A better look at the Michelson/Morley experiment

Roald C. Maximo
rc-maximo@uol.com.br
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Abstract
For over a century, the elusive nature of the outcome of the M/M experiment has baffled generations of physicists from all around the world. Indeed, the analysis has revealed some subtleties. I have already, for some time, had, intuitively, all the pieces of the puzzle in my mind but didn't know how to correctly join them. I tried twice without success but now, everything leads me to believe I could finally assemble the whole picture. So, I go back to the subject in a detailed way that seems to me absolutely clear and unambiguous.

To fully appreciate what happens in a M/M type of experiment, we should make a brief foray into the realm of light aberration since both are intimately connected as will be shown. Fig. 1 is a graphical representation showing a fixed light emitting source, and the aberration angle φ as seen by a distant observer traveling with velocity **v** at an angle θ in relation to the emitted light wave front

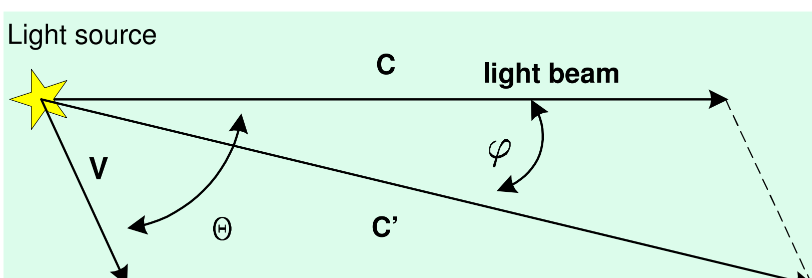

Fig. 1

where the aberration angle φ is given by equation (1) bellow

$$\varphi = \operatorname{asin}\left[\frac{v \cdot \sin(\theta)}{\sqrt{\left(c^2 + v^2\right) + 2 \cdot c \cdot v \cdot \cos(\theta)}}\right] \qquad (1)$$

But, as is already widely known, there is no light aberration when the source and observer are co-moving. Were it not so, there would arise perceptible topographic errors when objects at a distance on the ground were observed during different times and engineering would be in serious trouble. And this fact is of crucial importance in understanding the outcome of experiments of the M/M type.
In Fig. (2) we have split the speed **v** into two components, $\mathbf{v_o}$ and $\mathbf{v_s}$ corresponding respectively to the speed of the observer and the speed of the source. A glance at fig (2) shows what comes about when the source starts moving in the same direction. We see that as v_s increases, the relative velocity starts decreasing and **v** in Eq. (1) has to be substituted by $(v_o - v_s)$ and Eq. (1) now reads:

$$\varphi = \operatorname{asin}\left[\frac{(v_o - v_s)\cdot\sin(\theta)}{\sqrt{\left[c^2 + (v_o - v_s)^2\right] + 2\cdot c\cdot(v_o - v_s)\cdot\cos(\theta)}}\right] \qquad (2)$$

When finally v_s becomes equal to v_o, the tilt angle φ in Eq. (2) becomes zero and light aberration is completely absent.

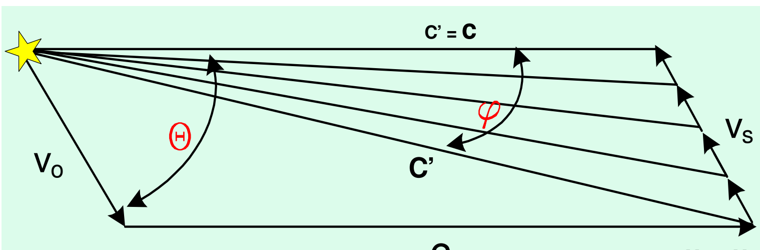

Fig. (2)

Let's see, now, what happens to the observed light rays.
In the presence of aberration, the value of c' in Figs.(1,2) is given by

$$c' = \vec{c} + \vec{v_o} = \sqrt{c^2 + v_o^2 - 2\cdot c\cdot v_o\cdot\cos(\theta)} \qquad (3)$$

which expands to a second order in **v** as $\quad c + v_o\cdot\cos(\theta) - \dfrac{v_o^2}{2\cdot c^2}\cdot\left(c\cdot\cos(\theta)^2 - c\right) \qquad (4)$

But, on a co-moving system (Fig 3) the tilt angle φ as seen above goes to zero and

$$\lim_{v_o \to 0} \sqrt{c^2 + v_o^2 - 2 \cdot c \cdot v_o \cdot \cos(\theta)} = c$$

and the vector sum reduces to

$$c' = \vec{c} + \vec{v} = c + v_o \cdot \cos(\theta) \qquad (5)$$

as can be inferred by direct inspection of Fig.(3) and is a first order effect. The second order term in (4) has vanished. Note that this must be so since otherwise light aberration would be still present in the co-moving system.

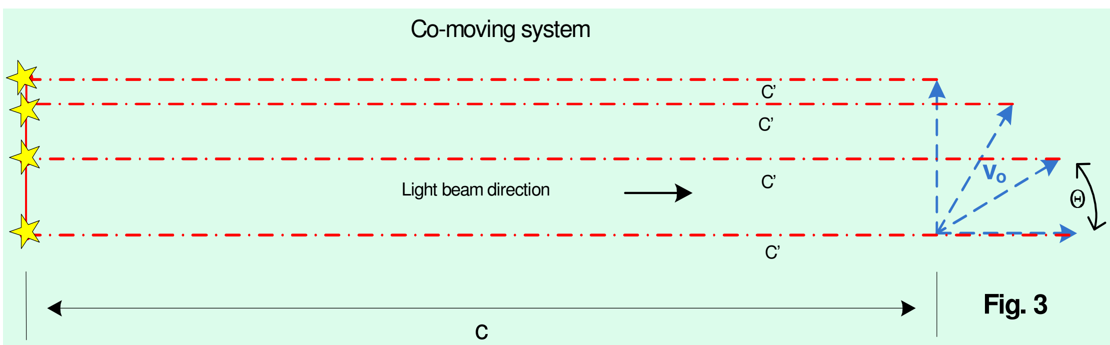

Fig. 3

Given a distance L between source and observer we can derive the phase shift for a freely propagating wave. From here on I will use v, the system speed, in place of v_o.

$$\psi = \frac{2 \cdot \pi}{\lambda} \cdot c' \cdot t \qquad \text{where} \qquad c' = c + v \cdot \cos(\theta) \qquad \text{and} \qquad t = \frac{L}{c}$$

and consequently

$$\psi = \frac{2 \cdot \pi}{\lambda} \cdot c' \cdot \frac{L}{c} = \frac{2 \cdot \pi \cdot L}{\lambda} \cdot \left(\frac{c + v \cdot \cos(\theta)}{c} \right) \qquad (6)$$

$\psi_0 = \frac{2 \cdot \pi}{\lambda} \cdot L$ is the phase for a system is at rest. The phase difference is given by

$$\Delta\psi = \psi_0 - \psi_0 \cdot \left(\frac{c + v \cdot \cos(\theta)}{c} \right) \qquad ===> \qquad \Delta\psi = -\psi_0 \cdot \frac{v}{c} \cdot \cos(\theta) \tag{7}$$

There are only three relevant situations to be considered in our analysis:

Situation 1: the ensemble and the light beam are moving in the same direction

$$\Delta\psi = \psi_0 \cdot \frac{v}{c} \cdot \cos(0) = \psi_0 \cdot \frac{v}{c} \qquad \text{for } \theta = 0$$

Situation 2: the ensemble and the light beam are moving in opposite directions

$$\Delta\psi = \psi_0 \cdot \frac{v}{c} \cdot \cos(\pi) = -\psi_0 \cdot \frac{v}{c} \qquad \text{for } \theta = \pi$$

situation 3: the ensemble and the light beam are moving in orthogonal directions

$$\Delta\psi = \psi_0 \cdot \frac{v}{c} \cdot \cos\left(\frac{\pi}{2}\right) = 0 \qquad \text{for } \theta = \pi / 2$$

The well known Sagnac effect takes advantage of the phase difference for situations **1** and **2** simultaneously. A simple Sagnac device is made in the form of two semi-circular paths with origin in the light source and ending in a phase comparator. It makes for a very sensitive first order turn detector now widely used in navigation. It becomes obvious that a linear configuration Fig.(4) would give the same results were it not for the difficulty of measuring the phase difference between two points separated by a linear distance equal to 2 L.

$$\left(\frac{2 \cdot \pi \cdot L}{\lambda} \cdot \frac{v}{c} \cdot \cos(0) \right) - \left(\frac{2 \cdot \pi \cdot L}{\lambda} \cdot \frac{v}{c} \cdot \cos(\pi) \right) \rightarrow \frac{4 \cdot \pi \cdot L \cdot v}{\lambda \cdot c} \tag{8}$$

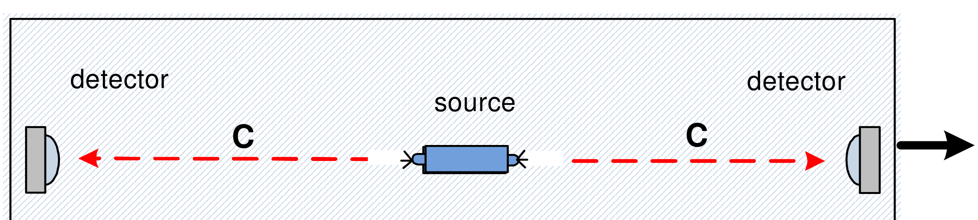

Fig. 4

Eq.(8) is the well known Sagnac formula in an extended arc or linear form.
On the other hand, the Michelson & Morley experiment compares the phase of an emitted wave with the phase of its reflected back one Fig.(5) and this makes for the sum of the outgoing and the incoming phases. In other words, a positive shifted and a negative shifted phase:

$$\left(\frac{2\cdot\pi\cdot L}{\lambda}\cdot\frac{v}{c}\cdot\cos(\theta)\right)+\left(\frac{2\cdot\pi\cdot L}{\lambda}\cdot\frac{v}{c}\cdot\cos(\theta+\pi)\right)\rightarrow 0 \qquad (9)$$

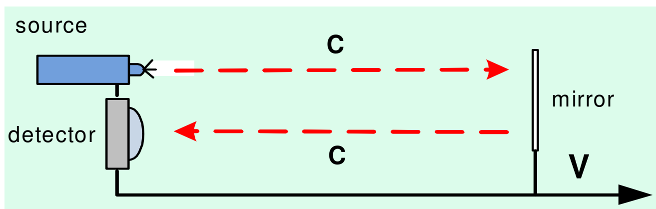

Fig-5

As here demonstrated and Eq.(9) makes clear, this type of experiments are bound to give an absolute null result for any angle θ of relative displacement.

Eq.(8) is pointing to the fact that the absolute velocity of the Earth may be measured if someone is able to idealize some means of measuring the phase difference between the extremities of a linear Sagnac device.

Criticisms are welcome