtz theorem the Hilbert field derived from the above potentials can be split into a divergence free part and a rotation free part. The Helmholtz decomposition theorem only concerns the static versions of the derived field. It is related to the fact that the Fourier transform of a vector field can be split in a longitudinal and a transversal version. There also exists a corresponding split of the multi-dimensional Dirac delta function in a longitudinal and a transversal version. If we use the position operator $\tilde{Q}$ as the coordinate operator, then the decomposition runs along straight lines. If we use the GPS operator $\breve{Q}$ then the decomposition runs along curved lines. In curved manifolds the Helmholtz decomposition theorem should be replaced by the Hodge decomposition theorem.

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[143] http://en.wikipedia.org/wiki/Noether%27s_theorem

A variation of **v** goes together with a variation of **A**. On its turn this goes together with a non-zero field $\dot{A}(r, t)$ which is a **dynamical** part of the derived Hilbert field.

Sciama uses a Maxwell equation to explain the relation between ∂**v**/∂t and $\dot{A}(r, t)$. Our setting differs, but the main reasoning is the same.

$$E(r, t) = -\nabla \Phi(r, t) - \frac{1}{c} \cdot \dot{A}(r, t) \tag{4}$$

$$\tilde{E}(k, \omega) = -k \cdot \tilde{\Phi}(k, \omega) - \frac{1}{c} \cdot \omega \tilde{A}(k, \omega) \tag{5}$$

If we exclude the first term because it is negligible small, we get:

$$E(r, t) = -\frac{\Phi}{c^2} \cdot \frac{\partial \mathbf{v}}{\partial t} = G \cdot \frac{\partial \mathbf{v}}{\partial t} \tag{6}$$

Remark: As soon as we turn to the dynamic version (4) an extra component $\dot{A}$ of field E appears that corresponds to acceleration ∂**v**/∂t. (See for derivation of Maxwell equations e.g. the online book http://www.plasma.uu.se/CED/Book; formula 3.25 or the section on Hilbert field equations in this e-paper)

As already claimed, in our setting the component $\nabla \Phi$ of the field E is negligible. With respect to this component the items compensate each other's influence. This means that if the influenced subject moves with uniform speed **v**, then $E \approx 0$. However, a vector potential A is present due to the movement of the considered item. Any acceleration of the considered item goes together with an extra non-zero E field. In this way the universe of items causes inertia in the form of a force that acts upon the accelerating item's charge.

We have used the coordinate space as a playground to implement an equivalent of Sciama's analysis. The analysis uses the fact that every item

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in universe causes an influence and that this influence reduces according to $f = –k/r$. (Compare this with [Bertrand's theorem](#)[144] in Wikipedia)

A uniform movement in Hilbert space does not on itself generate a reaction of the universe of items. Any alteration of that uniform movement will cause as reaction **a field**. The physical name for this reaction is **action**. It usually gets the symbol **S**. When the path of the item coincides with a **geodesic**, then it can be travelled field free.

Uniform movements do cause displacement of charges. On its turn it changes the configuration of the local field. Thus, indirectly the local field will also act on uniform displacements. As we see from inertia, any field change goes together with a corresponding acceleration.

It must be noticed that the original analysis of Sciama uses observable position space rather coordinate space and it uses a different notion of time. However, the general conclusion stays the same. Sciama's analysis is criticized because it uses infinite speed of information transfer. Since we do not work in observable position space, we do not encounter coordinate time. So for us, this criticism is misplaced. Most part of the story plays in a stationary condition. Even the uniform movement is stationary. The acceleration deviates from the stationary condition. This deviation goes together with an extra field component.

[Coordinate time](#)[145] relates to observations of position. It is a local player in the game, where the progression parameter is a global player.

The situation with electromagnetic fields is different, because with this field positive and negative charges compensate each other's long range influence. For that reason there exists no electromagnetic background influence. The masses of the gravitational and inertial fields only

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[144] http://en.wikipedia.org/wiki/Bertrand's_theorem
[145] Dynamics; Relativity

compensate each other's long range influences through geometrical circumstances. Still in combination, they create gigantic potentials.

The particles outside the information horizons also contribute to the inertia.

We may reverse the conclusion of the analysis:

An extra field component goes together with an acceleration of the local item.

Thus when through uniform movement the local field configuration changes, then that change goes together with an acceleration of the local item.

Nearby items

Items that are located nearby have a different effect. In general their influence will not have its strength equal to the average strength. Further these items are not uniformly distributed. Still at macroscopic distances their influence depends on inter-distance as $f = -k/r$. As a consequence their influences form a landscape of which the effects will become sensible in the action of the fields that surround the considered item. This landscape will form a curved action space. The considered item will try to follow a geodesic through that curved space.

Rotational inertia

Besides linear inertia there exists rotational inertia. In a non-rotating universe hold near the origin $\mathbf{A} = 0$ and $\Phi = -c^2/G$. We choose units such that c=G=1. In a universe rotating slowly with angular speed ω hold

$$A_x = \omega \cdot y \tag{1}$$

$$A_y = -\omega \cdot x \tag{2}$$

$$\tag{3}$$

$$A_z = 0$$

$$\Phi = -\sqrt{1 + (\omega \cdot r)^2} \tag{4}$$

A constant angular movement meets the fields that correspond to a centripetal force.

The field E has the form

$$E = \frac{\omega^2 r}{\sqrt{1 + \omega^2 r^2}} \tag{5}$$

An added uniform speed v meets the fields corresponding to a Coriolis force.

$$H = \nabla \times A = 2 \cdot \omega \tag{6}$$

$$v \times H = 2 \cdot v \times \omega \tag{7}$$

The forces are usually considered as *fictitious* but they are actually caused by inertia. Sciama treats them in section 5 of his paper. Like fields of linear inertia these rotation related fields correspond to actions of the manipulator.

Storage, sign selections and virtual items.
The static fields act as storage media for the location and the speed of the charges of the physical items.

When the values of the fields are stored in hyper complex numbers, then the sign choices for these numbers will also be reflected in these fields. Each of the n independent imaginary base vectors will introduce an independent sign selection. This will produce 2^n field sign flavors.

The fields can be interpreted as reflections of the presence of non-actual items that are ready to exchange roles with actual items.

The proposition

This finding indicates that when our interpretation of Sciama's analysis is correct, the original proposition

> *All items in universe influence each other's position*.

is not generally true. The universe of items does not influence position. It counteracts acceleration of individual items. Position is only influenced in an indirect way and presupposes an observation. If the item moves in a geodesic with uniform speed, then the position changes while the influences of all other items compensate each other. In such cases the summed influence is zero.

We may alter the original proposition (♠). If our analysis is correct, then the proposition

> *All items in universe influence each other's acceleration*.

is true.

The origin of dynamics

If we want to discover the origin of dynamics, we must first determine what the static structure of nature is. We already found an important ingredient of this skeleton: the lattice structure of quantum logic and the corresponding lattice structure of the closed subspaces of a Hilbert space. Both structures are only defined in a static way. Nothing is said about their dynamics. Besides of these static relations the concept of wave functions and density operators offer insight in the probability and information content of these relations. These subjects correspond to private fields, which are the constituents of a covering field. This covering field can be seen as the superposition of all private fields. For a selected coordinate system the static covering field can be decomposed into a rotation free and a divergence free part. Depending on the configuration of the anchors of the private fields the decomposition does not run along straight coordinate lines. This defines a local curvature that depends on the selected coordinate system. That curvature can be used to define a curvature field. This together defines the ingredients of the static status quo. It all fits in a model that we call *extended quantum logic* or equivalently *extended Hilbert space*.

In the previous part of the paper the added component of the static structure of nature is investigated: the static structure of the influences. It appears that this structure is identical with the structure of static Hilbert fields. Both the analysis of inertia and the study of Hilbert fields showed the static relation between divergence free fields and rotation free fields. These analyses also showed the influence of dynamics on the coupling of these static fields. The analysis of Hilbert fields explained how these fields change as a function of the progression parameter q_0. Inertia showed how these fields get coupled when the uniform movement of a physical item is disturbed. We also explained that uniform movement may cause a reconfiguration of the field. On its turn, this change may initiate extra movement.

Next we try to find a more precise formulation for these origins of dynamics.

Extended quantum logic

Wave functions represent the probability of finding properties of actual items. For example if a GPS type coordinate system is selected as its parameter space, then this property can be the position of the item. If it is a GMS type system, then the property can be momentum.

In quantum theory the values of fields are treated in equations of motion in a similar way as the wave functions of actual items are treated in such equations. The Hilbert book model interprets the wave function itself as part of the field.

When fields in general can be considered as representations of the probability of finding properties of actual as well as virtual items, then the fields get an interesting interpretation.

In quantum logic the realistic physical item is represented by a proposition in the form of a statement that says everything about that item.

For non-actual items the new interpretation would mean that in extended quantum logic the non-actual items are represented by potential propositions that are ready to become actual propositions or that were actual propositions in the past.

It means that traditional quantum logic is embedded in extended quantum logic such that it apart from propositions about actual physical items also contains preconditions about future physical items and post-conditions about past physical items.

This information is contained the Fourier transforms of the QPAD's that belong to elementary particles. It is already shown that the canonical

conjugated operators give information about changes of the original operators[146].

In this way, the set of propositions of extended quantum logic is much larger than the set of propositions of quantum logic.

Interpretation in logical terms

The results of the analysis of inertia mean that when the redefinition of the set of vectors that belongs to the representation of the item occurs such that this corresponds to a uniform movement of the physical item, then the influences of the universe of items tend to compensate each other. The whole is treated as a static set of relations. Otherwise, the universe of items reacts with a corresponding extra field component. This means an extra blur = extra divergence of the stochastic inaccuracy of properties of the considered item.

Besides of the universe wide response, a local variance in the distribution of items causes a variation in the influences. This local variance can partly be the consequence of a uniform displacement of particles.

It seems that quantum logic and Helmholtz decomposition together define an important part of the static relations that exist in physics. The fields appear to resist the disturbance of the interrelations in the lattice of quantum propositions. In dynamical sense this lattice might step from one static status quo to the next. After a step new conditions are established that again must fulfill the laws that govern the static situation. If this is a proper interpretation, then it is likely that the progression step is taken universe wide. After each step the positions of the physical items relative to the fields have changed, thus when the fields are not uniformly distributed, the items meet a different field configuration. The next step is taken with and due to these new conditions.

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[146] Functions and fields; Canonical conjugate; The quaternionic displacement generator

Quantum logic only defines a static skeleton in which the dynamics of quantum physics takes place. To make it a dynamic logic, the set of axioms must be extended. The new axioms must state that all propositions influence each other. The influence depends on their mutual (coordinate) distance. In stationary conditions, which include uniform motion, these influences compensate each other. When an atomic predicate that concerns an element of an ordered set is replaced in a non-ordered fashion, meaning that the distance between the replaced elements does not stay the same, then the universe of all propositions will react such that the influences of the other propositions no longer compensate each other. The disordered influences counteract the disordered replacement.

Besides of that the local variance in the distribution of the propositions, which corresponds to a variance of the distribution of the corresponding physical items, also cause a variation in the influences that propositions have with respect to each other.

In Hilbert space these influences are implemented in the actions of Hilbert fields. In quantum physics the influence appears as a set of physical fields.

Minkowski signature

One important step must still be taken. In physics observed spacetime has a Minkowski signature. Further we observe that space corresponds with the imaginary part of a position quaternion for which the real part seems to have no direct physical meaning. We must find an explanation for these facts. The Minkowski signature defines the following time-like relation between the space time step Δs, the space step Δq and the coordinate time step Δt

$$(\Delta s)^2 \;=\; (\Delta t)^2 - (\Delta q)^2/c^2 \tag{1}$$

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During inertial motion this corresponds for the [proper time](http://en.wikipedia.org/wiki/Proper_time)[147] τ to

$$(\Delta\tau)^2 \;=\; (\Delta t)^2 - (\Delta q)^2/c^2 \tag{2}$$

$$\boldsymbol{\Delta t} \;=\; \boldsymbol{\Delta\tau} + \boldsymbol{\Delta q}/c \tag{3}$$

This is a triangle relation where $\boldsymbol{\Delta t}$ is at the hypotenuse.
If we substitute the Planck-length for Δq and the Planck-time for Δt then $\Delta\tau$ equals zero.

Dynamics

Schrödinger or Heisenberg picture

For global rotations around its origin the Hilbert unit sphere acts as an affine space. It does not matter whether the eigenvectors of operators or the subspace that represents the item is moved. We can take the picture in which the subspace stays fixed, while the eigenvectors move and the operators change with them. This is the **Heisenberg picture.**
We can also take the picture in which the eigenvectors and operators stay fixed and the subspace moves. This is the **Schrödinger picture**.

We are only interested in the consequences. These are determined by the relative movement, not by the absolute movement. For a given physical item, in both pictures the expectation values of the operators vary in the same way.

Unitary transform

A unitary transform is a bounded normal operator. Unitary transforms keep the value of inner products untouched. If a unitary transform is applied to two vectors, then their inner product stays the same.

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[147] http://en.wikipedia.org/wiki/Proper_time

Unitary transforms need not have eigenvectors. For example Fourier transforms do not possess eigenvectors. In the rigged Hilbert space $\mathbf{H}$ functions exist that apart from a scaling factor are invariant under Fourier transformation.

If a unitary transform has eigenvectors then it has unit sized eigenvalues and to each of these eigenvalues correspond one or more eigenvectors that are mutually orthogonal. Unitary transforms are completely determined by their vector replacement characteristics or by their eigenvectors and the corresponding eigenvalues.

When a unitary operator U is applied to the eigenvector $|q>$ of an operator Q with eigenvalue q, then the eigenvector is transferred into another vector $|U\ q>$. In general $|U\ q>$ is not another eigenvector of Q. In quaternionic Hilbert space the expectation value for $|QU\ q>$ is no longer q, but

$$< q\ U|QU\ q> = < q|U^{\dagger}QU\ q> \tag{1}$$

Or, with other words the operator Q is **redefined** to $U^{\dagger}QU$.

The norm of the expectation value $< f\ U|QU\ f>$ for an arbitrary vector $|f>$ does not depend on U. It only depends on Q and $|f>$. However the expectation value is rotated and the rotation depends on U.

Trail of infinitesimal transforms

The effect of a single unitary transform U can also be achieved by a trail of infinitesimal unitary transforms $\{U_t\}_t$. This also holds for a set of unitary operators that operate in parallel.

The situation sketched above can be refined for any instant t occurring after t=0. We can treat it more generally by chopping the path from $\{|f_s>\}_s$ to $\{|g_{st}>\}_s$ into a **trail** of infinitesimal steps of size Δt that is achieved by a set of infinitesimal transforms $\{U_{st}\}_{st}$, where

$$|g_{st}> = |\prod_s U_{st} f_s > \tag{1}$$

and

$$U_{st} \approx 1 + \Delta S_{st} \tag{2}$$

The parameter t acts as the trail progression parameter. It is not identical with our common notion of time. If it has anything to do with time it will be confronted with a lower limit, which is set by the Planck-time. The infinitesimal transforms U_{st} work in parallel as well as in sequence. ΔS_{st} represents the current local infinitesimal action step. It is an imaginary operator. Like time the action step also has a lower limit that is determined by a corresponding Planck unit.

The Heisenberg picture conforms to the description with unitary transforms where operators are redefined. When this is done in small steps, then the redefined operator becomes a function of progression parameter t.

Unitary transform with full set of eigenvectors

When a unitary transformation U is applied to an arbitrary vector $|f>$, which is not an eigenvector, then that vector is transferred into another vector $|g> = |U f>$, which has the same norm. If $|f>$ is an eigenvector of U then $|f>$ is **not** transferred to a different vector, but it is multiplied with the corresponding eigenvalue. Also in this case the norm stays the same.

If a unitary transform contains a full set of eigenvectors, then multidimensional subspaces usually contain one or more eigenvectors of that unitary transform. In that case the transfer of a multidimensional closed subspace requires a set of parallel unitary transforms.

If we take a set of vectors $\{|f_s>\}_s$ that together span a closed subspace, then a set of suitable unitary transforms $\{U_s\}_s$, can in parallel transfer all

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vectors of this set such that after the transform $|g_s> = |U_s f_s>$ the set $\{|g_s>\}_s$ spans the new subspace. Each of the members U_s of the set $\{U_s\}_s$ can be split in a trail. $\{U_{st}\}_t$

Fourier transform as unitary transform

Unitary transforms exist that have no eigenvectors. For example in Hilbert space a Fourier transform has no eigenvectors and no eigenvalues. It does not leave a single Hilbert vector untouched.

The Fourier transform converts an orthonormal base into another orthonormal base, which is the canonical conjugate of the first.

Hilbert fields exist that apart from a scaling factor are invariant under Fourier transformation. They keep their form through Fourier transformation. For that reason they are often called eigenfunctions, but they do not correspond to eigenvectors. Their form stays the same, but their parameters change. So, the name eigenfunction is incorrect. The Hermite functions[148] are notorious examples of Fourier invariant functions. Even and odd functions have an indirect relation to functions that are invariant under Fourier transformation.

An invariant function is not an eigenfunction. In extended separable Hilbert space, every Fourier transform causes a resampling of the analyzed field or function.

Each Fourier transform means a complete replacement of the current orthonormal base. For that reason, a Fourier transform that resides in separable Hilbert space can never be an infinitesimal unitary transform. Stated in different words this means: The transform $U_{st} = 1 + \Delta S_{st}$ is not a Fourier transform. However, Fourier transforms $U_{\mathcal{F}}$ and reverse Fourier transforms $U_{\mathcal{F}}^{\dagger}$ can be member of a trail of unitary transforms, where each

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[148] Functions and fields; Functions that are invariant under Fourier transform.

trail step contains a move up and down to Fourier space, while in Fourier space only an infinitesimal action is taken.

$$|g_{st}> = |\prod_s U_{\mathcal{F}} U_{st} U_{\mathcal{F}}^{\dagger} f_s >$$

(1)

Stepping through the Fourier space has the advantage that there derivation turns into multiplication and multiplication with a factor close to unity reduces to addition.

Single infinitesimal step

The success of the [Feynman path integral formalism](http://en.wikipedia.org/wiki/Path_integral_formulation)[149] gives us guidance in the analysis of what happens during a single infinitesimal step. We analyze an arbitrary trail consisting of infinitesimal trajectory steps:

$$< q_{t_1} | U_{s_{t_1,t_0}} \, q_{t_0} > = \left\{ \prod_{t=t_0}^{t=t_1} (< q_t' | p_t > < p_t \, U_{s_t} | q_t >) \right\} < q_{t_1} | p_{t_1} >$$

(1)

During a single step the system moves from position q to $q' = q + \Delta q$? Let us evaluate $< q' | p > < p \, U_{st} | q >$ for a single trajectory. Here U_{st} is an infinitesimal unitary transform. It is a member of the set of parallel unitary transforms that act on a target subspace. In the following text we leave the parallel trajectory index s in U_{st} unspecified. We concentrate on the sequence index t, which represents the progression parameter. The infinitesimal sequence step comprises three sub-steps:

1. Goto Fourier space. This is achieved by part $< q' | p >$.
2. Perform the action. This is done by $< p \ U_t^{\dagger} |$.
3. Go back to configuration space. This is achieved by $< p | q >$.

The sense behind the first and the last part is a travel to and back from Fourier space. Step two means that in Fourier space the action of the operator is just a multiplication with factor $\exp(\Delta s_t)$.

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[149] http://en.wikipedia.org/wiki/Path_integral_formulation

$$< p \ U_t^\dagger| = \exp(\Delta s_t) < p| \tag{2}$$

First we split $< q'|p >$.

$$< q'|p > = < q|p > \ \exp\left(\frac{\Delta q \ p}{\hbar}\right) \approx < q|p > \left(1 + \frac{\Delta q \ p}{\hbar}\right) \tag{3}$$

$$< p|q > = < q|p >^* \tag{4}$$

$$< q'|p > \exp(\Delta s_t) < p|q > \tag{5}$$

$$\approx < q|p > \left(1 + \frac{\Delta q \ p}{\hbar}\right) (1 + \Delta s_t) < p|q >$$

$$= < q|p > \ C_t \ < q|p >^*$$

This is a quaternionic rotation of the central term C_t, which is close to unity. The quaternionic rotation manipulators stands for the route to Fourier space and the route back from Fourier space. The central term C_t stands for what is done during a single step by the action in Fourier space.

$$C_t \approx \left(1 + \frac{\Delta q \ p}{\hbar}\right)(1 + \Delta s_t) \approx 1 + \frac{\Delta q \ p}{\hbar} + \Delta s_t = 1 + \ \Delta C \tag{6}$$

$$\Delta C = \frac{\Delta q \ p}{\hbar} + \Delta s_t \tag{7}$$

$$\hbar \ \Delta C - \hbar \ \Delta s_t = \ \Delta q \ p \tag{8}$$

$$= \ \Delta q_0 \ p_0 - \ \langle \Delta q, p \rangle + \Delta q_0 \ p + \ \Delta q \ p_0 + \ \Delta q \times \ p$$

We study the step ΔC somewhat deeper. Since q and p are considered to be imaginary, we skip the parts containing Δq_0 or p_0.

$$\hbar \ \Delta C - \hbar \ \Delta s_t = \ - \langle \Delta q, p \rangle + \ \Delta q \times \ p \tag{9}$$

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We introduce K. It characterizes the infinitesimal step.

$$K \equiv -\hbar \frac{\Delta C}{\Delta t} \approx -\hbar \, \dot{C} \tag{10}$$

Both ΔC and K are functions of progression parameter t.

$$ \tag{11}$$

$$\hbar \, \Delta C_0 = -K_0 \, \Delta t = -\langle \Delta \boldsymbol{q}, \boldsymbol{p} \rangle$$

$$K_0 = \langle \frac{\Delta \boldsymbol{q}}{\Delta t}, \boldsymbol{p} \rangle \approx \langle \dot{\boldsymbol{q}}, \boldsymbol{p} \rangle \tag{12}$$

$$\hbar \, \Delta \boldsymbol{C} - \hbar \, \Delta \boldsymbol{s}_t = -\boldsymbol{K} \, \Delta t - \hbar \, \Delta \boldsymbol{s}_t = \Delta \boldsymbol{q} \times \boldsymbol{p} \tag{13}$$

$$\boldsymbol{K} = -\hbar \frac{\Delta \boldsymbol{s}_t}{\Delta t} + \frac{\Delta \boldsymbol{q}}{\Delta t} \times \boldsymbol{p} \approx -\hbar \, \dot{\boldsymbol{s}}_t + \dot{\boldsymbol{q}} \times \boldsymbol{p} \tag{14}$$

The steps Δq_t and Δs_t depend on the step Δt of the trajectory parameter t that is used to chop the unitary transform U_{t_1,t_0}.
In the trail the imaginary part of K is rotated.
If $\Delta \boldsymbol{q}$ is zero then

$$K = -\hbar \frac{\Delta \boldsymbol{s}_t}{\Delta t} \tag{15}$$

Δt is never zero. If Δt equals the Planck-time, then $|\Delta \boldsymbol{q}|$ is either zero or it equals the Planck-length. In that case

$$\left| \frac{\Delta \boldsymbol{q}}{\Delta t} \right| = c \tag{16}$$

Ray tracing
Following a trail has much in common with ray tracing in optics.
However in optics the use of characteristics that have their base in Fourier analysis seems to be more fruitful than ray tracing. Ray tracing follows

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the path of a sharp particle, while Fourier analysis is capable of following the life path of a blurred particle and include more of the influences of the environment in the analysis. It is sensible to expect that the advantages of Fourier analysis also hold for wave mechanics. As shown here, to a certain extent the path integral approach also makes use of Fourier analysis.

The dynamic of the private field

The private field represents the particle. When it is interpreted as a probability amplitude distribution, then it can be related to the expectation value of the position and the expectation value of the momentum of the particle. In this way the private field represents all potential paths with their corresponding probabilities. The Feynman path integral relates all these paths with the actual path, which corresponds to the path that uses minimal action.

Relativity

In advance Einstein's own explanation of the origin of relativity was: "*There is no logical way to the discovery of these elementary laws. There is only the way of intuition.*" Read more in:
http://www.time.com/time/magazine/article/0,9171,878733,00.html#ixzz15NlhpWDu

Transformations that describe displacements

The current explanation of the origin of special relativity is based on the properties of the generalized transformation that causes a displacement with uniform speed. We apply the corresponding reasoning on the Hilbert book model. This model represents static status quos of the universe by extended quantum logics or equivalently by extended separable Hilbert spaces. The complete model consists of a sequence of such extended separable Hilbert spaces.

The unit sphere of the separable Hilbert space is an affine space. It houses all unit length eigenvectors. This also holds for the eigenvectors of the position operator. This means that between two realizations of the Hilbert

space the eigenvector that corresponds to the origin of position can be freely selected. Or with other words the origin of position can be selected freely.

Differences between positions in subsequent members of the sequence of extended separable Hilbert spaces can be interpreted as displacements. The displacement is a coordinate transformation. For the properties of this transformation it does not matter where the displacement starts or in which direction it is taken. The same holds for displacements that concern sequence members that are separated further apart.

The corresponding displacements form a group. The displacement is a function of both the position and the sequence number. The displacement $z, t \rightarrow z', t'$ can be interpreted as a coordinate transformation and can be described by a matrix

$$\begin{bmatrix} t' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & \delta \\ \beta & \alpha \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} \tag{1}$$

The matrix elements are interrelated.

Uniform movement
When the displacement concerns a uniform movement, the interrelations of the matrix elements become a function of the speed v. The group properties <u>fix the interrelations</u>[150].

$$\begin{bmatrix} t' \\ z' \end{bmatrix} = 1/\sqrt{1 + kv^2} \begin{bmatrix} 1 & kv \\ -v & 1 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} \tag{2}$$

If k is positive, then there may be transformations with $kv^2 \gg 1$ which transform time into a spatial coordinate and vice versa. This is considered to be unphysical.

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[150] Appendix; Displacement in an isotropic medium

The condition k = 0 corresponds to a **Galilean transformation**

$$\begin{bmatrix} t' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -v & 1 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} \tag{3}$$

The condition $k < 0$ corresponds to a Lorentz transformation. We then can set $kc^2 = -1$, where c is an invariant speed that corresponds to the maximum of v.

$$\begin{bmatrix} t' \\ z' \end{bmatrix} = 1/\sqrt{1 - v^2/c^2} \begin{bmatrix} 1 & -v/c^2 \\ -v & 1 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} \tag{4}$$

Thus, when the displacement transformation group features a maximum speed, then it concerns Lorentz transforms.

Since in each progression step photons step with a non-zero space step and both step sizes are fixed, the speed of the photon at microscopic scale is fixed. No other particle goes faster, so in the model a maximum speed occurs. With other words when sequence members at different sequence number are compared, then the corresponding displacements can be described by Lorentz transformations.

Lorentz transformations introduce the phenomena that go together with relativity, such as length contraction, time dilatation and relativity of simultaneity that occur when two inertial reference frames are considered. These phenomena occur in the Hilbert book model when different members of the sequence of Hilbert spaces are compared.

Simultaneity
The restrictions set by the relativity of simultaneity limit the freedom of the selection of reference frames.

$$\Delta t_c = \left(\Delta t_p - \Delta z_p\, v/c^2 \right)/\sqrt{1 - v^2/c^2} \tag{17}$$

Here t_p is the proper time, which is measured by a clock that travels with the observed item and t_c is the coordinate time that is measured by the observer.

If $\Delta t_p = 0$ then depending on v and Δz_p the time difference Δt_c is non-zero.

This restriction of selection of reference frames means that the inertial reference frames cannot arbitrarily be taken from the sequence of extended Hilbert spaces. Usually at least one of them must be taken from a multi-sample range of extended Hilbert spaces.

For photons the proper time step is always zero.

Infinitesimal unitary transforms

Still another indication exists that the model supports special relativity.

The position operator $\boldsymbol{Q}$ is modified by the unitary operators of the trail into another operator $\boldsymbol{Q}_t$ that has different eigenvectors and different eigenvalues.

$$\boldsymbol{Q}_{t+\Delta t} = U_t \boldsymbol{Q}_t U_t^\dagger \tag{1}$$

$$U_t \approx 1 + \Delta \boldsymbol{S}_t \tag{2}$$

$$U_t^\dagger \approx 1 - \Delta \boldsymbol{S}_t \tag{3}$$

$$\boldsymbol{Q}_{t+\Delta t} \approx \boldsymbol{Q}_t + [\Delta \boldsymbol{S}_t, \boldsymbol{Q}_t] \tag{4}$$

$$\Delta \langle \boldsymbol{q} \rangle = \langle \boldsymbol{Q}_{t+\Delta t} \rangle - \langle \boldsymbol{Q}_t \rangle \approx [\Delta \boldsymbol{S}_t, \boldsymbol{Q}_t] = 2 \, \boldsymbol{Q}_t \times \Delta \boldsymbol{S}_t \tag{5}$$

This indicates that the step $\Delta \langle \boldsymbol{q} \rangle$ in the expectation value $\langle \boldsymbol{Q}_t \rangle$ of $\boldsymbol{Q}_t$ is perpendicular to both $\boldsymbol{Q}_t$ and $\Delta \boldsymbol{S}_t$. The steps $\Delta \langle \boldsymbol{q} \rangle$ and $\Delta \boldsymbol{S}_t$ form a right angular triangle with a hypotenuse: $c \, \Delta \boldsymbol{t}_c$, such that:

$$c \, \Delta \boldsymbol{t}_c = \Delta \langle \boldsymbol{q} \rangle + \Delta \boldsymbol{S}_t \tag{6}$$

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Here we introduced a new imaginary variable t_c.

With $\Delta\boldsymbol{\sigma} = \Delta\boldsymbol{S}_t/c$ the Minkowski signature of a new "observable" spacetime becomes visible.

$$\Delta t_c = \frac{\Delta\langle\boldsymbol{q}\rangle}{c} + \Delta\boldsymbol{\sigma} \tag{7}$$

$$\Delta\boldsymbol{\sigma} = \Delta t_c - \Delta\langle\boldsymbol{q}\rangle/c \tag{8}$$

$$|\Delta\boldsymbol{\sigma}|^2 = |\Delta t_c|^2 - |\Delta\langle\boldsymbol{q}\rangle|^2/c^2 \tag{9}$$

Thus, the analysis of what occurs during a single infinitesimal step gives us an indication how relativity enters physics. However, it asks for the introduction of a local notion of time $t_c = |t_c|$ that differs considerably from the (global) progression parameter t. This new parameter is the **coordinate time**[151] t_c.

Proper time

In relativity, **proper time**[152] t_p is time measured by a single clock between events that occur at the same place as the clock. It depends not only on the events but also on the motion of the clock between the events. An accelerated clock will measure a proper time between two events that is shorter than the coordinate time measured by a non-accelerated (inertial) clock between the same events.

$$\left|\Delta t_p\right|^2 = |\Delta t_c|^2 - |\Delta\langle\boldsymbol{q}\rangle|^2/c^2 \tag{1}$$

$$\Delta\boldsymbol{\sigma} = \frac{\Delta\boldsymbol{S}_t}{c} = \boldsymbol{e}_\sigma\,\Delta t_p \tag{2}$$

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[151] http://en.wikipedia.org/wiki/Coordinate_time
[152] http://en.wikipedia.org/wiki/Proper_time

Thus, proper time t_p is, via the action step $\Delta \boldsymbol{S}_t$ related to our notion of progression parameter t.

For a photon the proper time step is always zero. This also holds in the realm of general relativity. In the vicinity of a black hole this leads to the fact that the radial velocity of a photon approaches zero when the photon approaches the border of the black hole. The border is located at the Schwarzschild radius r_{sh}

$$r_{sh} = 2MG/c^2 \tag{3}$$

We use polar coordinates and the expression for the metric near the black hole

$$\left|\Delta t_p\right|^2 = (1 - \frac{r_{sh}}{r})|\Delta t_c|^2 - \left(\frac{|\Delta \boldsymbol{r}|^2}{1 - r_{sh}/r} + r^2 |\Delta \Omega|^2\right)/c^2 \tag{4}$$

Take $\Delta \Omega = 0$. Then with $\Delta t_p = 0$

$$\frac{dr}{dt_c} = c\left(1 - \frac{r_{sh}}{r}\right) \tag{5}$$

Discussion
We have successfully introduced special relativity into our model.
By introducing relativity this way we perform a few tricks.

- We neglect the real part of the quaternionic position observable. In our model it plays no essential part in dynamics.
- Clocks do not count progression steps. Instead they tend to measure coordinate time differences.
- We may shift from the global progression parameter t to the local coordinate time t_c.
- We may combine the resulting observed space with coordinate time into a Minkowski/Lorentzian space.

As a consequence
- We then shift from 2^n-on/Riemannian space to Minkowski/Lorentzian space.
- Most physicists will use Clifford, Jordan and Grassmann algebras rather than 2^n-on algebras.
- With these algebras they can use complex analysis instead of the more complicated 2^n-on analysis.
- But if they do so, they are confronted with unintuitive selection features.
- In the new space the quaternion waltz becomes an odd operation.
- Spinors can help in order to cope with these changes.

Can we do without relativity?
Yes.
- Skip coordinate time.
- Use clocks that measure the progression parameter.

However, you would have to fight existing conventions.

Inertia and progression step
The covering field represents the influence of the universe of all particles. According to the findings about [inertia][153], the change ΔE since the last progression step of the corresponding curvature field E determines the acceleration that a local particle senses during the current progression step.

This results in the acceleration $\frac{\partial \mathbf{v}}{\partial t}$ of the particle.

$$\Delta \boldsymbol{E}(\boldsymbol{r},t) = G \cdot \frac{\partial \mathbf{v}}{\partial t} \tag{1}$$

Redefinition
If we want to use the Schrödinger picture, rather than the Heisenberg picture, then it is better not to use unitary transforms, because they

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[153] Influence; Inertia

change the operators by acting on the eigenvectors of the operators. Instead the subspace should be redefined without touching eigenvectors.

Let us suppose that there exists a dynamical equivalent of the traditional quantum logic. The equivalent of a move of a physical item in the lattice of propositions is a redefinition of a subset of the propositions. The redefinition occurs in terms of atomic predicates that describe the properties of the physical items. In the Hilbert space this corresponds with a redefinition of a relevant part of the Hilbert subspace in terms of the eigenvectors that belong to the new eigenvalues.

The redefinition concerns the Hilbert space which represents the current static status quo. The step transforms the current version of the Hilbert space into a past version of the Hilbert space and it transforms a future version of the Hilbert space into the new current version. This is interesting in the light that a Hilbert field exists that controls the relation between the past, the current and the future versions of the Hilbert fields. For that reason we will call this special Hilbert field the **adventure field**. A transform that controls dynamics converts a future Hilbert space into the new current Hilbert spaces and it converts the current Hilbert spaces into a past Hilbert space. This transform will be called **progression transform**. The local blurs that characterize the adventure field form boundary conditions for the local transfer characteristics of the progression transform. Each item type is surrounded by a characteristic blur.

A progression transform that moves Hilbert subspaces without touching the eigenvectors of normal operators will be called a **redefiner**. The effect of the action of the redefiner on expectation values of operators must be similar to the effect of the trail of parallel unitary transforms treated in the previous paragraphs. While the set of parallel trails of unitary transforms act in the Heisenberg picture, the redefiner acts in the Schrödinger picture. As indicated earlier, the redefiner has an equivalent in the dynamic version of quantum logic.

In order to achieve the same effect as the Heisenberg picture, the Hilbert subspace redefiner must to a large degree have similar properties as the trails of parallel infinitesimal unitary transformations that are used to move the subspace in the Heisenberg picture. The redefinition keeps the inner products of vectors intact. Where unitary transforms rotate vectors around the origin of a Hilbert space, the redefiner takes subspaces of a potential future Hilbert space in order to redefine them into subspaces of the new current Hilbert space. In contrast to a unitary transform the redefiner does not change the eigenvectors of normal operators. Thus, it leaves the operators untouched. Like the trails of unitary transforms the redefinition works in infinitesimal steps. These infinitesimal actions also form trails. In this way the manipulated subspace can move close to continuously through Hilbert space. Where the redefiners act on subspaces, the trails of unitary transforms redefine operators.

During this process the subspace may change its configuration. This may include a change that corresponds to the change of type definitions of atomic predicates. The redefiner steps from one stationary situation to the next. The Schrödinger picture conforms to the description with a redefiner. The result for the position of the locator must be the same as it was under the influence of the set of parallel infinitesimal unitary operators in the Heisenberg picture. The redefiner moves the subspace such that the new locator position is similar to the value as was established by the redefined position operator. It means that during the redefiner step the position of the locator undergoes an infinitesimal number transform that is equivalent to the infinitesimal transform that is established by the redefined position operator. That redefinition was caused by the parallel infinitesimal unitary transforms.

Trails
In fact the Δt step characterizes the redefinition step. The subsequent replacement of vectors and the replacement of the corresponding eigenvalues can be interpreted as a rather continuous movement of the

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corresponding characteristic subjects. Here we encountered ten different trails.

1. The trail of subsequent manipulators (infinitesimal unitary transforms or infinitesimal redefiners) that each perform an infinitesimal action.
2. The trail of subspaces, which with respect to the manipulators are characteristic for the considered item.
3. The trail of corresponding "action values" of the redefiner.
4. The trails of corresponding "action values" of the unitary transforms.
5. The trail of eigenvectors $|q_t>$
6. The trail of corresponding observables Q_t.
7. The trail of corresponding observed expectation values q_t.
8. The trail of values $\psi(q_t)$ of a wave function.
9. This, on its turn corresponds to a trail of a state in coordinate space
10. And a trail of that state in Hilbert space.

Cycles

It is quite possible that locally subsequent steps are done in cycles of two or more steps. It is obvious that movements inside an item are cyclic. In ideal circumstances these movements are harmonic.

Redefiner

The concept of dynamic manipulator gives us reason to introduce a **new type of actuator**: the redefiner $Ɍ$. This actuator moves subspaces, but leaves vectors untouched. It works in infinitesimal steps. In the Hilbert book model its activity fits in the conversion from an actual Hilbert space to the next Hilbert space. It is easily interpreted as a function $Ɍ_t$ of the progression parameter t. Its scope spans the subsequent Hilbert spaces. The effect of each step on an item is similar to the effect of a set of parallel infinitesimal unitary transforms $\{U_{ts}\}_s$. The current "action value" of the redefiner is a number, which is close to unity. It is an "average" of the "actions values" of the parallel infinitesimal uniforms that are active in the same step. The redefiner accepts 2^n-ons as "action values".

The redefiner has an equivalent in a dynamic quantum logic, where it redefines propositions that concern the same objects as are represented by the closed subspaces of the Hilbert space that are moved by $\mathcal{R}_t$. There seems to be no objection against the assumption that $\mathcal{R}_t$ has a global scope. If we take that point of view, then the progression parameter t also has a global scope.

With this interpretation, the redefiner is a universe-wide stepper. It transforms the universe from one static situation to the next static situation. These static situations are governed by extended quantum logic, which combines traditional quantum logic, the blur of representations of physical items and the Helmholtz/Hodge decomposition theorems. After each step a new static status quo of subspaces and fields is established. After the step the conditions have been changed. After each step the position of the physical item relative to the fields has changed, thus when the fields are not uniformly distributed, the item meets a different field configuration. On the other hand the fields represent the blurs of the individual items. Thus, when the position or the type of the item has changed, then the local configuration of the field has changed. This is the way that macroscopic dynamics takes place in quantum physics.

Equations of motion

Private continuity equation

Existence, transport, generation and annihilation of information carrying quanta is governed by a continuity equation. In short this equation runs:

$$\text{Total change within V} = \text{flow into V} + \text{production inside V} \qquad (1)$$

This integral equation corresponds to differential equations in which an information quantum density ρ_0, an information quantum current $\boldsymbol{\rho}$ and an information quantum source s will play a role.

Particles act as sources and drains. Private fields represent the currents and the static density distributions. Wave functions are private fields that represent the situation in the direct environment of particles.

The rotation free part of the private field corresponds to the divergence of the information QPAD, whose squared modulus corresponds to the probability density. The transverse part of the private field corresponds to curl of the information current QPAD, whose squared modulus corresponds to the information current. Together the private fields form the covering field. The covering field is the superposition of all private fields. The curvature field is derived from the decomposition properties of the covering field.

Particles

Fields are superpositions of QPAD's. These QPAD's are typical for corresponding particles and are attached to one or a small set of Hilbert vectors. The anchor points of the fields are eigenvectors of the strand operator. Thus the eigenvalue of these vectors are positions. All other properties of the particle are properties of its private field.

Particle types

Boson dynamics is controlled by U(1). Four boson types exist: photons, gluons, W type and Z type bosons. They all have spin ± 1. However, apart from the W type, bosons do not carry a charge. They are the messengers that transfer interactions. Photons and gluons are massless bosons. Z and W type bosons have mass. They mediate weak field forces. The photons mediate EM field forces. Photons have no charge. Gluons have color charge. The gluons mediate color (strong) field forces. All bosons can be considered as a pair of fermions. For that reason it might be sensible to attach in the Hilbert book model two Hilbert vectors to a single boson that each attach to a fermion.

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Quark color dynamics is controlled by SU(3) and quark sign flavor is controlled by SU(2). Six quark types exist: up, down, charm, strange, top and bottom. These quarks are grouped into 3×2 sign flavors. They all have spin ±½ and fractional electric charge. Further, each quark has one of three color charges: red, green or blue. The quarks have mass. The Hilbert book model attaches a quark to a single Hilbert vector. The attached private field can be red, green or blue. An SU(3) group treats the corresponding color conversions.

The color neutral hadrons are aggregates and group into baryons and mesons. The baryons consist of three quarks and the mesons consist of a quark and an anti-quark. In this picture the gluons are not counted. For the aggregates the anchor points carry together a centralized probability distribution that represents the influence of a single charge.

Eight types of gluons exist. Gluons have color charge. The eight gluons correspond to the eight generators of the SU(3) group. They each attach to two Hilbert vectors.
Glueballs are aggregates that consist of gluons.

Six lepton types exist. All leptons have spin ±½. The three neutrino types have no charge and relative little mass. The electron, the muon and the tau particle are all massive particles. The leptons are attached to a single Hilbert vector. These leptons are grouped into 3×2 sign flavors. The sign flavors correspond to an SU(2) group.

Gen I		Gen II		Gen III		type
Up	Down	charm	Strange	top	Bottom	**quark**
2/3	-1/3	2/3	-1/3	2/3	-1/3	charge
low	high	low	high	low	high	sign flavor
						red
						green
						blue
ν_e	e electron	ν_μ	μ muon	ν_τ	τ tau	**lepton**
0	-1	0	-1	0	-1	charge

An SU(3) group treats colors for quarks. An SU(2) group treats sign flavor conversions for both quarks and leptons. The W and Z bosons mediate sign flavors.

type	mediates	#
photon	Leptons, hadrons, $W^{\pm}$ EM	1
gluon	Quark color	8
W^+, W^-	Fermion sign flavor	1,1
Z	Fermion sign flavor	1

Photons and gluons are massless. Neutrinos are nearly massless.

Usually the Higgs boson is also included into the standard model. The Hilbert book model does not do that. It takes the position that the reason for a particle to cause curvature is already contained in the structure of the private field.

In the strand model bosons are attached to a single strand, quarks are attached to two strands and leptons attach to three strands. In the Hilbert book model all elementary fermions attach to a single Hilbert vector. Bosons attach to two Hilbert vectors. One vector attaches to a fermion and the other vector attaches to an anti-fermion.

Interactions

QPAD's, which represent particles, move and rotate. That is interpreted as a movement / rotation of the corresponding item. Interactions may change the form of the QPAD's. Three types of change are discerned:

In strand theory the *first Reidemeister move*, or *type I move*, or *twist*, is the addition or removal of a twist in a corresponding strand. In Hilbert space it involves the approach of a single Hilbert vector into the realm of a particle. The twist, is related to the electromagnetic interaction. Two twist directions are possible. The twists form an SU(1) group.

In strand theory the *second Reidemeister move*, or *type II move*, or *poke*, is the addition or removal of a bend of one strand under (or over) a second strand. In Hilbert space it involves the interaction of two Hilbert vectors in the realm of a particle, where one Hilbert vector approaches the particle. The poke is related to electro-weak interaction. Three basic pokes exist. The pokes form an SU(2) group.

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In strand theory the ***third Reidemeister move***, or ***type III move***, or ***slide***, is the displacement of one strand segment under (or over) the crossing of two other strands. In Hilbert space it involves the simultaneous interaction of three Hilbert vectors in the realm of a particle, where one Hilbert vector approaches the particle. The slide is related to electro-strong interaction. The slides form an SU(3) group.

Each Reidemeister move generates a single corresponding observable quant or annihilates a single potentially observable quant.

Schrödinger equation

When the spin has a constant direction:
The first term on the left side signifies the quantum generation rate per time step.
The second term indicates the influence of the electric field on this rate.
The first term on the right signifies the generation rate per path length.
The second term indicates the influence of the vector potential on this rate.
The square dependence indicates the increasing alignment of spin with the movement.

$$(\hbar\omega - q\tilde{V})\tilde{\psi}(x,t) = (\hbar\boldsymbol{k} - q\tilde{\boldsymbol{A}})\tilde{\psi}(x,t) \tag{1}$$

$$(i\hbar\partial_t - qV)\psi = \frac{1}{2m}(-i\hbar\boldsymbol{\nabla} - q\boldsymbol{A})^2\psi \tag{2}$$

Pauli equation

When the spin has no constant direction:
The density $\rho(x,t)$ and the Euler angles α,β,γ define the Pauli equation:

$$\Psi(x,t) = \sqrt{\rho}\, e^{\frac{i\alpha}{2}} \begin{bmatrix} \cos\left(\frac{\beta}{2}\right) e^{\frac{i\gamma}{2}} \\ i\sin\left(\frac{\beta}{2}\right) e^{-\frac{i\gamma}{2}} \end{bmatrix} \tag{1}$$

Due to the half angles, the two-component matrix is not a vector, but a spinor.

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$$\sigma = \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right) \tag{2}$$

$$1 \mapsto I, \quad i \mapsto i\,\sigma_1, \quad j \mapsto i\,\sigma_2, \quad k \mapsto i\,\sigma_3 \tag{3}$$

Pauli's equation for the evolution of a free quantum particle with spin ½ is:

$$i\hbar\partial_t \Psi = -\frac{\hbar^2}{2m} (\sigma \nabla)^2 \Psi \tag{4}$$

$$(i\hbar\partial_t - qV)\Psi = \frac{1}{2m} (-i\hbar\nabla - qA)^2\Psi - \frac{q\hbar}{2m}\,\sigma B\Psi \tag{5}$$

The last term shows the influence of spin.

Dirac equation

The final and most detailed description of elementary fermions, the Dirac equation, results from combining all three ingredients:

1. the relation between the quantum of action and the phase of the wave function,
2. the relativistic mass–energy relation,
3. spin 1/2.

$$i\frac{\partial\psi}{\partial t} = (\alpha\nabla \pm i\beta m)\psi \tag{1}$$

$$\alpha = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}, \begin{bmatrix} 0 & j \\ -j & 0 \end{bmatrix}, \begin{bmatrix} 0 & k \\ -k & 0 \end{bmatrix} \tag{2}$$

$$\gamma_5 = \beta = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{3}$$

$$\psi = \sqrt{\rho}\, e^{i\delta}\, L(v) R\left(\frac{\alpha}{2}, \frac{\beta}{2}, \frac{\gamma}{2}\right) \tag{4}$$

$\rho(r, t)$ is the probability density.
δ is a phase which represents the relative importance of particle and antiparticle density.

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α, β *and* γ are Euler angles. They describe the average local orientation and phase of the spin axis.
(this defines a rotating spin vector)
$\boldsymbol{v}(r, t)$ is the average local Lorentz boost.
LR is an abbreviation for the boosted and rotated unit spinor.(quantum)

The probability amplitude ψ moves and rotates and individually the quanta carry position, momentum and angular momentum (including spin) information.

Fields

It is clear that the physical fields play an important role in nature. They form an indispensable ingredient in the establishment of dynamics. Each physical item follows a path through a set of universe wide fields. The static gravitational field, the electrostatic field and the electromagnetic field are all subjected to the Helmholtz decomposition theorem. The difference between the gravitational field and the electromagnetic field is that the masses are non-negative and the electric charges are, apart from a sign, always the same. All other fields also have charges that on the long range will compensate each other. The gravitation field can be seen as being derived from the curvature set by the decomposition of the covering field. The covering field is the superposition of all fields but the gravitation field.
When the path with respect to the gravitation field corresponds to a unit speed curve then that field executes no action onto that item. Only the gravitation field keeps its long range because its charges do not compensate each other's potentials. They only compensate each other's forces.

More fields

There exists a list of fields with shorter ranges than the range of the gravitation field and the range of the electromagnetic fields. The electro-weak field and the electro-strong field are not treated here in detail. The action represented by a complete Lagrangian indicates how fields appear in the argument of a manipulator. See [Lagrangian of the world](#)[154]

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[154] Appendix; Thoughts; The world's action

for a complete survey of terms. Mendel Sachs[155] has also found a way to bring all terms under the same hood.

Lagrangian

$$\mathcal{L} = \bar{\varphi}(i\hbar c\not{D} - mc^2)\varphi - \frac{1}{4\mu_0} F_{\mu\upsilon}F^{\mu\upsilon} - \tfrac{1}{4}\sum_{a=1}^{3} W_{\mu\upsilon}^a W_a^{\mu\upsilon} \tag{1}$$

$$- \tfrac{1}{4}\sum_{a=1}^{8} G_{\mu\upsilon}^a G_a^{\mu\upsilon}$$

Where $\mu, \upsilon = 1, 2, 3$
The first term concerns the affected particle.
The second term concerns electromagnetic interactions. Reidemeister twists. SU(1).
The third term concerns unbroken weak interactions. Reidemeister moves. SU(2).
The fourth term concerns unbroken strong interactions. Reidemeister slides. SU(3).

$$\not{D} = \gamma^\sigma D_\sigma = \gamma^\sigma(\partial_\sigma - iqA_\sigma) \tag{2}$$

$$F_{\mu\upsilon} = \partial_\mu A_\upsilon - \partial_\upsilon A_\mu \tag{3}$$

$$W_{\mu\upsilon}^a = \partial_\mu W_\upsilon^a - \partial_\upsilon W_\mu^a - g\, f^{abc}\, W_\mu^b W_\upsilon^c \tag{4}$$

$$G_{\mu\upsilon}^a = \partial_\mu G_\upsilon^a - \partial_\upsilon G_\mu^a - g\, f^{abc}\, G_\mu^b G_\upsilon^c \tag{5}$$

φ is the private field of the affected particle.
A_υ, W_υ^a and W_υ^a are vector potentials of the corresponding subfields
g is the gauge coupling constant[156]. The quantity f^{abc} is the structure constant[157] of the gauge group.

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[155] Appendix; Thoughts; Representing multiple fields

[156] http://en.wikipedia.org/wiki/Coupling_constant

[157] http://en.wikipedia.org/wiki/Algebra_over_a_field#Structure_coefficients

Path through field

The text in this section is borrowed from Wikipedia.

In a [Riemannian manifold](158) M with [metric tensor](159) g, the length of a continuously differentiable curve $\gamma : [a, b] \to M$ is defined by

$$L(\gamma) = \int_a^b \sqrt{g_{\gamma(t)(\dot{\gamma}(t), \dot{\gamma}(t))}} \, dt \qquad (1)$$

The distance $d(p, q)$ between two points p and q of M is defined as the [infimum](160) of the length taken over all continuous, piecewise continuously differentiable curves $\gamma : [a, b] \to M$ such that $\gamma(a) = p$ and $\gamma(b) = q$. With this definition of distance, geodesics in a Riemannian manifold are then the locally distance-minimizing paths, in the above sense.

The minimizing curves of L in a small enough [open set](161) of M can be obtained by techniques of [calculus of variations](162). Typically, one introduces the following [action](163) or [energy functional](164)

$$E(\gamma) = \tfrac{1}{2} \int_a^b g_{\gamma(t)(\dot{\gamma}(t), \dot{\gamma}(t))} \, dt \qquad (2)$$

It is then enough to minimize the functional E, owing to the [Cauchy–Schwarz inequality](165)

$$L(\gamma)^2 \leq 2(b - a)\, E(\gamma) \qquad (3)$$

with equality if and only if $|d\gamma/dt|$ is constant.

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[158] http://en.wikipedia.org/wiki/Riemannian_manifold

[159] http://en.wikipedia.org/wiki/Metric_tensor

[160] http://en.wikipedia.org/wiki/Infimum

[161] http://en.wikipedia.org/wiki/Open_set

[162] http://en.wikipedia.org/wiki/Calculus_of_variations

[163] http://en.wikipedia.org/wiki/Action_(physics)

[164] http://en.wikipedia.org/wiki/Energy_functional

[165] http://en.wikipedia.org/wiki/Cauchy%E2%80%93Schwarz_inequality

The [Euler–Lagrange](166) equations of motion for the functional E are then given in local coordinates by

$$\frac{d^2 x^\lambda}{dt^2} + \Gamma^\lambda_{\mu\nu} \cdot \frac{dx^\mu}{dt} \cdot \frac{dx^\nu}{dt} = 0 \tag{4}$$

where $\Gamma^\lambda_{\mu\nu}$ are the [Christoffel symbols](167) of the metric. This is the **geodesic equation**.

Calculus of variations

Techniques of the classical [calculus of variations](168) can be applied to examine the energy functional E. The [first variation](169) of energy is defined in local coordinates by

$$\delta E(\gamma)(\varphi) = \left.\frac{\partial}{\partial t}\right|_{t=0} E(\gamma + t\,\varphi) \tag{1}$$

The [critical points](170) of the first variation are precisely the geodesics. The second variation is defined by

$$\delta^2 E(\gamma)(\varphi, \psi) = \left.\frac{\partial^2}{\partial t^2}\right|_{t=0} E(\gamma + t\,\varphi + s\psi) \tag{2}$$

In an appropriate sense, zeros of the second variation along a geodesic γ arise along [Jacobi fields](171). Jacobi fields are thus regarded as variations through geodesics.

By applying variational techniques from [classical mechanics](172), one can also regard [geodesics as Hamiltonian flows](173). They are solutions of the

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[166] Appendix; Derivation of the one dimensional Euler Langrange equation

[167] Equations of motion; Path through field; Christoffel symbols

[168] http://en.wikipedia.org/wiki/Calculus_of_variations

[169] http://en.wikipedia.org/wiki/First_variation

[170] http://en.wikipedia.org/wiki/Critical_point_(mathematics)

[171] http://en.wikipedia.org/wiki/Jacobi_field

[172] http://en.wikipedia.org/wiki/Classical_mechanics

associated [Hamilton–Jacobi equations](http://en.wikipedia.org/wiki/Hamilton%E2%80%93Jacobi_equation)[174], with (pseudo-)Riemannian metric taken as [Hamiltonian](http://en.wikipedia.org/wiki/Hamiltonian_mechanics)[175].

Affine geometry

A **geodesic** on a smooth manifold M with an [affine connection](http://en.wikipedia.org/wiki/Affine_connection)[176] ∇ is defined as a curve $\gamma(t)$ such that [parallel transport](http://en.wikipedia.org/wiki/Parallel_transport)[177] along the curve preserves the tangent vector to the curve, so

$$\nabla_{\dot{\gamma}}\dot{\gamma}(t) = 0 \tag{1}$$

at each point along the curve, where $\dot{\gamma}$ is the derivative with respect to t. More precisely, in order to define the covariant derivative of $\dot{\gamma}$ it is necessary first to extend $\dot{\gamma}$ to a continuously differentiable imaginary Hilbert field in an [open set](http://en.wikipedia.org/wiki/Open_set)[178]. However, the resulting value of the equation is independent of the choice of extension.

Using [local coordinates](http://en.wikipedia.org/wiki/Local_coordinates)[179] on M, we can write the **geodesic equation** (using the [summation convention](http://en.wikipedia.org/wiki/Summation_convention)[180]) as

$$\frac{d^2 x^\lambda}{dt^2} + \Gamma^\lambda_{\mu\nu} \cdot \frac{dx^\mu}{dt} \cdot \frac{dx^\nu}{dt} = 0 \tag{2}$$

where $x^\mu(t)$ are the coordinates of the curve $\gamma(t)$ and $\Gamma^\lambda_{\mu\nu}$ are the [Christoffel symbols](http://en.wikipedia.org/wiki/Christoffel_symbol)[181] of the connection ∇. This is just an ordinary differential equation for the coordinates. It has a unique solution, given an initial position and an initial velocity.

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[173] http://en.wikipedia.org/wiki/Geodesics_as_Hamiltonian_flows
[174] http://en.wikipedia.org/wiki/Hamilton%E2%80%93Jacobi_equation
[175] http://en.wikipedia.org/wiki/Hamiltonian_mechanics
[176] http://en.wikipedia.org/wiki/Affine_connection
[177] http://en.wikipedia.org/wiki/Parallel_transport
[178] http://en.wikipedia.org/wiki/Open_set
[179] http://en.wikipedia.org/wiki/Local_coordinates
[180] http://en.wikipedia.org/wiki/Summation_convention
[181] http://en.wikipedia.org/wiki/Christoffel_symbol

From the point of view of classical mechanics, geodesics can be thought of as trajectories of free particles in a manifold. Indeed, the equation $\nabla_{\dot{\gamma}} \dot{\gamma}(t) = 0$ means that the acceleration of the curve has no components in the direction of the surface (and therefore it is perpendicular to the tangent plane of the surface at each point of the curve). So, the motion is completely determined by the bending of the surface. This is also the idea of the general relativity where particles move on geodesics and the bending is caused by the gravity.

Christoffel symbols

If x^i, $i = 1, 2, ..., n$, is a local coordinate system on a manifold M, then the tangent vectors

$$e_\mu = \frac{\partial}{\partial x_\mu}, \qquad \mu = 1, 2, ..., n \tag{1}$$

define a basis of the tangent space of M at each point. The Christoffel symbols $\Gamma_{\mu\nu}^\lambda$ are defined as the unique coefficients such that the equation

$$\nabla_\mu e_\nu = \Gamma_{\mu\nu}^\lambda \cdot e_\lambda \tag{2}$$

holds, where ∇_μ is the [Levi-Civita connection](http://en.wikipedia.org/wiki/Levi-Civita_connection)[182] on M taken in the coordinate direction e_μ.

The Christoffel symbols can be derived from the vanishing of the covariant derivative of the metric tensor g_{ik}:

$$0 = \nabla_\lambda g_{\mu\nu} = \frac{\partial g_{\mu\nu}}{\partial x_\lambda} - g_{\eta\mu} \cdot \Gamma_{\mu\lambda}^\eta - g_{\mu\eta} \cdot \Gamma_{\nu\lambda}^\eta \tag{3}$$

By permuting the indices, and re-summing, one can solve explicitly for the Christoffel symbols as a function of the metric tensor:

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[182] http://en.wikipedia.org/wiki/Levi-Civita_connection

$$\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2} \cdot g^{\mu\nu} \cdot \left(\frac{\partial g_{\eta\nu}}{\partial x^{\lambda}} + \frac{\partial g_{\eta\lambda}}{\partial x^{\nu}} - \frac{\partial g_{\nu\lambda}}{\partial x^{\eta}} \right) \qquad (4)$$

where the matrix $(g^{\mu\nu})$ is an inverse of the matrix $(g_{\mu\nu})$, defined as (using the Kronecker delta, and Einstein notation for summation)

$$g^{\lambda\mu} \cdot g_{\mu\nu} = \delta^{\lambda}_{\nu} \qquad (5)$$

Although the Christoffel symbols are written in the same notation as tensors with index notation, they are **not** tensors, since they do not transform like tensors under a change of coordinates.
Under a change of variable from $(x^1, \ldots, x^n)$ to $(y^1, \ldots, y^n)$, vectors transform as

$$\frac{\partial}{\partial y^i} = \frac{\partial x^k}{\partial y^i} \cdot \frac{\partial}{\partial x^k} \qquad (6)$$

and so

$$\Gamma^k_{ij} = \frac{\partial x^p}{\partial y^i} \cdot \frac{\partial x^q}{\partial y^j} \cdot \Gamma^r_{pq} \cdot \frac{\partial y^k}{\partial x^r} + \frac{\partial y^k}{\partial x^m} \cdot \frac{\partial^2 x^m}{\partial y^i \partial y^j} \qquad (7)$$

where the underline denotes the Christoffel symbols in the y coordinate frame. Note that the Christoffel symbol does **not** transform as a tensor, but rather as an object in the jet bundle.
At each point, there exist coordinate systems in which the Christoffel symbols vanish at the point. These are called (geodesic) normal coordinates, and are often used in Riemannian geometry.
The Christoffel symbols are most typically defined in a coordinate basis, which is the convention followed here. However, the Christoffel symbols can also be defined in an arbitrary basis of tangent vectors e_{μ} by

$$\nabla_{e_{\mu}} e_{\nu} = \Gamma^{\lambda}_{\mu\nu} \cdot e_{\lambda} \qquad (8)$$

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The action along the live path

The integrated action S_{ab} is performed over a distance along the action trail or equivalently over a period of coordination time

$$S_{ab} = -\int_a^b m \cdot c^2 \cdot ds + matter\ terms \tag{1}$$

$$= -\int_{\tau_a}^{\tau_b} m \cdot c^2 \cdot \sqrt{1 - \left(\frac{v}{c}\right)^2} \cdot d\tau + matter\ terms$$

$$= \int_{\tau_a}^{\tau_b} \mathcal{L} \cdot d\tau$$

m is the mass of the considered item.
v is the speed in Q space.
$\mathcal{L}$ is the Lagrangian.

The first line of this formula can be considered as an integral along the trail in coordinate space or equivalently over the trail in Hilbert space. The next lines concern integrals over the corresponding path in observed space combined with coordinate time. It must be noticed that these spaces have different signature.

$$\mathcal{L} = -m \cdot c^2 \cdot \frac{ds}{d\tau} + matter\ terms \tag{2}$$

In general relativity, the first term generalizes (includes) both the classical kinetic energy and interaction with the Newtonian gravitational potential. It becomes:

$$m \cdot c^2 \cdot \frac{ds}{d\tau} = -m \cdot c \cdot \sqrt{g_{\alpha\beta} \cdot \dot{q}_\alpha \cdot \dot{q}_\beta} \tag{3}$$

$g_{\alpha\beta}$ is the rank 2 symmetric metric tensor which is also the gravitational potential. Notice that a factor of c has been absorbed into the square root. The matter terms in the Lagrangian $\mathcal{L}$ differ from those in the integrated action S_{ab}.

$$S_{ab\_matter} = - \int_a^b e \cdot A_\gamma \cdot dq^\gamma + \text{other matter terms} \tag{4}$$

The matter term in the Lagrangian due to the presence of an electromagnetic field is given by:

$$\mathcal{L} = - m \cdot c^2 \cdot \frac{ds}{d\tau} + e \cdot \dot{q}^\gamma \cdot A_\gamma + \text{other matter terms} \tag{5}$$

A_γ is the electromagnetic 4-vector potential.

Black hole

Classical black hole

According to classical mechanics the no-hair theorem[183] states that, once a black hole achieves a stable condition after formation, it has only three independent physical properties:

- mass,
- charge, and
- angular momentum.

The surface gravity κ may be calculated directly from Newton's Law of Gravitation184, which gives the formula

$$\kappa = \frac{Gm}{r^2} \tag{2}$$

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[183] http://en.wikipedia.org/wiki/No-hair_theorem
[184] http://en.wikipedia.org/wiki/Newton%27s_Law_of_Gravitation

where m is the mass of the object, r is its radius, and G is the [gravitational constant](185). If we let $\rho = m/V$ denote the mean density of the object, we can also write this as

$$\kappa = \frac{4\pi}{3} G \rho r \tag{3}$$

For fixed mean density ρ, the surface gravity κ is proportional to the radius r. (4)

[Sciama](186) relates G to the potential that is raised by the community of particles. For fixed mean density ρ this is shown by

$$\Phi = -\int_V \frac{\rho}{r} dV = -\rho \int_V \frac{dV}{r} = \rho 2\pi R^2 \tag{5}$$

$$G \approx \frac{-c^2}{\Phi} = \frac{-c^2}{\rho 2\pi R^2} \tag{6}$$

Here R is the current radius of the universe.

Simple black hole

The Schwarzschild radius r_S for a non-rotating spherical black hole is

$$r_S = \frac{2Gm}{c^2} \tag{1}$$

General black hole

More generally holds

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[185] http://en.wikipedia.org/wiki/Gravitational_constant
[186] Influence;Inertia

$$dM = \frac{\kappa}{8\pi} dA + \Omega dJ + \phi dQ$$

where

- M is the mass/energy,
- A is the horizon area,
- Ω is the angular velocity,
- J is the angular momentum,
- ϕ is the electrostatic potential,
- κ is the [surface gravity](),
- Q is the electric charge.

For a stationary black hole, the horizon has constant surface gravity. It is not possible to form a black hole with surface gravity. $\kappa = 0$.

Quantum black hole

When quantum mechanical effects are taken into account, one finds that black holes emit thermal radiation ([Hawking radiation]()) at temperature

$$T_H = \frac{\kappa}{2\pi}$$

A quantum black hole is characterized by an entropy S and an area A. The entropy of a black hole is given by the equation:

$$S = \frac{c^3 kA}{4\hbar G}$$

The [Bekenstein-Hawking Entropy]() of three-dimensional black holes exactly saturates the bound

$$S = \frac{kA_P}{4}$$

where A_P is the two-dimensional area of the black hole's event horizon in units of the Planck area,

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$$A_P = l_p^2 = \frac{\hbar G}{c^3}.$$

In the Hilbert book model this equals the number of granules that covers the horizon of the black hole.

The horizon of the black hole is an event horizon because information cannot pass this horizon. (Near the horizon the speed of light goes to zero.)

Holographic principle

The holographic principle[187] states that the entropy contained in a closed surface in space equals the entropy of a black hole that has absorbed everything that is contained in this surface.

In the Hilbert book model it means that if the surface is considered as a sparsely covered horizon, then that sparse horizon contains as many granules as the densely covered horizon of the corresponding black hole. It also means that the maximum entropy that can be contained inside a surface corresponds to a dense coverage with granules of that surface. In the Hilbert book model, any dense or sparse horizon reflects via its contained entropy the number of granules that are contained in the corresponding volume.

We might extend this picture by stating that the number of granules in a volume corresponds with the entropy in the volume. In the Hilbert book model the number of granules corresponds to the number of Hilbert vectors that are attached to a QPAD. It also corresponds to the number of anchor points of the primary physical fields.

The eigenvectors of the strand operator correspond to quantum logical propositions that represent physical particles. These propositions have a binary yes/no value. In the extended model these propositions get extra content via the attached QPAD's.

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[187] http://en.wikipedia.org/wiki/Holographic_principle

Chandrasekhar limit

The [Chandrasekhar limit](http://en.wikipedia.org/wiki/Chandrasekhar_limit)[188] is an upper bound on the mass of a stable white dwarf star:

$$M_{limit} = \frac{\omega_3^0 \sqrt{3\pi}}{2} \left(\frac{\hbar c}{G}\right)^{3/2} \frac{1}{(\mu_e m_H)^2}$$

where:

- $\hbar$ is the reduced Planck constant
- c is the speed of light
- G is the gravitational constant
- μ_e is the average molecular weight per electron, which depends upon the chemical composition of the star.
- m_H is the mass of the hydrogen atom.
- $\omega_3^0 \approx 2.018236$ is a constant connected with the solution to the [Lane-Emden equation](#).

Approximately:

$$M_{limit} \propto \frac{M_P^3}{m_H^2}.$$

Where

$$M_P = \sqrt{\hbar c / G} \text{ is the Planck mass}$$

Birth of the universe

The unit sphere of the separable Hilbert space H is an affine space. All unit size eigenvectors end in this sphere.

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[188] http://en.wikipedia.org/wiki/Chandrasekhar_limit

The eigenvectors of the strand operator are exceptional. They are surrounded by a QPAD *that installs the tendency to keep these vectors together*. The parameter of these distributions is taken from a background coordinate system. This means that also the eigenvectors of the strand operator possess a position in this background coordinate system. The background coordinate system is formed by the eigenspace of an operator that houses in the Gelfand triple Ħ of the Hilbert space Ħ. The coupling between the eigenvectors of the strand operator and the eigenspace of the operator in the rigged Hilbert space that provides the background coordinate system is not precise. It is stochastic and of the order of the Planck-length. That is why the granules have this size.

The eigenvectors of the strand operator all touch a granule. The relation with quantum logic means that the Hilbert vector stands for a proposition that has a yes/no value. In case of the Hilbert vectors that are attached to the granules the yes/no value represents group membership. Thus each granule represents a bit of information.

For the eigenvectors vectors of the strand operator a densest packaging exists. It means that in that condition the QPAD's have shrunk to their smallest possible location difference.

Assumption 1: *In that condition, due to the properties of the QPAD's, the mutual tension works asymmetrically*.

This asymmetry means that in a surface that is formed by a set of densely packed granules the tension on one side is stronger than the surface tension at the other side. As a consequence the final configuration of a densest packaging becomes an empty bubble.

In the starting condition all eigenvectors of the strand operator are densely packed in one assembly.

Assumption 2: *After that moment the packaging density relaxes*.

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The number of granules does not change. Thus, during this spreading the total entropy does not change.

The package may fall apart in several separated subassemblies and a large series of single or more loosely packed granules. For the single and the more loosely packed granules the corresponding QPAD's fold out. The densely packed subassemblies take again a bubble shape.

This process may occur instantly or gradually, but most probably it will be done in a sequence of these two possibilities.

First occurs a sudden change of scale between the strand operator in the separable Hilbert space **Ħ** and the GPS operator that delivers the background coordinate system and that resides in the rigged Hilbert space **Ħ**. It is possible that originally the bubble covered the whole of the unit sphere of the Hilbert space **Ħ,** or it may just cover a finite dimensional subspace of **Ħ**. This means that the bubble contains an infinite or a finite amount of granules, which suddenly get diffused in a much larger space. That space is affine like the unit sphere of the Hilbert space **Ħ**. The diffusion takes place at every occupied location in the background coordinate system.

This kind of universe has no spatial origin or it must be the center of the outer horizon. With the aid of the background coordinate system, it will be possible to indicate a center of that universe. Each item in this universe has its own private information horizon. This horizon is set by the reach of the light that has been travelling since the birth of the universe. As long as this light does not reach the outer horizon that sub-universe looks isotropic. A multitude of such sub-universes exist that need not overlap. However, they all look at their border at an image of part of the start horizon. Such, sub-universes obey the [cosmological principle](http://en.wikipedia.org/wiki/Cosmological_principle)[189].

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[189] http://en.wikipedia.org/wiki/Cosmological_principle

In the next phase the further expansion occurs gradually. Because the QPAD's that are attached to the granules install a tendency for the granules to stay together, a different motor must be present behind this expansion. This motor can be found in the fact that with increasing radius the number of pulling granules grows faster than the decrease of the forces that are executed by the fields of these granules that is caused by the increasing distance. In an affine space this is always and everywhere true. This effect is also the source of inertia.

Due to local attraction, loosely packed and single granules may reassemble in bubble shaped subassemblies. These subassemblies are known as black holes. Single granules and small aggregates of granules are known as elementary particles, nuclei or atoms.

Much larger aggregates may be formed as well but these are not densely packed. Elementary particles can be categorized according to the configuration of their private fields. The private fields determine whether the particle is matter, with other words whether it has mass or not.

Inside the bubble the fact that the granule represents matter is not recognizable. It is only recognizable when the attached QPAD gets the chance to unfold. That condition is true when the granule is not part of a densely packed subassembly.

The requirements for the birth of the universe are:
1. The existence of a strand operator
2. The existence of QPAD's that install the tendency to keep these eigenvectors of the strand operator together
3. When the large numbers of eigenvectors are densely packed, then the assembly forms a bubble, because due to the properties of the QPAD's, the mutual tension works asymmetrically
4. In advance the eigenvectors of the strand operator are densely packed in one bubble.

5. A non-zero probability exists that the package density will be relaxed and the package falls apart. This may happen in a two stage process
   a. A sudden reduction of scale occurs
   b. Next a force that pulls the granules further away from each other exists

In the first episode of the universe the sudden scale change took place. This ripped the original bubble apart. Next a gradual further expansion took place.

The granules that move freely can at the utmost take one space step at every progression step. When the ratio of the space step and the progression step is fixed, then this determines a maximum speed of granules. A certain type of granules takes a space step at every progression step. That type transports information at the maximum possible speed.

When the path of these information transmitting particles is a straight line, then after a while, the other types of granules no longer get messages from the birth episode of the universe. But this need not be the case.

Since the messenger has a finite speed, it brings information from the past. First of all the speedy messenger and the slow addressee may have started from different locations. Further, due to curvature of space the path of the speedy messenger may take much longer than the duration of the much straighter path that the much slower addressee has taken. The information about the past that is included in the message might be close to the episode in which the granules were combined in one large bubble.

Thus despite the fact that most of the information that is generated during the birth of the universe is long gone, still some of that information may reach particles long after the instance of birth. When this information is

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interpreted it gives the impression of a [metric expansion of the universe](http://en.wikipedia.org/wiki/Metric_expansion_of_space)[190].

Optics

The optical Fourier transform (OTF) is an objective imaging quality characteristic for imaging devices in a similar way as the frequency transfer function qualifies the signal transfer function of a linearly operating electronic device. The transfer quality of a chain of linear signal transforming devices is characterized by the product of the frequency transfer functions of the elements of the chain. In a similar way the OTF of a chain of imaging devices is given by the product of the OTF's of the elements of the chain. However, this is a profound simplification of reality. The product rule only holds when the transfer characteristics of the imaging devices are spatially uniform over the complete input field of the separate imaging components. Further, the conditions in which the OTF's of the components are determined must be similar to the conditions in the chain. More in detail, this means that the angular distribution, the chromatic distribution and the homogeneity of the radiation must be identical.

In optics, the image sided spread function equals the convolution of the object sided spread function and the point spread function (PSF, the image of a point). The Fourier transform of the image sided spread function is equal to the product of the Fourier transform of the object sided spread function and the optical Fourier transforms (OTF's) of the imaging devices. When several imaging devices work in sequence, then the total optical transfer function of the imaging system equals the product of the transfer functions of the components.

If we restrict to a static situation and include the "depth" of the image, the static PSF is a three parametric function. Thus the OTF must have the

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[190] http://en.wikipedia.org/wiki/Metric_expansion_of_space

same number of parameters. Like the PSF the three dimensional OTF has a longitudinal component and a two dimensional transverse component. In most cases only the transverse component is used as an imaging quality characteristic. On-axis the transverse component is rotationally symmetric. Off-axis its modulus, the MTF, is symmetric but not rotationally symmetric. On-axis the MTF is presented as a one parametric curve in which only the positive axis is given. Off-axis the two extremes of the MTF are given. They correspond to radial and tangential directions.

Due to the fact that Hilbert fields are blurred Hilbert distributions, wave mechanics has much in common with wave optics. For each compact normal operator the Hilbert subspace that represents a physical item corresponds to a spread in Hilbert space and a corresponding spread in the eigenspaces of that normal operator. The distribution of this spread is represented in a [wave function](), or more correctly, in a probability distribution. For example the wave function that has the position as a variable corresponds to the triple consisting of a physical item, its Hilbert subspace representation and the position operator.

After a move of a physical item its position related wave function has much in common with the spread function that characterizes the blur of the image sided pictures in a linear operating imaging system. The physical fields that influence the physical item have an equivalent in the chain of imaging devices that transfers the image.

The product formula for the transfer functions relies on several preconditions. First of all it relies on the fulfillment of the requirement for sufficient spatial uniformity of the transfer. At all places where information is passed, the transfer characteristics must be sufficiently identical. The product formula has only validity in the spatial area where this requirement is fulfilled.

The transfer characteristics will be different for each Fourier component. Their quality will reduce with higher spatial frequencies.

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The final result can be computed in longitudinal direction by multiplication. In lateral direction these regions are restricted to areas where the transfer is locally sufficiently uniform. In the summation that is used to compute a sensible average the angular and chromatic distribution of the transferred information play a role. These distributions determine the summation coefficients. The extent of the region in which the considered transfer function is considered valid depends on the accuracy that is required for the result of the computation. Sign selections inside the radiation determine the polarization. Often in optics this feature and its influence is ignored. Coherence plays a role as well, but in practice optical imaging uses either nearly completely incoherent light or nearly completely coherent light.

In wave mechanics the wave function, which is taken just before the item moves, gets the role of the object. After a movement through a region of the fields the wave function has been changed. Its Fourier transform then equals the product of the Fourier transform of the original wave function and the wave transfer functions (WTF's) of the fields that influence the item. If several steps are taken in sequence, then the transfer functions of the passed field pieces must be multiplied in order to get the overall result. This transfer is affected in a similar way by spatial non-uniformity as the optical case.

In cylindrical imaging systems Seidel aberrations take their toll. When the system is folded or when lenses are not perfectly in line, also non-cylindrical influences will influence the imaging quality. The measurement and the specification of the OTF must cope with the spatial non-uniformity of the imaging characteristics of the imaging devices and with the angular and chromatic distribution of the radiation. The OTF also depends on the longitudinal location of the object and where the image is detected. This also occurs with the WTF of physical fields. Both in optics and in wave mechanics the precise locations of the "object" and the "image" are often not well determined. They are defined by spatial

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distributions in three dimensions. In both cases the angular and chromatic distributions of the contributing radiation influence the transfer. The final result is constituted by the weighted sum of all contributions.

With inhomogeneous (= incoherent) imaging the phases are ignored. These facts indicate the difference between the particle view and the wave view. From optics it is known that the modulation transfer function (MTF) is a proper imaging qualifier for inhomogeneous light imaging. In inhomogeneous imaging the imaging process can be properly described by ray tracing. Ray tracing has much similarities with the application of the path integral. However, ray tracing normally does not use arbitrary paths. In inhomogeneous imaging phases are scrambled. For holographic imaging the phase transfer function (PTF) or the whole OTF is the better measure. With holographic imaging the phases carry the depth information. Feynman's path integral can cover arbitrary paths because, according to Feynman's claim, interference via the phases eliminates the contributions of non-realistic paths. That is why in the path integral the angular distribution of the radiation plays no role.

In optics the image space is often a surface. In optics the OTF depends on the position in the object space. Off axis the OTF is not rotationally symmetric. The OTF also depends on the angular distribution and the chromatic distribution of the radiation. These dependencies also hold for the WTF in wave mechanics.

A longitudinal displacement of the image spread function with respect to the object spread function corresponds to an extra phase term in the longitudinal component of the Fourier transform of the image spread function. A lateral displacement corresponds to an extra phase term in the transverse component of the Fourier transform. In wave mechanics this holds for the respective components of the Fourier transform of the wave function after the move.

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The resemblance between optics and wave mechanics becomes striking when the discrete lens pack is replaced by a medium with a continuously varying refraction. In optics this happens with electron optical lenses that are used in imaging with charged particles.

When the point spread function is a function of three-dimensional position, then the OTF is also a three-parametric function of spatial frequency. The MTF is a symmetric function. However, the MTF is not rotationally symmetric (in 2D) or spherical symmetric (in 3D). On its vertical axis the MTF indicates the part of the energy of the radiation that is transferred by a given spatial frequency.

Veiling glare and halo

Due to reflections on refracting surfaces some of the energy of the radiation loses much of its spatial information content. As a consequence the MTF shows a sharp peak near zero spatial frequency. This phenomenon is called **veiling glare**. When the drop is not so fast the phenomenon is called **halo**.

Equivalents of veiling glare and halo can also occur in wave mechanics. In this way spurious radiation and a spurious halo can enter space. This can happen in the form of energy or in the form of matter. Spurious radiation contains no spatial information.

Strands

A strand is a notion introduced by [Christoph Schiller](http://www.motionmountain.net/research.html)[191]. In this e-paper I try to embed the notion of a strand into the context of a sequence of Hilbert spaces or similarly into the context of a sequence of traditional quantum logics. The strand interpretation used in this e-paper is a mixture of the interpretation of Christoph Schiller and my personal interpretation. Any difference with the original interpretation is for my account. The reader must take Schiller's interpretation as the most original. See also [braid theory](http://en.wikipedia.org/wiki/Braid_symmetry)[192].

In my interpretation:

> Strands are chains of granules in the eigenspace of a strand operator. One of the granules represents the current state of the chain. That granule is the anchor location of a QPAD. All granules in the chain obtain a position from a background coordinate system that is defined in the Gelfand triple. The anchor point coincides with a Hilbert vector, which is also an eigenvector of the strand operator.

Schiller's strands

The fundamental principle of the strand model is:

> **Planck units** are defined through crossing switches of strands.

An alternative fundamental postulate is:

> An **event** is the switch of a crossing between two strand segments.

The original interpretation of strands can be found in [Motion Mountain, volume VI](http://www.motionmountain.net/research.html)[193]. In Schiller's words:

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[191] http://www.motionmountain.net/research.html

[192] http://en.wikipedia.org/wiki/Braid_symmetry

[193] http://www.motionmountain.net/research.html

> **Strands** are one-dimensional curves in three-dimensional space that are closed or reach the border of space (or a horizon).
> Strands exist in relation to a 3D background coordinate system.
> Strands are fluctuating.
>
> The one-dimensional strand curve fluctuates and as a consequence the fields in its surround exist. Fluctuation of a strand becomes apparent when the strand is averaged over a short time range. In this way the strands cause a blur. That blur represents the field that goes together with the strand.

Simple elementary types are bosons. All tangles made of one strand are elementary particles of spin 1, thus, they are elementary vector bosons. Conversely, all elementary spin-1 particles are made of one strand, because other tangles do not reproduce the spin-1 behavior under rotations: only one-stranded tangles return to the original strand after a core rotation by 2π.

According to the strand model no Higgs boson are required.

The strand model predicts that apart from the six quarks and the graviton, no other two-stranded elementary particles exist in nature.

Leptons correspond with triples of tangled strands. The strand model predicts that apart from the six leptons, no other elementary particles made of three strands exist in nature.

More complex types are composed of the above mentioned elementary types.

Interaction is caused by one of three processes.
- The first process involves a single strand. It corresponds with normal electromagnetic interaction. It is characterized by the first Reidemeister move.
- The second process involves two strands. It corresponds with the electro-weak force. It is characterized by the second Reidemeister move.

- The third process involves three strands. It corresponds with the electro-strong force. It is characterized by the third Reidemeister move.

These interactions play in the direct environment of strand cores. According to Schiller, gravitational forces have their origin in the tails, relative far away of the cores. That is also the region where masses get their influence.

Strands and their fluctuations are *unobservable*. The only things that become observable from a strand are its crossing switches with itself or with other strands. Drawings of strands are made in order to clarify strand behavior. In that case strands are pictured in 3D space and the rotations are represented by rotating cores or knots.

The **tangle function** – the (short) time average of strand crossings – corresponds with a complex **QPAD.**

The strand theory does not say anything about the transfer of information to quanta.

Planck values
Schiller: Up to a numerical factor, the limit for every physical observable corresponds to the Planck value. (The limit values are deduced from the commonly used Planck unit values simply by substituting 4G for G.) According to Schiller these limit values are the true natural units of nature. In fact, the ideal case would be to redefine the usual Planck values for all observables to these extreme values, by absorbing the numerical factor 4 into the respective definitions. In the strand model, Schiller calls the limit values the corrected Planck units and assumes that the factors have been properly included.

Strand basics
A **crossing** between two strands has a position and a direction. It is the position where the distance between the strands has a minimum. The distance is measured

in terms of a selected background coordinate system. The distance is measured in corrected Planck-length units.

A **crossing switch** is a turn of the crossing over π radians. Via its infinitesimal geometry the crossing switch defines the action $\hbar/2$, the corrected Planck-length l_{Pl}, the corrected Planck-time t_{Pl} and the Boltzmann constant k.

Events are observable crossing switches of unobservable strands. Every event in nature is characterized by the corrected Planck-time, the corrected Planck-length, the Planck entropy, i.e., the Boltzmann constant k, and Planck's quantum of action $\hbar$ (for a full turn)

The *distance* between two particles is the maximum number of crossing switches that could appear between them. Length measurement is thus defined as counting corrected Planck-lengths.

The *time interval* between two events is the maximum number of crossing switches that could appear between them. Time measurement is thus defined as counting corrected Planck-times.

The physical ***action*** of a physical system evolving from an initial to a final state is the number of crossing switches that take place. Action measurement is thus defined as counting crossing switches. Physical action is thus a measure for the change that a system undergoes.

The *entropy* of any physical system is related to the total number of crossing switches that are possible. Entropy measurement is thus defined through the counting of potential crossing switches. The strand model thus states that any large physical system – be it made of matter, radiation, empty space or horizons – has entropy.

Strand table

Typical strand configurations:

Physical system	Strands	Tangle type
Vacuum	many infinite unknotted strands	unlinked
Dark energy	many fluctuating infinite strands	unlinked
Elementary vector boson	one infinite strand	knotted or unknotted curve
Quark	two infinite strands	rational tangle

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Lepton	three infinite strands	braided tangle
Meson, baryon	three or more infinite strands	rational tangle
Higher-order propagating fermion	two or more infinite strands	locally knotted or prime tangle
Virtual particles	open, unlinked and closed strands	trivial tangles, knots, links
Composed systems	many strands	separable tangles
Graviton	two infinite twisted strands	specific rational tangle
Gravity wave	many infinite twisted strands	many graviton tangles
Horizon	many tightly woven infinite strands	web-like tangle
Earliest form of the universe	Single closed strand	No tangles

See the Toolkit for more details on strands.

Unique aspects of the model

Fundament

- The model takes as basis the axioms of traditional quantum logic.
- It exploits the isomorphism between the set of propositions in this logic and the set of closed subspaces of an infinite dimensional separable Hilbert space H in which the inner product is defined over the division ring of the quaternions.
- A proposition that treats everything that can be said about a physical item represents that item. Thus, the model represents physical items.
- Traditional quantum logic and its partner the separable Hilbert space H cannot represent physical fields and they cannot represent dynamics.
- However, this basic model can be extended such that fields are attached to it. However, this extended model only represents a static status quo.
- A sequence of such extended models can represent dynamics.
- The separable Hilbert space H does not contain a useable GPS coordinate operator. Due to the granularity of its eigenspace, such a normal operator would introduce preferred directions in the imaginary part of that eigenspace.
- Instead the corresponding continuous GPS operator that resides in the corresponding rigged Hilbert space H can act as a background coordinate operator. Its eigenspace can be used to indicate the location of the field values. However, this operator cannot directly be used in order to locate the Hilbert vectors that represent particles.

- Instead a special normal operator whose eigenspace contains a set of freely located chains of granules can deliver the position observables. This operator is a strand operator.
- In each chain one granule represents the current position. It divides the chain in a past part and a future part.
- A QPAD takes care of the smoothness in the surround of the current granule. This attachment extends the separable Hilbert space.
- Particles are represented by a single Hilbert vector or by a small set of Hilbert vectors. These vectors are eigenvectors of the strand operator they correspond with the current granule of a corresponding chain and are blurred by a spread function that can be interpreted as a QPAD. The blur of the set of Hilbert vectors represents the private field of the particle and describes the cloud of quanta that carry the observable information about the particle. The quanta represent positions where the particle can be detected.
- The particle acts as the source or as the drain of these quanta. The cloud moves and rotates around a rotation axis.
- The superposition of all private fields constitutes a covering field.
- For a given coordinate system the static decomposition of the covering field into a rotation free part and one or two divergence free parts runs along curved lines. The local curvature value can be used to define a derived partner field of the covering field. This curvature field has all the characteristics of the gravitation field.
- The private fields of bosons are attached to a single unit size Hilbert vector and touch all other unit size Hilbert vectors.
- The private fields of quarks are attached to a pair of unit size Hilbert vectors and touch all other unit size Hilbert vectors.

- The private fields of leptons are attached to a triple of unit size Hilbert vectors and touch all other unit size Hilbert vectors.
- In interactions bosons take care of the transfer.
- Each electromagnetic interaction involves only one extra Hilbert vector.
- Each weak interaction involves an internal and an extra Hilbert vector.
- Each strong interaction involves two internal and one extra Hilbert vector.
- There are no more elementary kinds of interactions.
- The progression parameter that counts the subsequent Hilbert spaces is not our common notion of time, but it has certainly some relation with it.

Insights
- The Minkowski signature of spacetime must have its explanation in what occurs during a progression step.
- The Minkowski signature of spacetime forbids that coordinate time acts as the fourth dimension that goes together with 3D coordinate space.
- Momentum acts as a precondition of the next displacement step.
- The fourth dimension must be as granular as the 3D displacement.
- Fields act as a precondition for the next action step.
- The displacement, measured in Planck-length units, the progression step measured in Planck-time units, the action step measured in Planck constant sized units and the entropy step in Boltzmann constant sized units form the basic steps during an observable event.
- Action represents change. Entropy represents potential change.
- A five-fold coverage of the separable Hilbert space exists

- A double coverage is done by the two extra members of the Gelfand triple that forms the corresponding rigged Hilbert space. This coverage delivers the background coordinate system.
- Another double coverage is done by the primary fields. Together they form the covering field. However, the static covering field can be decomposed into a rotation free part and a divergence free part.
- For a given coordinate system a curvature field can be derived from this covering field. This delivers the fifth cover.

Together these six elements deliver a sandwich that can characterize a static status quo of the universe.

Discussion

Macro and micro

Up to so far, the treatise confines to macroscopic dynamics. Micro dynamics concerns movements that occur inside the representation of small physical items. For each small physical item, it concerns the movements that occur inside the subsequent subspaces that represent this item.

In order to stay inside the item, the internal movements must be quasi periodical. They can be combinations of oscillations and rotations. The harmonics oscillator and the spherical harmonics are well known examples.

The local manipulator can be seen as a complicated (Fourier?) transform. The functions, which describe quantum harmonic movements, seem to be invariant under the action of this manipulator. Thus it appears that micro dynamics occurs via a different process than macro dynamics.

Dynamic logic

The current trend in quantum logic development is to add axioms that change the static character of quantum logic in a more dynamic and operational logic. Logic of quantum actions (LQA[194]) adds unitary transforms as the source of dynamics. As we see in this article these transforms are not the real fundamental causes of dynamics. The fields that accompany the physical items form the more fundamental reason for the existence of dynamics. They control the redefinition of the actual propositions. To my knowledge the influences of physical fields are not yet covered by any dynamic logic theory.

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[194] http://www.vub.ac.be/CLWF/SS/BethPaper_Final.pdf

Conclusion

The fact that the set of propositions in traditional quantum logic is lattice isomorphic with the set of closed subspaces of an infinite dimensional separable Hilbert space H, offers the possibility to study with mathematical means what happens with the propositions.

Quantum logic is only a partial description of the fundamentals of quantum physics. It only describes the static skeleton in which the quantum dynamics takes place. It does not treat physical fields. However, traditional quantum logic can be extended into a wider logic, such that fields are also included. When this is done, the task of the fields appears to bring coherence between past, current and future versions of extended quantum logics and dynamics can simply be considered as the simultaneous step from a future version, to a current version and from the current version to a past version.

An important ruler of quantum dynamics is the influence that is exposed by the universe of items in the phenomenon inertia. It indicates the laws that govern the exchange of atomic predicates from enveloping propositions. It characterizes the fields as the sticky resistance of the universe of quantum logical propositions against unordered redefinition of their members. This shapes the dynamics of the logic that describes dynamic quantum physics.

The fields consist of basic constituents that can be interpreted as QPAD's. The covering field is the superposition of these basic constituents and the gravitation field is a curvature field that can be derived from the static decomposition characteristics of the covering field. This picture carries on the assumption that the configuration of the covering field causes the curvature of the coordinate system.

Blurred Hilbert distributions form Hilbert fields. Blurred elementary Hilbert distributions form the private fields of corresponding elementary particles. The blurs can be interpreted as probability distributions and as

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such they describe the sources of quantum noise. This means that the blurs also represent the probability of the generation, presence and annihilation of actual and virtual elementary particles.

The dynamics of the life path of an item can be described by a geodesic equation. The live environment can be considered as sets of 2^n-ons that locally resemble quaternion spaces or in a still smaller region resemble complex number spaces. These numbers constitute the values of the fields that influence the dynamics of the items. The analysis of the local infinitesimal dynamic step also reveals the origin of special relativity.

In our model a universe wide progression stepper exists. This is the progression parameter clock. Due to this fact the redefinitions are universe wide synchronized. It also means that in the model universe is controlled by a single dynamic redefiner. However, its actions are locally influenced by fields, which are directly connected to the items that are present in this environment.

Inertia influences macroscopic dynamics. Microscopic movements are governed by a different process. They are directly controlled by the local manipulator and relate to its invariant functions.

Trying to implement a complex quantum logical proposition in Hilbert space is indeed an elucidating experience.

In the Hilbert book model, fields have several functions and interpretations:

- From the analysis of inertia you can derive that they represent the sticky resistance of the community of propositions/physical-items against unordered change. A uniform movement is still considered as a well ordered change. Acceleration is considered as unordered change and goes together with field activity.

- Fields are constituted of blurred sets of Hilbert vectors. With other words Hilbert fields are blurred Hilbert distributions.
- The blur renders the field differentiable.
- The blur can be interpreted as a QPAD.
- Wave functions are probability amplitudes. No difference with private fields exists.
- Blurs can be squeezed and can be looked at in another coordinate representation, such as the canonical conjugated coordinates.
- Fields can be interpreted as the storage place of the conditions of future, present and past Hilbert spaces or equivalently as the storage place of the conditions of future, present and past versions of quantum logic systems.
- Like the Hilbert spaces and the quantum logics, the static fields describe a static status quo.
- Fields can be interpreted as the housing of annihilation and creation operators that act on actual or virtual particles.
- The probalistic nature of the fields invites their interpretation as clouds of quanta. These quanta represent potential realizations of Hilbert vectors that on their turn represent the anchor points of actual or virtual particles in past, present or future versions of traditional quantum logic propositions.
- In the view that uses the canonical conjugated coordinates the quantum cloud can be interpreted as a wave package.

Appendix

History of quantum logic

Around 1930 John von Neumann and Garrett Birkhoff were searching for an acceptable explanation of the results of experiments that showed that the execution of an observation of a very small object can completely destroy the validity of an earlier observation of another observable of that object. The Schrödinger equation that agreed with the dynamic behaviour of the particles already existed. Not much later Heisenberg's matrix formulation became popular as well. Quite soon the conclusion was made that something was fundamentally wrong with the logic behind the behaviour of small particles. These small objects show particle behaviour as well as wave behaviour and they show quantization effects. It was found that the distribution axiom of classical logic had to be changed. Soon it became apparent that the lattice structure of classical logic must be weakened from an orthocomplementary modular form to an orthocomplementary weakly modular lattice. The quantum logic was born. The next step was to find a useful mathematical presentation of this new logic. A historic review of what happened can be found in: "Quantum Theory: von Neumann" vs. Dirac; http://www.illc.uva.nl/~seop/entries/qt-nvd/. It includes extensions of the concept of Hilbert space and application of these concepts to quantum field theory. Another source is: http://www.quantonics.com/Foulis_On_Quantum_Logic.html.

Quantum logic

Elementary particles behave non-classical. They can present themselves either as a particle or as a wave. A measurement of the particle properties of the object destroys the information that was obtained from an earlier measurement of the wave properties of that object.
With elementary particles it becomes clear that that nature obeys a different logic than our old trusted classical logic. The difference resides in the modularity axiom. That axiom is weakened. The classical logic is congruent to an orthocomplemented modular lattice. The quantum logic

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is congruent to an orthocomplemented weakly modulare lattice. Another name for that lattice is orthomodular lattice.

Lattices

A subset of the axioms of the logic characterizes it as a half ordered set. A larger subset defines it as a lattice.

A lattice is a set of elements $a, b, c, ...$that is closed for the connections $\cap$ and $\cup$. These connections obey:

- The set is partially ordered. With each pair of elements a, b belongs an element c, such that $a \subset c$ and $b \subset c$.
- The set is a $\cap$ half lattice if with each pair of elements a, b an element c exists, such that $c = a \cap b$.
- The set is a $\cup$ half lattice if with each pair of elements a, b an element c exists, such that $c = a \cup b$.
- The set is a lattice if it is both a $\cap$ half lattice and a $\cup$ half lattice.

The following relations hold in a lattice:

$$a \cap b = b \cap a \tag{1}$$

$$(a \cap b) \cap c = a \cap (b \cap c) \tag{2}$$

$$a \cap (a \cup b) = a \tag{3}$$

$$a \cup b = b \cup a \tag{4}$$

$$(a \cup b) \cup c = a \cup (b \cup c) \tag{5}$$

$$a \cup (a \cap b) = a \tag{6}$$

The lattice has a partial order inclusion $\subset$:

$$a \subset b \Leftrightarrow a \subset b = a \tag{7}$$

A complementary lattice contains two elements n and e with each element a an complementary element a' such that:

$$a \cap a' = n \tag{8}$$

$$a \cap n = n \tag{9}$$

$$a \cap e = a \tag{10}$$

$$a \cup a' = e \tag{11}$$

$$a \cup e = e \tag{12}$$

$$a \cup n = a \tag{13}$$

An orthocomplemented lattice contains two elements n and e and with each element a an element a'' such that:

$$a \cup a'' = e \tag{14}$$

$$a \cap a'' = n$$

$$(a'')'' = a \tag{15}$$

$$a \subset b ⮀ b'' \subset a'' \tag{16}$$

e is the unity element; n is the null element of the lattice

A distributive lattice supports the distributive laws:

$$a \cap (b \cup c) = (a \cap b) \cup (a \cap c) \tag{17}$$

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$$a \cup (b \cap c) = (a \cup b) \cap (a \cup c) \tag{18}$$

A modular lattice supports:

$$(a \cap b) \cup (a \cap c) = a \cap (b \cup (a \cap c)) \tag{19}$$

A weak modular lattice supports instead:

There exists an element d such that

$$a \subset c \Leftrightarrow (a \cup b) \cap c = a \cup (b \cap c) \cup (d \cap c) \tag{20}$$

where d obeys:

$$(a \cup b) \cap d = d \tag{21}$$

$$a \cap d = n \tag{22}$$

$$b \cap d = n \tag{23}$$

$$[(a \subset g) \, and \, (b \subset g) \Leftrightarrow d \subset g \tag{24}$$

In an atomic lattice holds

$$\exists_{p \, \epsilon \, L} \, \forall_{x \, \epsilon \, L} \, \{x \subset p \Rightarrow x = n\} \tag{25}$$

$$\forall_{a \, \epsilon \, L} \, \forall_{x \, \epsilon \, L} \, \{(a < x < a \cap p) \Rightarrow (x = a \, or \, x = a \cap p)\} \tag{26}$$

p is an atom

Both the set of propositions of quantum logic and the set of subspaces of a separable Hilbert space H, have the structure of an orthomodular lattice. In this respect these sets are congruent.

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In Hilbert space, an atom is a pure state (a ray spanned by a single vector).

Classical logic has the structure of an orthocomplemented distributive modular and atomic lattice.
Quantum logic has the structure of an orthomodular lattice. That is an orthocomplented weakly modular and atomic lattice. The set of closed subspaces of a Hilbert space also has that structure.

Proposition
In Aristotelian logic a proposition is a particular kind of sentence, one which affirms or denies a predicate of a subject. Propositions have binary values. They are either true or they are false.

Propositions take forms like "*This is a particle or a wave*". In quantum logic "*This is a particle.*" is not a proposition.

In mathematical logic, propositions, also called "propositional formulas" or "statement forms", are statements that do not contain quantifiers. They are composed of well-formed formulas consisting entirely of atomic formulas, the five [logical connectives](http://en.wikipedia.org/wiki/Logical_connective)[195], and symbols of grouping (parentheses etc.). Propositional logic is one of the few areas of mathematics that is totally solved, in the sense that it has been proven internally consistent, every theorem is true, and every true statement can be proved. Predicate logic is an extension of propositional logic, which adds variables and quantifiers.

In Hilbert space a vector is either inside or not inside a closed subspace. A proper quantum logical proposition is "*Vector |f> is inside state s*".

In Hilbert space, an atomic predicate corresponds with a subspace that is spanned be a single vector.

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[195] http://en.wikipedia.org/wiki/Logical_connective

Predicates may accept attributes and quantifiers. The predicate logic is also called first order logic. A dynamic logic can handle the fact that predicates may influence each other when atomic predicates are exchanged.

Observation

In physics, particularly in quantum physics, a system **observable** is a property of the system state that can be determined by some sequence of physical operations. This paper distinguishes between measurements and observations.

- With an observation the state is considered as a linear combination of eigenvectors of the observable. An observation returns the statistical expectation value of the eigenvalue of the observable.
- A measurement transforms the observed state to one of the eigenvectors of the observable. What happens depends on the characteristics of the measuring equipment. The measurement can be seen as a combination of a transformation and an observation.

Depending on the characteristics of the measuring equipment a measurement and a clean observation can give the same result.

With this interpretation of the concept of observation it is possible to let states observe other states. A state might do a transformation before doing an observation but in general it fails the equipment to arrange that transformation. In nature observations are far more common than measurements.

Displacement in an isotropic medium

The coordinate transformations between inertial frames that correspond to [displacements in an isotropic medium](#)[196] form a group. They can be represented by a matrix.

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[196] http://en.wikipedia.org/wiki/Lorentz_transformation#Derivation

$$\begin{bmatrix} t' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & \delta \\ \beta & \alpha \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} \tag{1}$$

The group membership corresponds to relations between the elements α, β, γ and δ

Consider the uniform motion of the origin of the frame K'. In the K' frame it has coordinates $(t', z' = 0)$, while in the K frame it has coordinates $(t, z = vt)$. This leads to

$$\beta = -v\alpha \tag{2}$$

The motion of the origin of the frame K gives

$$\beta = -v\gamma \tag{3}$$

With $\alpha = \gamma$ the inverse transform will be

$$\begin{bmatrix} t \\ z \end{bmatrix} = \frac{1}{\gamma^2 + v\delta\gamma} \begin{bmatrix} \gamma & -\delta \\ v\gamma & \gamma \end{bmatrix} \begin{bmatrix} t' \\ z' \end{bmatrix} \tag{4}$$

This inverse transform is similar to moving K with negative velocity. This means

$$\gamma^2 + v\delta\gamma = 1 \tag{5}$$

Define

$$k = \frac{\delta}{v\gamma} \tag{6}$$

$$\gamma^2 + v\delta\gamma = \gamma^2(1 + kv^2) = 1 \rightarrow \gamma = 1/\sqrt{1 + kv^2} \tag{7}$$

This reduces the transform to

$$\begin{bmatrix} t' \\ z' \end{bmatrix} = 1/\sqrt{1 + kv^2} \begin{bmatrix} 1 & kv \\ -v & 1 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} \tag{8}$$

If k is positive, then there may be transformations with $kv^2 \gg 1$ which transform time into a spatial coordinate and vice versa. This is considered to be unphysical.

The condition k = 0 corresponds to a Galilean transformation

$$\begin{bmatrix} t' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -v & 1 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} \tag{9}$$

The condition $k < 0$ corresponds to a Lorentz transformation. We can set $kc^2 = -1$, where c is an invariant speed that corresponds to the maximum of v.

$$\begin{bmatrix} t' \\ z' \end{bmatrix} = 1/\sqrt{1 - v^2/c^2} \begin{bmatrix} 1 & -v/c^2 \\ -v & 1 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} \tag{10}$$

The Lorentz transform can also be written as a hyperbolic transform

$$\begin{bmatrix} ct' \\ z' \end{bmatrix} = \begin{bmatrix} \cosh \phi & -\sinh \phi \\ -\sinh \phi & \cosh \phi \end{bmatrix} \begin{bmatrix} ct \\ z \end{bmatrix} \tag{11}$$

The Lorentz transform corresponds to a length contraction

$$L' = \left(L - v\Delta t_p\right)\sqrt{1 - v^2/c^2} \tag{12}$$

Where $L = \Delta z$ is the proper length (the length of the object in its rest frame) and $L' = \Delta z'$ is the length observed by an observer in relative motion with respect to the object.

The Lorentz transform corresponds to a time dilatation

$$\Delta t_c = \left(\Delta t_p - \Delta z_p \, v/c^2\right)/\sqrt{1 - v^2/c^2} \tag{13}$$

Where t_p is the proper time (the time of the object in its rest frame) and t_c is the coordinate time observed by an observer in relative motion with respect to the object.

$$dt_p{}^2 = dt_c{}^2 - \frac{dx^2 + dy^2 + dz^2}{c^2} \tag{14}$$

This determines the Minkowski signature $(1, -1-1-1)$.

In the neighborhood of large masses this is no longer correct.

$$dt_p{}^2 = \left(1 - \sum_i \frac{2GM_i}{r_i c^2}\right) dt_c{}^2 - \frac{dx^2 + dy^2 + dz^2}{c^2} \tag{15}$$

where:

dt_p is a small increment of proper time t_p;
dt_c is a small increment in the coordinate time t_c
dx, dy and dz are small increments in the three coordinates x, y, z of the clock's position;
$\sum_i \frac{GM_i}{r_i}$ represents the sum of the Newtonian gravitational potentials due to the masses M_i in the neighborhood, based on their distances r_i from the clock.

The coordinate velocity of the clock is

$$v = \sqrt{\frac{dx^2 + dy^2 + dz^2}{dt_c{}^2}} \tag{16}$$

The coordinate time t_c is the time that would be read on a hypothetical "coordinate clock" situated infinitely far from all gravitational masses and stationary in the system of coordinates ($v=0$).

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Simultaneity

$$\Delta t_c = \left(\Delta t_p - \Delta z_p \, v/c^2\right)/\sqrt{1 - v^2/c^2} \tag{17}$$

If $\Delta t_p = 0$ then depending on v and Δz_p the time difference Δt_c is non-zero.

Relativistic momentum

The relativistic classical momentum is

$$P = mv = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} \tag{18}$$

The rest mass is m_0. The relativistic mass is

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \tag{19}$$

Relativistic energy

For $v \ll c$ a Taylor expansion gives

$$mc^2 = m_0 c^2 \left[1 + \tfrac{1}{2}\left(\frac{v}{c}\right)^2 + \tfrac{3}{8}\left(\frac{v}{c}\right)^4 + \tfrac{5}{16}\left(\frac{v}{c}\right)^6 + \cdots\right] \tag{20}$$

$$\approx m_0 c^2 + \tfrac{1}{2}m_0 v^2$$

The first term at the right side is the rest energy. The second term is the Newton kinetic energy. The term at left is the relativistic energy.

Quaternion coordinates

This part of the appendix describes candidates for the coordinates on the coordinate sphere.

Polar coordinates

The equivalent to rectangular coordinates in quaternion space is (a_τ, a_x, a_y, a_z)

$$a = a_\tau + \underline{\mathbf{i}} \cdot a_x + \underline{\mathbf{j}} \cdot a_y \pm \underline{\mathbf{i}} \cdot \underline{\mathbf{j}} \cdot a_z \qquad (1)$$

The equivalent to polar coordinates in quaternion space is

$$a_\tau = \|a\| \cos(\psi) \qquad (2)$$

$$a_x = \|a\| \sin(\psi) \sin(\theta) \cos(\varphi) \qquad (3)$$

$$a_y = \|a\| \sin(\psi) \sin(\theta) \sin(\varphi) \qquad (4)$$

$$a_z = \|a\| \sin(\psi) \cos(\theta) \qquad (5)$$

$sin(\psi)$, where $\psi = (0, \pi)$, is known as the (imaginary) amplitude of the quaternion. Angle $\theta = (0, \pi)$ is the (co-)latitude and angle $\varphi = (0, 2\pi)$ is the longitude.
For any fixed value of ψ, θ and φ parameterize a 2-sphere of radius $sin(\psi)$, except for the degenerate cases, when ψ equals 0 or π, in which case they describe a point.

This suggests the following structure of the argument $\underline{\boldsymbol{\Lambda}}$

$$a = \|a\| \exp(\tilde{\imath} \cdot \psi) \qquad (6)$$

$$= \|a\| (\cos(\psi) + \tilde{\imath} \sin(\psi)) \qquad (7)$$

$$= a_\tau + \|a\| \tilde{\imath} \sin(\psi) = a_\tau + \boldsymbol{a} \qquad (8)$$

The imaginary number $\underline{\tilde{\imath}}$ may take any direction.

3 sphere

A 3-sphere is a compact, connected, 3-dimensional manifold without boundary. It is also simply-connected. What this means, loosely speaking, is that any loop, or circular path, on the 3-sphere can be continuously shrunk to a point without leaving the 3-sphere. The [Poincaré conjecture](http://en.wikipedia.org/wiki/Poincar%C3%A9_conjecture)[197] proposes that the 3-sphere is the only three dimensional manifold with these properties (up to homeomorphism)[198].

The round metric on the 3-sphere in these coordinates is given by

$$ds^2 = d\psi^2 + sin^2(\psi)(d\theta^2 + sin^2(\theta)d\varphi^2) \qquad (1)$$

The volume form is given by

$$dV = sin^2(\psi) sin(\theta) \, d\psi \wedge d\theta \wedge d\varphi \qquad (2)$$

The 3-dimensional volume (or **hyperarea**) of a 3-sphere of radius r is

$$2\,\pi^2\, r^3 \qquad (3)$$

The 4-dimensional **hypervolume** (the volume of the 4-dimensional region bounded by the 3-sphere) is

$$\tfrac{1}{2}\,\pi^2\, r^4 \qquad (4)$$

The 3-sphere has constant positive sectional curvature equal to $1/r^2$.

The 3-sphere has a natural Lie group structure SU(2) given by quaternion multiplication.

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[197] http://en.wikipedia.org/wiki/Poincar%C3%A9_conjecture
[198] http://en.wikipedia.org/wiki/3-sphere

The 3-sphere admits non-vanishing vector fields (sections of its tangent bundle). One can even find three linearly-independent and non-vanishing vector fields. These may be taken to be any left-invariant vector fields forming a basis for the Lie algebra of the 3-sphere. This implies that the 3-sphere is parallelizable. It follows that the tangent bundle of the 3-sphere is trivial.

There is an interesting action of the circle group $\mathbb{T}$ on $\mathbb{S}^3$ giving the 3-sphere the structure of a principal circle bundle known as the Hopf bundle. If one thinks of $\mathbb{S}^3$ as a subset of $\pmb{C}^2$, the action is given by

$$(z_1, z_2) \, \lambda \; = \; (z_1 \, \lambda, z_2 \, \lambda) \; \forall_{\lambda \, \in \mathbb{T}}. \tag{5}$$

The orbit space of this action is homeomorphic to the two-sphere $\mathbb{S}^2$. Since $\mathbb{S}^3$ is not homeomorphic to $\mathbb{S}^2 \times \mathbb{S}^1$, the Hopf bundle is nontrivial.

Hopf coordinates

Another choice of hyperspherical coordinates, (η, ξ_1, ξ_2), makes use of the embedding of $\mathbb{S}^3$ in $\pmb{C}^2$. In complex coordinates $(z_1, z_2) \in \pmb{C}^2$ we write

$$z_1 = exp(\tilde{\imath} \, \xi_1) \, sin(\eta) \tag{1}$$

$$z_2 \; = \; exp(\tilde{\imath} \, \xi_2) \, cos(\eta) \tag{2}$$

Here η runs over the range 0 to $\pi/2$, and ξ_1 and ξ_2 can take any values between 0 and 2π. These coordinates are useful in the description of the 3-sphere as the Hopf bundle

$$\mathbb{S}^1 \to \mathbb{S}^3 \to \mathbb{S}^2 \tag{3}$$

For any fixed value of η between 0 and $\pi/2$, the coordinates (ξ_1, ξ_2) parameterize a 2-dimensional torus. In the degenerate cases, when η equals 0 or $\pi/2$, these coordinates describe a circle.

The round metric on the 3-sphere in these coordinates is given by

$$ds^2 = d\eta^2 + sin^2(\eta)\,(d\zeta_1^2 + cos^2(\eta)\,d\,\zeta_2^2) \qquad (4)$$

and the volume form by

$$dV \;=\; sin(\eta)\,cos(\eta)\;d\eta{\wedge}d\zeta_1{\wedge}d\zeta_2 \qquad (5)$$

Group structure

Because the set of unit quaternions is closed under multiplication, $\mathbb{S}^3$ takes on the structure of a group. Moreover, since quaternionic multiplication is smooth, $\mathbb{S}^3$ can be regarded as a real Lie group. It is a non-abelian, compact Lie group of dimension 3. When thought of as a Lie group $\mathbb{S}^3$ is often denoted $Sp(1)$ or $U(1, \mathbb{H})$.

It turns out that the only spheres which admit a Lie group structure are $\mathbb{S}^1$, thought of as the set of unit complex numbers, and $\mathbb{S}^3$, the set of unit quaternions. One might think that $\mathbb{S}^7$, the set of unit octonions, would form a Lie group, but this fails since octonion multiplication is non-associative. The octonionic structure does give $\mathbb{S}^7$ one important property: *parallelizability*[199]. It turns out that the only spheres which are parallelizable are $\mathbb{S}^1$, $\mathbb{S}^3$, and $\mathbb{S}^7$.

By using a matrix representation of the quaternions, $\mathbb{H}$, one obtains a matrix representation of $\mathbb{S}^3$. One convenient choice is given by the Pauli matrices:

$$\left(a_\tau \,+\, a_x \cdot \mathbf{i} \,+\, a_y \cdot \mathbf{j} \,+\, a_z \cdot \mathbf{k}\right) = \begin{bmatrix} a_\tau + \tilde{\imath}\cdot a_x & a_y + \tilde{\imath}\cdot a_z \\ -a_y + \tilde{\imath}\cdot a_z & a_\tau - \tilde{\imath}\cdot a_x \end{bmatrix} \qquad (1)$$

This map gives an injective algebra homomorphism from $\mathbb{H}$ to the set of 2×2 complex matrices. It has the property that the absolute value of a quaternion q is equal to the square root of the determinant of the matrix image of q.

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[199] http://en.wikipedia.org/wiki/Parallelizability

The set of unit quaternions is then given by matrices of the above form with unit determinant. This matrix subgroup is precisely the special unitary group SU(2). Thus, $\mathbb{S}^3$ as a Lie group is isomorphic to SU(2).

Using our hyperspherical coordinates (η, ξ_1, ξ_2) we can then write any element of SU(2) in the form

$$\begin{bmatrix} \exp(\tilde{\imath} \cdot \xi_1) \cdot \sin(\eta) & \exp(\tilde{\imath} \cdot \xi_2) \cdot \cos(\eta) \\ -\exp(\tilde{\imath} \cdot \xi_2) \cdot \cos(\eta) & \exp(-\tilde{\imath} \cdot \xi_1) \cdot \sin(\eta) \end{bmatrix} \tag{2}$$

Another way to state this result is if we express the matrix representation of an element of SU(2) as a linear combination of the Pauli matrices. It is seen that an arbitrary element $U \in SU(2)$ can be written as

$$U = \alpha_\tau \cdot 1 + \sum_{n=x,y,z} \alpha_n \, I_n \tag{3}$$

The condition that the determinant of U is +1 implies that the coefficients α_n are constrained to lie on a 3-sphere.

Versor
Any **unit quaternion** q can be written as a **versor**:

$$u = exp(\tilde{\imath} \ \psi) = cos(\psi) + \tilde{\imath} \ sin(\psi) \tag{1}$$

This is the quaternionic analogue of Euler's formula. Now the unit imaginary quaternions all lie on the unit 2-sphere in Im $\mathbb{H}$ so any such $\tilde{\mathbf{\imath}}$ can be written:

$$\tilde{\imath} = \boldsymbol{i} \, cos(\varphi) \, sin(\theta) + \boldsymbol{j} \, sin(\varphi) \, sin(\theta) + \boldsymbol{k} \, cos(\theta) \tag{2}$$

Symplectic decomposition
Quaternions can be written as the combination of two complex numbers and an imaginary number k with unit length.

2ⁿ-on construction

The 2^n-ons use the following doubling formula

$$(a, b)(c, d) = (a\, c - (b\, d^*)^*, (b^* c^*)^* + (b^*(a^*((b^{-1})^* d^*)^*)^*)^*) \tag{1}$$

Up until the 16-ons the formula can be simplified to

$$(a, b)(c, d) = (a\, c - b\, d^*, c\, b + (a^* b^{-1})\,(b\, d)) \tag{2}$$

Up to the octonions the Cayley Dickson construction delivers the same as the 2^n-on construction. From n>3 the 2^n-ons are 'nicer'.

2ⁿ-ons

Table of properties of the 2^n-ons. See
www.math.temple.edu/~wds/homepage/nce2.ps.

Type	name	Lose
1-ons	*Reals.*	
2-ons	*Complex numbers*	$z^* = z$ (the * denotes conjugating); the ordering properties that both {z > 0, -z > 0, or z = 0} and {w > 0, z > 0 implies w + z > 0, wz > 0}.
4-ons	*Quaternions*	commutativity ab = ba; the algebraic closedness property that every univariate polynomial equation has a root.
8-ons	*Octonions*	associativity ab · c = a · bc.
16-ons	(*not Sedenions!*)	right-alternativity x · yy = xy · y; right-cancellation x = xy · y⁻¹ ; flexibility x · yx = xy · x; left-linearity (b + c)a = ba + ca; anti-automorphism ab = ba, (ab)⁻¹ = b⁻¹ a⁻¹ ; left-linearity (b + c)a = ba + ca; continuity of the map x → xy; Moufang and Bol identities; diassociativity
32-ons		generalized-smoothness of the map x → xy; right-division properties that xa = b has (generically) a solution x, and the uniqueness of such an x; the "fundamental theorem of algebra" that every

polynomial having a unique "asymptotically dominant monomial" must have a root; Trotter's formula:

$$\lim_{n\to\infty}\left[e^{x/n}e^{y/n}\right]^{n} = \lim_{n\to\infty}\left(1 + \frac{x+y}{n}\right)^{n} = e^{x+y}$$

Type	*name*	Retain
2^n-ons		Unique 2-sided multiplicative & additive identity elements 1 & 0; Norm-multiplicativity $|xy|^2 = |x|^2 \cdot |y|^2$; Norm-subadditivity $|a + b| \le |a| + |b|$; 2-sided inverse $a^{-1} = a^*/|a|^2$ (a # 0); $a^{**} = a$; $(x \pm y)^* = x^* \pm y^*$; $(a^{-1})^{-1} = a$; $(a^*)^{-1} = (a^{-1})^*$; $|a|^2 = |a|^2 = a^*a$; Left-alternativity $yy \cdot x = y \cdot yx$; Left-cancellation $x = y^{-1} \cdot yx$; Right-linearity $a(b + c) = ab + ac$; r^{th} power-associativity $a^n\, a^m = a^{n+m}$; Scaling $s \cdot ab = sa \cdot b = as \cdot b = a \cdot sb = a \cdot bs = ab \cdot s$ (s real); Power-distributivity $(ra^n + sa^m)b = ra^n\, b + sa^m\, b$ (r, s real); Vector product properties of the imaginary part: **ab** - re(**ab**) of the product for pure-imaginary 2^n-ons **a,b** regarded as $(2^n - 1)$-vectors; $\langle xa,b\rangle = \langle a,x^*b\rangle$, $\langle xa,xb\rangle = |x|^2 \cdot \langle a,b\rangle$ and $\langle x,y\rangle = \langle x^*,y^*\rangle$ Numerous weakened associativity, commutativity, distributivity, antiautomorphism, and Moufang and Bol properties including 9-coordinate ``niner'' versions of most of those properties; contains 2^{n-1}-ons as subalgebra.

The most important properties of 2^n-ons

If a,b,x,y are 2^n-ons, n ≥ 0, and s and t are scalars (i.e. all coordinates are 0 except the real coordinate) then

unit: A unique 2^n-on 1 exists, with $1 \cdot x = x \cdot 1 = x$.

zero: A unique 2^n-on 0 exists, with $0 + x = x + 0 = x$ and $0 \cdot x = x \cdot 0 = 0$.

additive properties: x+y = y+x, (x+y)+z = x+(y+z);

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$-x$ exists with $x + (-x) = x - x = 0$.

norm: $|x|^2 = xx^* = x^*x$.

norm-multiplicativity: $|x|^2 \cdot |y|^2 = |x \cdot y|^2$.

scaling: $s \cdot x{\cdot}y = s{\cdot}x \cdot y = x{\cdot}s \cdot y = x \cdot s{\cdot}y = x \cdot y{\cdot}s$.

weak-linearity: $(x + s){\cdot}y = x{\cdot}y + s{\cdot}y$ and $x{\cdot}(y + s) = x{\cdot}y + x{\cdot}s$.

right-linearity: $x{\cdot}(y + z) = x{\cdot}y + x{\cdot}z$.

inversion: If $x \neq 0$ then a unique x^{-1} exists, obeying $x^{-1}{\cdot}x = x{\cdot}x^{-1} = 1$. It is $x^{-1} = x{\cdot}|x|^{-2}$.

left-alternativity: $x \cdot xy = x^2{\cdot}y$.

left-cancellation: $x \cdot x^{-1}{\cdot}y = y$.

effect on inner products: $\langle x{\cdot}a, b \rangle = \langle a, x^*{\cdot}b \rangle$, $\langle x, y \rangle = \langle x^*, y^* \rangle$, $\langle x^*{\cdot}a, x^{-1}{\cdot}b \rangle = \langle a, b \rangle$,

and $\langle x{\cdot}a, x{\cdot}b \rangle = |x|^2 \cdot \langle a, b \rangle$.

Conjugate of inverse: $(x^{-1})^* = (x^*)^{-1}$.

Near-anticommutativity of unequal basis elements: $e_k^2 = -1$ and $e_k{\cdot}e_l^* = -e_l{\cdot}e_k^*$ if $k \neq l$.

(Note: the case $k; l > 0$ shows that unequal pure-imaginary basis elements anticommute.)

Alternative basis elements: $e_k{\cdot}e_l \cdot e_k = e_k \cdot e_l{\cdot}e_k$, $e_l{\cdot}e_k \cdot e_k = e_l \cdot e_k{\cdot}e_k$, and $e_k{\cdot}e_k \cdot e_l = e_k \cdot e_k{\cdot}e_l$. (However, when $n \geq 4$ the 2^n-ons are not flexible i.e. it is not generally true that $x{\cdot}y \cdot x = x \cdot y{\cdot}x$ if x and y are 16-ons that are not basis elements. They also are not right-alternative.)

Quadratic identity: If x is a 2^n-on (over any field F with charF $\neq 2$), then $x^2 + |x|^2 = 2{\cdot}x$ re x

Squares of imaginaries: If x is a 2^n-on with re $x = 0$ ("pure imaginary") then $x^2 = -|x|^2$ is nonpositive pure-real.

Powering preserves im$x$ direction

Niners

Niners are 2n-ons whose coordinates with index > 8 are zero. The index starts with 0.

9-**flexibility** $xp \cdot x = x \cdot px$, $px \cdot p = p \cdot xp$.

9-**similitude unambiguity** $xp \cdot x^{-1} = x \cdot px^{-1}$, $px \cdot p^{-1} = p \cdot xp^{-1}$.

9-**right-alternativity** $xp \cdot p = x \cdot p^2$, $px \cdot x = p \cdot x^2$.

9-**right-cancellation** $xp^{-1} \cdot p = x$, $px^{-1} \cdot x = p$.
9-**effect on inner products** $\langle x, yp \rangle = \langle xp, y \rangle$, $\langle xp, yp \rangle = |p|^2 \langle x, y \rangle$.
9-**left-linearity** $(x + y)p = xp + yp$, $(p + q)x = px + qx$.
9-**Jordan-identity** $xp \cdot xx = x(p \cdot xx)$, $py \cdot pp = p(y \cdot pp)$.
9-**coordinate-distributivity** $([x + y]z)_{0;...;8} = (xz + yz)_{0;...;8}$.
9-**coordinate-Jordan-identity** $[xy \cdot xx]_{0;...;8} = [x(y \cdot xx)]_{0;...;8}$.
9-**anticommutativity for orthogonal imaginary 2^n-ons**
If $\langle p, x \rangle = \text{re } p = \text{re } x = 0$ then $px = -xp$.
9-**reflection** If $|a| = 1$ and the geometric reflection operator is defined
below then $-(\text{refl}[a](y))_{0;...;8} = (a \cdot y^*a)_{0;...;8}$, and $-\{\text{refl}[a](y)\}^*_{0;...;8} = (a^*y \cdot a^*)_{0;...;8}$,
and
if either a or y is a niner then $-\text{refl}[a](y) = a \cdot y^*a$ and $-\text{refl}[a](y) = a^*y \cdot a^*$.

$$\text{refl}[\vec{x}](\vec{t}) \overset{\text{def}}{=} \vec{t} - \frac{2\langle \vec{x}, \vec{t} \rangle}{|\vec{x}|^2}\vec{x} \tag{3}$$

What holds for the niners, also holds for the octonions.

Regular quaternionic functions

See: http://www.zipcon.net/~swhite/docs/math/quaternions/analysis.html
and http://world.std.com/~sweetser/quaternions/ps/Quaternionic-analysis-memo.pdf).
The differential df is given by

$$df = \frac{\partial f}{\partial q_0}dq_0 + \frac{\partial f}{\partial q_i}dq_i + \frac{\partial f}{\partial q_j}dq_j + \frac{\partial f}{\partial q_k}dq_k \tag{1}$$

A regular function f obeys:

$$\frac{\partial f}{\partial q_0} + i\frac{\partial f}{\partial q_i} + j\frac{\partial f}{\partial q_j} + k\frac{\partial f}{\partial q_k} = 0 \tag{2}$$

In addition the regular function f obeys:

$$\int_C D_q f = 0 \tag{3}$$

where C is any smooth closed 3-manifold in $\mathbb{H}$. D_q is the quaternion representing an element δC of the 3-manifold, its magnitude being equal to the volume of δC and its direction being normal to δC.

$$f(p) = \frac{1}{2\pi^2} \int_{\mathfrak{D}} \left\{ \frac{(q-p)^{-1}}{|q-p|^2} D_q f(q) \right\} \tag{4}$$

where $\mathfrak{D}$ is a domain in $\mathbb{H}$ in which f is regular and p is a point inside $\mathfrak{D}$.

$$D_q = dq_i \wedge dq_j \wedge dq_k - i\, dq_0 \wedge dq_j \wedge dq_k - j\, dq_0 \wedge dq_k \wedge dq_i \\ - k\, dq_0 \wedge dq_i \wedge dq_j \tag{5}$$

$$d(dq \wedge dq\, f) = Dq\, f'(q) \tag{6}$$

$$dq \wedge dq = i\, dq_j \wedge dq_k + j\, dq_k \wedge dq_i + k\, dq_i \wedge dq_j \tag{7}$$

Here $a \wedge b$ is the external vector product between vectors a and b. It is not the quaternionic external product.

$$d(Dq\, f) = 0 \tag{8}$$

$$d\left[\frac{(q-p)^{-1}}{|q-p|^2} \right] Dq\, f(q) = \Delta \left(\frac{1}{|q-p|^2} \right) f(q)\, dq_0 \wedge dq_i \wedge dq_j \wedge dq_k \tag{9}$$

where Δ is the Laplacian on $\mathbb{R}^4$ and $dq_0 \wedge dq_i \wedge dq_j \wedge dq_k$ is the standard volume 4-form. Since $\frac{1}{|q-p|^2}$ is the Green's function for the Laplacian in $\mathbb{R}^4$, (4) follows from (9).

$$\Delta f = \frac{\partial f^2}{\partial q_0^2} + \frac{\partial f^2}{\partial q_i^2} + \frac{\partial f^2}{\partial q_j^2} + \frac{\partial f^2}{\partial q_k^2} \tag{10}$$

If f is regular in an open set U, then it has a power series expansion about each point of U. Thus, point-wise differentiability, together with the four

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real conditions (2) on the sixteen partial derivatives of f, is sufficient to ensure analyticity.

The set of homogeneous regular functions of degree n forms a quaternionic vector space of dimension $(n + 1)(n + 2)/2$;
This is true for any integer n if for negative n it is understood that the functions are defined and regular everywhere except at 0. The functions with negative degree of homogeneity correspond to negative powers of a complex variable, and occur in the quaternionic Laurent series which exists for any regular function which is regular in an open set except at one point.

On the unit sphere in $\mathbb{H}$ the homogeneous regular functions form a group isomorphic to SU(2). The harmonic analysis of these functions bears the same relation to quaternionic analysis as the theory of complex Fourier series does to complex analysis.

Because the quaternions are four-dimensional, there is no counterpart to the geometrical description of complex analytic functions as conformal mappings. The zeros of a quaternionic regular function are not necessarily isolated, and its range is not necessarily open; neither of these sets needs even be a sub-manifold of $\mathbb{H}$.

Definition: A function $f : \mathbb{H} \rightarrow \mathbb{H}$ is quaternion-differentiable on the left at q if the limit

$$\frac{df}{dq} = \lim_{h \to 0} \frac{f(q + h) - f(q)}{h} \tag{11}$$

exists.

Theorem: Suppose the function f is defined and quaternion-differentiable on the left throughout a connected open set U. Then on U, f has the form:

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$$f(q) = a + q\,b \tag{12}$$

for some $a, b \in \mathbb{H}$.

Even if f is quaternion-differentiable, it will not in general satisfy Cauchy's theorem in the form

$$\int d_q f = 0 \tag{13}$$

where the integral is round a closed curve; in fact the only functions satisfying this equation for all closed curves are the *constant* functions.

Definition 2: A function f $:$ $\mathbb{H}$ $\rightarrow$ $\mathbb{H}$ is left-regular at $q \in \mathbb{H}$ if it is **real-differentiable** at q and there exists a quaternion $f_l'(q)$ such that

$$d(dq \wedge dq\, f) = -2D_q f_l'(q) \tag{14}$$

It is right-regular if there exists a quaternion $f_r'(q)$ such that

$$d(f\, dq \wedge dq) = -2f_r'(q)D_q:$$

Clearly, the theory of left-regular functions will be entirely equivalent to the theory of right-regular functions. For definiteness, we will only consider left-regular functions, which we will call simply **regular**. We will write

$$f'(q) = f_l'(q) \tag{15}$$

and call it the **derivative** of f at q.

Theorem2: (about the Cauchy-Riemann-Fueter equations)
A real-differentiable function f is regular at q if and only if

$$\frac{\partial f}{\partial q_0} + i\frac{\partial f}{\partial q_i} + j\frac{\partial f}{\partial q_j} + k\frac{\partial f}{\partial q_k} = 0 \tag{16}$$

This is formula (2)

Theorem 3: A differentiable function f is regular at q if and only if

$$D_q \wedge df = 0 \tag{17}$$

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Theorem 4: If f is regular at every point of the 4-parallelepiped C,

$$\int_C D_q f = 0 \tag{18}$$

This is equivalent to formula (3).

$$G(q) = \frac{q^{-1}}{|q|^2} \tag{19}$$

Note that

$$G(q) = -\partial_l \frac{1}{|q|^2} = -\partial_r \frac{1}{|q|^2} \tag{20}$$

It follows that $\bar{\partial}_l G = 0$, i.e. G is regular except at 0.

Theorem 5: A function which is regular in an open set U is real-analytic in U
This follows from (4).

Theorem 6: (Cauchy's theorem for a differentiable contour)
Suppose f is regular in an open set U, and let C be a differentiable 3-chain in U, which is homologous to 0 in the differentiable singular homology of U, i.e. $C = \partial C'$ for
some differentiable 4-chain C' in U. Then

$$\int_C D_q f = 0 \tag{21}$$

In order to state the general form of the integral formula, we need an analogue of the notion of the winding number of a curve round a point in the plane.

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Definition 3: Let q be any quaternion, and let C be a closed 3-chain in $\mathbb{H}\backslash\{q\}$. Then C is homologous to a 3-chain $C' : \partial I^4 \rightarrow S$, where S is the unit sphere with center q.

The wrapping number of C about q is the degree of the map C'.

Theorem 7: (The integral formula for a differentiable contour)

Suppose f is regular in an open set U. Let $p \in U$, and let C be a differentiable 3-chain

in $U\backslash\{p\}$ which is homologous, in the differentiable singular homology of $U\backslash\{p\}$, to

a 3-chain whose image is ∂B for some ball $B \subset U$. Then

$$\frac{1}{2\pi^2} \int_C \left\{ \frac{(q-p)^{-1}}{|q-p|^2} D_q f(q) = n f(p) \right\} \tag{22}$$

where n is the wrapping number of C about p.

Formulas (21) and (22) also hold for a rectifiable 3-chain C.

Since regular functions are harmonic, they satisfy a maximum-modulus principle and a Liouville theorem. As with functions of a complex variable, Liouville's theorem follows immediately from the Cauchy-Fueter integral formula.

Theorem 8: (Morera's theorem) Suppose that the function f is continuous in an open set U and that

$$\int_C D_q f = 0 \tag{21}$$

for every 4-parallelepiped C contained in U. Then f is regular in U.

Theorem 9: Let u be a real-valued function defined on a star-shaped open set $U \in \mathbb{H}$.

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If u is harmonic and has continuous second derivatives, there is a regular function f defined on U such that $Re\ f = u$.

This shows that there are as many regular functions of a quaternion variable as there are harmonic functions of four real variables. However, these functions do not include the simple algebraic functions, such as powers of the variable, which occur as analytic functions of a complex variable.

The separable Hilbert space Ħ

Notations and naming conventions
{f_x}$_x$ means ordered set of f_x . It is a way to define functions.

The use of bras and kets differs slightly from the way Dirac uses them.

|f> is a ket vector, f> is the same ket
<f| is a bra vector, <f is the same bra

A is an operator. |A is the same operator
A† is the adjoint operator of operator A. A| is the same operator
| on its own, is a nil operator
|A| is a self-adjoint (Hermitian) operator

We will use capitals for operators and lower case for quaternions, eigenvalues, ket vectors, bra vectors and eigenvectors. Quaternions and eigenvalues will be indicated with *italic* characters. Imaginary and anti-Hermitian objects are often underlined and/or indicated in **bold** text.

$\sum_k$ means: sum over all items with index k.
$\int_x$ means: integral over all items with parameter x.

Quaternionic Hilbert space
The Hilbert space is a **linear space**. That means for the elements |f>, |g> and |h> and numbers a and b:

Ket vectors

For **ket** vectors hold

$$|f> + |g> = |g> + |f> = |g + f> \tag{1}$$

$$(|f> + |g>) + |h> = |f> + (|g> + |h>) \tag{2}$$

$$|(a + b)\, f > = |f>\cdot a + |f>\cdot b \tag{3}$$

$$(|f> + |g>)\cdot a = |f>\cdot a + |g>\cdot a \tag{4}$$

$$|f>\cdot 0 = |0> \tag{5}$$

$$|f>\cdot 1 = |f> \tag{6}$$

Depending on the number field that the Hilbert space supports, a and b can be real numbers, complex numbers or (real) quaternions.

Bra vectors

The **bra** vectors form the dual Hilbert space $\mathcal{H}^{\dagger}$ of $\mathcal{H}$.

$$<f| + <g| = <g| + <f| = |g + f> \tag{1}$$

$$(<f| + <g|) + <h| = <f| + (<g| + <h|) \tag{2}$$

$$<f\,(a + b)> = <f|\cdot a + <f|\cdot b = a^{*}\cdot<f| + b^{*}\cdot<f| \tag{3}$$

$$(<f| + <g|)\cdot a = <f|\cdot a + <g|\cdot a = a^{*}\cdot<f| + a^{*}\cdot<g| \tag{4}$$

$$0\cdot<f| = <0| \tag{5}$$

$$1\cdot<f| = <f| \tag{6}$$

Scalar product

The Hilbert space contains a **scalar product,** also called **inner product,** <f|g> that combines **Ħ** and **Ħ†** in a direct product that we also indicate with **Ħ**.
The scalar product <f|g> satisfies:

$$\langle f | g + h \rangle = \langle f | g \rangle + \langle f | h \rangle \tag{1}$$

$$\langle f | \{ | g \rangle \cdot a \}_g = \{ \langle f | g \rangle \}_g \cdot a \tag{2}$$

With each ket vector |g> in **Ħ** belongs a bra vector <g| in **Ħ†** such that for all bra vectors <f| in **Ħ†**

$$\langle f | g \rangle = \langle g | f \rangle^* \tag{3}$$

$$\langle f | f \rangle = 0 \text{ when } | f \rangle = | 0 \rangle \tag{4}$$

$$\langle f | a \, g \rangle = \langle f | g \rangle \cdot a = \langle g | f \rangle^* \cdot a = \langle g \, a | f \rangle^* = (a^* \cdot \langle g | f \rangle)^* = \langle f | g \rangle \cdot a \tag{5}$$

In general is <f|a g> ≠ <f a|g>. However for real numbers r holds <f|r g> = <f r|g>

Remember that when the number field consists of quaternions, then also <f|g> is a quaternion and a quaternion q and <f|g> do in general not commute.

The scalar product defines a **norm**:

$$|| f || = \sqrt{(\langle f | f \rangle)} \tag{6}$$

And a **distance**:

$$D(f,g) = || f - g || \tag{7}$$

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The Hilbert space **H** is closed under its norm. Each converging row of elements of converges to an element of this space.

Separable

In mathematics a topological space is called separable if it contains a countable dense subset; that is, there exists a sequence $\{x_n\}_{n=1}^{\infty}$ of elements of the space such that every nonempty open subset of the space contains at least one element of the sequence.

Every continuous function on the separable space **H** is determined by its values on this countable dense subset.

Base vectors

The Hilbert space **H** is **separable**. That means that there exist a countable row of elements $\{f_n>\}$ that **spans** the whole space.

If $<f_n | f_m> = \delta(m,n) = [1$ when $n = m$; 0 otherwise]
then $\{| f_n>\}$ forms an **orthonormal base** of the Hilbert space.

A ket base $\{| k>\}$ of **H** is a minimal set of ket vectors $| k>$ that together span the Hilbert space **H**.
Any ket vector $| f>$ in **H** can be written as a linear combination of elements of $\{| k>\}$.

$$| f> = \sum_k (| k> \cdot <k | f>) \tag{1}$$

A bra base $\{<b |\}$ of **H**† is a minimal set of bra vectors $<b |$ that together span the Hilbert space **H**†.
Any bra vector $<f |$ in **H**† can be written as a linear combination of elements of $\{<b |\}$.

$$<f | = \sum_b (<f | b> \cdot <b |) \tag{2}$$

Usually base vectors are taken such that their norm equals 1. Such a base is called an othonormal base.

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Operators

Operators act on a subset of the elements of the Hilbert space.

Linear operators

An operator Q is linear when for all vectors |f> and |g> for which Q is defined and for all quaternionic numbers a and b:

$$|Q \cdot a\ f> + |Q \cdot b\ g> = |a \cdot Q\ f> + |b \cdot Q\ g> = |Q\ f> \cdot a + |Q\ g> \cdot b = \tag{1}$$

$$Q\ (|f> \cdot a + |g> \cdot b) = Q\ (|a\ f> + |b\ g>) \tag{2}$$

B is **colinear** when for all vectors |f> for which B is defined and for all quaternionic numbers *a* there exists a quaternionic number *c* such that:

$$|B \cdot a\ f> = |a \cdot B\ f> = |B\ f> \ c \cdot a \cdot c^{-1} \tag{3}$$

If |f> is an eigenvector of operator A with quaternionic eigenvalue *a*, then is |*b* f> an eigenvector of A with quaternionic eigenvalue $b \cdot a \cdot b^{-1}$.

A| = A† is the **adjoint** of the **normal** operator A. |A is the same as A.

$$<f\ A|\ g> = <fA^\dagger|g>^* \tag{4}$$

$$A^{\dagger\dagger} = A \tag{5}$$

$$(A \cdot B)^\dagger = B^\dagger \cdot A^\dagger \tag{6}$$

|B| is a **self adjoint** operator.

| is a nil operator.

The construct |f><g| acts as a linear operator. |g><f| is its adjoint operator.

$$\sum_n \{|f_n> \cdot a_n \cdot <f_n|\}, \tag{7}$$

where a n is real and acts as a density function.

The set of eigenvectors of a normal operator form an orthonormal base of the Hilbert space.
A self adjoint operator has real numbers as eigenvalues.

$\{<q|f>\}_q$ is a function $f(q)$ of parameter q.
$\{<g|q>\}_q$ is a function $g(q)$ of parameter q.

When possible, we use the same letter for identifying eigenvalues, eigenvalues and the corresponding operator.
So, usually $|q>$ is an eigenvector of a normal operator Q with eigenvalues q.

$\{q\}$ is the set of eigenvalues of Q.
$\{q\}_q$ is the ordered field of eigenvalues of q.
$\{|q>\}_q$ is the ordered set of eigenvectors of Q.
$\{<q|f>\}_q$ is the **Q view** of $|f>$.

Normal operators
The most common definition of continuous operators is:

> A **continuous** operator is an operator that creates images such that the inverse images of open sets are open.

> Similarly, a **continuous** operator creates images such that the inverse images of closed sets are closed.

A normal operator is a continuous linear operator.
A normal operator in **H** creates an image of **H** onto **H**. It transfers closed subspaces of **H** into closed subspaces of **H**.

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Normal operators represent continuous quantum logical observables.

The normal operators N have the following property.

$$N: \mathbf{H} \Rightarrow \mathbf{H} \tag{1}$$

N commutes with its **(Hermitian) adjoint** $N^\dagger$

$$N \cdot N^\dagger = N^\dagger \cdot N \tag{2}$$

Normal operators are important because the spectral theorem holds for them.
Examples of normal operators are

- **unitary** operators: $U^\dagger = U^{-1}$, unitary operators are bounded;
- **Hermitian** operators (i.e., self-adjoint operators): $N^\dagger = N$;
- **Anti-Hermitian** or anti-self-adjoint operators: $N^\dagger = -N$;
- **Anti-unitary** operators: $I^\dagger = -I = I^{-1}$, anti-unitary operators are bounded;
- **positive operators**: $N = MM^\dagger$
- **orthogonal projection** operators: $N = N^\dagger = N^2$

Spectral theorem

For every compact self-adjoint operator T on a real, complex or quaternionic Hilbert space $\mathbf{H}$, there exists an orthonormal basis of $\mathbf{H}$ consisting of eigenvectors of T. More specifically, the orthogonal complement of the kernel (null space) of T admits, either a finite orthonormal basis of eigenvectors of T, or a countable infinite orthonormal basis $\{e_n\}$ of eigenvectors of T, with corresponding eigenvalues $\{\lambda_n\} \subset \mathbb{R}$, such that $\lambda_n \rightarrow 0$. Due to the fact that $\mathbf{H}$ is separable the set of eigenvectors of T can be extended with a base of the kernel in order to form a complete orthonormal base of $\mathbf{H}$.

If T is compact on an infinite dimensional Hilbert space $\mathbf{H}$, then T is not invertible, hence $\sigma(T)$, the spectrum of T, always contains 0. The spectral theorem shows that $\sigma(T)$ consists of the eigenvalues $\{\lambda_n\}$ of T, and of 0 (if 0 is not already an eigenvalue). The set $\sigma(T)$ is a compact subset of the real line, and the eigenvalues are dense in $\sigma(T)$.

 A normal operator has a set of eigenvectors that spans the whole Hilbert space $\mathbf{H}$.
In quaternionic Hilbert space a normal operator has quaternions as eigenvalues.

The set of eigenvalues of a normal operator is NOT compact. This is due to the fact that $\mathbf{H}$ is separable. Therefore the set of eigenvectors is countable. As a consequence the set of eigenvalues is countable. Further, the eigenspace of normal operators has no finite diameter.

A continuous bounded linear operator on $\mathbf{H}$ has a compact eigenspace. The set of eigenvalues has a closure and it has a finite diameter.

Eigenspace
The set of eigenvalues $\{q\}$ of the operator Q form the eigenspace of Q

Eigenvectors and eigenvalues
For the eigenvector $|q\rangle$ of normal operator Q holds

$$|Q\, q\rangle = |q\, q\rangle = |q\rangle \cdot q \tag{1}$$

$$\langle q\, Q^\dagger | = \langle q\, q^* | = q^* \cdot \langle q| \tag{2}$$

$$\forall_{|f\rangle\, \in\, \mathbf{H}}\, [\{\langle f|Q\, q\rangle\}_q = \{\langle f|q\rangle q\}_q = \{\langle q\, Q^\dagger|f\rangle^*\}_q = \{q^*\langle q|f\rangle^*\}_q] \tag{3}$$

The eigenvalues of 2^n-on normal operator are 2^n-ons

$$\tag{4}$$

$$Q = \sum_{j=0}^{n-1} I_j Q_i$$

The Q_j are self-adjoint operators.

Generalized Trotter formula

For bounded operators $\{A_j\}$ hold:

$$\lim_{n\to\infty} \left(\prod_{j=1}^{p} e^{A_j/n} \right)^n = \exp\left(\sum_{j=1}^{p} A_j \right) = \lim_{n\to\infty} \left(1 + \frac{\sum_{j=1}^{p} A_j}{n} \right)^n \tag{1}$$

In general

$$\exp\left(\sum_{j=1}^{p} A_j \right) \neq \prod_{j=1}^{p} e^{A_j} \tag{2}$$

Unitary operators

For unitary operators holds:

$$U^{\dagger} = U^{-1} \tag{1}$$

Thus

$$U \cdot U^{\dagger} = U^{\dagger} \cdot U = 1 \tag{2}$$

Suppose $U = I + C$ where U is unitary and C is compact. The equations $U U^* = U^* U = I$ and $C = U - I$ show that C is normal. The spectrum of C contains 0, and possibly, a finite set or a sequence tending to 0. Since $U = I + C$, the spectrum of U is obtained by shifting the spectrum of C by 1.

The unitary transform can be expressed as:

$$U = \exp(\tilde{\mathbf{I}} \cdot \Phi / \hbar) \tag{3}$$

$$\hbar = h/(2 \cdot \pi) \tag{4}$$

Φ is Hermitian. The constant h refers to the granularity of the eigenspace. Unitary operators have eigenvalues that are located in the unity sphere of the 2^n-ons field.

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The eigenvalues have the form:

$$u = \exp(\underline{\mathbf{i}} \cdot \phi/\hbar) \tag{5}$$

ϕ is real. $\underline{\mathbf{i}}$ is a unit length imaginary number in 2^n-on space. It represents a direction.
u spans a sphere in 2^n-on space. For constant $\underline{\mathbf{i}}$, u spans a circle in a complex subspace.

Polar decomposition

Normal operators N can be split into a real operator A and a unitary operator U. U and A have the same set of eigenvectors as N.

$$N = ||N|| \cdot U = A \cdot U \tag{1}$$

$$N = A \cdot U = U \cdot A \tag{2}$$

$$= A \cdot \exp(\tilde{\underline{\mathbf{I}}} \cdot \Phi)/\hbar)$$

$$= \exp(\Phi_r + \tilde{\underline{\mathbf{I}}} \cdot \Phi)/\hbar)$$

Φ_r is a positive normal operator.

Ladder operator

General formulation

Suppose that two operators X and N have the commutation relation:

$$[N, X] = c \cdot X \tag{1}$$

for some scalar c. If $|n\rangle$ is an eigenstate of N with eigenvalue equation,

$$|N\,n\rangle = |n\rangle \cdot n \tag{2}$$

then the operator X acts on $|n\rangle$ in such a way as to shift the eigenvalue by c:

$$|N \cdot X\, n\rangle = |(X \cdot N + [N, X])\, n\rangle \qquad (3)$$
$$= |(X \cdot N + c \cdot X)\, n\rangle$$
$$= |X \cdot N\, n\rangle + |X\, n\rangle \cdot c$$
$$= |X\, n\rangle \cdot n + |X\, n\rangle \cdot c$$
$$= |X\, n\rangle \cdot (n+c)$$

In other words, if $|n\rangle$ is an eigenstate of N with eigenvalue n then $|X\, n\rangle$ is an eigenstate of N with eigenvalue $n + c$.

The operator X is a *raising operator* for N if c is real and positive, and a *lowering operator* for N if c is real and negative.

If N is a Hermitian operator then c must be real and the Hermitian adjoint of X obeys the commutation relation:

$$[N, X^\dagger] = - c \cdot X^\dagger \qquad (4)$$

In particular, if X is a lowering operator for N then $X^\dagger$ is a raising operator for N and vice-versa.

Unit sphere of Ħ
The ket vectors in **Ħ** that have their norm equal to one form together the **unit sphere** Θ of **Ħ**.
Base vectors are all member of the unit sphere. The eigenvectors of a normal operator are all member of the unit sphere.
The end points of the eigenvectors of a normal operator form a **grid** on the unit sphere Θ of **Ħ**.

Closure
The closure of **Ħ** means that converging rows of vectors converge to a vector of **Ħ**.

In general converging rows of eigenvalues of Q do not converge to an eigenvalue of Q.

Thus, the set of eigenvalues of Q is open.

At best the density of the coverage of the set of eigenvalues is comparable with the set of 2^n-ons that have rational numbers as coordinate values.

With other words, compared to the set of real numbers the eigenvalue spectrum of Q has holes.

The set of eigenvalues of operator Q includes 0. This means that Q does not have an inverse.

The rigged Hilbert space **H** can offer a solution, but then the direct relation with quantum logic is lost.

Canonical conjugate operator P

The existence of a canonical conjugate represents a stronger requirement on the continuity of the eigenvalues of canonical eigenvalues.

Q has eigenvectors $\{|q>\}_q$ and eigenvalues q.

P has eigenvectors $\{|p>\}_P$ and eigenvalues p.

For each eigenvector $|q>$ of Q we define an eigenvector $|p>$ and eigenvalues p of P such that:

$$< q|p > = < p|q >^* = exp\,(\hat{\imath} \cdot p \cdot q / \hbar) \tag{1}$$

$\hbar = h/(2\pi)$ is a scaling factor. $< q|p >$ is a quaternion. $\hat{\imath}$ is a unit length imaginary quaternion.

Displacement generators

Variance of the scalar product gives:

$$i\,\hbar\,\delta < q|p > = -p < q|p > \delta q \tag{1}$$

$$i\,\hbar\,\delta < p|q > = -q < p|q > \delta p \tag{2}$$

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In the rigged Hilbert space **H** the variance can be replaced by differentiation.

Partial differentiation of the function <q|p> gives:

$$i\,\hbar\,\partial/\partial q_s < q|p > = -p_s < q|p >\tag{3}$$

$$i\,\hbar\,\partial/\partial p_s < p|q > = -q_s < p|q >\tag{4}$$

Gelfand triple

The rigged Hilbert space **H** that belongs to a separable Hilbert space Ħ is a Gelfand triple.

A rigged Hilbert space is a pair (H, Φ) with Ħ a Hilbert space, Φ a dense subspace, such that Φ is given a <u>topological vector space</u> structure for which the <u>inclusion map</u> i is continuous.

Identifying Ħ with its dual space Ħ*, the adjoint to i is the map

$$i^*\!: Ħ = Ħ^* \rightarrow \Phi^*\tag{1}$$

The duality pairing between Φ and Φ^* has to be compatible with the inner product on Ħ, in the sense that:

$$\langle u, v \rangle_{\Phi \times \Phi^*} = (u, v)_Ħ\tag{2}$$

whenever $u \in \Phi \subset Ħ$ and $v \in Ħ = Ħ^* \subset \Phi^*$.

The specific triple $(\Phi \subset Ħ \subset \Phi^*)$ is often named after the mathematician <u>Israel Gelfand</u>).

Note that even though Φ is isomorphic to Φ^* if Φ is a Hilbert space in its own right, this isomorphism is *not* the same as the composition of the inclusion i with its adjoint i^*

$$i^*i\!: \Phi \subset Ħ = Ħ^* \rightarrow \Phi^*\tag{3}$$

Gamma matrices

A sign inversion of all three imaginary base vectors of quaternion number space is equivalent to a switch from right handedness to left handedness. The sign of the real base vector need not be affected by this inversion. The two sign selections and the number space form three items. Together n items have n(n-1) relations. Thus the three items have four relations. Thus with respect to these relations, four types of fields exist.

$$\psi = \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix}, \qquad \psi_L = \begin{bmatrix} \psi_{L\uparrow} \\ \psi_{L\downarrow} \end{bmatrix}, \qquad \psi_R = \begin{bmatrix} \psi_{R\uparrow} \\ \psi_{R\downarrow} \end{bmatrix} \tag{1}$$

The fact that the quaternionic imaginary base vectors are represented by the 2×2 Pauli σ^k matrices[200], indicates the properties of their external vector product $\boldsymbol{k} = \boldsymbol{i} \times \boldsymbol{j}$. However, the sign selection of the handedness is reflected by the combination of the $\boldsymbol{\alpha}$ matrix and the β matrix in the γ matrices.

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \qquad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{2}$$

$$\alpha_1 = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}, \qquad \alpha_2 = \begin{bmatrix} 0 & j \\ -j & 0 \end{bmatrix}, \qquad \alpha_3 = \begin{bmatrix} 0 & k \\ -k & 0 \end{bmatrix} \tag{3}$$

$$\beta = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{4}$$

In Dirac representation, the four contravariant gamma matrices are

$$\gamma^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \qquad \gamma^1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \tag{5}$$

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[200] http://en.wikipedia.org/wiki/Pauli_matrices

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$$\gamma^2 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}, \qquad \gamma^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

It is useful to define the product of the four gamma matrices as follows:

$$\gamma^5 = i\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{6}$$

The gamma matrices as specified here are appropriate for acting on Dirac spinors written in the Dirac basis; in fact, the Dirac basis is defined by these matrices. In the Dirac basis[201]:

$$\gamma^0 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}, \qquad \gamma^k = \begin{bmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{bmatrix}, \qquad \gamma^5 = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \tag{7}$$

This corresponds with $\alpha_k = \gamma^k, \beta = \gamma^5$.
Apart from the Dirac basis, a Weyl basis exists

$$\gamma^0 = \gamma^\beta = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \qquad \gamma^k = \begin{bmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{bmatrix}, \qquad \gamma^5 = \begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix} \tag{8}$$

The Weyl basis has the advantage that its chiral projections[202] take a simple form:

$$\psi_L = \tfrac{1}{2}(1 - \gamma^5)\psi = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\psi, \quad \psi_R = \tfrac{1}{2}(1 + \gamma^5)\psi = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}\psi \tag{9}$$

$$\psi^* = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\psi \tag{9}$$

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[201] http://en.wikipedia.org/wiki/Gamma_matrices#Dirac_basis
[202] http://en.wikipedia.org/wiki/Chirality_(physics)

Lagrangian general principles

The least action principle leads to an equation of motion of the type

$$\frac{\partial S}{\partial \varphi_i} = 0 \tag{1}$$

where the action, $\mathcal{S}$, is a functional of the dependent variables $\varphi_i(s)$ with their derivatives and s itself

$$S\left[\varphi_i, \frac{\partial \varphi_i}{\partial s}\right] = \int \mathcal{L}\left[\varphi_i[s], \frac{\partial \varphi_{i[s]}}{\partial s^\alpha}, s^\alpha\right] d^n s \tag{2}$$

and where $s = \{s^\alpha\}$ denotes the set of n independent variables of the system, indexed by $\alpha = 1.2.3, \dots, n$

The Euler–Lagrange equations of this action are

$$\frac{\partial}{\partial s^\alpha} \frac{\partial \mathcal{L}}{\partial \frac{\partial \varphi_i}{\partial s^\alpha}} - \frac{\partial \mathcal{L}}{\partial \varphi_i} = 0 \tag{3}$$

The energy tensor $\mathcal{T}_{\alpha\beta}$ is

$$\mathcal{T}_{\alpha\beta} = \sum_i \left[\frac{\partial \mathcal{L}}{\partial \frac{\partial \varphi_i}{\partial s_\alpha}} \frac{\partial \varphi_i}{\partial s^\beta} - \mathcal{L} \, g_{\alpha\beta i} \right] \tag{4}$$

$\mathcal{T}_{00i}$ is regarded as an expression for the Hamiltonian density $\mathcal{H}$. With t as a special parameter, we define

$$\mathcal{H} = \dot{\varphi}^i \frac{\partial \mathcal{L}}{\partial \dot{\varphi}_i} - \mathcal{L} = \dot{\varphi}^i \wp_i - \mathcal{L} \tag{5}$$

$$\wp_i = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}_i} = -\frac{\partial \mathcal{H}}{\partial \varphi_i} \tag{6}$$

$$\frac{\partial \mathcal{H}}{\partial \wp_{i_i}} = \dot{\varphi}_i \tag{7}$$

$$\frac{\partial \mathcal{H}}{\partial t} = -\frac{\partial \mathcal{L}}{\partial t} \tag{8}$$

The Euler-Lagrange equations are

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\varphi}_i} = \dot{\wp}_i = \frac{\partial \mathcal{L}}{\partial \varphi_i} \tag{9}$$

For an elementary particle with private field ψ and the field ϕ in its environment this means

$$S(\psi, \dot{\psi}, \phi, \dot{\phi}) = \int L(\psi, \dot{\psi}, \phi, \dot{\phi})\, dt = \int \mathcal{L}\,(\psi, \dot{\psi}, \phi, \dot{\phi})dx^4 \tag{10}$$

$$L = \int \mathcal{L}\,(\psi, \dot{\psi}, \phi, \dot{\phi})\, dx^3 \tag{11}$$

For the action S to be Lorentz invariant the Lagrangian density $\mathcal{L}$ must be a Lorentz scalar.

The equations of motion obtained from this functional derivative are the Euler–Lagrange equations of this action. Dynamical systems whose equations of motion are obtainable by means of an action principle on a suitably chosen Lagrangian are known as Lagrangian dynamical systems. Examples of Lagrangian dynamical systems range from the classical version of the Standard Model, to Newton's equations, to purely mathematical problems such as geodesic equations and Plateau's problem

Continuity equation
Density must have the dimension $[L^{-3}]$ and be a 0-component of a 4-vector satisfying the continuity equation

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$$\frac{\partial j^\mu}{\partial x_\mu} = 0 \qquad (1)$$

The zero index term is charge density. (Mass is a version of charge).

$$\rho_0 = j^0 \qquad (2)$$

The other terms concern current density.

Derivation of the one dimensional Euler Lagrange equation

This is taken from [Wikipedia](#)[203].

Equation

The Euler–Lagrange equation is an equation satisfied by a function, q, of a [real](#)[204] argument, t, which is a stationary point of the [functional](#)[205]

$$S(q) = \int_a^b L\big(t, q(t), \dot{q}(t)\big)\ dt \qquad (1)$$

where:

q is the function to be found:

$$q: [a, b] \subset \mathbb{R} \to X \qquad (2)$$
$$t \mapsto v = \dot{q}(t)$$

such that q is differentiable, $q(a) = x_a$, and $q(b) = x_b$;

$\dot{q}$ is the derivative of q:

$$\dot{q}: [a, b] \to T_{q(t)} X \qquad (3)$$
$$t \mapsto v = \dot{q}(t)$$

[203] http://en.wikipedia.org/wiki/Euler%E2%80%93Lagrange

[204] http://en.wikipedia.org/wiki/Real_number

[205] http://en.wikipedia.org/wiki/Functional_(mathematics)

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TX being the [tangent bundle](#) of *X* (the space of possible values of derivatives of functions with values in *X*);

L is a real-valued function with [continuous](#)[206] first [partial derivatives](#)[207]:

$$L:[a,b] \times X \times TX \to \mathbb{R}$$
$$t, x, v \mapsto L(t, x, v)$$

(4)

The Euler–Lagrange equation, then, is given by

$$L_x\big(t, q(t), \dot{q}(t)\big) - \frac{d}{dt}L_v\big(t, q(t), \dot{q}(t)\big) = \frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial v} = 0$$

(5)

where L_x and L_v denote the partial derivatives of *L* with respect to the second and third arguments, respectively.

If the dimension of the space *X* is greater than 1, this is a system of differential equations, one for each component:

$$\frac{\partial L}{\partial x_i} - \frac{d}{dt}\frac{\partial L}{\partial v_i} = 0; i = 1, \ldots, n$$

(6)

Derivation

Given a functional

$$J = \int_a^b F\big(t, y(t), y'(t)\big)\, dt$$

(1)

on $C^1([a,b])$ with the boundary conditions $y(a) = A$ and $y(b) = B$, we proceed by approximating the extremal curve by a polygonal line with n segments and passing to the limit as the number of segments grows arbitrarily large.

Divide the interval $[a,b]$ into $n + 1$ equal segments with endpoints $t_0 = a, t_1, t_2, \ldots, t_n, t_{n+1} = b$ and let $\Delta t = t_k - t_{k-1}$. Rather than a smooth function $y(t)$ we consider the polygonal line with vertices

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[206] http://en.wikipedia.org/wiki/Continuous_function

[207] http://en.wikipedia.org/wiki/Partial_derivatives

$(t_0, y_0), \ldots, (t_{n+1}, y_{n+1})$, where $y_0 = A$ and $y_{n+1} = B$. Accordingly, our functional becomes a real function of n variables given by

$$J(y_1, \ldots, y_n) \approx \sum_{k=0}^{n} F\left(t_k, y_k, \frac{y_{k+1}-y_k}{\Delta t}\right) \Delta t \tag{2}$$

Extremes of this new functional defined on the discrete points $t_0, \ldots, t_{n+1}$ correspond to points where

$$\frac{\partial J(y_1, \ldots, y_n)}{\partial y_m} = 0 \tag{3}$$

Evaluating this partial derivative gives

$$\frac{\partial J}{\partial y_m} = F_y\left(t_m, y_m, \frac{y_{m+1}-y_m}{\Delta t}\right) \Delta t + F_{y'}\left(t_{m-1}, y_{m-1}, \frac{y_m-y_{m-1}}{\Delta t}\right) \tag{4}$$
$$- F_y\left(t_m, y_m, \frac{y_{m+1}-y_m}{\Delta t}\right)$$

Dividing the above equation by Δt gives

$$\frac{\partial J}{\partial y_m \Delta t} = F_y\left(t_m, y_m, \frac{y_{m+1}-y_m}{\Delta t}\right) \tag{5}$$
$$+ \frac{F_{y'}\left(t_{m-1}, y_{m-1}, \frac{y_m-y_{m-1}}{\Delta t}\right) - F_y\left(t_m, y_m, \frac{y_{m+1}-y_m}{\Delta t}\right)}{\Delta t}$$

and taking the limit as $\Delta t \to 0$ of the right-hand side of this expression yields

$$F_y - \frac{dF_{y'}}{dt} = 0 \tag{6}$$

The left hand side of the previous equation is the [functional derivative](http://en.wikipedia.org/wiki/Functional_derivative)[208] $\frac{\delta J}{\delta y}$ of the functional J. A necessary condition for a differentiable functional to have an extremum on some function is that its functional derivative at that function vanishes, which is granted by the last equation.

Euler Lagrange equations of field

First order equations
The Dirac Lagrangian density is

$$\mathcal{L} = \psi^* \left[\gamma^\mu \left(i \frac{\partial}{\partial x_\mu} - eA_\mu \right) - m \right] \psi \tag{1}$$

The corresponding Euler-Lagrange equation

$$\left[\gamma^\mu \left(i \frac{\partial}{\partial x_\mu} - eA_\mu \right) - m \right] \psi = 0 \tag{2}$$

The Dirac 4-current is

$$j^\mu = \psi^* \gamma^\mu \psi \tag{3}$$

The density is the 0-component

$$\rho_{Dirac} = \psi^* \psi \tag{4}$$

The Dirac Hamiltonian density is

$$\mathcal{H} = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} \dot{\psi} - \mathcal{L} = \psi^* \left[\alpha^\mu \left(i \frac{\partial}{\partial x_\mu} - eA_\mu \right) + \beta m + eV \right] \psi \tag{5}$$

$$\mathcal{H} = \psi^* [-\langle \boldsymbol{\alpha}, \boldsymbol{\nabla} + e\boldsymbol{A} \rangle + \beta m + eV] \psi \tag{6}$$

The Dirac equation runs

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[208] http://en.wikipedia.org/wiki/Functional_derivative

$$\mathcal{H}\psi = [-\langle \boldsymbol{\alpha}, \boldsymbol{\nabla} + e\boldsymbol{A} \rangle + \beta m + eV]\psi \tag{7}$$

$$\mathcal{H}\begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} = [-\langle \boldsymbol{\alpha}, \boldsymbol{\nabla} + e\boldsymbol{A} \rangle + eV]\begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} + m\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} \tag{8}$$

$$= [-\langle \boldsymbol{\alpha}, \boldsymbol{\nabla} + e\boldsymbol{A} \rangle + eV]\begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} + m\begin{bmatrix} \psi_R \\ \psi_L \end{bmatrix}$$

$$\mathcal{H}\psi_L = (-\boldsymbol{\nabla} - e\boldsymbol{A} + eV)\psi_L + m\psi_R = (-\boldsymbol{D} + eV)\psi_L + m\psi_R \tag{9}$$

$$\mathcal{H}\psi_R = (\boldsymbol{\nabla} + e\boldsymbol{A} + eV)\psi_R + m\psi_L = (\boldsymbol{D} + eV)\psi_R + m\psi_L \tag{10}$$

The mass m couples ψ_L and ψ_R. The fact $m = 0$ decouples ψ_L and ψ_R.

The Dirac Hamiltonian density (6) as well as the Dirac Hamiltonian (7) *do not contain a derivative of ψ with respect to time.*

Now, the form of an energy eigenfunction is

$$\psi(\boldsymbol{x}, t) = \chi(\boldsymbol{x}) \exp(-\boldsymbol{E}t) \tag{11}$$

$$\mathcal{H}\psi = [-\langle \boldsymbol{\alpha}, \boldsymbol{\nabla} + e\boldsymbol{A} \rangle + eV]\psi + m\psi^* \tag{12}$$

Lagrangians in quantum field theory

Dirac Lagrangian
The Lagrangian density for a [Dirac field](http://en.wikipedia.org/wiki/Fermionic_field#Dirac_fields)[209] is:

$$\mathcal{L} = \frac{i\hbar c}{2}\left(\bar{\psi}\,\gamma^\mu\,\frac{\partial \psi}{\partial x_\mu} - \psi\,\gamma^\mu\,\frac{\partial \bar{\psi}}{\partial x_\mu} \right) - m\,c^2\bar{\psi}\,\psi \tag{1}$$

where ψ is a [Dirac spinor](http://en.wikipedia.org/wiki/Dirac_spinor)[210] (annihilation operator), $\bar{\psi}$ is its [Dirac adjoint](http://en.wikipedia.org/wiki/Dirac_adjoint)[211] (creation operator)

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[209] http://en.wikipedia.org/wiki/Fermionic_field#Dirac_fields

[210] http://en.wikipedia.org/wiki/Dirac_spinor

[211] http://en.wikipedia.org/wiki/Dirac_adjoint

$$\bar{\psi} = \psi^{\dagger}\,\gamma^0 \tag{2}$$

Quantum electrodynamic Lagrangian

The Lagrangian density for [quantum electro dynamics](http://en.wikipedia.org/wiki/Quantum_electrodynamics)[212] is:

$$\mathcal{L}_{QED} = \frac{i\hbar c}{2}\left(\bar{\psi}\,\gamma^{\mu}D_{\mu}\psi - \psi\,\gamma^{\mu}D_{\mu}\bar{\psi}\right) - m\,c^2\bar{\psi}\,\psi - \frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu} \tag{3}$$

where $F_{\mu\nu}$ is the [electromagnetic tensor](http://en.wikipedia.org/wiki/Electromagnetic_tensor)[213], D is the gauge covariant derivative.

$$F^{\mu\nu} = \frac{\partial A_{\nu}}{\partial x_{\mu}} - \frac{\partial A_{\mu}}{\partial x_{\nu}} = \begin{bmatrix} 0 & -\dfrac{E_x}{c} & -\dfrac{E_y}{c} & -\dfrac{E_z}{c} \\ \dfrac{E_x}{c} & 0 & -B_z & B_y \\ \dfrac{E_y}{c} & B_z & 0 & -B_x \\ \dfrac{E_z}{c} & -B_y & B_x & 0 \end{bmatrix} \tag{4}$$

$$F_{\mu\nu}F^{\mu\nu} = B^2 - E^2/c^2 \tag{5}$$

This is a Lorentz scalar.

The equation of motion is

$$i\gamma^{\mu}\partial_{\mu}\psi - m\psi = e\gamma^{\mu}(A^{\mu} + B^{\mu})\psi \tag{6}$$

The left-hand side is like the original Dirac equation and the right-hand side is the interaction with the electromagnetic field.

$$\partial_{\nu}F^{\mu\nu} = e\bar{\psi}\,\gamma^{\mu}\,\psi \tag{7}$$

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[212] http://en.wikipedia.org/wiki/Quantum_electrodynamics
[213] http://en.wikipedia.org/wiki/Electromagnetic_tensor

Now, if we impose the Lorenz-Gauge condition, i.e., that the divergence of the four potential vanishes then we get

$$\Box A^\mu = e\bar{\psi}\,\gamma^\mu\,\psi \tag{8}$$

The d'Alembert operator $\Box$ is defined as:

$$\Box = \partial_\mu\partial^\mu = g_{\mu\nu}\partial^\mu\partial^\nu = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \tag{9}$$

$$= \frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{\partial^2}{\partial t^2} - \Delta$$

Quantum chromodynamic Lagrangian

The Lagrangian density for [quantum chromodynamics](http://en.wikipedia.org/wiki/Quantum_chromodynamics)[214]:

$$\mathcal{L}_{QCD} = \frac{i\hbar c}{2}\left(\bar{\psi}_n\,\gamma^\mu D_\mu\psi_n - \psi_n\,\gamma^\mu D_\mu\,\bar{\psi}_n\right) - m_n\,c^2\bar{\psi}_n\psi_n - \frac{1}{4}G^\alpha_{\mu\nu}G^{\mu\nu}_\alpha \tag{1}$$

where D is the QCD gauge covariant derivative, $n = 1\dots6$ counts the quark types, and $G^\alpha_{\mu\nu}$ is the gluon field strength tensor.

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[214] http://en.wikipedia.org/wiki/Quantum_chromodynamics

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Zak transform
(See also http://eom.springer.de/Z/z130030.htm)

Definition
The Weil-Brezin-Zak transform $Z_\alpha(f)$ of a function f is defined by

(1)

$$Z_\alpha[f](t,\omega) = Z_\alpha f(t,\omega) = \sqrt{\alpha} \sum_{k=-\infty}^{\infty} f(\alpha\, t + \alpha\, k)\, \exp(-2\pi i k\omega)$$

Where $\alpha > 0$ and t and ω are real. When $\alpha = 1$, one denotes $Z_\alpha f$ by Zf.

If f represents a signal, then its Zak transform can be considered as a mixed time-frequency representation of f, and it can also be considered as a generalization of the discrete Fourier transform of f in which an infinite sequence of samples in the form $f(\alpha\, t + \alpha\, k)$, k = 0, ±1, ±2,…, is used

Elementary properties.

(2)

   1) (linearity): for any complex numbers α and β,

$$Z[\alpha\, f(t) + \beta\, g(t)](t,\omega) = \alpha\, Z[\,f(t)](t,\omega) + \beta\, Z[\,g(t)](t,\omega)$$

   2) (translation): for any integer m,   (3,4)

$$Z[\,f(t+m)](t,\omega) = \exp(2\,\pi\, \boldsymbol{k}\, m\, \omega)\, Z[\,f](t,\omega)$$

in particular,

$$(Z\,f)(t+1,\omega) = \exp(2\,\pi\, \boldsymbol{k}\; \omega)Z\,f(t,\omega)$$

   3) (modulation):   (5)

$$Z\,[\exp(2\,\pi\, \boldsymbol{k}\, m\, t)f](t,\omega) = \exp(2\,\pi\, \boldsymbol{k}\, m\, t)(Z\,f)(t,\omega)$$

   4) (periodicity): The Zak transform is periodic in w with period one, that is,   (6)

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$$(Z f)(t, \omega + 1) = (Z f)(t, \omega)$$

5) (translation and modulation): By combining 2) and 3) one obtains   (7)

$$Z \left[\exp(2 \pi \mathbf{k} m t) f(t + n)\right](t, \omega) = \exp(2 \pi \mathbf{k} m t)\exp(2 \pi \mathbf{k} n \omega)(Z f)(t, \omega)$$

6) (conjugation):   (8)

$$\left(Z \bar{f}\right)(t, \omega) = \overline{(Z f)}(t, -\omega)$$

7) (symmetry): If f is even, then

$$(Z f)(t, \omega) = (Z f)(-t, -\omega) \tag{9}$$

and if f is odd, then

$$(Z f)(t, \omega) = -(Z f)(-t, -\omega) \tag{10}$$

From 6) and 7) it follows that if f is real-valued and even, then

$$(Z f)(t, \omega) = \overline{(Z f)}(t, -\omega) = (Z f)(-t, -\omega) \tag{11}$$

Because of 2) and 4), the Zak transform is completely determined by its values on the unit square $Q = [0,1] \times [0,1]$.

8) (convolution): Let

$$h(t) = \int_{-\infty}^{\infty} R(t - s) f(s) ds \tag{12}$$

then

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$$(Z\,h)(t,\omega) = \int_0^1 (Z\,R)(t-s,\omega)\,(Z\,f)(s,\omega)\,ds \tag{13}$$

Analytic properties.

If f is a continuous function such that

$$f(t) = O((1+|t|)^{-1-\epsilon}) \text{ as } t \to \infty \text{ for some } \epsilon > 0 \tag{1}$$

Then $Z\,f$ is continuous on Q. A rather peculiar property of the Zak transform is that if $Z\,f$ is continuous, it must have a zero in Q. The Zak transform is a unitary transformation from $L^2(\mathbb{R})$ onto $L^2(Q)$.

Inversion formulas.

The following inversion formulas for the Zak transform follow easily from the definition, provided that the series defining the Zak transform converges uniformly:

$$f(t) = \int_0^1 (Z\,f)(t,\omega)\,d\omega\,;\, -\infty < t < \infty \tag{1}$$

$$\tilde{f}(-2\,\pi\,\omega) = \frac{1}{\sqrt{2\,\pi}} \int_0^1 exp(-2\,\pi\,k\,\omega\,t)\,(Z\,f)(t,\omega)\,dt \tag{2}$$

and

$$f(2\,\pi\,\omega) = \frac{1}{\sqrt{2\,\pi}} \int_0^1 exp(-2\,\pi\,k\,x\,t)\,(Z\,\tilde{f})(x,t)\,dx \tag{3}$$

where $\tilde{f}$ is the Fourier transform of f, given by

$$\tilde{f}(\omega) = \frac{1}{\sqrt{2\,\pi}} \int_{-\infty}^{\infty} f(x)\,exp(k\,\omega\,x)\,dx \tag{4}$$

Applications.

The Gabor representation problem can be stated as follows: Given $g \in L^2(\mathbb{R})$ and two real numbers, α, β, different from zero, is it possible to represent any function $f \in L^2(\mathbb{R})$ by a series of the form

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$$f = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} c_{mn} \; g_{m\beta,n\alpha} \, , \qquad (1)$$

where $g_{m\beta,n\alpha}$ are the Gabor functions, defined by:

$$g_{m\beta,n\alpha}(x) = \exp(2 \, \pi \, \beta \, x) \, g(x - n\alpha) \qquad (2)$$

and c_{mn} are constants? And under what conditions is the representation unique?

Fix a coordinate x in a line $\mathbb{R}$; the family of functions in $\mathbb{R}$

$$e_\lambda\,(x) \; = \; \sqrt[4]{2} \exp \, \left(-\pi \, (x \, - \, p)^2 \, + \, 2 \, \pi \, \boldsymbol{k} \, \theta \, x\right) \qquad (3)$$

are called Gabor functions. Here $\lambda = (p, \theta)$ is a point in the phase space $\Phi = \mathbb{R} \otimes \mathbb{R}$'

The operators

$$\mathcal{A} \; = \; \frac{1}{2 \, \pi} \frac{d}{dx} \, + \, x \qquad (4)$$

$$\mathcal{A}^\dagger \; = \; -\frac{1}{2 \, \pi} \frac{d}{dx} \, + \, x \qquad (5)$$

in L^2 are adjoint one to another. They are called the *annihilation* and the *creation*
operators.

Any Gabor function is an eigenvector of the annihilation operator:

$$\mathcal{A} e_\lambda \; = \; \lambda e_\lambda \qquad (6)$$

where

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$$\lambda = (p, \theta) \tag{7}$$

and

$$\lambda = p + \boldsymbol{k}\theta \tag{8}$$

For any φ in the domain of the operator a we have

$$Z(\mathcal{A}\varphi) = AZ\varphi \tag{9}$$

$$A = \frac{1}{2\pi\boldsymbol{k}} \left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial y} \right) + y \tag{10}$$

Thoughts

The following texts represent collections of thoughts that still have to be brought in proper order and in mutual consistency.

Spin and dyadic product

As factors of the dyadic product we consider imaginary quaternionic numbers or vectors in $\mathbb{R}_3$. The product corresponds to a matrix. This matrix acts as an operator.

$$u \otimes v \to \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \begin{bmatrix} u_1 v_1 & u_1 v_2 & u_1 v_3 \\ u_2 v_1 & u_2 v_2 & u_2 v_3 \\ u_3 v_1 & u_3 v_2 & u_3 v_3 \end{bmatrix} \tag{1}$$

The product of quaternions contains sign selections. For the imaginary parts this selection has to do with the handedness of the external product. Dyadic products are well suited to store the product such that the sign selections are stored as well. The sign selection plays its role in the dyad **ij**, which consists of two imaginary base numbers. The dyad **ij** $=-$**ji**, and **k** can be $\pm$ **ij**. Let us apply this to the definition of S_z.

$$\tag{2}$$

$$S_{z=} -i\hbar \begin{bmatrix} 0 & \boldsymbol{e_x e_y} - \boldsymbol{e_y e_x} & 0 \\ \boldsymbol{e_y e_x} - \boldsymbol{e_x e_y} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= i\hbar \begin{bmatrix} 0 & -2\boldsymbol{e_x e_y} & 0 \\ 2\boldsymbol{e_x e_y} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This shows that the definition of S_z via the dyadic product reflects the choice in handedness of the external product of $\mathbf{e}_x$ and $\mathbf{e}_y$.

Wave package

The linear momentum is interpretable as a displacement operator. This operator is better treated in Fourier space than in configuration space. In Fourier space a particle becomes a wave package. The Fourier transforms of the fields describe the wave package.

Operator $\boldsymbol{P}$ has eigenfunctions $\tilde{f}(p)$ with eigenvalues p:

$$< q|p > = \tilde{f}(p) = < p|q >^* = f^*(q) = \exp\left(\boldsymbol{k} \cdot \frac{p\, q}{\hbar}\right)) \tag{1}$$

$$< p|\boldsymbol{P}\, p > = p \tag{2}$$

A pure particle can be represented by a single Hilbert vector |f>. Its wave function is given by:

$$\psi(q) = < \psi|q > \tag{3}$$

Or by:

$$\tilde{\psi}(p) = < p|\psi > \tag{4}$$

A mixed particle takes a <u>Hilbert distribution</u>[215] in order to define its presence.

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[215] Functions and fields; Distributions in quaternionic Hilbert space

$$\rho(q) = <\rho|q> \qquad (5)$$

A blurred Hilbert distribution is a Hilbert field.

$$\phi(q) = \rho(q) \circ \varphi(q) \qquad (6)$$

A different type of blur gives a different type of Hilbert field.
The <u>wave functions</u>[216] and private Hilbert fields represent particles. Their Fourier transforms represent wave packages. A very particular Hilbert field is a probability density that is based on a <u>probability density operator</u>[217].
A single wave mode represents a plane wave. Look at the linear momentum of the field contained in a volume V surrounded by surface S:

$$\boldsymbol{P}_{field} = \int_V \boldsymbol{g}_{field}\, dV = \int_V \rho_0 \boldsymbol{\phi}\, dV + \int_V \langle \nabla\boldsymbol{\phi}, \boldsymbol{E} \rangle\, dV + \oint_S \langle \hat{\boldsymbol{n}}, \boldsymbol{E}\boldsymbol{\phi} \rangle dS \qquad (7)$$

For each temporal Fourier mode of the field in free space (vanishing charge density ρ_0, no variance of scalar potential ϕ_0), where $\boldsymbol{E}\boldsymbol{\phi}$ falls off rapidly, we can neglect the first and the third term.

$$\boldsymbol{P}_{field} \approx \int_V \langle \nabla\boldsymbol{\phi}, \boldsymbol{E} \rangle\, dV \qquad (8)$$

Further:

$$\boldsymbol{E} = \nabla\phi_0 - \nabla_0\boldsymbol{\phi} \approx -\nabla_0\boldsymbol{\phi} \qquad (9)$$

$$\qquad (10)$$

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[216] States
[217] States; State definition; Probability density

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$$P_{field} \approx - \int_V \langle \nabla \phi, \nabla_0 \phi \rangle \, dV$$

$$= \int_{V_p} \langle \mathbf{p} \widetilde{\phi}, \nabla_0 \widetilde{\phi} \rangle \, dV_p$$

$$= \int_{V_p} \mathbf{p} \langle \widetilde{\phi}, \nabla_0 \widetilde{\phi} \rangle \, dV_p$$

$$= \int_{V_p} \omega(p) \, \mathbf{p} \langle \widetilde{\phi}, \widetilde{\phi} \rangle \, dV_p$$

If the function $\langle \widetilde{\phi}(p), \widetilde{\phi}(p) \rangle$ gives the probability density for eigenvalue p. Then, this gives reason to interpret $\langle \phi(q), \phi(q) \rangle$ as probability density for the position q of the particle.

Fourier mode
A Fourier mode is a single frequency wave. It can be interpreted as a "particle" or as a train of particles whose charge is blurred by a very wide spread function. The corresponding current is blurred by that same spread function. It means that the divergence along the wave reduces to zero.

Often waves of the same frequency that belong to different mutually perpendicular fields combine to form polarized waves. The waves may differ in their phase shifts. The combination then forms a polarized wave. Depending on the phase difference it may be an elliptical polarized wave, a circular polarized wave or a linearly polarized wave.

Systems
A system is a local assembly of physical items that act as a single physical item. Its state[218] is mixed. When a redefinition of physical items in terms of

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[218] States

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atomic predicates goes together with influences between items in the form of fields, then a redefinition of a system in terms of its components will certainly also have such effects. The redefinition may take different forms. It may be represented by an emission or absorption of a component or it may be a reshuffling of the components. The simplest case of reshuffling is a permutation of items that belong to the same category. A more complex situation is a periodic movement of one or more components within the realm of a system. In addition each sequence of creation and annihilation is a form of redefinition.

The system has its own characteristic vectors. The wave function may depend on the permutation state of the system. For example for fermions an odd permutation changes the sign of the (position related) wave function. For bosons a permutation does not affect the wave function. Permutations of different categories of components go together with their own type of influence. Thus, there are fermionic fields and there are bosonic fields. Each of these fields has its own type of creation and annihilation. Being fermion or boson relates to the spin type of the component. The annihilation and creation operators are closely related to the type of components involved and are also closely related to the type of fields involved. The annihilation/creation operators of fermions anti-commute and the annihilation/creation operators of bosons commute.

Entropy

A [system](#) is a local assembly of [physical items](#)[219] that act as a single physical item. The Density operator ϱ relates to the currently considered observable Q. A pure state is a ray spanned by an eigenvector of the operator Q.

[The von Neumann entropy](#)[220] $S(\varrho)$ of a physical system that is characterized by a [state](#)[221] $|\psi>$ is given by

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[219] Logic; Items

[220] http://en.wikipedia.org/wiki/Von_Neumann_entropy

[221] States

$$\rho = \sum_q \{|q > \lambda_q < q|\} = \sum_q \{\lambda_q \cdot \rho_q\} \tag{1}$$

$$\rho_q = |q >< q| \tag{2}$$

$$\lambda_q = |< \psi|q >|^2$$

$$S(\rho) = -k_B \cdot \sum_q \{\lambda_q \cdot \ln(\lambda_q)\} \tag{3}$$

The entropy $S(\varrho)$ describes the departure of the system from a pure state. In other words, it measures the degree of mixture ([entanglement[222]](http://en.wikipedia.org/wiki/Quantum_entanglement)) of the state $|\psi>$.

Some properties of the von Neumann entropy:

- $S(\varrho)$ is only zero for pure states.
- $S(\varrho)$ is maximal and equal to $\log_2 N$ for a maximally mixed state, N being the dimension of the Hilbert space.
- $S(\varrho)$ is invariant under changes in the basis of ϱ, that is, $S(\varrho) = S(U\varrho U^\dagger)$, with U a unitary transformation.
- $S(\varrho)$ is concave, that is, given a collection of positive numbers λ_q which sum to unity ($\Sigma_q \lambda_q = 1$) and density operators ϱ_q, we have

$$S\left(\sum_q \lambda_q \rho_q\right) \geq \sum_q \lambda_q S(\rho_q) \tag{4}$$

- $S(\varrho)$ is additive. Given two density matrices ϱ_A, ϱ_B describing independent systems A and B, then

[222] http://en.wikipedia.org/wiki/Quantum_entanglement

$$S(\rho_A \otimes \rho_B) = S(\rho_A) + S(\rho_B) \tag{5}$$

Instead, if ϱ_A, ϱ_B are the reduced density operators of the general state ϱ_{AB}, then

$$|S(\rho_A) - S(\rho_B)| \leq S(\rho_{AB}) \leq S(\rho_A) + S(\rho_B) \tag{6}$$

While in Shannon's theory the entropy of a composite system can never be lower than the entropy of any of its parts, in quantum theory this is not the case, i.e., it is possible that $S(\varrho_{AB}) = 0$ while $S(\varrho_A) > 0$ and $S(\varrho_B) > 0$.

Intuitively, this can be understood as follows: In quantum mechanics, the entropy of the joint system can be less than the sum of the entropy of its components because the components may be [entangled](http://en.wikipedia.org/wiki/Quantum_entanglement)[223]. The left-hand inequality can be roughly interpreted as saying that entropy can only be canceled by an equal amount of entropy. If system A and system B have different amounts of entropy, the lesser can only partially cancel the greater, and some entropy must be left over. Likewise, the right-hand inequality can be interpreted as saying that the entropy of a composite system is maximized when its components are uncorrelated, in which case the total entropy is just a sum of the sub-entropies.

- The von Neumann entropy is also *strongly sub-additive.* Given three Hilbert spaces, A, B, C,

$$S(\rho_{ABC}) + S(\rho_B) \leq S(\rho_{AB}) + S(\rho_{BC}) \tag{7}$$

Isolated systems
With isolated systems we mean systems in a geometrically compound environment where influences from the environment compensate each other, possibly including the influences on the environment that are caused by the system under consideration. This includes e.g. the gravitation field. The gravitation potential cannot be zero, but the

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[223] http://en.wikipedia.org/wiki/Quantum_entanglement

influence of other items can be negligible. Internal influences are internally compensated such that they are not felt by other systems. For example the sum of the charges, which are related to electromagnetic fields is zero. It means that the Fourier transforms of the local fields consist of linear combinations of discrete terms. This holds for the electrostatic fields and the magneto-static fields. It holds for rectangular components as well as for polar components. These components are the germs of quanta and are the source of creations and annihilations.

For example consider the vector potential A. Its Fourier transform can be written as:

$$\boldsymbol{A}(\boldsymbol{r},t) =$$

$$\sum_{k} \sum_{\mu=-1,1} \left\{ \boldsymbol{e}_\mu \cdot a_{\mu\boldsymbol{k}}(t) \cdot \exp\big(i(\boldsymbol{k},\boldsymbol{r})\big) + \ \bar{\boldsymbol{e}}_\mu \cdot \bar{a}_{\mu\boldsymbol{k}}(t) \right. \tag{1}$$
$$\left. \cdot \exp\big(-i(\boldsymbol{k},\boldsymbol{r})\big) \right\}$$

Where $\mathbf{e}_\mu$ are unit sized polarization vectors. They depend on the orthonormal vectors $\mathbf{e}_x$ and $\mathbf{e}_y$ that represent quaternionic imaginary base numbers. The index μ labels the photon spin. The product $\mathbf{e}_\mu{\cdot}a_\mu$ represents a quaternionic imaginary number. The number i can be interpreted as a base imaginary number in the direction of $\boldsymbol{k}$.

$$\boldsymbol{e}_1 \stackrel{\text{def}}{=} \frac{-1}{\sqrt{2}} \big(\boldsymbol{e}_x + i \cdot \boldsymbol{e}_y\big) \tag{2}$$

$$\boldsymbol{e}_{-1} \stackrel{\text{def}}{=} \frac{1}{\sqrt{2}} \big(\boldsymbol{e}_x - i \cdot \boldsymbol{e}_y\big) \tag{3}$$

$$(\boldsymbol{e}_x, \boldsymbol{k}) = 0 \tag{4}$$

$$(\boldsymbol{e}_y, \boldsymbol{k}) = 0 \tag{5}$$

$$\big[a_\mu(\boldsymbol{k}), a_{\mu'}(\boldsymbol{k}')\big] = 0 \tag{6}$$

$$\left[a^{\dagger}{}_{\mu}(\boldsymbol{k}), a^{\dagger}{}_{\mu'}(\boldsymbol{k'})\right] = 0 \tag{7}$$

$$\left[a_{\mu}(\boldsymbol{k}), a^{\dagger}{}_{\mu'}(\boldsymbol{k'})\right] = \delta_{\mu\mu'} \cdot \delta_{kk'} \tag{8}$$

Here the $\sqrt{\dfrac{\hbar}{2\omega V\epsilon_0}}\, a_{\mu}(\boldsymbol{k})$ are the operator equivalents of the coefficients $a_{\mu k}$ and $\omega = c\ |\boldsymbol{k}| = ck$.
This results in:

$$\boldsymbol{A}(\boldsymbol{r}, t) = \sum_{k,\mu} \sqrt{\frac{\hbar}{2\omega V\epsilon_0}} \{\boldsymbol{e}_{\mu(k)} \cdot a_{\mu}(\boldsymbol{k}, t) \cdot \exp(i(\boldsymbol{k}, \boldsymbol{r})) + \bar{\boldsymbol{e}}_{\mu(k)} \cdot a^{\dagger}{}_{\mu}(\boldsymbol{k}, t) \cdot \exp(-i(\boldsymbol{k}, \boldsymbol{r}))\} \tag{9}$$

$$\boldsymbol{E}(\boldsymbol{r}, t) = i \cdot \sum_{k,\mu} \sqrt{\frac{\hbar}{2\omega V\epsilon_0}} \{\boldsymbol{e}_{\mu(k)} \cdot a_{\mu}(\boldsymbol{k}, t) \cdot \exp(i(\boldsymbol{k}, \boldsymbol{r})) - \bar{\boldsymbol{e}}_{\mu(k)} \cdot a^{\dagger}{}_{\mu}(\boldsymbol{k}, t) \cdot \exp(-i(\boldsymbol{k}, \boldsymbol{r}))\} \tag{10}$$

$a_{\mu}(\boldsymbol{k}, t)$ is an annihilation operator and $a^{\dagger}{}_{\mu}(\boldsymbol{k}, t)$ is a creation operator.

$$a^{\dagger}{}_{\mu}(\boldsymbol{k}, t)\ |n> = |n+1> \sqrt{n+1} \tag{11}$$

$$a^{\dagger}{}_{\mu}(\boldsymbol{k}, t)\ |0> = |1> \tag{12}$$

$$a_{\mu}(\boldsymbol{k}, t)\ |n> = |n-1> \sqrt{n} \tag{13}$$

$$a_{\mu}(\boldsymbol{k}, t)\ |0> = 0 \tag{14}$$

$$\left[a_{\mu}(\boldsymbol{k}), \left(a^{\dagger}{}_{\mu}(\boldsymbol{k})\right)^{n}\right] = \left(a_{\mu}(\boldsymbol{k})\right)^{n} \tag{15}$$

The Hamiltonian is:

$$H(t) = \hbar\omega \sum_{k,\mu} \{a_{\mu}{}^{\dagger}(\boldsymbol{k}, t) \cdot a_{\mu}(\boldsymbol{k}, t) + \tfrac{1}{2}\} \tag{16}$$

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The number operator N_μ gives the number of quanta:

$$N_\mu(\boldsymbol{k}, t) = a_\mu{}^\dagger(\boldsymbol{k}, t) \cdot a_\mu(\boldsymbol{k}, t) \tag{17}$$

The quanta discussed here are bosons. With the electromagnetic field they are photons. Photons have integer spin 1. With the dyadic product $\otimes$ follows:

$$S_z \overset{\text{def}}{=} -i\hbar(\boldsymbol{e}_x \otimes \boldsymbol{e}_y - \boldsymbol{e}_y \otimes \boldsymbol{e}_x) \text{ and cyclically for } x \to y \to z \to x \tag{18}$$

$$[S_x, S_y] = i\hbar S_z \tag{19}$$

$$S_z \cdot \boldsymbol{e}_\mu = \mu \cdot \boldsymbol{e}_\mu \tag{20}$$

Fermions have half integer spin. With fermions the creation and annihilation operators a and a† have different commutation relations. Instead of commuting, these operators anti-commute.

Measurement

We differentiate between a measurement using a piece of equipment and an observation as is done between items in universe. In the particle view the measuring equipment scrambles the phases. After that scrambling an observation is done. In the wave view the measuring equipment takes care that the phases stay intact, while the amplitudes are ignored during the next observation.

In measurement terms the scramble of the phases is called **de-coherence**. In the same sense the care to keep phases pure and the neglecting of the amplitudes could be called re-coherence. Both actions can be related with the Fourier transforms that convert the wave view into the particle view or vice versa.

Measurement preparation

In a measurement the observation follows after a preparation phase by the measuring equipment. Such a preparation may squeeze the shape of

the private field that represents the item. For example, a preparation for precise position measurement may squeeze the private field and change the item's subspace such that its range of covered position eigenvectors becomes very short and that its range of covered momentum eigenvectors extends very far. Similarly, when a preparation is made for precise momentum measurement then the item's private field is squeezed and its subspace is changed in the other direction, such that it covers a huge range of position eigenvectors and a very short range of momentum eigenvectors. A Fourier transform does not change the item's subspace. It changes the private field of the item from position based coordinates to momentum based coordinates or vice versa.

Changing the item's subspace such that its range of covered position eigenvectors becomes very short and that its range of covered momentum eigenvectors extends very far is called **decoherence**. In case of a system it reduces the entanglement of that system.

Hamilton-Jacobi

The Hamilton-Jacobi equation shows how the Hamiltonian relates to the action S of the current manipulator. In this section we consider t to be the manipulator time!

$$H \cdot U_t = \tilde{I}_t \cdot \hbar \cdot \frac{\partial U_t}{\partial t} \tag{1}$$

For the eigenvalues holds

$$\Delta u_t \approx \Delta S_{lt} \cdot u_t \tag{2}$$

Thus, we can put

$$H \cdot U_t = - \left(\frac{\partial S_t}{\partial t} \right) \cdot U_t \tag{3}$$

$$\tag{4}$$

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$$H = - \left(\frac{\partial S_t}{\partial t} \right)$$

For the expectation values s_t of the action operator S_t holds

$$\tilde{\imath}_t \cdot \hbar \cdot \frac{\Delta s_t}{\Delta t} = \boldsymbol{e}_{t0} + \boldsymbol{e}_{t1} \cdot \Delta t \cdot \frac{\chi_{t1}}{2} + \boldsymbol{e}_{t2} \cdot \Delta t^2 \cdot \frac{\chi_{t1} \cdot \chi_{t2}}{6} \tag{5}$$

$$- \boldsymbol{e}_{t3} \cdot \Delta t^3 \cdot \frac{\chi_{t1} \cdot \chi_{t2} \cdot \chi_{t3}}{24} + \mathcal{O} \left(\Delta t^3 \right)$$

This derivation is completely independent from the observation of Q. Thus S_t has nothing to do with the Minkowski metric that appears during observations of position.

The Lagrangian
The Lagrangian is equivalent to the local geodesic equation.

The Lagrangian $\mathcal{L}_\tau$ is related with the action s_t.

$$s_t = \int_a^b \mathcal{L}_\tau d\tau \tag{1}$$

The integral is taken over the trail with the observed path. The index t of the action S_t is the trail progression parameter. The integration parameter stands for the coordinate time. The right side of the equation plays in Lorentzian space.

The Euler Lagrange equations explicitly use observations. For that reason the Lagrangian is considered to be a function of the observed q, the velocity $\dot{q}$ and the coordinate time τ. The velocity is measured with the coordinate time.

$$\mathcal{L}_\tau = \mathcal{L}_\tau(\tau, q, \dot{q}) \tag{2}$$

$$\dot{q} = \frac{dq}{d\tau} \tag{3}$$

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The Euler-Lagrange equations are:

$$\frac{\partial \mathcal{L}_\tau(\tau, q, \dot{q})}{\partial q_i} - \frac{d}{d\tau} \frac{\partial \mathcal{L}_\tau(\tau, q, \dot{q})}{\partial \dot{q}_i} = 0 \qquad (4)$$

for i = x, y, z

When the Lagriangian does not vary with one or more of its parameters, then this corresponds with a corresponding symmetry of the system. By Noether's theorem[224], such symmetries of the system correspond to conservation laws[225]. In particular, the invariance of the Lagrangian with respect to time τ implies the conservation of energy.

By partial differentiation of the above Lagrangian, we find:

$$\frac{\partial \mathcal{L}_\tau(\tau, q, \dot{q})}{\partial q_i} = \frac{\partial U}{\partial q_i} = F_i \qquad (5)$$

$$\frac{\partial \mathcal{L}_\tau(\tau, q, \dot{q})}{\partial \dot{q}_i} = m \cdot \dot{q}_i = p_i \qquad (6)$$

where the force is $F = -\nabla U$ (the negative gradient of the potential, by definition of conservative force), and p is the momentum. By substituting these into the Euler–Lagrange equation, we obtain a system of second-order differential equations for the coordinates on the particle's trajectory,

$$F_i = \frac{d(m\dot{q}_i)}{dt} = m \cdot \ddot{q}_i = \dot{p}_i \qquad (7)$$

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[224] http://en.wikipedia.org/wiki/Noether%27s_theorem
[225] http://en.wikipedia.org/wiki/Conservation_law

which is Newton's second law.

The world's action
The action S_t represents the influences that the rest of the world via unitary operator U_t release onto the state $\{|f>_s\}_s$.

In his book about quantum gravity Rovelli writes:
"In the general relativistic parlance 'matter' is anything which is not the gravitational field. As far as current physics knows, the world is made up of the gravitational field, Yang Mills fields, fermion fields and, presumably, scalar fields."[226]

All these fields give a contribution to the action S.

$$S(e, \omega, A, \psi, \phi) \tag{1}$$

$$= S_{GR}[e, \omega] + S_{matter}[e, \omega, A, \psi, \phi] = S_{GR}[e, \omega] + S_{YM}[e, A] + S_f(e, \omega, A, \psi) + S_{sc}[e, A, \phi]$$

e is the gravitational field.
A(q) is the electromagnetic field.
ω(q) is the spin connection. It is a one form in the Lie algebra of the Lorentz group so(3,1)
ψ(q) is a scalar field, possibly with values in the representation of the Yang Mills group.
ϕ(q) is a field in the spinor representation of the Lorenz group.
A(q) has a non Abelian connection to the Yang Mills group.

The local characteristics of these fields must be represented in the eigenvalue of the current manipulator.

Representing multiple fields
Professor Mendel Sachs recently wrote a few books in which he promotes the inclusion of more terms in the metric than Einstein did. Sachs uses a

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[226] Carlo Rovelli, book: Quantum gravity, 2004, chapter 2, paragraph 2.1.2

four vector with quaternionic coefficients in order to specify the metric. Sachs uses all sixteen terms, while Einstein skipped six due to symmetry considerations. The argument of Sachs is that the symmetry is broken due to the characteristics of the quaternion number space. See: http://www.compukol.com/mendel/publications/publications.html.

16-ons contain the required 16 real numbers that can be arranged as a four vector with quaternion coefficients. Sachs still uses the Minkowski metric. So, his view concerns observed spacetime.

Planck limits for all physical observables

This is taken from: http://www.motionmountain.net/research.html .

Basic measures

The basic measures of physics are:

$$\text{The Planck-length, } l_{Pl} = \sqrt{\hbar G/c^3} = 1.6 \cdot 10^{-35} \text{ m} \tag{1}$$

$$\text{The Planck-time, } t_{Pl} = \sqrt{\hbar G/c^5} = 5.4 \cdot 10^{-44} \text{ } s \tag{2}$$

$$\text{The Planck energy is given by: } E_{Pl} = \sqrt{\hbar c^5/G} = 2.0 GJ \tag{3}$$

Fundamentals

A large part of modern physics can be summarized in four simple and fundamental statements on motion:

quantum theory on action:	$W \geqslant \hbar$
thermodynamics on entropy:	$S \geqslant k$
special relativity on speed:	$v \leqslant c$
general relativity on force:	$F \leqslant \dfrac{c^4}{4G}$

These limits are valid for all physical systems, whether composite or elementary, and for all observers. Note that the limit quantities of special relativity, thermodynamics, quantum theory and general relativity can also be seen as the right-hand sides of the respective indeterminacy relations:

length l and acceleration a	$\Delta l \, \Delta a \leqslant c^2$
the displacement d and momentum p	$\Delta d \, \Delta p \geqslant \dfrac{\hbar}{2}$
temperature T and energy U	$\Delta \dfrac{1}{T} \Delta U \geqslant \dfrac{k}{2}$
Energy flow E and size l	$\Delta E \Delta l \leqslant \dfrac{c^4}{4G}$

By combining the three fundamental limits, we can obtain limits on a number of physical
observables.The following limits are valid generally, for both composite and elementary
systems:

time interval:
$$t \geqslant \sqrt{\frac{4G\hbar}{c^5}} = 1.1 \cdot 10^{-43} \text{s}$$

time-distance product:
$$td \geqslant \frac{4G\hbar}{c^4} = 3.5 \cdot 10^{-78} \text{ ms}$$

acceleration:
$$a \leqslant \sqrt{\frac{c^7}{4G\hbar}} = 2.8 \cdot 10^{51} \text{ m/s}^2$$

angular frequency:
$$\omega \leqslant 2\pi \sqrt{\frac{c^5}{2G\hbar}} = 5.8 \cdot 10^{43} \text{ /s}$$

Adding the knowledge that space and time can mix, we get:

distance:
$$d \geqslant \left(\frac{4G\hbar}{c^3}\right)^{1/2} = 3.2 \cdot 10^{-35} \text{m}$$

area:
$$A \geqslant \frac{4G\hbar}{c^3} = 1.0 \cdot 10^{-69} \text{ m}^2$$

volume:
$$V \geqslant \left(\frac{4G\hbar}{c^3}\right)^{3/2}$$
$$= 3.4 \cdot 10^{-104} \text{ m}^3$$

curvature:
$$K \leqslant \frac{c^3}{4G\hbar} = 1.0 \cdot 10^{69} \text{ /m}^2$$

mass density:
$$\rho \leqslant \frac{c^5}{16G^2\hbar} = 3.2 \cdot 10^{95} \text{ kg/m}^3$$

Elementary particles

A particle is elementary if the system size l is smaller than any conceivable
dimension, thus for elementary particles:
$$l \leqslant \frac{\hbar}{mc} \tag{1}$$

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Using this limit, we find the well-known mass, energy and momentum limits, valid only for elementary particles:

$$m \leqslant \sqrt{\frac{\hbar c}{4G}} = 1.1 \cdot 10^{-8} \text{ kg} = 0.60 \cdot 10^{19} \text{ GeV/c}^2 \tag{2}$$

$$E \leqslant \sqrt{\frac{\hbar c^5}{4G}} = 9.8 \cdot 10^8 \text{ J} = 0.60 \cdot 10^{19} \text{ GeV} \tag{3}$$

$$p \leqslant \sqrt{\frac{\hbar c^3}{4G}} = 3.2 \text{ kgm/s} = 0.60 \cdot 10^{19} \text{ GeV/c} \tag{4}$$

Virtual particles
Virtual particles do not obey the mentioned limits.

EM limits
Our discussion of limits can be extended to include electromagnetism. Using the (lowenergy)
electromagnetic coupling constant α, we get the following limits for physical systems interacting electromagnetically:

electric charge $\quad q \geqslant \sqrt{4\pi\varepsilon_0 \alpha c \hbar} = e = 0.16 \, aC$

electric field

$$E \leqslant \sqrt{\frac{c^7}{64\pi\varepsilon_0 \alpha \hbar G^2}} = \frac{c^4}{4Ge} = 1.9 \cdot 10^{62} \text{ V/m}$$

magnetic field

$$B \leqslant \sqrt{\frac{c^5}{64\pi\varepsilon_0 \alpha \hbar G^2}} = \frac{c^3}{4Ge} = 6.3 \cdot 10^{53} \text{T}$$

Voltage

$$U \leqslant \sqrt{\frac{c^4}{16\pi\varepsilon_0 \alpha G}} = \frac{1}{e}\sqrt{\frac{\hbar c^5}{4G}} = 6.1 \cdot 10^{27}\text{V}$$

Inductance

$$L \geqslant \frac{1}{4\pi\varepsilon_0 \alpha}\sqrt{\frac{4G\hbar}{c^7}} = \frac{1}{e^2}\sqrt{4G\hbar^3 c^5}$$
$$= 4.4 \cdot 10^{-40}\text{H}$$

With the additional assumption that in nature utmost one particle can occupy one Planck volume, we get

charge density

$$\rho_e \leqslant \sqrt{\frac{\pi\varepsilon_0\alpha}{16G^3}\frac{c^5}{\hbar}} = e\sqrt{\frac{c^9}{64G^3\hbar^3}}$$

$$= 4.7 \cdot 10^{84} \text{ C/m}^3$$

Capacitance

$$C \geqslant 4\pi\varepsilon_0\alpha\sqrt{\frac{4G\hbar}{c^3}} = e^2\sqrt{\frac{4G}{c^5\hbar}}$$

$$= 2.6 \cdot 10^{-47} F$$

For the case of a single conduction channel, we get

electric resistance

$$R \geqslant \frac{1}{4\pi\varepsilon_0\alpha c} = \hbar\, e^2 = 4.1 \text{ k}\Omega$$

electric conductivity $G \leqslant 4\pi\varepsilon_0\alpha c = e^2\,\hbar = 0.24 \text{ mS}$

electric current

$$I \leqslant \sqrt{\frac{\pi\varepsilon_0\alpha c^6}{G}} = e\sqrt{\frac{c^5}{4\hbar G}} = 1.5 \cdot 10^{24} \text{ A}$$

Indeterminacy relations:

C capacity and U potential difference $\qquad \Delta C\,\Delta U \geqslant e$

Electric current I and time $t \qquad \Delta I\,\Delta t \geqslant e$

Derived limits

The ratio of angular momentum D to energy E times length L has the dimensions of inverse speed. Since speeds are limited by the speed of light, we get

$$D_{system} \leqslant \frac{1}{c}\,LE \qquad\qquad (1)$$

The action limit

$$W \leqslant LE/c \tag{2}$$

is not exceeded in any physical process.

Since action values in nature are limited from below by ħ, we get a limit for the speed of a system:

$$v_{system} \geqslant \hbar c^2 \, \frac{1}{LE} \tag{3}$$

This is not a new result; it is just a form of the indeterminacy relation of quantum theory.

Thanks to the connection $W = FLT$ between action W, force F, distance L and time T, we can deduce

$$F_{system} \geqslant \frac{\hbar}{2c} \, \frac{1}{T^2} \tag{4}$$

The power P emitted by a system of size L and mass M is limited by

$$c^3 \, \frac{M}{L} \geqslant P_{system} \geqslant 2\hbar G \, \frac{M}{L^3} \tag{5}$$

In 1973 Bekenstein discovered a famous limit that connects the entropy S of a physical system with its size and mass. No system has larger entropy than one bounded by a horizon. The larger the horizon surface, the larger the entropy.

$$\frac{S}{S_{limit}} \leqslant \frac{A}{A_{limit}} \tag{6}$$

which gives

$$S \leqslant k \frac{c^3}{4G\hbar} A \tag{7}$$

where A is the surface of the system. Equality is realized only for black holes. We assume that the limits for vacuum are opposite to those for matter. We can then write

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$$c^2/4G \leqslant M/L \tag{8}$$

for the vacuum.
Using

$$\frac{S}{S_{c.Planck}} \leqslant \frac{M}{M_{c.Planck}} \frac{A}{A_{c.Planck}} \frac{L_{c.Planck}}{L} \tag{9}$$

we get

$$S \leqslant \frac{\pi k c}{\hbar} ML = \frac{2\pi k c}{\hbar} MR \tag{10}$$

This is called Bekenstein's entropy bound.

A lower limit for the temperature T of a thermal system can be found using the idea that the number of degrees of freedom of a system is limited by its surface, or more precisely, by the ratio between the surface and the Planck surface. We get the limit

$$T \geqslant \frac{4G\hbar}{\pi k c} \frac{M}{L^2} \tag{11}$$

Lower limit for the electric field E:

$$E \geqslant 4Ge \frac{M^2}{Q^2 L^2} \tag{12}$$

Lower limit for the magnetic field B:

$$B \geqslant \frac{4Ge}{c} \frac{M^2}{Q^2 L^2} \tag{13}$$

Cosmological limits
Cosmology is characterized via the cosmological constant Λ by the inequality:

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$$l \lessapprox \frac{1}{\Lambda} \tag{1}$$

For single particles, the absolute lower speed limit, the cosmological speed limit, is given by:

$$v_{particle} \geqslant \frac{\sqrt{4G\hbar/c}}{L_{universe}} = L_{corr.Planck}\sqrt{\Lambda}\, c \approx 7 \cdot 10^{-53} \; m/s \tag{2}$$

The negative energy volume density $-\Lambda c^4/4\pi G$ corresponds to a force value

$$F = \frac{\Lambda \hbar c}{2\pi} = 4.8 \cdot 10^{-79} \; N \tag{3}$$

This is also the gravitational force between two corrected Planck masses located at the cosmological distance $\sqrt{\pi/4\Lambda}$.

In nature there is a minimum time interval, $l_{Pl}/c = t_{Pl}$, the Planck-time.

A recent prediction derived from the standard model of elementary particles give as an upper limit for the electron dipole moment d_e a value of

$$\frac{|d_e|}{e} < 3 \cdot 10^{-21} \; m \tag{4}$$

The mass m of any elementary particle is constrained by the Planck mass m_{Pl}

$$m < \frac{\hbar}{c \, l_{Pl}} = \sqrt{\frac{\hbar c}{G}} = m_{Pl} = 2.2 \cdot 10^{-8} \; kg = 1.2 \cdot 10^{19} \; GeV/c^2. \tag{5}$$

The maximum possible value for mass density ρ_{Pl} is

$$\rho_{Pl} = \frac{c^5}{G^2 \hbar} = 5.2 \cdot 10^{96} \; kg/m^3 \tag{6}$$

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Within a factor of order one, we find

$$K \; < \; \frac{c^3}{G\hbar} \; = \; 0.39 \cdot 10^{70} \, m^{-2} \tag{7}$$

as a limit for the surface curvature K in nature. In other words, the universe has never been a point, never had zero age, never had infinite density, and never had infinite curvature.

Limit quality
Nature provides two limits for each observable: a Planck limit and a cosmological limit.
All measurements are limited in precision.
Because of the fundamental limits to measurement precision, the measured values of physical observables do not require the full set of real numbers. In fact, limited precision implies that observables cannot be described by the real numbers.
At Planck scales it is impossible to distinguish between matter and vacuum. Vacuum and matter do not differ at Planck scales. Similarly, at the Planck-length it is impossible to distinguish between positive and negative time values: so particles and antiparticles are not clearly distinguished at Planck scales.

The strictest upper limits are those with the smallest exponent for length, and the strictest lower limits are those with the largest exponent of length.
The accuracy of time measurements is limited by the Planck-time t_{Pl}.
The accuracy of length measurements is limited by the Planck-length l_{Pl}.
All measurements – be they measurements of position, speed, mass or any other observable – are electromagnetic. In other words, all measurements in nature are detection of photons. And in strand theory photon absorption and detection are intimately related to the crossing switch.
All electromagnetic information is communicated by directed information carrying quanta in the form of shot noise. However, secondary information can be derived from the shape of the quantum cloud.

References:
More useful stuff is collected in the toolkit
Axiomatic Quantum Theory, W. Lücke, http://arxiv.org/PS_cache/quant-ph/pdf/9510/9510024v2.pdf

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An overview of gravity theories:
http://arxiv.org/PS_cache/arxiv/pdf/0909/0909.4672v2.pdf.

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Part three

On the Origin of Physical Fields

On the Origin of Physical Fields

Abstract

Physical fields form the solution of nature for the problem that the set of observations is overwhelming the set of underlying variables.

On the origin of physical fields.

The Hilbert book model is a simple model of physics that is strictly based on traditional quantum logic and on the lattice isomorphic model; the set of subspaces of an infinite dimensional separable Hilbert space for which the inner product is specified by using quaternions[227].

This restriction results in the fact that all sets of variables are countable. At the same time most observations are taken from a continuum. As a result the set of potential observations overwhelms the set of variables[228]. The situation is comparable to the situation in which the number of equations is far larger than the number of variables that should form the result. In such cases, probably, the set of equations will appear to be inconsistent. In order to cure the situation, it is common to assume that the observations are inaccurate. The inaccuracy must be stochastic or with other words the observation result must be blurred.

Nature applies a similar solution, but instead of a simple spread function in the form of a probability density distribution, nature applies a quaternionic probability amplitude distribution (QPAD). This QPAD can be split into a real part that represents a "charge" density distribution and an imaginary part that represents a corresponding "current" density distribution. The "charge" represents the set of properties of the thing that is being observed. The parameter of the distribution represents the location at which the "charge" is observed. The squared modulus of the

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[227] See: http://www.crypts-of-physics.eu/HilbertBookModelEssentials.pdf

[228] A continuum has a higher cardinality than a countable set.

QPAD represents the probability density of the presence of the "charge" at the location that is specified by the parameter.

This approach transfers the dynamics of the observation into a streaming problem. The equation of motion of the "charge" becomes a continuity equation[229].

The properties of particles move according to the above principle. With each elementary particle belong one or more QPAD's that act as private fields of the particle and that determine its dynamic behavior when it moves freely. However, these fields overlap. In this way these fields and the corresponding particles interact.

A subset of the elementary particles is massless. These particles correspond to a single QPAD. That does not say that their fields cannot overlap with other QPAD's.

All other elementary particles are identified by an ordered pair of QPAD's that are two field sign flavors of the same base field. The coordinate system, whose values are used as field parameter, has its own field sign flavor and acts as a sign flavor reference.

Categories of fields

Two categories of fields exist; QPAD's and administrator fields.

Primary fields

The first category consists of quaternionic probability amplitude distributions (QPAD's). The QPAD's may overlap and through this superposition they may form covering fields. The QPAD's exist in four sign flavors. The same holds for the covering fields. The QPAD's may interact. When different sign flavors interact the strength of the local interaction is characterized by a coupling factor. The members of this category will be called primary fields.

Secondary fields

The second category consists of administrator fields. These fields administer the effect of interactions on the local curvature of the

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[229] Another name for "continuity equation" is "balance equation".

positioning coordinate system. For all properties that characterize a coupling of sign flavors of primary fields an administrator field exist that registers the influence of that property during interactions on the local curvature.

One of these administrator fields is the gravitation field. It administers the influence of the strength of the coupling between sign flavors of primary fields on the local curvature.
The electromagnetic fields administer the influence of the electric charge on the local curvature.

The angular momentum including the spin also influences the local curvature. Also this effect is administered

The members of this category will be called secondary fields or administrator fields.

Metric

The local metric is a tensor. It intends to register the influence of fields on the local curvature. In order to do this it requires a coordinate system and a way to qualify the influence that the local value of the fields has on the selected coordinate system. It can do this via "charges", thus the properties that characterize the local QPAD's. Or it can use the values of the administrator fields. For example the Kerr Newman metric uses the local mass (density), the local electric charge (density) and the local angular momentum (density) in order to relate these to the local curvature[230].

Geo-cavities

The massive elementary particles correspond to two shearing QPAD's, which are sign flavors of the same base field. This combination is capable of generating a geo-cavity at the center location of the particle[231].

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[230] See next part.
[231] See: http://www.crypts-of-physics.eu/OriginOfMass.pdf

Nothing exists in universe, but QPAD's and geo-cavities.

Part four

On the Origin of Mass

On the Origin of Mass

Abstract

Mass is caused by fields of elementary particles that are able of creating geo-cavities at their center. Another cause is the presence of a different geometric anomaly such as a black hole.

Geo-cavities

Geo-cavities are geometrical abnormalities in the form of holes in the local geometry of a (pseudo)-Riemannian manifold.

A Riemannian manifold [232]is a real differentiable manifold in which each tangent space is equipped with an inner product, a Riemannian metric, which varies smoothly from point to point. On a pseudo-Riemannian manifold[233] the metric tensor need not be positive-definite. Instead a weaker condition of non-degeneracy is imposed.

The environment of a geo-cavity is described by a metric. The value of the metric depends on a selected coordinate system. Spherical geo-cavities require that the local geometry is specified using spherical coordinates.

Inside the hole no coordinates exist. Thus, the coordinates must circumvent the geo-cavity.

As a consequence, geo-cavities are surrounded by a very strong local curvature that follows the skin of the geo-curvature.

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[232] http://en.wikipedia.org/wiki/Riemannian_manifold
[233] http://en.wikipedia.org/wiki/Lorentzian_manifold

Nothing is present inside a geo-cavity. A quaternionic probability amplitude distribution[234] (QPAD) can be interpreted as the combination of a charge density distribution and a current density distribution. The squared modulus of this QPAD is a distribution of the presence of the load of properties that the QPAD transports. Thus QPAD that is defined in the neighborhood of the geo-cavity goes to zero when its parameter approaches the geo-cavity.

Information can neither enter nor leave the geo-cavity.

Geo-cavities have a skin and that skin has an area.

All geo-cavities have a virtual mass. This mass relates to the area of its skin.
The curvature in the surround of the geo-cavity corresponds to a gravitational potential that depends on the mass of the geo-cavity.

When the area of the skin is large enough, then a geo-cavity has entropy. In that case the entropy is proportional to the area of its skin. Entropy has an integer value. The unit of entropy is set by Boltzmann's constant.

Geo-cavities may have electrical charge. This charge corresponds to an electrostatic potential. Electric charge has an integer value. The unit of electrical charge is fixed. Some elementary particles (quarks) have a charge which is ⅓ or ⅔ of that unit value.

The metric is a function of the properties of the geo-cavity.

Geo-cavities generate a gravitation field and when appropriate a Coulomb field. Both fields are administrators. The gravitation field administers the curvature that corresponds with the metric. The Coulomb

[234] http://www.crypts-of-physics.eu/OriginOfPhysicalFields.pdf

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field administers the scalar and vector potential that is caused by the electric charge.

Reference coordinates

The situation can be described by two coordinate systems. One is a flat reference system the other is mostly flat, but it can locally be strongly curved. The reference system can be used to locate the center of the geo-cavities. The values of the curved coordinate system are used as positioning parameters.

Classes of geo-cavities

Several classes of geo-cavities exist.

One class of geo-cavities is generated and supported by a set of shearing fields. These fields are quaternionic probability amplitude distributions (QPAD's). Their squared modules are probability density distributions that describe the probability of the presence of a load of properties that characterize the set of coupled fields. We indicate the density distribution of the probability of presence with the shorthand PPDD. Inside the skin of the geo-cavity the PPDD does not exist and on approach of the skin the PPDD goes to zero.

The elementary particle geo-cavities share their properties with the fields that generate and support them. In this way, the geo-cavities relate to the gravitation field and the Coulomb field. Both fields are administrators.

Another class of geo-cavities is formed by the black holes. The event horizon of the black hole forms the skin of the geo-cavity.

As an alternative to the Big Bang, the start of the universe can also be thought to be implemented by a start-cavity.

Elementary particles

The equation of motion of an elementary particle is a continuity equation. It means that describing the motion of elementary particles is in fact a streaming problem. The general form of the equation of free motion of a massive elementary particle is[235]:

$$\nabla \psi^x \;=\; m \, \psi^y \tag{1}$$

Here the quaternionic nabla operator is the transporter. ψ^x indicates the transported quaternionic field sign flavor[236]. ψ^y indicates the coupled quaternionic field sign flavor and m is the coupling factor.
The ordered pair $\{\psi^x, \psi^y\}$ identifies the quantum type. The field configuration determines the coupling factor.
The coupling factor m follows from:

$$\int_V (\psi^{y*} \nabla \, \psi^x) \, dV \;=\; m \int_V (\psi^{y*} \, \psi^y) \, dV \;=\; m \; \int_V |\psi^y|^2 \, dV = m \, g \tag{2}$$

The two fields shear. At the location of the sign switch the fields produce a geo-cavity. The size of this geo-cavity is determined by the strength of the coupling factor m.
This geo-cavity is surrounded by a curvature of the geometry that is so strong that information cannot pass the skin of the geo-cavity. Outside of the geo-cavity the curvature follows a pattern that corresponds to the rest mass of the particle.
The gravitation field administrates this curvature[237].
The surround of the geo-cavity is described by a metric, which is a function of the properties of the ordered pair $\{\psi^x, \psi^y\}$. Many of these properties are combined in the "charge" that is transported by the field ψ^x. These properties are:

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[235] http://www.crypts-of-physics.eu/EssentialsOfQuantumMovement.pdf
[236] Quaternionic fields have sign flavor. Elementary particles have flavor.
http://en.wikipedia.org/wiki/Flavour_(particle_physics)
[237] http://www.crypts-of-physics.eu/TheCauseOfGravitation.pdf

- The coupling factor m
- The electric charge.
- The color charge.
- The spin.

The coupling factor and the electric charge are isotropic properties.
The color charge and the spin are anisotropic properties.
The coupling factor and the spin are integral properties.

The geo-cavity that is produced by an electron does not have the values of the Kerr–Newman metric that characterize black holes. Already the structure of the pair of field sign flavors that surround the geo-cavity differs from the structure of the EM field that is supposed to surround a charged black hole.

The heaviest top quark has a mass of 177 GeV/c^2
In physics, the Planck mass (m_P) is the unit of mass in the system of natural units known as Planck units[238]. It is defined as

$$m_P = \sqrt{\frac{h\,c}{G}} = 1.2209 \times 10^{19} \; GeV/c^2 = 2.17651(13) \times 10^{-8} \; kg \qquad (3)$$

The Planck mass is approximately the mass of the Planck particle[239], a hypothetical minuscule black hole who's Schwarzschild radius equals the Planck length.

$$l_{Pl} = \sqrt{\hbar G/c^3} = 1.6 \cdot 10^{-35} \; m. \qquad (4)$$

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[238] http://en.wikipedia.org/wiki/Planck_units
[239] http://en.wikipedia.org/wiki/Planck_particle

The Planck mass is the smallest possible mass for an observable black hole.

This also leads to the conclusion that the elementary particle geo-cavity differs from the black hole geo-cavity.

The elementary particle geo-cavity cannot be limited by the Planck mass. At a significant distance from its skin it must still deliver the proper curvature and the corresponding gravitation field. This means that the elementary particle geo-cavity is a pure classical geo-cavity. On radii below the Planck scale the curvature k is specified by the strictly geometric formula:

$$k = 1/r^2 \qquad\qquad (5)$$

The elementary particle geo-cavity is considered to have a radius defined by:

$$r_s = \frac{2GM}{c^2} \qquad\qquad (6)$$

The radius is far below the Planck length.

Thus, the unit of entropy does not fit into the elementary particle geo-cavity.

For deeper investigation, see: http://arxiv.org/abs/0802.2914 .

Black holes

When considered as an observed item, a black hole fulfills the specification of a geo-cavity[240]. The main difference with the usual notion of a black hole is that a geo-cavity is per definition empty. Within its skin nothing is present. A black hole is surrounded by a very strong curvature field such that information can no longer pass the skin of the hole. Therefore the hole can as well be completely empty. What happens to the

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[240] Usually a black hole is not considered to be empty. However, it is impossible to check this fact.

material that is sucked up by the black hole? Well, that will be ripped apart into its smallest possible parts. Part of the debris is used to widen the skin of the hole. The other part escapes from the absorption process and is reflected back. The hole gets bigger, but that only becomes visible via the enlargement of the skin. The surface of the skin gives an indication of the mass, which the hole represents. The curvature around the black hole is in correspondence with this mass. However, the hole itself is empty.

The black hole fulfills the definition of a geo-cavity. However, it is a rather large one.

The skin of the black hole can be seen as a collection of ground states of absorbed particles. Each of these ground states occupies a very small part of the surface and each represents a minimum amount of information. In this way is the entropy of a black hole relates to the surface of the skin. Black holes fulfill the no-hair theorem[241].

The surround of a black hole is described by a Kerr-Newman metric[242].

Black hole creation

Black holes are believed to be created when a huge star collapses under its own weight. However this is a far too simple picture. Indeed the enormous pressure near the center of a huge stage is required to generate the germ of a black hole that in different environmental conditions would easily fall apart. However this germ is surrounded by plenty of food such that it can grow quickly into a more durable black hole. As long as there is sufficient supply of star-matter the size of the black hole will increase. After a while, all of the star-matter is consumed or pushed away, out of reach of the black hole. The consumed matter is ripped apart and attached to the skin of the black hole. That skin grows. It is the carrier of all properties of the black hole. Instead of the consumed star, now the black hole is attracting matter from its environment.

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[241] http://en.wikipedia.org/wiki/No-hair_theorem
[242] http://en.wikipedia.org/wiki/Kerr%E2%80%93Newman_metric

So, the center of the star does not collapse, it is converted into a black hole that eats the star from inside out. During the process the outer part of the star collapses in the direction of the much smaller black hole.

Creating a black hole germ

Even when creating a black hole germ would mean that it is very short lived under normal conditions, it may hold the opportunity that under the proper conditions the germ can grow into a more durable form before it normally should collapse.

This fact makes the creation of black hole germs an interesting subject.

The start cavity

At its start the universe may have consisted of space that was empty except for a large geometric abnormality. It was a geo-cavity with nothing outside its skin and nothing inside its skin.

The skin consisted of ground states of particles. This cavity appeared to be instable and imploded[243]. The debris spread through the space that came available. The ground state obtained energy and their fields unfolded.

The size of the start cavity was huge and so was the mass that it represented. This mass was converted to energy, which became attached to the ground states.

The start cavity fits in the definition of a geo-cavity.

Metrices

Kerr-Newman metric

The Kerr–Newman metric describes the geometry of spacetime in the vicinity of a rotating mass M with charge Q. The formula for this metric depends upon what coordinates or coordinate conditions are selected.

Spherical coordinates

The line element $d\tau$ in spherical coordinates is given by:

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[243] This differs from the picture that corresponds to the Big Bang.

$$c^2\, d\tau^2 = -\left(\frac{dr^2}{\Delta} + d\theta^2\right)\rho^2 + (c\, dt - \alpha\, sin^2(\theta)\, d\phi)^2\, \frac{\Delta}{\rho^2} \qquad (1)$$

$$-\left((r^2 + \alpha^2)\, d\phi - \alpha\, c\, dt\right)^2\, \frac{sin^2(\theta)}{\rho^2}$$

where the coordinates r, θ and ϕ are the parameters of the standard spherical coordinate system. The length-scales α, ρ and Δ have been introduced for brevity.

$$\alpha = \frac{J}{M\,c} \qquad (2)$$

$$\rho^2 = r^2 + \alpha^2\, cos^2(\theta) \qquad (3$$

$$\Delta = r^2 - r_s\, r + \alpha^2 + r_Q^2 \qquad (4)$$

r_s is the Schwarzschild radius[244] (in meters) of the massive body, which is related to its mass M by

$$r_s = \frac{2GM}{c^2} \qquad (5)$$

where G is the gravitational constant[245].

Compare this with the Planck length, $l_{Pl} = \sqrt{\hbar G/c^3}$
The Schwarzschild radius is radius of a spherical geo-cavity with mass M. The escape speed from the surface of this geo-cavity equals the speed of light. Once a stellar remnant collapses within this radius, light cannot escape and the object is no longer visible. It is a characteristic radius associated with every quantity of mass.

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[244] http://en.wikipedia.org/wiki/Schwarzschild_radius
[245] http://en.wikipedia.org/wiki/Gravitational_constant

r_Q is a length-scale corresponding to the electric charge Q of the mass

$$r_Q^2 = \frac{Q^2 G}{4\pi\varepsilon_0 c^4} \tag{6}$$

where $\frac{1}{4\pi\varepsilon_0}$ is Coulomb's force constant[246].

Cartesian coordinates

The Kerr–Newman metric can be expressed in "Kerr–Schild" form, using a particular set of Cartesian coordinates

$$g_{\mu\nu} = \eta_{\mu\nu} + f\, k_\mu\, k_\nu \tag{1}$$

$$f = \frac{G\, r^2}{r^4 + a^2\, z^2}\left[2\, M\, r - Q^2\right] \tag{2}$$

$$k_x = \frac{r\, x + a\, y}{r^2 + a^2} \tag{3}$$

$$k_y = \frac{r\, y - a\, x}{r^2 + a^2} \tag{4}$$

$$k_0 = 1 \tag{5}$$

Notice that **k** is a unit vector. Here M is the constant mass of the spinning object, Q is the constant charge of the spinning object, η is the Minkowski tensor, and a is a constant rotational parameter of the spinning object. It is understood that the vector **a** is directed along the positive z-axis. The quantity r is not the radius, but rather is implicitly defined like this:

$$1 = \frac{x^2 + y^2}{r^2 + a^2} + \frac{z^2}{r^2} \tag{6}$$

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[246] http://en.wikipedia.org/wiki/Coulomb%27s_law

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Notice that the quantity r becomes the usual radius $R = \sqrt{x^2 + y^2 + z^2}$ when the rotational parameter a approaches zero. In this form of solution, units are selected so that the speed of light is unity ($c = 1$).

In order to provide a complete solution of the Einstein–Maxwell Equations, the Kerr–Newman solution not only includes a formula for the metric tensor, but also a formula for the electromagnetic potential:

$$A_\mu = \frac{Q\,r^3}{r^4 + a^2\,z^2}\,k_\mu \tag{7}$$

At large distances from the source (R>>a), these equations reduce to the Reissner-Nordstrom metric[247] with:

$$A_\mu = \left(-\phi, A_x, A_y, A_z\right) \tag{8}$$

The static electric and magnetic fields are derived from the vector potential and the scalar potential like this:

$$\boldsymbol{E} = -\nabla\phi \tag{9}$$

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \tag{10}$$

Schwarzschild metric

Schwarzschild coordinates

Specifying a metric tensor[248] is part of the definition of any Lorentzian manifold[249]. The simplest way to define this tensor is to define it in compatible local coordinate charts and verify that the same tensor is defined on the overlaps of the domains of the charts. In this article, we

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[247] http://en.wikipedia.org/wiki/Reissner%E2%80%93Nordstr%C3%B6m_metric
[248] http://en.wikipedia.org/wiki/Metric_tensor
[249] http://en.wikipedia.org/wiki/Lorentzian_manifold

will only attempt to define the metric tensor in the domain of a single chart.

In a Schwarzschild chart[250] (on a static spherically symmetric spacetime), the line element ds takes the form

$$ds^2 = -\big(f(r)\big)^2 dt + \big(g(r)\big)^2 dr + r^2(d\theta^2 + \sin^2(\theta)\, d\phi^2) \tag{1}$$

$$-\infty < t < \infty,\ r_0 < r < r_1, 0 < \theta < \pi, -\pi < \phi < \pi$$

In the Schwarzschild chart, the surfaces $t = t_0, r = r_0$ appear as round spheres (when we plot loci in polar spherical fashion), and from the form of the line element, we see that the metric restricted to any of these surfaces is

$$d\sigma = r_0^2(d\theta^2 + \sin^2(\theta)\, d\phi^2), \qquad 0 < \theta < \pi, -\pi < \phi < \pi \tag{2}$$

That is, these nested coordinate spheres do in fact represent geometric spheres with
surface area

$$A = 4\pi r_0^2 \tag{3}$$

And Gaussian curvature
$$K = 1/r_0^2$$

That is, they are geometric round spheres. Moreover, the angular coordinates θ, ϕ are exactly the usual polar spherical angular coordinates: θ is sometimes called the colatitude and ϕ is usually called the longitude. This is essentially the defining geometric feature of the Schwarzschild chart.

With respect to the Schwarzschild chart, the Lie algebra of Killing vector fields is generated by the time-like irrotational Killing vector field $\boldsymbol{\partial_t}$ and

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[250] http://casa.colorado.edu/~ajsh/schwp.html

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three space-like Killing vector fields

$\partial_\phi, \sin(\phi)\, \partial_\theta + \cot(\theta)\cos(\phi)\, \partial_\phi, \cos(\phi)\, \partial_\theta - \cot(\theta)\sin(\phi)\, \partial_\phi$

Here, saying that ∂_t is irrotational means that the vorticity tensor of the corresponding time-like congruence vanishes; thus, this Killing vector field is hyper-surface orthogonal. The fact that our spacetime admits an irrotational time-like Killing vector field is in fact the defining characteristic of a static spacetime. One immediate consequence is that the constant time coordinate surfaces $t = t_0$ form a family of (isometric) spatial hyper-slices. (This is not true for example in the Boyer-Lindquist chart for the exterior region of the Kerr vacuum, where the time-like coordinate vector is not hyper-surface orthogonal.)

It may help to add that the four Killing fields given above, considered as abstract vector fields on our Lorentzian manifold, give the truest expression of both the symmetries of a static spherically symmetric spacetime, while the particular trigonometric form which they take in our chart is the truest expression of the meaning of the term Schwarzschild chart. In particular, the three spatial Killing vector fields have exactly the same form as the three non-translational Killing vector fields in a spherically symmetric chart on E3; that is, they exhibit the notion of arbitrary Euclidean rotation about the origin or spherical symmetry.

However, note well: in general, the Schwarzschild radial coordinate does not accurately represent radial distances, i.e. distances taken along the space-like geodesic congruence which arise as the integral curves of ∂r. Rather, to find a suitable notion of 'spatial distance' between two of our nested spheres, we should integrate $g(r)dr$ along some coordinate ray from the origin:

$$\Delta\rho = \int_{r_1}^{r_2} g(r)dr \tag{4}$$

Similarly, we can regard each sphere as the locus of a spherical cloud of idealized observers, who must (in general) use rocket engines to accelerate radially outward in order to maintain their position. These are static observers, and they have world lines of form $r = r_0, \theta = \theta_0, \phi = \phi_0$, which of course have the form of vertical coordinate lines in the Schwarzschild chart.

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In order to compute the proper time interval between two events on the world line of one of these observers, we must integrate $f(r)dt$ along the appropriate coordinate line:

$$\Delta\tau = \int_{t_1}^{t_2} f(r)dt \tag{5}$$

Schwarzschild metric

In Schwarzschild coordinates[251], the Schwarzschild metric has the form:

$$c^2\,d\tau^2 = \left(1 - \frac{r_s}{r}\right)c^2\,dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2(\theta)\,d\phi^2) \tag{6}$$

where:
- τ is the proper time (time measured by a clock moving with the particle) in seconds,
- c is the speed of light in meters per second,
- t is the time coordinate (measured by a stationary clock at infinity) in seconds,
- r is the radial coordinate (circumference of a circle centered on the star divided by 2π) in meters,
- θ is the colatitude (angle from North) in radians,
- φ is the longitude in radians, and
- r_s is the Schwarzschild radius (in meters) of the massive body.

Lemaître coordinates

In Schwarzschild coordinates the Schwarzschild metric has a singularity. Georges Lemaître was the first to show that this is not a real physical singularity but simply a manifestation of the fact that the static Schwarzschild coordinates cannot be realized with material bodies inside the gravitational radius[252]. Indeed inside the gravitational radius everything falls towards the center and it is impossible for a physical body to keep a constant radius.

A transformation of the Schwarzschild coordinate system from $\{t, r\}$ to the new coordinates $\{\tau, \rho\}$,

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[251] http://en.wikipedia.org/wiki/Schwarzschild_coordinates
[252] http://en.wikipedia.org/wiki/Lemaitre_coordinates

$$d\tau = dt + \frac{\sqrt{r_s/r}}{\left(1 - \frac{r_s}{r}\right)}\, dr \tag{1}$$

$$d\rho = dt + \frac{\sqrt{r/r_s}}{\left(1 - \frac{r_s}{r}\right)}\, dr \tag{2}$$

leads to the Lemaître coordinate expression of the metric,

$$ds^2 = d\tau^2 - \frac{r_s}{r}\, d\rho^2 - r^2(d\theta^2 + \sin^2(\theta)\, d\phi^2) \tag{3}$$

Where

$$r = r_s^{\frac{1}{3}} \left[\frac{3\,(\rho - \tau)}{2}\right]^{\frac{2}{3}} \tag{4}$$

In Lemaître coordinates there is no singularity at the gravitational radius, which instead corresponds to the point $\frac{3\,(\rho-\tau)}{2} = r_s$. However, there remains a genuine gravitational singularity at the centrum, where $\rho - \tau = 0$, which cannot be removed by a coordinate change.

The Lemaître coordinate system is synchronous, that is, the global time coordinate of the metric defines the proper time of co-moving observers. The radially falling bodies reach the gravitational radius and the center within finite proper time.

Along the trajectory of a radial light ray,

$$dr = \left(\pm 1 - \sqrt{r_s/r}\right) d\tau \tag{5}$$

therefore no signal can escape from inside the Schwarzschild radius, where always $dr < 0$ and the light rays emitted radially inwards and outwards both end up at the origin.

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Part five

How the brain works

How the brain works

Abstract

This paper analyses the associative capabilities of the brain and takes the consequences of that capability.

Pre-processing

A study on how the environment is observed and interpreted should start with an investigation of how the sense-organs and the brain cooperate. Between the sense-organs and the brain exists a series of pre-processors that encode and pre-interpret the incoming signals. This process also performs some noise filtering, such that later stages of the processing are not bothered by misinformation. For that reason the pre-processors act as decision centres where the signal transfer is blocked when the signal to noise ratio stays underneath a given level, e.g. 2.3 (Crozier's law. The level may differ in different persons.). In this way the visual trajectories run via a cross-over to the cortex. The cross-over encodes and adds depth information. After a series of additional pre-processing steps the signal arrives in the fourth cortex layer. Here about four square millimetres is devoted to the direct environment of each receptor of the fovea. In this area a complete geometric encoding of the local geometry and dynamics of the perceived picture is presented. This includes whether the detected detail is a line or an edge or another form, in which direction it is positioned and whether the detail moves. (See the papers of Hubel and Wiesel on the visual trajectory and the visual cortex for more detailed information).

Processing

Thus, the brain does not work with a pictorial copy of the picture that is received on the fovea. In further steps the encoded map is interpreted.

That part of the brain tries to associate the details of the map with remembered and recognized items. When dynamics is considered then it must also be considered that the eyes are continuously scanning the input scene.

Image intensification

I studied visual perception because I needed this to specify useful measuring standards for night vision and X-ray imaging equipment (~1975). Many of the known visual illusions are due to the pre-processing in the visual trajectory. The viewing chain includes lenses, image intensifier tubes and either a camera or the human visual system. This last component includes the eye ball. The object is noisy and can be considered as a Poisson process. With respect to the noise the optical components act as binomial processes. Their point spread functions act as integration area. Image intensification is usually a Poisson process, but channel plates are characterized by an exponential distribution rather than by a Poisson distribution. Chains that include Poisson processes and binomial processes can be considered as one generalized Poisson process. Thus, imaging chains that include channel plates are more difficult to characterize.

Imaging quality characteristics

When the imaging chain can be characterized by a Poisson process, then its quantum detection efficiency can be characterized by the Detective Quantum Efficiency (DQE). Its optical imaging quality can be characterized by the Optical Transfer Function (OTF). With inhomogeneous light imaging it is sufficient to use the modulus, the Modulation Transfer Function (MTF). The MTF of the chain is the product of the MTF's of the components of the imaging chain.

Vision of noisy images

The intensification of image intensifiers is such that at low radiation levels the output image is formed by large numbers of separate light dots that together give the impression of a snowy picture. The visual trajectory contains a sequence of pre-processors that each performs a part of the encoding of the object. At its input the visual cortex gets an encoded image rather than an optical image of the perceived scene. This encoded image is further encoded and interpreted in channels higher in the brain. This is done by associating the elements of the encoded image that is entering the visual cortex. The folded visual cortex offers about four square millimetres for the encoding of the environment of each separate receptor in the fovea. The pre-processors act as decision centres. When the offered signal to noise ratio is too low then nothing is passed. This is a general principle in the encoding process and also governs the association of encoded data in other parts of the brain.

The research resulted in a significant contribution of our laboratory to the world standards for the measurement of the OTF and the DQE.

Information association

The associative nature of the process is common for al kinds of objects and parts of objects. That includes objects that did not enter through one of the sense-organs. For example a house is not stored in the brain as a complete concept. It is stored as a series of details that can be associated to the concept. If a sufficient number of these details are detected then a decision centre in the brain decides that the whole concept is present. In this way not only a particular house can be recognized, the process can recognize a series of objects that resemble the original house. It classifies houses. By adding details that can be associated with it, the concept of a house can be widened. The resulting information, i.e. the information that passed the decision centre, is used for further reasoning. Together with other details the same details can also be used to detect other concepts by a different association. When the association act still produces too much

noise, then the information is not produced and further reasoning is neither disturbed nor triggered by this fact. High enough in the hierarchy individuals can be discerned. The brain is not static. The network of communication paths and decision centres is dynamically adapted to the changing needs.

Noise filter

The decision level for the signal to noise ratio may vary from person to person. If the level becomes too low the person may start hallucinating. Further, the level may be influenced by body owned messenger stuff, drugs, poisons and medicines.

Brain waves

On the other hand one must not underestimate the positive value of noise. Noise may be the stimulus of new thoughts. These are built upon existing association networks to which via noise or via a new impression a new aspect is added. This may lead to several conclusions that did not exist before.

Reasoning

The brain is capable to perform complex reasoning. However it must be trained to perform the reasoning in a logical way. For example, it must learn that the start from a false presumption can cause the deduction of any conclusion, just or false. When a path of reasoning is helpful, then it is stored in a similar way as an observation. Not the reasoning itself is stored, but the details that are part of the reasoning path. Also here association of the details and a suitable noise threshold plays its role. The reasoning can be identified as a theory and its concept can be widened. The brain can also generate new details that together with existing details can act as a reasonable theory. Even noise can generate such signals. These details can be perceived as a dream or as a newly invented theory. It depends whether the theory is accepted as realistic. That means that the

brain must be capable of testing the realism of a theory. This testing can be improved by training. The brain can forget stored details and stored concepts. This holds for objects as well as theories. Valuable concepts are regularly refreshed and become better remembered.

Other species

Hubel and Wiesel did their experiments on several kinds of vertebrates, such as goldfishes, cats and humans. Their main target was visual perception. Where the handling of the signals of sense organs in the brains is quite similar for all vertebrates, the handling of paths of reasoning by humans is superior in comparison to other vertebrates.

Humans

Humans have an advantage over other vertebrates. Apart from direct observation the theories and the concepts of things can also be retrieved by communication with other parties. This occurs by education, discussion, reading books, papers or journals, seeing films or videos or surfing the internet. These media can also act as a reference medium that extends the storage capacity of the brain.

Science

Mathematics is a particularly helpful tool that extends the capability of the brain to perform reasoning in a logical and precise way. Physics extends this capability further with focus on observables. Philosophy adds self-reflection and focuses on the why and how of existence. Every branch of science adds to the capabilities of the individuals and to the effectiveness of the community.

Physical reality

Our brain has a limited storage capability. We cannot comprehend things that have an enormous complexity. However we can detect regularities.

Our brain is optimized to detect regularities. The laws of physics appear regularly in our observations or can be deduced from regularly returning observations. More complex laws are derived using tools and in combination with other people. Nature is not only controlled by laws. It is also controlled by boundary conditions. These boundary conditions may be caused by the influence of items that lay beyond the reach of our direct observations. The number and complexity of boundary conditions far outgrows the number of recognized laws of nature. The laws of nature play a role in our theories. However, the boundary conditions play a much smaller role. This is because the laws of nature that we detect treat a simplified version of the environment. In this abstraction the boundary conditions play no realistic role. This is another reason why our theories differ from physical reality.

Theories

These deliberations learn that theories are a product of our mind. They can be used as a looking glass that helps in the observation and interpretation of physical reality. However, it is false to interpret the theories as or as part of physical reality. When a theory fits, then it is congruent, to some extent, with physical reality. That does not say that we as human beings and the environment from which we take our observations are not part of reality. It says that what our brain produces is another thing than physical reality.

Inventions of the human mind

Infinity is typically an invention by the human mind. There exist strong indications that nature does not support infinity. In the same sense unlimited precision real numbers are prohibited in the physical universe by the holographic principle and the Bekenstein bound. However, we can embed the results of our observations in a model that includes infinities and unlimited precision. For example classical mechanics and field theories use these concepts. Quantum mechanics shows us that as soon as

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we introduce unlimited precision we are immediately confronted with Heisenberg's uncertainty principle. We need infinity and unlimited precision in order to resolve the paradoxes that otherwise creep into our theories. We use theories that are in direct conflict with each other. One forbids infinity, the other theory uses and requires it. This says at least one thing; none of the theories describes physical reality correctly. Thus none of the theories can replace the concept of physical reality. Still it appears useful to use both theories side by side. It means that great care must be taken with the interpretation of the theories.

History

Mathematical theories and physical theories tend to build upon the results of other exact theories. After some generations a very complex building is obtained. After a while it becomes humanly impossible to check whether the building elements are correct and whether the binding is done correctly. So, complex exact theories should be questioned.

Dreams

In this sense, only when we study our own dreams, fantasies or theories, then we observe these items and the dreams; fantasies and theories become part of "physical reality". If the theory is congruent with a part of physical reality, it will become useful as a view on physical reality.

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Part six

A Law of Nature

A Law of Nature

Abstract

Complexity plays an important role in all kinds of human activity as well as in nature. Complexity can be defined in terms of the number of relations that must be handled accounted relative to the number of potential relations. Modularization is an efficient methodology that helps reducing the number of relevant relations. It has the property that it becomes more efficient when the availability and the diversity of modules that can be coupled increase. Its efficiency can grow exponentially when modules can be generated out of simpler modules. In nature this effect leads to the generation of very complex creatures, such as intelligent species. In fact it is possible to interpret this tendency as a new law of nature.

Complexity

Relations can be encountered in several areas of physics and in human interactions.
We will define complexity in terms of the number potential relations divided by the number of relevant relations.

There exists a tendency in nature to reduce complexity via modularization.

This tendency grows when more suitable modules become available. Finally this tendency enables nature to create very sophisticated and intelligent creatures.

Potential complexity

Potential complexity of a set of objects is a measure that is defined by the number of potential relations that exist between the members of that set. Actual complexity of a set of objects is a measure that is defined by the number of relevant relations that exist between the members of the set. It takes time and other resources to determine whether a relation is relevant or not. Only an expert has the knowledge that a given relation is relevant. Thus it is advantageous to have as little irrelevant potential relations as is possible, such that mainly relevant and preferably usable relations result.

If there are n elements in the set, then there exist n*(n-1) potential relations.

Physical relations

Physics is based on relations. Quantum logic is a set of axioms that restrict the relations that exist between quantum logical propositions. Via its isomorphism with Hilbert spaces quantum logic forms a fundament for quantum physics. Classical logic is a similar set of restrictions that define how we can communicate logically. Quantum logic only describes static relations. Traditional quantum logic does not treat physical fields and it does not touch dynamics. However, traditional logic can be extended such that physical fields are included as well and by assuming that dynamics is the travel along subsequent versions of extended quantum logics, also dynamics will be treated. The set of propositions of traditional logic is isomorphic with the set of closed subspaces of a Hilbert space. This is a mathematical construct in which quantum physicists do their investigations and calculations. In this way fundamental physics can be constructed. Here holds very strongly that only relevant relations have significance.

Modularization

Actual complexity can be reduced by modularization, by applying standard module interfaces and standard inter-module communications, by increasing the diversity of modules and by increasing the availability of modules. Another important factor is the ease of modular system configuration.

Modularization can easily reduce the relational complexity of large complex systems by several orders of magnitude. The resources needed to devise and generate that system reduce by the same factor.

Nature secures its modules literally by brute force or when those binding forces are not available by enveloping the module in a skin such that only controlled access must be accepted by the module.

Modular system design

The system configuration process profits most from modularization, but reuse of modules and interfaces also saves resources and building time. The capability to create modules out of simpler modules gives the improvement an enormous boost.

The complexity of a modular system can be many orders of magnitude less than the complexity of a monolith.

Systems can be atoms, molecules, (in-) organic stuff, organs, living objects, products, modules, modular subsystems, organizations, governments, stars, galaxies, et cetera.

Two kinds of modular system design exist. Modularization can be done randomly, as is done in nature by the evolution process, or it can be done in an intelligent way as is done by human system designers.

1. Stochastic module and modular system design. In this way modules and their coupling are created via stochastic processes. This way of system creation is used by nature.
2. Intelligent module and modular system design. In this way modules and their coupling are designed via well planned processes that are controlled by intelligent individuals.

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The efficiency of modular system generation is influenced by the following factors:

- Resources needed to generate modules
- Availability of modules
- Diversity of modules
- Availability of provide interfaces
- Availability of require interfaces
- Diversity of interfaces
- Mutual static adaptation of provide and require interfaces
- Mutual dynamic adaptation of modules via their interfaces

In intelligent modular system design the following extra factors play an important and stimulating role:

- Public knowledge of available modules and their static and dynamic specification
- Public knowledge of available interfaces and their specification
- Availability of standardized module design tools
- Availability of standardized interface design tools
- Availability of standardized modular system configuration tools
- A market of standardized modules
- Publicly accessible repositories where automatically and humanly readable modules are published
- Publicly accessible repositories where automatically and humanly readable interfaces are published

A large availability, couple-ability and diversity of modules ease and stimulate the system configuration both in the random trial and error approach as well as in the case of intelligent system configuration.

Too much diversity works negative. So there exists an optimum diversity both for modules and for interfaces.

The same holds for availability. Too much availability exhausts precious resources, which are not used effectively and could be better used otherwise.

Intelligent modularization

Generation, management, improvement and support of modular systems normally cost a fraction of the time and the resources that the equivalent for a monolith or a layered system takes.

In intelligent modularization publication of the capabilities of modules and interfaces will reduce the complexity and increase the efficiency of system configuration and will increase the effectiveness of module and interface design.

Intelligent designers of modular systems secure the integrity of their modules by power such as a patent mechanism or by proper encapsulation. The first measure secures the intelligence property that went into the design. The second measure guards against unwanted access that may hamper the integrity of the module. Still the module must have publicly accessible interfaces.

Providing tools for component and interface generation and providing system configuration tools will also reduce the complexity of system generation. Via modularization the complexity of system configuration can be reduced so strongly that it becomes possible to automate the configuration process.

In intelligent modularization interfaces make only sense when they are standardized, when they are well known and when their use is widely spread.

When they are well known, connected interfaces can be seen as a single relevant relation. In this way they replace a number of potential relations.

Coupling

Provide interfaces couple the server module with the require interface(s) of one or more client modules. Communication occurs via relations and via relation paths. It must be done via a well-defined protocol. Otherwise the communication makes no sense. Interfaces can couple modules directly or they can couple via communication channels. Another way of coupling occurs via broadcasting controls or messages.

Intelligent communication is usually regulated via standardized protocols. The regulation may concern a handshake, a send with immediate return or a single send or receive. A broadcast is a special type of communication.

In high urgency communication and in message streaming measures must be taken against deadlocks and race conditions.

An important aspect of functioning of interfaces is the quality of coupling. It is important with which partner they couple. It might even be a matter of life and death. If the serving partner has no intention to save the integrity of the coupled module and the module has insufficient defense against such attacks, than that client module may get disturbed.

When the modules have the capability to generate siblings, as is the case in biologic systems, then the quality of mating is important for the success of the siblings. Thus in evolution not the survival of the fittest, but the quality of coupling is of crucial importance for the survival of species. In more abstract sense, the quality of coupling is important for the abundance of module types.

Interfaces

Interfaces have a type definition. Not only the static aspects of the interface coupling play a role, also the dynamic properties may be important.

Interfaces have several general aspects. Interfaces have static characteristics and dynamical characteristics that mostly can be specified separately. An interface can be a require-interface or it is a provide-interface. Often a require-interface and a provide-interface are combined in a single physical realization. In intelligent system design the require-interface is often just a link with a type definition. It may fit at several types of provide-interfaces. The capabilities of this server may be larger than required by the client.

For example in software there exist SW-SW interfaces, SW-HW interfaces, streaming interfaces and notification interfaces. A given environment may ask for an adapted physical implementation of the interface.

Communication-channels may connect require interfaces to provide interfaces.

In nature in most cases physical fields play the role of interfaces. In biology and technology all kinds of interfaces are explored.

Thus, there is a lot to be said about abstract interfaces.

Principle of minimal action

The principle of minimal action is based on the tendency of the fields to keep coherence between subsequent dynamical steps.

If you accept that action stands for change, thus for change of properties of particles, thus for change in the relations between particles and that entropy stands for potential change, then the ratio of action and entropy stands for the relational complexity of nature. Thus the principle of minimal action stands for minimizing the relational complexity. Thus, even in its fundaments, nature has a built-in tendency to minimize relational complexity.

The power of modularization

Modularity enables and stimulates nature to create sophisticated creatures via a random process called evolution.

The intelligent modular system creation process enables humans to create very sophisticated systems in much shorter time than nature took in order to create smart individuals.

The laws of entropy are directed towards the increase of chaos. The tendency to reduce complexity via modularization works in the reverse direction. When more capable modules come into being, then it becomes possible to generate more complex systems and it becomes also possible to construct more complex modules out of these modules.

Both nature and intelligent system designers make use of the advantages of modularization. It enabled nature to create intelligent and very sophisticated creatures, such as human beings.

Evolution is a stochastic process. Set against the fact that the system generation process acted by human individuals profits from the intelligence of the actor, nature has enormous resources like time, energy and matter. This gives nature an enormous potential. If one starts from the assumption that universe is its own creator, then it creates all of its parts. That also holds for such immensely complex creatures as human beings. The law of increasing complexity reduction via advancing modularization forces nature to create such hyper-complex things. Modularization is a very complex concept. Its aspects may easily fill a thick book.

It would be best to put the influence of modularization into a law of nature, but due to the complexity of the modularization process it is difficult to formulate such a law.

Abusing modularization

The fact that the advantages of modularization can be denied is shown by several human system generation processes. The most prominent are the ways software is generated and the way that fusions of large organizations are promoted.

See:

[Story of the war against software complexity.pdf](); part six of this book
[Managing the generation process.pdf]() ;part seven of this book

Software

The fact that modularization is not or hardly applied in software is responsible for the exponential growth of the costs of the development and generation of complex software systems. The software industry is responsible for this deficiency. Their customers suffer.

A demo project showed that if modularization is fully implemented in embedded software generation, then:

- The whole world can contribute in the design of a large variety of modules.
- The operating systems can be generated by the system configuration tool from dedicated modules that are adapted to the needs of the collected application modules.
- The configuration tool can simulate and test a large part of the envisioned system that is generated from such a dedicated operating system and skeleton components that are generated on the basis of specifications that are taken from machine readable repositories. The skeletons can be replaced one by one by real binaries and the system can be tested at each phase.
- Such a generation process can be largely automated and can be controlled by a creative system designer rather than by a genial system architect that takes a high risk to get a burnout when the project fails.
- The overall system generation will take a few weeks of a few specialists compared to the hundreds of man years that are currently spent on comparable projects.
- Compared to the current way of system generation the process is very democratic. Everybody who knows a niche of expertise can contribute his modules.
- Compared to the open software solution, everybody who generates modules is allowed to earn money from the investment of his intelligent property.

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Part seven

Story of a War against Software Complexity

Story of a War
against Software Complexity

Abstract

This is the account of the course of a project that had the aim to improve the efficiency of embedded software generation with several orders of magnitude. All factors that determined the success of the project are treated honestly and in detail.

Prelude

In 1995 the physicist Hans van Leunen was employee of an internal software house of a large electronics company. His speciality was the creation of scientific software. Then he got the invitation from a software strategist Henk de Vries to join the semiconductor department in order to resolve a quickly emerging problem. The costs of complex embedded software were growing exponentially and this would cause severe problems in the next future. The reasons why the costs of software generation grow exponentially are the growing size and the growing complexity of the embedded software that goes into high-tech appliances. One of the reasons of the non-linear growth of costs is the growth of complexity. But the exponential growth of costs is mainly caused by surpassing of the available resources, which on its turn required measures against expected internal and external damage claims. In many cases software projects were stopped when the costs were expected to explode, or when they did not seem to reach the expected result.

Analysis

There exist several possible solutions to this dilemma. One is to move the software development to low wage countries. Another is to apply open source software. A third possibility is to increase the quality of the

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software generation process. One way to do this is to improve the control of the flow of the generation process. Another way is to improve the way that software is generated. The management of the electronics company tried all these possibilities. Improving the control of the flow of the software generation process has little sense when the generation process itself has severe defects.

Setting

The electronics company is successful in the generation of hardware. This is for a large part due to the fact that hardware is generated via a modular approach. This is one of the reasons that the research lab of the firm created a dedicated way to create modular embedded software. However, this is a rather closed system and it is directed to the direct need, the generation of software for consumer appliances. Still it is intuitively felt that a modular approach will improve the effectiveness of software generation. There are also many objective reasons for this point of view.

History

The electronics firm was not very successful with its software projects. Many software projects were stopped after having burned hundreds of man years and millions of dollars, leaving the project leaders, system architects and the software designers back in despair. For that reason relief was sought in outsourcing of the software generation. One form of it is the use of open source software. Parallel to it the internal software generation was moved for a significant part to low wage countries like India. This was only a short time solution. The exponential growth of costs took its toll there as well.

Also the switch to open source software was no smart decision. The electronics firm had no control over the way that the open source software was evolving and the open source software generation suffered the same bad habits as the present-day commercial software generation

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process does. Commercial software generation and open source software generation are both non-modular. There exists no healthy and lively software modules market that stimulates the diversity, availability, accessibility and favourable quality/price ratio that characterizes the hardware modules market. Thus, the high-tech hardware appliances industry is still confronted with the negative aspects of the current software generation technology. It drives their costs high and the fragility of the software is transferred to the hardware products that include the software. The ineffectiveness of the software generation affects the affordability and the time to market of the hardware products.

Strategy

A small group of experts consisting of the software specialist Hans van Leunen, the software strategist Henk de Vries and a software marketing specialist William Vanderboon Ringer studied the resulting possibilities and concluded that a drastic change in the way that software is generated is a promising solution of the problem. The way that hardware is generated was taken as an example. Hardware is generated mostly in a modular way. Modularization reduces the relational complexity of the design and construction process. It also enables partition and delegation of the design and construction work. It even enables a flourishing modules market.

Approach

The group tried to interest vendors of embedded software generation tools to join the enterprise. It was obvious that international standards would play a crucial role. So, the group stimulated the management of the electronics firm to involve other electronics firms and the OMG. All these measures lacked sufficient success. The tool vendors were interested, but used the opportunity to monitor whether their current way of operation was endangered. They did not really take part in the development. The other electronics companies took the role of an observer and asked for a

convincing demo of the concept. OMG lets standards create by the interested parties. It is not usage that the standard is introduced by a single company.

What happened

The group encountered severe resistance against their intentions from internal software development groups, because it was expected that the generation of the modules would be outsourced to the suppliers of the software modules market. This fear is realistic. On the other hand it became more and more clear that the internal software generation capabilities were not measured up against the task to create large and complex embedded software systems. Several costly debacles proved this. Especially managers, including the managers of software groups, showed that they lacked a proper feeling for the factors that influence complex software generation.

Attack

The group decided to create a demo version of the modular software generation system that included major parts of the envisioned system. This includes software module development tools, system configuration tools, web and local file based repositories that act as searchable exchange places for machine and humanly readable specifications of modules and interfaces and central services that act as a marketplace for software modules. The module development tool can generate skeleton modules and it can generate the interface definitions from specifications that are retrieved from web based or local repositories. The tool helps filling the skeletons with working code. The configuration tool retrieves specifications of modules and interfaces from the repositories. It can retrieve the binaries of modules from the market place or from a local store. It enables the mostly automatic assembly of modules into target systems. It adds a dedicated RTOS that consists of automatically generated modules. The RTOS provides automatic memory garbage

collection. The central service collects specifications from the module developers and distributes these to the web based repositories. The central service also collect the binaries of the modules and stores the specifications and the binaries in its banks. The central service acts as a modules market.

Set-back

The project of the group was severely hampered by the dot com crisis in 2001. This stopped all long term research projects and brought the funding of the group to a minimum. The development of the demo continued at a low pace and stopped in 2004. At that time most planned parts of the demo worked at least for a large part.

Remnants

The central service worked partly. The development tools are functioning. Modules can be generated and the configuration tool can assemble systems from these modules and add a service layer that consists of automatically generated dedicated modules. The service layer includes garbage collection. It uses connection schemes and scheduling schemes that dynamically control the switch between system modes. The created system does not contain a HAL and it does not contain interrupt services. Instead it relies on the services of a virtual machine or a POSIX OS. This is not the target to work on top of hardware but it is good enough for most demonstration purposes. The tools generate software in C++, but as a bonus it can deliver C# code. That code works on top of a dotNet virtual machine. The tools and central services are written in C#.

Apart from SW/SW interfaces the modules may contain HW/SW interfaces. Streaming interfaces and the notification interfaces that handle interrupts were planned. The skeleton of the modules are modelled after Microsoft's Component Object Model (COM), but the IUnknown interface is replaced by the IAccessor interface. That interface replaces the AddRef

and Release functions with a ResetInstance routine. Instead of the designer, the system is made responsible for the garbage collection. For that reason the new module skeleton is named Robust Component Object Model (RCOM).

Goal

The demo was planned to demonstrate the generation of real-time embedded software. That goal is not reached. However, many aspects of the planned target are shown in the completed part of the demo. That part offers trust in the feasibility of the final goal.

Lessons

The project also learned many valuable lessons.

- The current suppliers of software generation tools are not interested in a drastic change in the way that software is generated.
- Despite the fact that embedded software is causing major problems, the companies that produce high-tech appliances or high-tech systems are hesitating to cooperate in improving the software generation process. Software generation is not their strength.
- This world is not good in organizing actions that are rather complex. For that reason it is difficult to arrange standards on new subjects.
- It is difficult to motivate management to enter new inroads when the reasons are not very simple and require insight in the topic.
- Managers of these days are interested in short term low risk solutions. They are not interested in long term solutions even when they promise high profits.
- The same holds for today's investors.
- Although most involved people intuitively see that a modular approach provides a better effective generation process and easier

support management, most of these people forget that without a suitable modules market the modules are too expensive and too scarce to make the assumption true.

- A modular system generation approach has no sense when it does not include an integrated and well functioning modules market. This also means that a system of web based and local repositories that contain the specifications of modules and specifications must be involved as well.
- Given enough resources, even a tiny group of determined software experts can design and construct a working version of a modular software generation system that includes all essential parts.

Conclusions

This world is not good at creating new standards. However, we are good in accepting default standards. Large electronic firms seem incapable of creating a suitable software generation system. Understandably, the existing software industry appears not willing to give up the profits that they retrieve from the current deplorable way of software generation.

Way out

There still exists a possible way to get out of this misery. When a small group of enthusiastic software developers and venture capital investors start with a project that establishes a working version of a modular software generation system that includes all ingredients to get a successful result, then they may cause the seed that will extend like an oil drop and smother the current way of software generation.

In a world where such a system exists the complex software assemblies are no longer created by a genial system architect and hundreds of man years of expensive programmers but instead by a creative modular system assembler that uses automated tools to construct his target in a fraction of the time, with a fraction of the resources and with a fraction of

the costs compared to his present-day colleague. He retrieves his modules from a modules market and he may also design and produce some missing modules. In a later phase he may decide to offer these new modules on the market.

His present-day colleague produces software systems, whose structure resembles a layered set of patchwork blankets. Even the most ingenious architect cannot oversee the details of this complex architecture. Therefore the system cannot be completely described properly. Thus, it cannot be tested fully and nobody can guarantee its proper functioning. Modular systems are inherently less complex. Especially its system configuration is orders of magnitude less complex. This results in a better manageability of the complexity and a higher robustness. On its turn it results in a better chance to be able to guarantee its proper functioning.

Discussion

The modules market is very democratic. Everybody that owns an appropriate modules development system can participate and fill a niche of the modules market. The modules market is a good replacement of the market for open source software. It has the advantage that the module developers can earn money for the intellectual property that they invested in the design and construction of the module. Still the products stay very affordable. In contrast the open source software community is non-democratic. In many cases the community forbids the contributing software developers to earn money from their intellectual property investments.

Part eight

Managing the Software Generation Process

Managing the Software Generation Process

Abstract

The current software generation process is rotten. This paper analyses why that is the case and what can be done about it.

Software complexity

It is no secret that the generation of complex software poses great problems for its producers. The cost is growing exponentially with their size and the time from conception to finalization grows likewise. The resulting products are fragile and force the vendors to reserve sufficient resources to cope with future warranty and damage claims. Buyers are aware of this situation but without reasonable alternative they are ready to live with the situation. The source of the misery is the complexity of the software and this complexity is mainly due to the relational complexity of its constituents. A radical modular approach as is applied in hardware system generation would cure the problem, but that requires a completely different way of software generation and software marketing.

Introduction

First the factors that hamper efficient system generation are treated independent of the application area. Then the solutions for eliminating these factors are given. Next the differences between the hardware area and the software area are shown in an historic view. Finally a possible improvement of the software case is sketched.

Managing complexity

Breaking level

Managing simple projects hardly ever poses problems. However, a situation in which complexity surpasses the boundary where a quick view no longer reveals potential problems requires special methods. These measures compensate or cure the lack of overview. The level of the boundary depends on the number of items involved in the process and on the nature of the relations between these items.

Measure of complexity

The number of potential relations between the items involved in the process explains a close to quadratic growth of potential complexity with the number of items involved. Between N items exist N×(N-1) potential relations. Usually only a small percentage of the potential relations are truly relevant relations. Dynamically relevant relations are the potential carriers of communication and control signals. They carry the activity and determine the capabilities of the considered system. It takes expertise knowledge to decide whether a potential relation is dynamically relevant. Gaining this expertise takes time and other resources. This explains why all potential relations have a direct impact on manageability. For that reason, the number of potential relations may act as a rough measure of potential complexity. Similarly the number of dynamically relevant relations may act as a rough measure of the actual complexity of the system. More precise measures will also consider the type of the relations. The type of the relation determines how that relation must be treated.

Procedures such as modularization of the system and categorization and grouping of the interrelations into interfaces significantly reduce the actual complexity of system design and creation. Each interface represents a well defined group of dynamically relevant relations. Well known interfaces contribute significantly to the reduction of complexity. They reduce a set of interrelations to a single relation. Modules can be assembled into systems by connecting them via compatible interfaces.

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Both the modules and the interfaces that couple these modules are of crucial importance for managing the complexity of system generation.

Extreme complexity

Very high degrees of complexity may introduce secondary effects that impair manageability far more severely than can be explained by the number of potential relations between the items involved in the system generation process. This occurs when it becomes humanly impossible to properly specify the activity of all dynamically relevant relations.

The inability to specify the product, implicates the inability to test it and as a consequence it implicates the inability to guarantee the proper functionality of the system. The implications of the lack of resources that are required to cope with complexity and the inability to specify the situation in sufficient detail can easily raise costs in an exponentially increasing way. Apart from causing unacceptably growing costs, the system generation process yields fragile results. The resulting product may even endanger the environment where it is applied. This requires reserving resources to insure resistance against future claims.

The modular approach

Modularization

The reasons why modularization significantly improves manageability of the generation process are manifold. For example, it may be possible to delegate the design or the creation of modules to other parties. Potential reuse of existing modules or their design is another important reason. However, the most important reason for applying modularity is the fact that proper encapsulation of the modules and the use of well known interfaces significantly reduce the number of dynamically relevant relations.

A simple example may explain this. A monolithic system consisting of 1000 items contains 999.000 potential relations. Its relational complexity

can be characterized by this number. A comparable modular system that consists of ten modules contains far less potential relations. Let the modules be coupled by well-known interfaces and let part of the interfaces be similar. Not every module connects to every other module. Let the largest module contain 200 items and let the total number of interfaces between any pair of modules be less than 5. The largest module has a potential relational complexity of 39.800. The complexity of the other modules is less. Thus the relational complexity met by the module designers is less than 40.000 and for most modules the relational complexity is less than 10.000. Between modules the interfaces take the role of the relations that are the internal members of these interfaces. The system designer is confronted with a relational complexity that is less than 100. The benefits of the reuse of interfaces and the advantages of the possible reuse of modules should also be considered. Thus compared to the monolithic case there is an increase in manageability of several orders of magnitude. Modularization of larger systems may offer benefits that are much higher. Diminishing relational complexity translates directly in lower man costs and in shorter time to realization. Further it has a very healthy effect on the robustness and reliability of the end product.

Modular system design

The system designer gets the strongest benefit from the modularization. Modularization simplifies system assembly significantly. This opens the possibility to automate the system integration process.

Modularization reaches its highest effectiveness when the design and creation process enables the assembly of modules out of other modules. In this way the microelectronics industry reaches the exponential growth of the capabilities of integrated components that is known as Moore's law.

Interfaces

In the design of a system the introduction of an interface increments the number of potential relations. However, because the interface encapsulates a series of dynamically relevant relations, the total relational

complexity will decrease. The new relation will only play a dynamic role when the corresponding modules are coupled or decoupled. This coupling can be done at system assembly time or during the operation of the system. At instances where no coupling or decoupling is performed the new relation acts as a static relation. It relays the communication and control signals to the dynamically relevant relations that are members of the interface. In the count for complexity a well known interface replaces the combined contributions of its members. In that view, it can be considered as a single dynamically relevant object.

Dynamically relevant relations are carriers of information or control signals. Depending on the direction of the control signal the corresponding interface member belongs to the require part of the interface or to the provide part of the interface. In the first case it acts as the sender of control signals. The require part of the interface contains members that belong to the current client module. If the interface member acts as the receiver of control signals, then the interface member belongs to the provide part of the interface. In return the interface member causes the module to deliver corresponding services. The provide part of the interface belongs to the module that acts as the current server. In order to become active the require part of the interface of the client module must be connected to the provide part of the interface of the module that acts as the server.

A module may act as a server at one instance and it may act as a client at other instances. In each of its roles it will use the appropriate provide or require interface parts. Multitasking modules may provide parallel actions.

In the assembly the coupling of the require interface part and a corresponding provide interface part may be stationary or it may be temporally. The provide interface part of an interface may serve one or more other interfaces. The service may be presented in parallel or in sequential order. The specifications of the provide interface part must at

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least cover the requirements of each of its customers. With respect to its potential capabilities, the provide interface part may offer more than is requested by a coupled require interface part. The specification of the provide interface part must be in accordance with the specification of the require interface part, but this only holds for the part that covers the services that the require interface part may demand.

In many cases the trigger of a provide interface member by a connected require interface member will not only result in an action of the server module. It may also cause the return of a response via the same connection. The response can be used for synchronization purposes and it may contain requested information.

In general an interface may contain both a provide part and a require part and the partition may change dynamically. It is difficult to understand and handle such mixed interfaces. When manageability is strived for, then mixed interfaces must be avoided. An exception exists when the communication requires a handshaking process. Preferably pure interfaces should be used. A pure interface contains either a require part or a provide part but not both. In the simplest case the specification of a require interface closely matches the specification of the corresponding provide interface.

Proper modules

Proper modules are properly encapsulated. A proper module hides its internals. Securing the intelligent property that went in its design is one of the reasons for this strict measure. Preventing unwanted access to the module is another reason. Proper modules can only be accessed through publicly known and well specified provide interfaces. A module is a part of an actual system or it is targeted as a part of one or more possible future systems. Proper modules take care that each access through an interface keeps the functional integrity of the module intact. An exception may be that the module signals to its environment that it is no longer in a

valid state. The environment may then decide to ignore the module in future actions or it may reset the module to a valid state.

A proper module must be able to perform one or more actions. These actions may be controlled via one or more of its provide interfaces. Purely static objects are never considered as proper modules.

Properties and actions

Each proper module has a set of properties that together describe its status. Besides of that, each proper module provides a series of actions. Each module interface offers indirect access in order to control the members of a well defined and ordered subset of these actions. The properties cannot be accessed directly. However, a given action may enable the reading of the value of a property or it may enable the direct or indirect setting of one or more properties.

Costs of modularization

Modularization has its price. The design and generation of modules and the organization of compatible interfaces is relatively expensive. Only extensive reuse of modules may render modularization economic. Reuse of modules and the availability of compatible well known interfaces between modules may significantly improve the manageability of the design and creation of complex systems. However, reuse implicates standardization and it asks for actions that promote availability, accessibility and diversity. These requirements are best provided by a healthy and lively modules market and media that publish the specification of the characteristics of available modules and interfaces. An open market may ensure a healthy price to quality ratio. It also stimulates the continuous improvement of the quality of the modules that become available. Preparing modules for an open market requires the hiding of the intellectual property that is invested in the design and creation of the module. On the other hand the specification of provide interfaces must be publicly known. Promoting other uses of the provide interfaces and the require interfaces that are applied in a given module will in its turn

promote the use of that module. It will increase the chance that other modules will become compatible with the considered module.

Abuse

Modularization can also be abused. Wrong access to a module may rupture its integrity. In that case the module is no longer trustworthy. A proper modularization technology must prevent improper access to modules. It means that access that bypasses the official interfaces of the module must be prevented. Clients of a module may be systems or other modules. During its actions a module may run through a sequence of states. A client of a module must only access the module while the module is in a state that is known to be save for this access. Properly created modules will then take care that their integrity will not be impaired. If the state of a module is not known, then the client may decide to reset the module to a save known state.

Abuse of modularity is stimulated by the misuse of the terms 'module' and 'component'. It often occurs that a system part is called 'component' or 'module' while it is far from properly encapsulated. Such system parts are not designed to preserve their integrity. People that do not have sufficient expertise may fall into this trap and may think that by assembling such improper components a similar reduction of complexity can be achieved as can be achieved with proper modules.

Modularization success cases

The success of modularization is widely demonstrated in the design and generation of hardware. Electronic appliances, autos, buildings, clothes, in fact most assembled products are not affordable without the fact that they are constructed from components. Many of the constituting components are themselves assembled from components. More important, the price, quality, diversity and availability of these components depend strongly on the corresponding lively components markets. The beneficial effects of the open market depend strongly on trustable specification of the

characteristics of the components and on media that report on availability and quality of these products.

Even nature relies on modularization. Most living creatures contain organs and are constituted from multiple cells. Human communities use modularity in the hierarchical structure of their organizations. This is best shown in a town hall or a post office where dedicated counters belonging to corresponding departments offer publicly known services to their customers.

The application of modularization in the software industry is far from a great success. Proper software modules exist, but their application is sparse. The current software development tools do not support the assembly of systems from modules. The software components rely on the support that is offered by the operating system that embeds these components. Most software components are designed to operate as singles in a larger non-modular environment. Generally, these modules do not couple with other modules. Currently the software industry does not offer a technology that enables the construction of modules out of other modules.

Requirements for success

When applied properly, modularization may significantly improve the system design and creation process. Keywords are the standardization, the diversity and the availability of modules and interfaces and the ease of the system integration process. The existence of a lively and effective modules market is also a very important aspect. System integration may be automated, but this requires the proper tuning of component specification, the system design tools and the matching components market. The technology must enable the construction of modules out of simpler modules. Using these preconditions the microelectronics industry provides very complex and tremendously capable integrated circuits.

With a proper automated design and assembly organization in place the modular system creation time will shrink to a small fraction of the time required by the manual non-modular equivalent. Where manual design and assembly of a complex monolithic target requires a genius as the system architect, a creative human operator may burn far less resources and achieve a similar or even better result by using an appropriate automated modular approach. Automation of the system design and creation process puts high demands on trustworthy and machine readable specifications of modules and interfaces.

Difficulties posed by modularization

The requirements posed by modularization are also the reasons why modularization is never a straightforward solution.

Diversity

The requirement of a high degree of diversity is in direct conflict with the requirement of sufficient standardization. An interface has both static and dynamic aspects. Dynamic requirements may ask for different interfaces that have similar static characteristics but different dynamic behavior. Environmental requirements may ask for specially adapted interfaces. Interfaces may be replaced by other interfaces that have a wider scope or a better performance. Similar considerations hold for modules.
In order to increase market profits, to simplify component discovery and to ease system integration the diversity of similar interfaces must be kept within sensible bounds. The same holds for modules.

Compatibility

In order to enable successful assembly, the selected modules must be mutually compatible. This translates to the requirement that the interfaces that couple the modules must be compatible. Provide interfaces must cover the demands of the coupled require interfaces. The requirements include both static and dynamic characteristics.

Real time behavior of modules may require measures that prevent or cure deadlock and race conditions. The design tools must enable the installation of these measures. Other measures must prevent that the system runs out of essential resources. The modules must be designed to support these measures. When all relevant data of the constituting modules are known, then the system design tools can help the system designer to implement sufficient resources and to take the appropriate measures.

Platforms

Components may be designed for different application areas. For example software may be designed for desktop purposes, for servers or for embedding in electronic appliances. In each of these cases there exists a choice of hardware platforms. Electronic hardware platforms require adapted software components and will certainly influence the dynamical characteristics of the interfaces of the software components. Mechanical modules may target automotive systems, avionics, nautical systems, stationary instruments or other mechanical systems. Each application area and supporting platform may require its own range of modules and interfaces. Each application area requests an adapted components market and an adapted system assembly technology.

Hiding intellectual property

In some application areas the hiding of the intellectual property that went into the design and the creation of modules is provided by their physical form or by market conditions such as a patent system. However, some application areas currently lack sufficient means to hide the design of the components. Without proper IP hiding a component's creator can never make profit in an open components market. In the past, this fact has certainly prevented that the software industry developed a healthy and lively software components market. This does not say that it is impossible to generate an effective IP hiding system for software modules.

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Availability

Availability is assured when several suppliers exist for popular modules. An easily accessible publication organization must promote and enable the discovery and the selection of existing modules.

Specification

The specification must be accurate and complete. The specification must contain sufficient details such that the system integrator can determine how the considered module can be assembled with other modules into a target system. Automated assembly asks for a machine readable and therefore well standardized specification format. This requires a dedicated XML format. The format can be defined in an XSD document. For humans, an XML document is not easily readable. The XML document can be made readable for humans via one or more XSL documents. The specification of the statical characteristics of an interface is well established. Currently there exists much less support for standardized specification of the dynamical characteristics of interfaces.

Hardware versus software

History

The hardware industry booked far more success with the application of modularization than the software industry. Partly the volatile nature of software is responsible for this fact. However, the differences in the evolution of the corresponding design and creation technologies had more influence on the success of modularization.

Long before the birth of electronic computers, modularization took its position in hardware industry. Computer hardware became affordable through far reaching application of modularization. The early computer programmers used machine code as the language to communicate with the computers. Soon the burden of inputting all these codes separately was eased by an assembly compiler that translated assembly terms into corresponding machine code sentences. Program parts could become

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reusable routines. Libraries of these routines became products that could be applied in different programming projects. The next step was the introduction of the third generation languages. These tools offered a better readable and much more flexible coding of the functionality that the programmers had to write. Powerful compilers translated the source code and combined it with the precompiled library members that were called by the written program.

Basic architecture trends

Up to so far this was no more than easing the process of producing machine code. The growing complexity of the programs demanded software development tools that enable a better overview of the architecture of the design. At this point two trends developed.

Functional analysis

The first trend, phrased 'structural analysis' created a split between the handling of properties and the handling of the actions that influence these properties. The methodology collected properties in 'data stores', actions in 'processes', data messages in 'data flows' and control messages in 'control flows'. The graphical representation of the result of the analysis was called a 'data flow diagram'. In advance, the approach proved very successful. It led to the introduction of several important software development items such as, routine libraries, file systems, communication systems and data bases. Most third generation programming languages and the early software development tools supported the 'structural analysis' approach.

Abstract data types

The second trend promoted the modular approach. It used 'abstract data types' introduced by David Parnas as its modules. In design phase the 'abstract data type' acted as an individual. It was well encapsulated and could only be accessed through one or more interfaces. In the seventies of the last century the complexity of most software projects did not enforce a modular approach. For that reason this modular design methodology was

not well supported by programming languages and by corresponding modular software development tools.

Object orientation

In a later phase the complexity of the software design increased such that a more modular approach became necessary. Instead of taking the proper modular approach of the 'abstract data type' the main software development turned to object orientation. Here the objects resemble 'abstract data types', but the objects are not properly encapsulated. Access via interfaces is possible, but the client of the object may also access the actions of the objects more directly. More severely, often the internal properties of the object can be altered directly by external actors. The possibility to inherit functionality from an object with a simpler design was given much more attention. The result was the development of libraries of classes of objects with a deep inheritance hierarchy.

Currently, object orientation is well supported by software languages and software development tools. Pity enough, current object oriented software development tools do not promote the use of popular interfaces.

Object orientation has some severe drawbacks. Without sufficient precautions, classes taken from different class libraries cannot be combined in programs. A class library with a deep inheritance hierarchy may become obsolete when its top classes contain services that are no longer up to date with current technology.

Current software components

The software industry also came with more proper software modules. Examples are Microsoft's COM components and the Java Beans. COM components are supported by some operating systems and Java Beans are supported by the Java virtual machine.

The support for COM in software languages and in software development tools is small. The design of the architecture of the COM skeleton

prevents trustworthy memory garbage collection management in cases where the module can be removed dynamically. COM is supported on some embedded systems that use UNIX or an operating system that supports POSIX.

Both Java Beans and COM components are not designed to construct components from components and need the support of an operating system or a virtual machine.

There exists a small open market for these software components. Most of them target desktop applications.

State of affairs

At this moment the software industry does not apply modularization in a serious way. There exists no theoretical reason why modularization in software system generation can not be as successful as the current modularization in hardware system generation currently is. However, effective modular software generation asks for a completely different way of software generation than is accomplished by the present software development industry.

Implementing proper modularization will offer chances to parties that are now excluded by the power of companies that control software development tools and software development processes. With the appropriate services in place, everybody who has access to a software component development environment can produce products that fill a market need. Future institutions that support software component development and component based system assembly will help the component developer in marketing the created components. In that case the current powers in the software industry will endanger losing market control. It is to be expected that they will battle to stay in control.

Coupling the market and the design and creation of software modules and interfaces

Standardization and marketing

Modularization asks for a dedicated and powerful standardization of specifications, interfaces and coupling procedures. A globally accessible service must support the distribution of the public documents. For example, dedicated web based repositories may contain standardized and categorized specification documents that can be discovered by an appropriate search mechanism. The development tools must be able to access the specification contents contained in these documents. Another globally accessible service must support the gathering, the sale and the delivery of the corresponding components. Both services must cooperate.

The tools and the services must intimately interact to enable the quick and efficient design of interfaces, components and target systems. At the same time the services must ensure that the intelligent property that is invested in the uploaded components keeps hidden from the public world. It must also be guaranteed that the component designers will get their rightful fee. It is very difficult to organize a properly controllable pay per copy of the components binary. It is suggested that the customers pay per project for each used binary.

Designing and generating components

The component designer collects the required interfaces from web based or local repositories or he designs one or more new interfaces. Then he designs and creates one or more components. He must test these thoroughly. When ready he uses the components for local system design or he packs one or more components into a package and sends this together with the appropriate documents to the institute that will market his products. The institute checks the contributions and after a positive conclusion the institute puts the binaries and documents in its banks. The institute will put the documents in the appropriate repositories where they become publicly accessible. Users of the components may buy the

components from the institute. The institute will ensure the payment of the developer that has put the product in the bank.

Versions and diversity

Versions and diversity of components both impede and support the manageability of the system integration process. Therefore the number of versions must be limited. Diversity of components must be made manageable by reducing the number of supported platforms and by limiting the number of supported environments. Development and creation of close copies of existing components must be avoided. Breaking these rules can easily destroy the advantages of modular system design.

Hiding intelligent property

Hiding intelligent property that is invested in the design of the component is one of the most difficult points of software component technology. It can be arranged by power: excluding customers from future membership when they offense the 'rules'. Or it can be ensured by a combination of encryption en recompilation supported by a hardware decryption. Every project gets its own encryption key. It must be ensured that a system designer can still use components that he himself has designed and created.

Automating system integration

The system integrator starts with collecting the required application components and with creating the necessary connection and scheduling scripts. The components are put in packages and a project document defines the target. Because of the fact that at the start of the system integration practically all relevant data are known, the system integration tool can automatically add a dedicated supporting operating system that includes automatic memory garbage collection. The retrieved component specifications suffice to enable the construction of skeleton systems. After linking, these skeleton components can already be tested. However, the 'empty' components do not produce much activity. During system development the skeleton components can be replaced step by step by fully operational binaries.

Publishing

Publications related to modularization comprise specifications, market promotion media and product quality comparison reports. The internal code of components is normally hidden. If the institution that designed the component wants this, it is possible to make this code public as part of the component specification.

A fully fledged software components industry

Sketch

There exists no theoretical reason why proper modularization cannot be achieved for software as it is done for hardware. The realization of some aspects will be easier while the achievement of other aspects will be harder. It is easier to send software products over internet. It is easy to search the document repositories of the component shops for interesting components and compatible interfaces. Using XML it becomes feasible to automate the design and creation process that makes use of these web based repositories, which contain machine readable specification documents that describe components and interfaces. A local file based equivalent of such a repository may store retrievals and new designs and serve both the system designer and the components developer. The repositories contain a search machine that looks for categorization terms that classify the specification documents for specific application areas. New designs can be uploaded to a central service that will check the information and store it in the worldwide accessible repositories. A webservice that acts as a dedicated web based shop may offer the corresponding modules. In the background of the webservice, binary banks will hold the binaries of the modules. The webservice will use a dedicated money bank to support the financial part of its activity. Via the webservice the component designers may upload their results to the central institution that will then market their products. Component development tools and system assembly tools interact with the

repositories and the webservices to implement an integrated design, assembly and marketing environment.

The demo

This is a very sketchy view of a possible implementation of an integrated software components creation and marketing system. In order to investigate the feasibility of this sketchy picture a demonstration system is built that contains working versions of all important constituents.

The demonstration system supports:
- Embedded software and desktop software[1]
- Provide interfaces
- Require interfaces[2]
- Memory mapped hardware interfaces
- Streaming interfaces
- Notification interfaces[3]
- Package[4] of a coherent set of components.
- Components[5] that consist of simpler components.
- Automatic creation of the supporting operating system from dedicated modules[6]
- Stepwise system build-up from a mix of skeleton components, partially functional components and fully functional components
- Automatic memory management
- System modes[7]

[1]In embedded software the generated system interacts directly with the hardware. The system assembly tool adds the HAL.

[2]Require interfaces are implemented as placeholders for special types that represent a reference to a provide interface.

[3]Notification interfaces accept hardware triggers.

[4]A package is a library of a coherent set of components. A component supplier will preferably deliver his products in the form of packages. A system designer will save his subsystems in the form of packages.

[5]A composed component is a dedicated package accompanied by a dedicated (fixed) connection scheme and a dedicated (fixed) scheduling scheme.

[6]In embedded software the system integration tool generates operating system modules in C++ source code. In desktop software the system design tool generates a layer that interacts with the virtual machine. This layer is generated in source code that corresponds with that virtual machine (C# or java).

[7]System modes are controlled by connection schemes and scheduling schemes. Dynamic removal or creation of modules should be restricted to the instances where the system mode changes. Memory management is also restricted to these instances.

A standard RTOS schedules threads by stopping and starting routines. In a component based environment the real time scheduler must stop, reset and start modules. Eventually the modules must be reconnected according to the currently valid connection scheme.

The demonstration system consists of the following components:
- An example of a web based repository
  - This repository exists of a hierarchy of directories that contain
    - XML documents, which contain structured specifications. Each document contains a series of categorization tags.
    - XSD documents, which define the structure of the specifications

- XSL documents, which help convert XML documents into humanly readable documents
  - The repository has a hierarchical structure. Components and interfaces are assembled in separate directories.
  - The repository is publicly accessible. Using the XSL files the XML documents are humanly readable via a modern web browser.
  - The repository contains a search machine that uses the attached category tags to find corresponding documents.
- An example of a local file based repository
  - This repository exists of a hierarchy of directories and has the same structure as the web based repository. This includes the search capability.
  - The local repository contains a larger variety of documents than the web based repository.
  - It acts as a local store for information that is retrieved from one or more web based repositories.
  - It acts as a local store for documents that are prepared to be send to a general institute that may put these documents on a web based repository.
  - The XML documents specify:
    - Component
    - Interface
      - Require interface
      - SW/SW
      - HW/SW
      - Streaming
      - Notification
    - Types
      - Plain type
      - Enum type
      - Interface type
      - Sequence type
      - Structure type

- Package description
- Connection scheme
- Scheduling scheme
- State chart
- Project description
- An example of a webservice that may act as the representative of a central institute. This institution serves the community that creates or uses software components. Components may appear as packages of simpler components.
  - The institute owns a local repository that contains all specifications of interfaces that exist in the domain of the webservice.
  - The institute owns a binary database that holds the binaries of all available software components.
  - The institute owns a local repository that contains all specifications of software components that exist in the domain of the webservice.
  - The webservice uses the binary databases and the local repositories to automatically serve the customers of the institute. Customers have no direct access to these stores.
  - The webservice helps partners of the central institute to distribute documents to their specialized web based repositories.
  - The webservice helps customers in buying software components and retrieving the corresponding binaries from the binary bank
  - The webservice helps software component developers to upload the binaries and corresponding specifications of their products.
  - The central institute takes care that the software component developers get paid for products that are downloaded via the webservice.
- A repository browser tool

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- o The tool helps with searching local or web based repositories for existing interfaces and components. Selected documents can be transferred from the web based repository to the local repository.
- An interface and component design tool
  - o The tool helps with specifying new interfaces. This includes:
    - Software-software interfaces
    - Software-hardware interfaces
    - Streaming interfaces
    - Notification interfaces
  - o The tool helps in specifying other design documents that go into the repositories.
  - o The tool helps with searching local or web based repositories for existing interfaces.
  - o The tool helps designing and creating the skeleton of a software component
  - o The tool helps with filling the skeleton with dedicated code
  - o The 'internal' code is normally hidden. However, it is possible to make this code public with the rest of the specification.
- A system assembly tool
  - o The tool helps with searching local or web based repositories for existing software components. It can retrieve the corresponding binaries from web based or local binary banks.
  - o The tool can work with components that are still in skeleton form.
  - o The tool can check whether components can fit together.
  - o The tool assembles selected components and adds a dedicated component based operating system.

Some hard rules must be obeyed.

- All components and all interfaces have a globally unique identifier.
- Any binary and any specification document that is uploaded to the central institute and that is accepted by this institute must never be changed or removed.
- New versions of an item are related to the previous version via a relation document that is attached to the specification document.
- The number of new versions of an item must not surpass 4.
- Close copies of items, that are not new versions, will not be accepted.

Software availability

The project that is discussed in this paper produced a large amount of software. The author of the paper owns this software and makes the C# source code available for free access at:

https://docs.google.com/leaf?id=0B8ZNOnNHFrbrODU3YmNkNzQtMDE0ZC00MzNkLWJiZTktN2U2MGVkOTAzYTkw&hl=en

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Part nine

Physics and Religion

Physics and Religion

I am not knowledgeable in religions. However, I know something of physics and of mathematics. It is easy for me to find a creator that is supported by the laws of physics and the theorems of mathematics. I soon give you a sketch. It is much more difficult if not impossible to find the thing that created and controls the creator. I will also explain why.

Universe as a creator

Let us first take the easy part and start with the habits of multi-dimensional Fourier transforms. A Fourier transform converts something that is very localized into something wide spread and vice versa. At the same time the parameter space turns into its canonical conjugate equivalent. Now take the universe as a subject and do the Fourier transform. It may be necessary to do some coordinate transformations in order to get a proper Fourier transform, but that is a secondary consideration. The universe is something that is very wide spread in its position coordinate system. Thus in the corresponding canonical conjugate (momentum) coordinate system the result of the transformation is very much localized. You could view it as an individual located in momentum space. If you do the same thing for the time domain then finally you get an individual in the momentum-energy domain.

We know that universe is evolving and during that evolution very complex items are created. Thus, universe as an individual is its own creator. A mechanism called

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modularization achieves that when conditions are favorable, the result is that very complex systems arise. If modules can be generated out of modules this process can go very far. It even achieves the generation of intelligent species. Those specimens can use their intelligence to speed up the evolution process. What in the beginning was a purely stochastic process then turns into an intelligence driven creation process. Thus nature builds conglomerates; starting from pure energy it creates particles, then atoms and molecules, then living species, then intelligent species and then God knows what. Nature appears to possess a built in tendency to reduce complexity and at the same time it builds more and more complicated systems.

The upper creator

Now comes the hard part. Consider the creator of the discussed creator. He is the creator of the universe. He tolerates that the first creator uses a very harsh way to achieve his goal. The evolution process is very hard for the subjects that fall off in the creation process. They are torn apart and the debris is used as resource for new creations. We humans do not see that as a big problem as long as it concerns dead stuff, but as soon as it concerns living species and especially when it concerns animals or humans, then we feel pity for the victims.

Religions nearly all suggest that this upper creator of the primary creator shows concern with what happens to the living species. At least he must take care of the intelligent species. However, if he does that, then he must intervene in the first

creation process. With other words, he must change the laws that govern the first creation process. Despite the fact that many humans believe that this is possible, physics does not show that such things actually happen. Which law of nature has built in controls such that the upper creator can control those laws?

However, laws are one part of the story. What actually happens is not only controlled by the laws of nature. It is also controlled by (boundary) conditions. A much larger amount of influential boundary conditions exists than the number of laws of nature. The upper creator can control the boundary and starting conditions of the primary creation process. That is a huge task, but he is almighty. It means that from the moment that it started, everything is already set. So it has little sense to pray to this almighty upper creator, then he already planned everything. He even planned your prayer.

Still religion can be good. It keeps communities together. It can give their members a happy and secure feeling. On the other hand religions can also be intolerant with respect to other religions. Wars are fought for economic reasons but they are also fought for reasons of religious nature. That is not good. Animals and even plants fight wars for economic reasons. Only humans fight for religious purposes, which if you think about it, is pure madness. Intelligent species should know better.

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Part ten

The Hilbert Book Model

The Hilbert Book Model

Abstract

This paper introduces a new model of physics. It is based on logic. It uses the congruence between the logic of quantum physics and a mathematical construct that got its name from David Hilbert. The Hilbert book model extends this construct such that fields and dynamics also fit in the new model.

Introduction

Every time when I read an article about the phenomena, which occur far from us in the universe, I'm surprised about the attention that this Farawayistan gets compared to the phenomena in the world of the smallest. Everything that happens there is dismissed with collective names such as "quantum mechanics" and "field theory". Rarely or never the treatise goes deeper. In this sub-nano-world spectacular images, such as appear in stories about the cosmos are not available.

What's playing?

Still, this part of our environment is at least as interesting and mysterious as the cosmos. What makes it even more interesting is that the fundamentals of physics can largely be found in this area. This gets enforced by the growing awareness that our knowledge of these foundations contains a lot of gaps.

Quantum Theory

Quantum mechanics and the corresponding quantum field theory have been developed mainly in the beginning of the last century.

This development occurred fairly violently and in many cases, scientists were already happy with a limited understanding that nevertheless brought enough usable formulas so that one could analyze quantum phenomena and could construct useful applications.

History

In the early days of quantum mechanics the approach was based on adapting equations of motion that were in use in classical mechanics. These equations were quantified via an intuitive process. In Schrödinger's approach the time dependence is placed in the state function of the particle. The operators that act on these state functions are kept static. The approach of Heisenberg positions the time dependence in the operators that act on the (static) state function. This difference in approach ultimately has no consequence for the properties of the physical particles. That means that the state function and the operators only play a background game. In contrast, the properties of the particles play the foreground act. The discovered indifference means that time does not belong to the properties of a particle. With respect to the state functions and the operators time only plays the role of a parameter. Apparently it does not matter whether you place this role in the state functions or in the operators. This parameter characterizes the progress of the dynamics[253]. On the other hand, position belongs to the properties of a particle. This indicates a fundamental difference between the role of space and the role of time.

The biggest confusion arose when it became clear that the smallest things could behave both as a particle and as a wave package. This confusion continues because it also means that nature is

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The role of time becomes clear in the paragraph about the Hilbert book.[253]

unpredictable in the behavior of its smallest parts. Many are unable or unwilling to accept this fundamental property.

Clarification

Already early on in the last century some solid explanations were given. Garret Birkhoff and John von Neumann showed that nature is not complying with the laws of classical logic. Instead nature uses a logic in which exactly one of the laws is weakened when it is compared to classical logic. As in all situations where rules are weakened, this leads to a kind of anarchy. In those areas where the behavior of nature differs from classical logic, its composition is a lot more complicated. That area is the site of the very small items. Actually, that area is in its principles a lot more fascinating than the cosmos. The cosmos conforms, as far as we know, nicely to classical logic. In scientific circles the weakened logic that is mentioned here is named traditional quantum logic.

Hilbert space model

Birkhoff and von Neumann went a step further. They discovered the fact that a mathematical structure, which more than a century earlier was discovered by mathematician David Hilbert, is in many respects similar to the structure of this quantum logic. This structure is a space with infinitely many dimensions. A position in this Hilbert space can be specified by using numbers. For each position that must be done with infinitely many numbers. Fortunately, that what is happening in this *infinite dimensional separable Hilbert space* can also be specified with functions. Luckier wise a lot was already known about functions that suit this purpose.

Numbers

The numbers that can be used, need not be limited to the real numbers, which we use in order to measure our three dimensional living environment. Constantin Piron found that these numbers at least must be members of a so-called division ring. There are only three division rings: the real numbers, the complex numbers and the quaternions. Virtually no one still knows the quaternions. William Rowan Hamilton discovered quaternions already in the nineteenth century. They are hyper complex numbers with a one-dimensional real part and a three-dimensional imaginary part.

Hilbert operators

Here you see appear an immediate reason for our three dimensional world. It also delivers a mystery, because the structure of Einstein's space-time differs from the structure of the quaternions. However, there are more puzzles.

Although the Hilbert space has an infinite number of dimensions, this infinity is countable. Countable means that in principle, a label with an ascending integer can be attached to each dimension. The set of real numbers is uncountable, but the set of rational numbers is countable and the set of rational quaternions is that too. So, to each dimension of the Hilbert space a rational quaternion can be attached. Mathematicians use the name operators for the mathematical things that can do this. The real numbers describe a continuum and the set of quaternions does that too. But the set of rational quaternions does not do this. This means that it is impossible to accurately describe smooth phenomena with the model obtained so far.

Graininess

The reality is even worse. There is increasing evidence that in its smallest form nature is grainy. So-called Planck units exist. These are unit sizes for time, place, action and entropy. It is basically impossible to measure the corresponding quantities more accurately than these Planck unit sizes indicate. It is as if within these limits the world does not exist or else, that nature steps over these regions.

GPS

Now suppose that we want to design a three-dimensional GPS system for nature by using the three-dimensional part of quaternions. This system would have to take into account the graininess of length. However, this is a great problem. A lattice consisting of a tightly packed collection of grains is afflicted with preferential directions. Such directions appear in nature in solids but they are not omnipresent in the universe. Therefore we need to find a different solution for the customized GPS system.

This solution must not use multidimensional collections of grains, because that would pose the same problem. This restriction only concerns the assignment of positions to particles. The horizons that also belong to the eigenspace of the operator do consist of densely packed granules.

Grain chains

A potential solution is a GPS that works with one-dimensional chains of grains. The chains represent paths. Not actual paths, but hypothetical paths. They can freely move in 3D space. There is one grain in the chain that represents the current position on this path. Only the direct environment of this grain corresponds to an actual

path. Now remains the problem to give each grain in the chain its own position.

In addition to the Hilbert space with countable dimension the mathematicians developed a Gelfand triple. As a kind of sandwich the two outer parts of this triple attach to the previously described separable Hilbert space. Because this triple directly associates to the separable Hilbert space, this sandwich is also known as a "rigged Hilbert space". In fact this name is incorrect because the triple is not a proper Hilbert space.

Fortunately, the rigged Hilbert space has an uncountable number of dimensions and can easily deliver a GPS system that can act as a continuum background coordinate system. The grain chains also have an equivalent in this rigged Hilbert space and this fact can be used to attach a position in the background coordinate system to each of the grains of a selected chain. However, this coupling is not accurate. The inaccuracy is stochastic and in its ground state it is of the order of the Planck-length. That is why the size of the granules has this magnitude. (In this region the QPAD is effectively zero).

Anchor Points

The grains of the chains that occupy the current position in the chain's "path" are in fact anchor points of elementary particles. Depending on its type an elementary particle has one or more of these anchor points.

Per time step the anchor point can **at the utmost** take one space step. If it does that, then it lands in the next grain of the chain. It can also stay at rest. That is why the chain represents a kind of path. The ratio between space step and time step is fixed and is equal to a constant c. That number equals the speed of a freely moving light

particle. In each time step a photon invariably takes a space-step. It also means that no particle can go faster than such a freely moving light particle.

Fields

The chains are not allowed to move arbitrarily. There is something that ensures that the chain keeps its smooth shape. This is provided by a probability distribution that is associated with the anchor point. In fact, it's a hyper complex function whose squared modulus equals the mentioned probability distribution. This function has quaternions as its function values and accepts quaternions as a parameter. The three-dimensional imaginary part of the hyper complex parameter may indicate a position. In that case, the probability distribution gives the probability that the next grain will be located at the value of the parameter. Our common notion of time does not fit into the real part of the quaternion. Our common notion of time is the time of the observer and not the time of the observed particle. In the Hilbert space the real part of the position operator is not used[254].

The form of the probability distribution ensures that only minimal changes occur. The quaternionic function contributes to the local field. It is the part of the field that corresponds to the considered grain chain.

Private Fields

An elementary particle can have one or more anchor points. In this way the corresponding hyper complex functions together form the private field of the elementary particle. This private field has the

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[254] The Hilbert space does not represent dynamics. However, the Hilbert book model does represent dynamics.

same properties as the wave function of the particle. Quantum mechanical scholars use this wave function in order to describe the behavior and the properties of the elementary particle.

Together, all the private fields of particles form a joint covering field that, like the separate private fields, covers the whole Hilbert space. In our model, this joint covering field is part of the physical fields in the environment of the particles.
The private fields overlap and because they are all probability amplitude distributions (QPAD's) their superposition causes an interaction between the particles that anchor on these fields.

Field Theory

According to field theory each static field can be split in a rotation free (longitudinal) part and a solenoidal (divergence free, transverse) part. Due to the configuration of the field, this split may run along curved lines. This defines a local curvature. Due to the curvature the paths of particles that should be straight get curved and shapes that should stay fixed become spatially variant.

The curvature value can be used to define a new field. It is derived from the joined covering field. We can call that new field the curvature field. It has all the aspects of the gravitational field. We can take the part of the curvature field that belongs to a particle as its private curvature field. From this private curvature field the mass of the particle can be computed. Physicists usually apply this relationship in the reverse order.

The field model

The field model, which is applied here, differs significantly from the common field model. Usually the electromagnetic

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fields and the gravitational field are assumed to be independent of each other and the gravitational field is assumed to cause a curvature in the coordinate system that must be taken into account in the treatment of the electromagnetic fields.

In this new model the cause of the local curvature is laid down in the properties and the configuration of the covering field, which consists out of the superposition of all fields except the gravitational field. The covering field also contains the fields that match the wave functions of particles. The curvature field is then derived from the local curvature. In other words, in this new model the gravitational field is a derived field. This approach causes an immediate unification of field theories.

Creation and annihilation

Chains can split and they can merge. The corresponding creation and annihilation occurs during a progression step and is controlled by QPAD's that are attached to the current granules.

Hilbert sandwich

The Hilbert space itself has no place for fields. Each private field covers the whole Hilbert space. However, in the same manner as described above for the Gelfand triple, it is possible to expand the aforementioned sandwich with three additional layers, which respectively represent the two decomposition parts of the covering field and the curvature field. Therefore, all in all, the expanded Hilbert sandwich consists of six layers.

Hilbert book

Each sandwich describes a static condition. Thus, this combination can still not describe any dynamics. This lack can be solved by putting a whole series of these sandwiches in an ordered sequence. In this way, a Hilbert book can be formed, in which each page represents a Hilbert sandwich. Glancing through this book then gives a picture of the dynamics of our universe. The page number acts as a progression step counter. This counter is not our common notion of time, but it has certainly something to do with it.

Other view

It might seem that the pages of the Hilbert book are mutually independent. However, this is not so. Each page contains the conditions that determine the contents of the following page. These preconditions are contained in another view of the data. That view can be obtained via a so-called Fourier transformation. This transfers the original coordinate system into an associated coordinate system. The function of position is replaced by the function of displacement. This displacement indicates where the relevant position in the next page moves. In jargon this second coordinate system is called the canonical conjugate of the first coordinate system. The original view gives a good picture of the "particle behavior" of the considered subject, while the new view gives a good picture of the wave behavior of the same subject.

Also the quaternionic fields store the current condition and the prerequisites for the next page. The real part of the field value stores the probability density. The squared modulus of the

value stores another probability density and the imaginary part stores a probability current density.

Discussion

What is described here is only a model. It is not more than a reflection of reality. The events we see in the cosmos are largely determined by the curvature field. The new GPS operator knows an outside horizon beyond which no chains exist. That operator also has internal horizons inside of which no chains exist. We know these internal horizons as the exterior of black holes. These horizons are covered with granules that are attached to eigenvectors of the position operator. Each granule carries a bit of information. That bit belongs to the Hilbert vector and indicates membership of the set of eigenvectors of the position operator. This corresponds to the fact that the Hilbert vector represents a quantum logical proposition and that proposition has a binary yes/no value.

The most controversial aspect of the Hilbert book model is the fact that this book consists of pages, each of which represents a static state of the universe. When displacements Lorentz transformations are considered, then usually both inertial frames are distributed over a range of Hilbert book pages. It is possible to locate maximally one of the two uniformly moving frames of reference entirely within a single page of the Hilbert book. Phenomena such as length contraction and time dilation and relativity of simultaneity fall within the Hilbert book model.

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Part eleven

A Tall Quantum Tale

A Tall Quantum Tale

I state you a proposition
and that proposition indicates
how the world works

Story

Prelude

A group of elderly Magi sit in a circle and discuss what happens around
them. That is not much. The youngest of them gets bored and starts

considering their discussion. The chat appears regulated, because if they start from a false proposition they will be able to draw any inference, whether true or not true, and then the conversation ends only in balderdash ad infinitum.

After some time, he has collected the rules. These rules prevent the conversations from getting out of control. He proposes these rules to his companion discussers. They are very pleased. From this moment on, every conversation runs fluently. The inventor writes his finding in a book and calls that book "Logic".

However, in their environment still little occurs that is worth a proper discussion. Since the talks no longer get out of control, most of the time passes in silence. The inventor feels bored again and therefore he tries to invent something else. He realizes that if he changes the rules in his book a little, then as a result, the discussions could be become much more interesting. He writes a new book that contains the changed rules. Next he changes the forest that exists in their neighbourhood in order to reflect the discussion rules.

After finishing this book and the forest, the situation has completely changed. Continuously, things appear in the forest around them that keep their conversations for ever alive. The writer calls the second book "Quantum Logic" and he renames his first book "Classical Logic". The toolkit that he uses to create the new structure of the forest also has a name. It is called "Mathematics".

M

S

The encounter

An old, very experienced senior meets a young curious guy, which is full of questions about the things that he has observed during his trip through his world. The youngster asks the elder whether he can ask him a few of his most urging questions. The senior reacts positively by nicking shortly. However, already the first question of the studious guy startles him:

S: Mister, can you explain me how the world works?

The elder thinks a while very deeply and comes then with his answer:

M: That would be a hell of a job, but I can at least give it a try. Please, sit down on that stone, because this will take some time.

The lad sits down and looks expectantly to his narrator. The old man takes a breath and starts:

M: This can be done in the form of a tale. It could be done better in the form of a truck load of formulas, but I doubt that you would understand these formulas. Do you accept that I pack the story in a tale?

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S: Well I like a tale much better than a truck load of formulas. I probably would not understand one of them. So please start with your tale.

The elder takes a breath and starts his tale.

M: The world is governed by a book of laws. It must conform to these laws. There is no punishment in not following the laws, but the world cannot do anything else then operate according to the rules that are written in the book of laws.

S: Where is that book and how is it called?

M: It is in the possession of the governor of Hilbert's bush. The book's name is "The rules of quantum logic".

S: What is in that book?

M: The book contains a small set of rules that regulate what the relations are between propositions that can be made about things that live in our world.

S: What things?

M: Well, anything that has an identity and that stores the condition it is in. Let us call such a thing an item or a particle and let us use the name state for the condition it is in. Mostly the concerned things are very small. However, these things can be very large.

S: What is different with that logic? I know only one kind of logic.

M: You know the kind of logic that humans base their reasoning on. They use the rules of logic in their discussions when they start with truth and want to stay with truth. Nature uses a kind of logic that has a much richer structure. However, in that logic only one rule is different.

S: How many rules contains the book and what do these rules mean?

M: The book contains somewhat more than twenty rules and they specify the structure of the relations between the allowable propositions.

S: There are not much rules in the book! How can that book rule the world?

M: You are right about this, but these rules are very powerful.

S: Please explain that.

M: Well, the structure of the propositions is reflected in the structure of Hilbert's bush. Hilbert's bush is a huge and dense forest and is connected to our world. Via these connections Hilbert's bush controls how the world works.

S: Thus, if I visit Hilbert's bush, then I can see how the world works?

M: No, if you visit Hilbert's bush, then you can see how the world is controlled.

S: How, can I visit Hilbert's bush?

M: Well, you can join me on a virtual trip to Hilbert's bush. I will be your guide.

S: Fine. How does Hilbert's bush look?

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The man describes a very strange environment. The chap follows the old man in his mind and shows astonished. However, in advance his guide warned that he would present a tale. So, he must belief what the man tells.

M: It is like a huge forest of poles. All poles have the same length and the feet of all poles are hooked at the same point in the centre of the bush. In this way the poles form an enormous sphere.

S: Where do these poles stand for?

M: The poles are the axes of a multidimensional cube that has an enormous dimension. First think of a three dimensional cube. Take a corner of it and take the three axes at that corner. You can identify the position of all points in the cube by three positions on rulers that are taken along the three axes.
Now, as in an umbrella, fold these axes together, such that they form a small bundle. Next add a large amount of axes to that bundle. Give every axis a unique label in the form of one or more numbers. Add a ruler to each of these axes. You can still define the position of each point in the multidimensional cube by stating the corresponding positions on the rulers. Next increase the number of dimensions until it reaches infinity. The axes now form a dense ball and they all are numbered with a unique label. Finally unfold in your imagination the "umbrella" again until all axes are again perpendicular to each other. You can start counting the dimensions of the cube, but you will never finish counting.

S: Thus the poles are a plain set of axes.

M: Yes, but the space between the perpendicular axes can also be filled with poles. In this way several sets of mutually perpendicular axis poles can be found.

S:What is the function of these axis poles?

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M: The axis poles have colours. Some axis poles are green poles. Together they form a base in which the position of all other poles can be expressed. Another set of axis poles are red. Also they form a base. Some of the poles are silver white. They are not necessarily axis poles. The silver white poles appear in bundles.

S: That is a strange kind of forest!

M: Indeed, but it is not the only thing that is strange about Hilbert's bush. Let me tell more about the silver white poles. The bundles of white poles represent and at the same time control the items in our world.

S: How is that arranged?

M: The items in our world are reflections of the bundles of white poles in Hilbert bush. What happens to the bundles will happen to the items.

The student tries to imagine the strange situation. Apparently two worlds exist. One in which he lives and one from where his live is controlled. He visualizes the forest in his brain.

S: What is the function of the green and red poles?

M: At their top these other poles contain a data store in the form of a label. The data stores of the green poles contain position data. They are a kind of kilometre indications that you find along our roads. Instead of a single number the stores contain all three coordinates. It works like a kind of primitive GPS system.

S: With some trouble I can understand what you paint for me.

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M: The data stores of the red poles contain speed data, or better said momentum data. In this way a bundle of silver white poles can determine the current position and the momentum of the moves of its pupil in the real world.

S: Why are there two types of data poles?

M: The governor arranged it that way. In this way the bundle cannot determine both types of data at the same time. It is another detail of how the governor models our world. The stores of the poles contain the values of the properties of the type observation to which the pole belongs. Mathematicians call these values eigenvalues and the corresponding poles eigenvectors. With this trick the governor leaves us uncertain about our exact condition.

S: What are mathematicians?

M: Mathematicians are scientists that amongst other things study the mechanisms, which determine the structure and behaviour of Hilbert's forest. The creator of the forest used mathematics to give it its functionality.

S: Can white poles read data?

M: No, in fact a shepherd that takes care of the silver white bundle does that. The forest is very dense. So, the shepherd can walk on top of these poles and guard his herd of sheep. From now on, I will call the silver white poles the shepherd's sheep.

S: How does the shepherd read the data?

M: The shepherd must turn to the data pole in order to read its data. If he is close to a green pole, then he is rather far from a red pole. In fact he

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may be at nearly the same distance from a series of red poles. He will usually read the nearest data pole. The same holds when the shepherd looks at other colours. Thus, the governor plays a strange trick with our world.

For the insiders: This is the source for the existence of Heisenberg's uncertainty principle. It is the cause of the quantum behaviour of small particles.

S: I must say, that is a strange situation!

M: Yes, let me proceed. It will become even much stranger.

S: Please, go on.

M: The shepherd drives his sheep through Hilbert's bush. He does that guided by the smells that he receives from other silver white bundles. The smells are mixtures of perfumes that are attractive and perfumes that are repellent. The shepherd reacts on these smells.

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S: What is causing these smells?

M: These smells are caused by the properties of the sheep. They hang as a blurring mist around each white pole, thus around each individual sheep. The sheep may also move inside the scope of the herd. That movement may also be caused by the influence of the emitted smells.

S: How does the shepherd keep his sheep together?

M: Well, that happens in a particular way. The bush is so dense, that it is impossible to let the poles move. Instead at each of his steps the shepherd redefines the poles that belong to his herd. These poles turn silver white. The poles that get outside of the herd obtain their original green or red colour. The smells create a tendency to minimize action of the cheap. Further there exists another mechanism, which is called inertia.

S: What is inertia?

M: The smells invoke a sticky resistance of the system of all herds against change. Inertia represents the combined influence of all other herds. The most distant herds together form the largest part of the set of herds. So, they have the largest effect. The influence of each individual herd decreases with distance. However, the number of herds increases faster with distance. The difference between the distant herds averages away. As a consequence the distant herds form a uniform background influence.

S: What is the effect of inertia on a herd?

M: Locally the inertia produces an enormous smell pressure. A smooth uniform movement does not disturb this potential. When the herd accelerates it stirs the perfumes and in this way the inertia produces a smell that goes together with this movement.

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S: I understand now how position is treated. What about time?

M: The shepherd owns a simple clock. That clock counts his steps. His steps are all the same size. When he drives his sheep around, he follows a track in Hilbert's bush. All shepherds take their steps in synchrony. In facts at each of their steps the complete forest is redefined. In this process the smells act as a guide. They store the current condition of the forest and these represent the preconditions for the new version of the forest. You can say that the smells represent potential versions of the forest. This includes potential versions of sheep. These potential sheep are virtual sheep.

S: So, compared to space, time is handled quite differently.

M: You understand it quickly and perfectly! You understand it better than the physicists of the last few centuries. Most of them were wrong with this subject. They think that time and space belong in one inseparable observable characteristic.

S: How many of these herds exist?

M: As many as there are particles in our world. So, there exist an enormous number of herds, but they are still countable. They can all be identified. All shepherds take their own track through Hilbert's bush.

S: That must make Hilbert's bush very large!

M: It is. Let me proceed. It must be obvious now that the herds influence each other's movements via their smells.

The lad reflects and pictures the forest in his mind as an enormous sphere. On top of that sphere a large number of shepherds push their own herd of silver white lights forward on curving tracks that are determined by the smells that other herds produce. At each of the shepherd's steps Hilbert's

forest is reconfigured. The old man must have a strange image of the world. Nonetheless, he must have his reasons.

S: So, the shepherds play a crucial role!

M: Yes, they manipulate their own herd. However, the smells of their sheep influence for other shepherds the observation of the position and momentum of other herds.

S: How do the smells influence that observation?

M: They give the data that are transmitted in the smell an extra turn. It means that other shepherds do not get a proper impression of the position and momentum data that are sent by other herds.

S: Is there a good reason for this confusing behaviour?

M: No, there is no reason. It is just a built in habit of all sheep. On the other hand, the governor established that habit when he designed mathematics. He designed mathematics such, that Hilbert's bush and its inhabitants behave according to the rules in his book.

S: What is the consequence of this strange behaviour?

M: The consequence is that the particles in the world get the wrong impression of the position and momentum of other items. For them it appears that there exists a maximum speed. And these items think that they live in a curved space.
For the insiders: This is the source of the existence of relativity as it was discovered, but not explained by Einstein.

S: Do they think that?

M: For them, it is the truth!

S: So, I live in a curved space and for me there exists a maximum speed.

M: That is right. You properly understand how the world is controlled. As long as you do not interpret that maximum speed as the limit set by your local police officer.

S: What happens inside a herd?

M: The sheep inside a well-shaped herd perform rhythmic movements. You could say that they are dancing. Physicists call it harmonic movements. These dances occur under the control of the shepherd. He considers them as his own possession.

S: What do you mean with a well-shaped herd?

M: A well-formed herd represents in our world a well-formed object, such as an atom.

S: Why is everything set up in such a strange way?

M: The governor of Hilbert's bush is very intelligent, but also very lazy. He does not want to create many rules, so that he does not have to write much in his law book. That is why he invented Hilbert's bush. He builds the consequences of all his rules into the structure and the dynamics of Hilbert's bush. That structure is in principle very simple. The same holds for the dynamics. In this way he does not have to take care on how the world evolves. However, this leaves an enormous freedom for what happens in the world that is controlled by Hilbert's bush. That on itself results in an enormous complexity of the world we live in. That renders the governor very, very smart and very, very lazy.

S: How did Hilbert's bush get its name?

M: Hilbert was the first human that discovered the governor's bush. So people give it his name.

S: Can everybody visit Hilbert's bush?

M: In principle yes. Everybody that possesses sufficient imagination can visit Hilbert's bush. There exist two guides. A mister Schrödinger tells the story as we did. He tells the story as if the bundle of silver white poles moves through the bush of green and red poles. The other guide, mister Heisenberg tells the story as if the bundle of white poles is stationary and the bush of green and red poles moves around. For the world it does not matter what moves. It only senses the relative motion.

S: How did intelligent creatures like us enter that world?

M: The governor installed a tendency to reduce complexity by means of modularization into his forest. When more compatible modules become available it becomes easier to construct more capable modules and more capable items from these modules. Given enough time, more and more capable items are created, which finally result in intelligent creatures. Scientists call this process evolution. It is a chaotic process, but it possesses a powerful tendency.

S: Uch. Can I tell this to my friends?

M: Yes, you can. And if you have learned to read formulas and work with them you can come back and I will tell you the same story in a cart load of formulas.

S: Thanks. I will come back when I am grown up. Can I still ask a final question?

M: You are a sauce-box, but you are smart. Go ahead.

S: What are you going to do after this?

M: I will visit a very old and very wise scientist, called Mendel. He claims that he has a cohesive explanation for all smells that shepherds react to.

S: Why is that important?

M: If his claim is right, then he has found the Holy Grail of physics.

S: Gosh!

After this the boy departs. Later he will become a good physicist.

Interpretation

The book of laws contains a number of axioms that define the structure of traditional quantum logic as an orthomodular lattice.

Hilbert's bush stands for an infinite dimensional separable Hilbert space that is defined over the number field of the quaternions. The set of the closed subspaces of the Hilbert space has the same lattice structure as traditional quantum logic.

The green poles represent an orthonormal base consisting of eigenvectors of the normal operator Q. This operator represents an observable quantity, which indicates the location of the item in space.

The red poles represent an orthonormal base consisting of eigenvectors of the normal operator P. This operator is the canonical conjugate of Q and represents an observable quantity, which indicates the momentum of the item.

The bundle of silver white poles and the herd of sheep represent a closed subspace of the Hilbert space that on its turn represents a particular quantum logical statement. This statement concerns a particle or a wave packet in our surroundings. Q describes the thing as a particle. P describes the thing as a wave packet.

The shepherd represents a complicated operator U_t that pushes the subspace, which is represented by his herd, around in the Hilbert space. The operator U_t may be seen as a trail of infinitesimal unitary operators. It is a function of the trail progression parameter t. The progression parameter differs from our common notion of time, which is the coordinate time.

Traditional quantum logic defines only the stationary structure of what happens in Hilbert's bush. The dynamics are introduced by the shepherds that react on the smells.

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The smells correspond to physical fields. The fields transport information about the conserved quantities that characterize the movements of the item and its elements. Each type of preserved quantity has its own field type. The operators U_t react on these fields. Inertia shows how these operators reflect the actions of the fields. Any acceleration of the item goes together with a reconfiguration of the fields.

The operator U_t transforms the observation operators Q and P into respectively

$$Q_t = U_t^{-1} \cdot Q \cdot U_t$$

and

$$P_t = U_t^{-1} \cdot P \cdot U_t$$

.This distorts the correct observation and ensures that the observer experiences a speed maximum and a curved space.

The eigenvalues of Q and P and the trail progression parameter t characterize the space-time in our live space. As already indicated t is not the same as our common coordinate time.

De eigenfunctions of U_t control the (harmonic) internal movements of the particles.

The sheep represent the elements/properties of the particle.

The effect of modularization is treated in http://www.crypts-of-physics.eu/ThereExistsATendencyInNatureToReduceComplexity.pdf.; part four of this book

HvL

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On the origin of physical dynamics
Author: Hans van Leunen

This book reports the current state of a running project that investigates the origin of physical dynamics.
It starts from the axioms of traditional quantum logic and extends this model such that it incorporates physical fields as well as dynamics.
It uses the isomorphism between the set of propositions of traditional quantum logic and the set of closed subspaces of an infinite dimensional separable Hilbert space that uses quaternions in order to specify its inner products.
The book finds solutions for the anomalies that are raised by the countability of the eigenspaces of normal quaternionic operators. It also takes the consequence of the observation that all information about nature becomes available in the form of clouds of information carrying quanta.
The book unifies all fields, such that except for the curvature field, all fields including the wave functions are considered as QPAD's. The curvature field is derived from the curvature of the superposition of all these primary fields. The curvature follows from the decomposition of this covering field in rotation free and divergence free parts.
In order to implement dynamics, the developed model applies a sequence of extended quantum logics or equivalently a sequence of extended separable Hilbert spaces. Each of the members of the sequence represents a static status quo of the universe. This leads to a new model of physics:

The Hilbert book model

Apart from this main subject the book contains a series of related papers.

http://vixra.org/author/Ir_J_A_J_van_Leunen
http://vixra.org/author/Ir_J_A_J_Hans_van_Leunen

http://www.crypts-of-physics.eu

The crypt is under a friend's house in Nesle, France

[Geef tekst op]