Rigidly rotating disk dust in the special relativity

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Abstract

A mathematical derivation of geometry is rigidly rotating disk dust, taking into account special relativity. Based on this formula are defined geometric forms a rotating disk, sphere and torus.

Keywords: Special relativity

In 1909, Ehrenfest proposed a thought experiment, in which the disc rotates at a relativistic speed. He showed that such a disc cannot be absolutely rigid according to Born [1]. Still don't have the formula for calculating the geometry of the disk [1-5].

Let's consider a disk that rotates relative $\boldsymbol{x} = 0$, $\boldsymbol{y} = 0$, $\boldsymbol{z} = 0$ around the $\boldsymbol{z}$ axis with an angular acceleration – $\boldsymbol{w}$ (Fig.1).

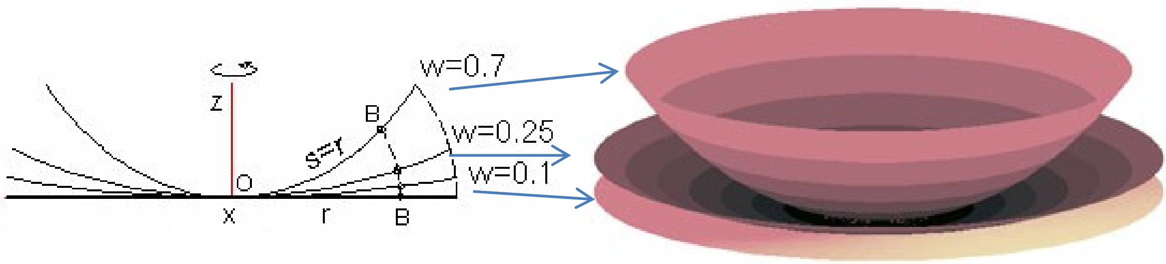

*Fig. 1. At rotation disc with any angular velocity W curve length remains constant, **OB = s = r**.*

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Speed of each element of a circle moves on a tangent, hence, concerning the motionless observer who is in the beginning of coordinates perimeter of a circle decreases. The distance from the origin to the element of the disc remains constant. As a result, for a stationary observer, the disc must change shape, deform.

In view of symmetry, it suffices to consider the change in shape of a disc in the plane X, Z (Fig. 1).

Any point (***B***) at a distance - $\boldsymbol{x_B}$, from the center of rotation moves with linear velocity - $\boldsymbol{v = wx}$ and relative to the observer describes a circle with perimeter:

$$\boldsymbol{P(x) = 2\pi x_B} \qquad (1)$$

If the distance to a point B - is equal in rest $\boldsymbol{x_B}$, at rotation of a disk concerning the motionless observer, the condition should be satisfied:

$$\boldsymbol{P(x) = P(x_B)\sqrt{1 - v^2/c^2}}$$

Hence, considering (1), we obtain

$$\boldsymbol{x = x_B\sqrt{1 - v^2/c^2}},$$

Or (***c***=1):

$$\boldsymbol{x = x_B\sqrt{1 - w^2x^2}}.$$

Hence,

$$\boldsymbol{x = x_B/\sqrt{1 + w^2x_B{}^2}} \qquad (2)$$

Here:

$\boldsymbol{x_B}$ -distance from the axis to point ***B*** at rest;

x - Distance from the axis of rotation to the same point on the disc when it rotates relative to a stationary observer at the center of the disk;

w- Angular velocity of the disk.

We introduce the parameter ***r*** to describe the curve connecting the origin to any point on the circumference of the disk. ***r*** - is numerically equal to the length of the curve, and the length of the curve does not change during the rotation disk.

From (2) we have:

$$\boldsymbol{x = \frac{r}{\sqrt{1+w^2r^2}}} \qquad (3)$$

Now we need to find the dependence of the position of a point on the ***Z***-axis rotating circle, depending on the - ***x (r)***.

The differential of arc length ***s=f (x (r))*** is equal to:

$$\boldsymbol{ds = \sqrt{(dx/dr)^2 + (dz/dr)^2}dr}$$

But ***ds*** = ***dr***, therefore,

$$\boldsymbol{(dx/dr)^2 + (dz/dr)^2 = 1} \qquad (4)$$

$$\boldsymbol{z = \int \sqrt{1-(dx/dr)^2}dr} \qquad (5)$$

We find from (3):

$$\boldsymbol{dx/dr = 1/(1+w^2r^2)^{3/2}} \qquad (6)$$

Then:

$$\boldsymbol{z = \int \sqrt{1-1/(1+w^2r^2)^3}\, dr} \qquad (7)$$

From equation (3) for any point on the disk are:

$$\boldsymbol{r = x/\sqrt{1 - w^2x^2}} \qquad (8)$$

And:

$$\boldsymbol{dr = 1/(1 - w^2x^2\,)^{3/2}\,dx} \qquad (9)$$

After substituting (8) and (9) in (7) and discarding small quantities higher than the fourth order, we obtain:

$$\boldsymbol{z = \sqrt{3}\int \frac{wx}{1-w^2x^2}dx} \qquad (10)$$

Solution of the integral:

$$\boldsymbol{z = -\frac{\sqrt{3}}{2w}\ln(1 - x^2w^2)} \qquad (11)$$

$$(\,0 \le x \le R/\sqrt{1 + w^2R^2})$$

Where ***R*** is the radius of the disc.

The equation shows that the rotating disk is turned off in a “cup”. Rotating sphere has also reshaped its form (Figure 2), and rotation of the torus (fig. 3) in addition to the deformation "raises" torus above the horizontal plane.

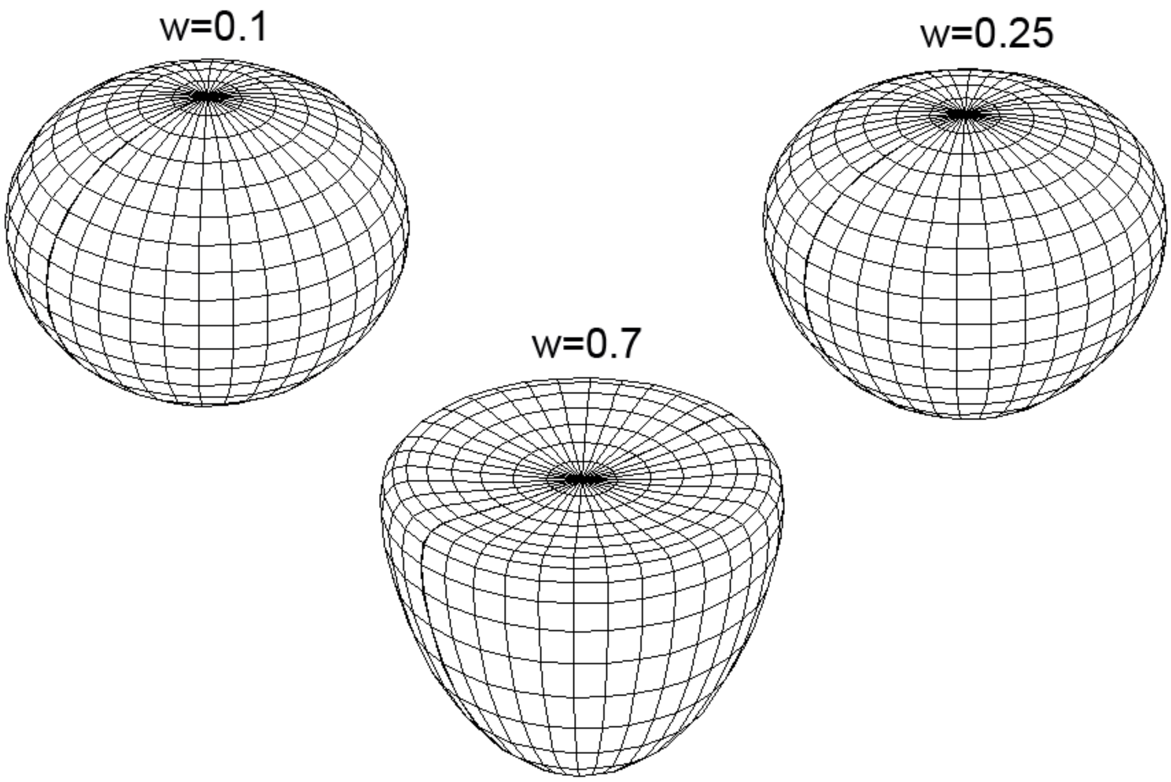

Fig. 2. Change of the form of sphere.

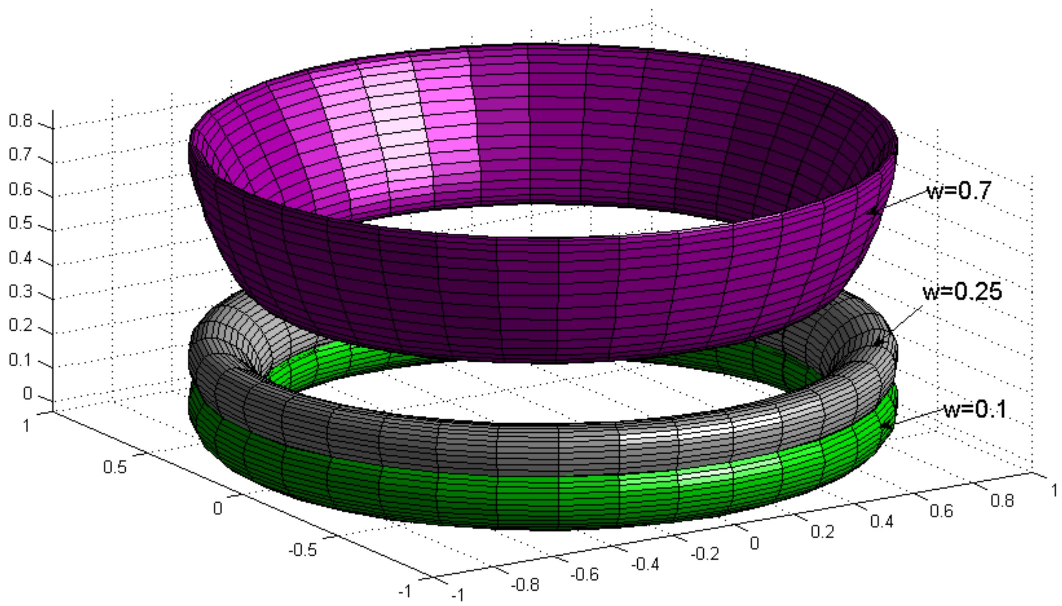

Fig. 3. Rotation of the torus in addition to the deformation "raises" torus above the horizontal plane.

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