A Lagrangian which models Lambda CDM cosmology and explains the null results of direct detection efforts.

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Abstract

The purpose of this paper is to reconcile observations of dark matter effects on the galactic and cosmological scales with the null results of astroparticle physics observations such as CDMS and ANTARES. This paper will also provide a candidate unified and simpler mathematical formulation for the Lambda CDM model. Unification is achieved by a combination of the f(R) approach, with the standard LCDM approach and inflationary models. It is postulated that dark matter-energy fields depend on the Ricci curvature R. Standard methods of classical and quantum field theory on curved space time are applied. When this model is treated as a quantum field theory in curved space-time, the dark matter-dark matter fermion annihilation cross section grows as the square of the Ricci scalar. It is proposed and mathematically demonstrated that in this model dark matter particles could have shorter lifetimes in regions of relatively strong gravity such as near the sun, near the Earth, or any other large mass. The unexpected difficulties in directly observing fermionic particles of dark matter in Earth based observatories are explained by this theory. The gravitational field of the Sun and Earth may effect them in ways the standard WIMP models would never predict.

Keywords: dark matter theory, dark energy, inflation, quantum field theory, modified gravity

1. Introduction

The ΛCDM model or "concordance model" is the standard model of modern cosmology. This model contains a number of separate theories with different mathematical formulations. This model has a total of fourteen free parameters, which are tightly constrained by observation. ΛCDM models the universe to the best degree which we can measure. While a minority search for alternatives, ΛCDM is the most accepted model. There are still certain details which need to be reconciled. The precise nature of dark matter and dark energy are chief among those concerns.

To understand dark matter, scientist have tried to observe it in a variety of experiments. There have been tantalizing hints of dark matter, but not a discovery. Dark

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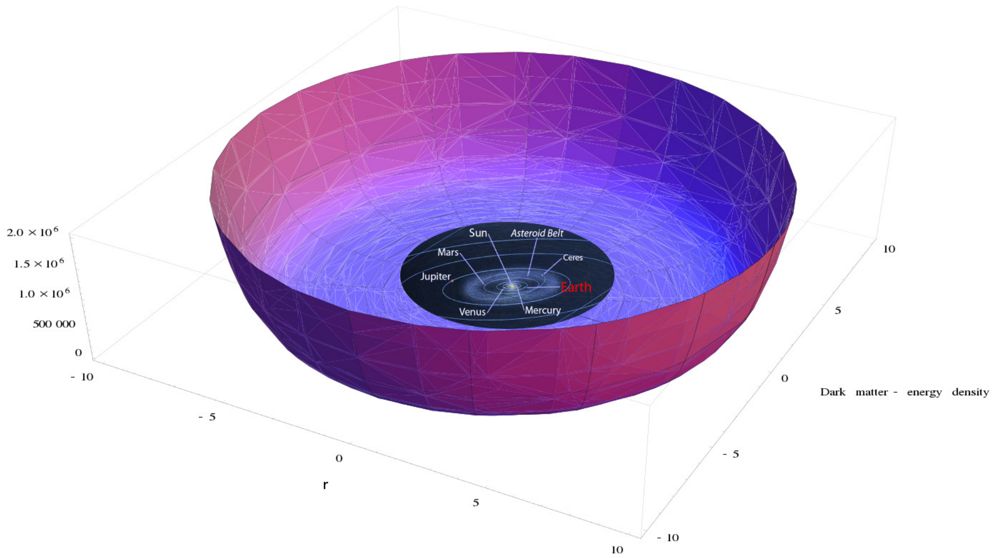

Figure 1: Graphical abstract: A 3D surface of revolution with the solar system, not to scale, embedded. Decreasing dark matter-energy density with decreasing distance from a center of mass, such as the Sun. Decreasing distance from a center of mass means increasing Ricci curvature. The lower dark matter-energy density would explain why dark matter is seen in astronomical observations in regions where the Ricci curvature would be small, but never near dense centers of mass. The lower density would explain why direct detection efforts have so far shown null results.

matter has proven more difficult to detect in ground based experiments than initially thought. This paper will try to explain why dark matter has been so hard to observe in Earth based experiments while the astronomical evidence for it is, practically, incontrovertible.

1.1. Observations in the literature.

Numerous astronomical observations confirm the existence of dark matter halos around galaxies [11]. While searches for dark matter particles in earthbound observatories have not found definitive results. The results for [10, 9], for instance, have been inconclusive. Observations of stellar motions within 13,000 light years of the sun found less dark matter than expected[16], but those results are not without controversy [6, 5].

Assuming dark matter has particles and anti-particles and behaves under gravity as normal matter would, there should be a concentration of said matter in and around astronomical centers of mass. Observations were made in search of dark matter - dark matter annihilation inside the sun [2], no dark matter annihilation signal was observed from the galactic core. A gamma ray halo around the galactic center suggest the possibility of dark matter- dark anti-matter annihilation to gamma rays at some distance from the galactic core itself [7, 15]. Precise measurements of the energy loss in binary pulsar PSR J0348+0432 due to gravitational radiation showed that it can be modeled with General Relativity without the addition of any dark matter [3]. These lines of evidence point to the conclusion that dark matter may not interact gravitationally in the exact same way as baryonic matter.

1.2. Hypothesis

These observations hint at the pattern indicated in the visual abstract Figure (1). The dark matter is most long lived where gravity is weak, and has a much shorter life where gravity is strong. If this is the case, then it would explain why dark matter has not been observed near dense concentrations of matter.

The fields precise behavior will depend on which metric and hence which R is in effect. In the case of a galaxy, the Ricci curvature corresponding to Schwarzchild's metric would be used. In the case of the universe the Friedman-Lemaitre-Robertson-Walker R would be used. In the following formulation the Ricci scalar is simply a parameter.

Hypothesis of Ricci curvature dependence: *Dark matter-energy fields depend on the Ricci curvature, R, in such a way that dark energy fields weaken as (R) increases and strengthen as R decreases.*

2. Classical field theory.

Based on the above observations and the hypothesis and using well known field theories for massive scalar and massive vector bosons, and massive spinor fields a candidate action can be written.

With the hypothesis of Ricci curvature dependence in mind we propose the following (Equation 1) .

$$s = \int \sqrt{-g} \Big(-\frac{R}{16\pi} - \nabla^\mu \phi \nabla_\mu \phi - \frac{1}{2} m_\phi^2 \phi^2 - \frac{R}{6}\phi^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} m_A^2 A^\mu A_\mu - \frac{R}{6} A^\mu A_\mu + \bar{\psi} \left(\imath \gamma^\mu D_\mu - m_\psi \right) \psi \Big) d^4x \tag{1}$$

In Equation 1 $\nabla_\mu = \partial_\mu + \Gamma^\lambda_{\mu\lambda}$, and $D_\mu = \nabla_\mu - i\beta A_\mu$.

Equation (1) contains the following fields. ϕ is a massive scalar field similar to the fields found in scalar inflation theory. The field A^μ is a massive dark photon field similar to fields found in previous literature on dark matter[1], inflation[13, 14, 12], and alternative gravity[4]. The spinor field ψ is a simple fermionic "dark matter" field which would couple to the massive dark photon. Last but not least the Ricci curvature R which will be used to parameterize the other fields is present. The scalar and vector fields have conformal coupling to the Ricci scalar which is itself a field. This action tells us how dark sector fields may interact with eachother, and gravity. By convention fermionic fields and massive fields are termed matter. Massless fields, and or bosonic fields are termed energy. ϕ, and A^μ, are dark energies, ψ is matter. Current direct detection efforts are tuned to search for dark fermions, WIMPS, and would not detect the scalar or vector fields.

To uniquely describe our universe one constraint is required. *The stress-energy needs to be at least proportional to the cosmological constant times the metric.* This results in the equation of constraint (equation 3), which is not derivable from the Lagrangian. In the following λ is simply a constant of proportionality. This constraint is imposed by observations, and ensures that this model will not stray to far from the ΛCDM model in its observed effects.

Equation 2 gives the stress-energy tensor.

$$
\begin{aligned}
T^{\mu\nu} = -2\nabla^{\mu}\phi\nabla^{\nu}\phi - F^{\mu\nu}g_{\lambda\delta}F^{\lambda\delta} - \left(m_A^2 + \tfrac{R}{3}\right)A^{\mu}A^{\nu} + \tfrac{i}{2}\bar{\psi}\gamma^{\mu}D^{\nu}\psi \\
-g^{\mu\nu}\left(\nabla_{\mu}\phi\nabla^{\mu}\phi + F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{2}\left(m_{\phi}^2\phi^2 + m_A A^{\mu}A_{\mu}\right)\right. \\
\left. -\tfrac{R}{6}\left(\phi^2 + A^{\mu}A_{\mu}\right) + \bar{\psi}\left(\imath\gamma^{\mu}D_{\mu} - m_{\psi}\right)\psi\right)
\end{aligned} \tag{2}
$$

$$
T^{\mu\nu} = \lambda g^{\mu\nu}\Lambda \tag{3}
$$

Following all the standard steps of classical field theory, the Euler-Lagrange equations can be derived. The classical field equations are as given in (Equation4).

$$
\left\{
\begin{array}{c}
R^{\mu\nu} - Rg^{\mu\nu} = \frac{8\pi G}{c^2}T^{\mu\nu} \\
\nabla_{\alpha}F^{\alpha\mu} - \left(\frac{m_A^2}{2} + \frac{R}{6}\right)A^{\mu} = 0 \\
\nabla_{\mu}\nabla^{\mu}\phi - \left(\frac{m_{\phi}^2}{2} + \frac{R}{6}\right)\phi = 0 \\
\left(D_{\mu}\gamma^{\mu} - m_{\psi}\right)\psi = 0
\end{array}
\right\} \tag{4}
$$

Now we will rewrite the field equations, Equation 4, in a form that is unambiguously valid in curved space time, and parameterized by the Ricci curvature scalar R. The equations will be written making use of the vierbein formulation where a vector field is represented using a non coordinate basis, and a transformation from the non-coordinate basis to the coordinate basis. Details about this formalism are available in Appendix J of [8]. What we need to know for this paper is how the coordinate basis is related to the non-coordinate basis by way of the metric tensor (Equation 5) .

$$
g_{\mu\nu}e_a^{\mu}e_b^{\nu} = \eta_{ab} \tag{5}
$$

In this formalism the Einstein field equations can be written as shown in equation 6.

$$
e_a^{\mu}R^{ab}e_b^{\nu} - Re_a^{\mu}e_b^{\nu}\eta^{ab} = \lambda e_a^{\mu}e_b^{\nu}\eta^{ab}\Lambda \tag{6}
$$

The equation of constraint 3, and the derivablility of the Einstein field equation from the action (Equation1) ensure that the proposed model will satisfy the solar system test of General Relativity, and model the large scale cosmology of the accepted ΛCDM theory any deviations from those models mus be small.

The scalar, vector, and spinor fields with R as a parameter will be notated as shown in Equation 7.

$$
A^{\mu} = e_a^{\mu}A^a(R), \phi = \phi(R) \tag{7}
$$

The derivatives are simplified by using the Ricci curvature as a parameter. Taking a derivative with respect to the Ricci curvature itself makes the connection coefficients in the covariant derivatives redundant. Those connection coefficients are, after all, corrections for the fact that a derivative is defined for a flat space but is being applied to a curved space. If the variable of differentiation is the Ricci curvature; then connection coefficients will vanish. The equations parameterized by the Ricci scalar become effectively one dimensional. The equations of motion simplify considerably.

The result is a set of four equations 8.

$$
\left\{
\begin{array}{c}
e_a^\mu R^{ab} e_b^\nu - R e_a^\mu e_b^\nu \eta^{ab} = 8\pi G \lambda \Lambda e_a^\mu e_b^\nu \eta^{ab} \\
e_a^\mu \frac{d}{dR} e_\mu^a \frac{d}{dR} \phi - \left(\frac{m_\phi^2}{2} + \frac{R}{6} \right) \phi = 0 \\
e_a^\mu \frac{d}{dR} e_\mu^a \frac{d}{dR} e_a^\mu A^a - \left(\frac{m_A^2}{2} + \frac{R}{6} \right) e_a^\mu A^a = 0 \\
\left(\imath e_a^\mu \gamma^a \left(e_\mu \frac{d}{dR} - i\beta e_\mu^b A_b \right) - m_\psi \right) \psi = 0
\end{array}
\right\} \tag{8}
$$

These equations, in particular the equation for the scalar field can be made simple enough to solve by Mathematica.

$$FullSimplify[DSolve[\{\backslash[Phi]''[R] - (m\hat{}2/2 + R/6) * \backslash[Phi][R] = 0\}, \backslash[Phi][R], R]] \tag{9}$$

Mathematica produces the following result (Equation10).

$$\{\{\backslash[Phi][R]- > AiryAi[(3m\hat{}2 + R)/6\hat{}(1/3)]C[1] + AiryBi[(3m\hat{}2 + R)/6\hat{}(1/3)]C[2]\}\} \tag{10}$$

The result is that the explicit Ricci curvature dependence is an Airy function. This could either be an Airy function of the first kind, denoted by Ai or of the second kind denoted Bi. The Airy function of the second kind approaches infinity as R grows. This behavior would not explain the null results of experiments on dark matter particles. The Airy function of the first kind is oscillatory at negative curvatures, and decreases exponentially with R at positive curvatures.

The final results are as follows. The details of the solutions are in a Mathematica CDF file, and handwritten notes in the supplementary materials. The scalar field is an Airy function in terms of R. The solution is 11.

$$\phi(R) = Ai((3m_\phi^2 + R)/6^{1/3}) \tag{11}$$

The vector field12 is quite similar to 11 but for the addition of the basis vector e^a.

$$A^a(R) = Ai((3m_A^2 + R)/6^{1/3}) e^a \tag{12}$$

For the spinor field the solution is expressed using Dirac spinors ($u(e_a^\mu P^a)$) as found in Peskin and Schroeder [17]. Mathematica produces13.

$$\psi(R) = u_p Exp\left(-\imath(F(R) + e_a^\mu p^a e_\mu^b x_b)\right) \tag{13}$$

Where u_p is a killing spinor and $F(R)$ in Equation 13, is a very complicated function shown in equation14. It contains a hypergeometric function (Hyper) of the Ricci curvature.

$$
\begin{aligned}
\psi[R] = Exp\Bigg(-\frac{1}{24} i \Bigg(24mR &+ \frac{8\ 3^{1/3}\beta\left(3m^2 \text{Hyper}\left[\left\{\frac{1}{3}\right\},\left\{\frac{2}{3},\frac{4}{3}\right\},\frac{m^6}{2}\right] - (3m^2+R)\text{Hyper}\left[\left\{\frac{1}{3}\right\},\left\{\frac{2}{3},\frac{4}{3}\right\},\frac{1}{54}(3m^2+R)^3\right]\right)}{\text{Gamma}\left[\frac{2}{3}\right]} \\
&+ \frac{2^{2/3}3^{5/6}\beta\text{Gamma}\left[\frac{2}{3}\right]\left(-9m^4\text{Hyper}\left[\left\{\frac{2}{3}\right\},\left\{\frac{4}{3},\frac{5}{3}\right\},\frac{m^6}{2}\right] + (3m^2+R)^2\text{Hyper}\left[\left\{\frac{2}{3}\right\},\left\{\frac{4}{3},\frac{5}{3}\right\},\frac{1}{54}(3m^2+R)^3\right]\right)}{\pi}\Bigg)
\end{aligned} \tag{14}
$$

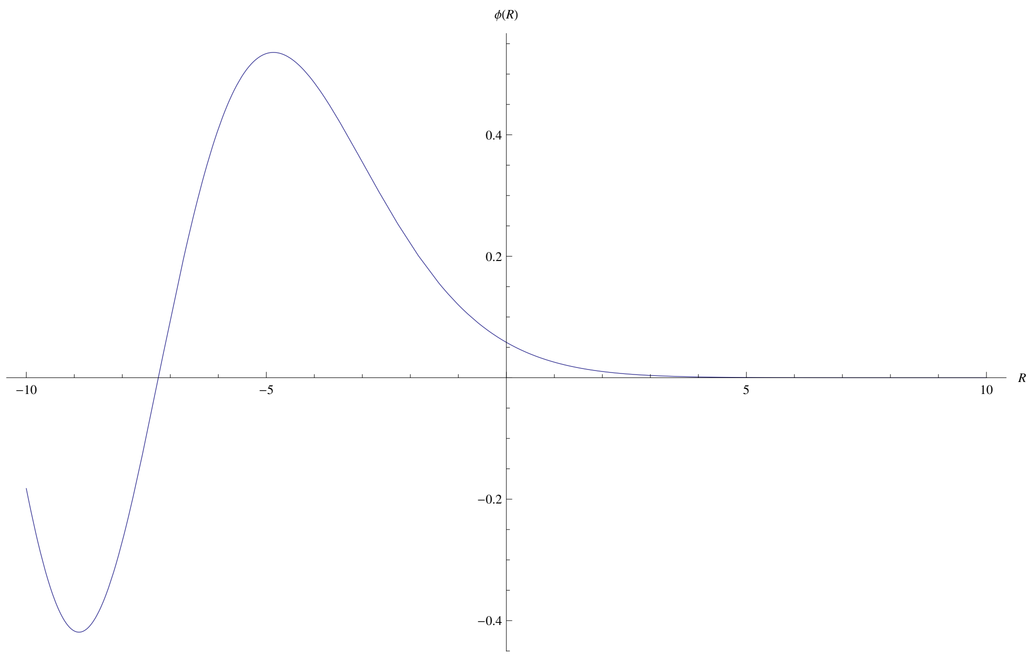

Figure 2: This is a plot of the solution found by Mathematica for the classical equations of motion for this theory for the scalar field. These fields decay towards zero as the curvature increases. The stronger the gravity - the weaker the fields.

2.1. Discussion of the classical fields.

In summary the classical solutions to the field equations are given in equation 15. These solutions are completely general and will apply to any valid Ricci curvature derived from classic General Relativity. The corrections these introduce will be small and, like any theory of modified gravity they only take effect at cosmological distances.

$$\left\{ \begin{array}{c} R \\ \phi(R) = Ai((3m_\phi^2 + R)/6^{1/3}) \\ A^a(R) = Ai((3m_A^2 + R)/6^{1/3})e^a \\ \psi(R) = u_p Exp\left(-\imath(F(R) + e_a^\mu p^a e_\mu^b x_b)\right) \end{array} \right\} \tag{15}$$

The scalar and vector fields become weaker as the curvature of space time increases. The spinor field oscillates rapidly with increasing Ricci curvature scalar.

2.2. Probability of fermion-fermion annihilation and direct detection efforts.

In terrestrial experiments which search for dark matter we have assumed that the dark matter will be fermionic. Let us consider the amplitude and cross section for the annihilation of four of these fermions into Ricci curvature, gravitational dark energy. In this interaction four spinors interact to annihilate to two vectors. Two vectors interact an annihilate to curvature. This is shown in figure (3) and equation (16). Figure (3) shows the annihilation of four spin one half particles into two spin one particles, which in turn annihilate to one spin two particle. On the face of it, this sounds like a very improbable interaction. To figure out the probability we will start with equation (16).

$$< R|\bar{\psi}\psi\bar{\psi}\psi > = < R|A^\mu A_\mu >< A^\mu A_\mu|\bar{\psi}\psi\bar{\psi}\psi > \tag{16}$$

After some tedious computation the answer works out to the following.

$$< R|\bar{\psi}\psi\bar{\psi}\psi > = \frac{(A_0^\mu A_{0\mu})\left(\bar{\psi}_{dir}\psi_{dir}\right)e^{G[R]}}{s'[R]e^{S[R]}}\left(\frac{R}{G'[R]} - 1\right) \tag{17}$$

In equation 17 the term $G[R]$ is a functional of the Ricci curvature scalar R which results from multiplying these fields together,$S[R]$ is the action as a functional of the Ricci curvature scalar R . The terms A_0^μ are constant, and $\bar{\psi}_{dir}$ is the standard solution for the Dirac spinor fields. G[R] and S[R] will oscillate. The interesting part of the squared probability is given by equation 18.

$$\left|< R|\bar{\psi}\psi\bar{\psi}\psi >\right|^2 \approx (R-1)^2 = R^2 - 2R + 1 \tag{18}$$

Equation 18 shows us that the cross section for these particles simply annihilating increases in area as the curvature of space time increases, and decreases as the curvature of space time decreases. Therefore as gravity becomes stronger, the particles lifetime becomes shorter. This behavior would partially explain why dark matter hasn't been directly detected in Earth bound experiments, or near centers of mass in general.

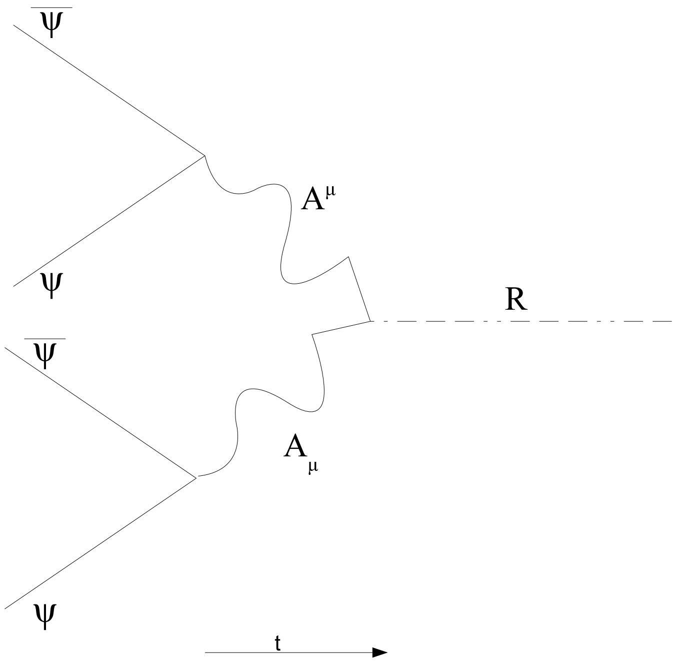

Figure 3: Feynman style diagram which shows the scenario where four dark fermions of spin 1/2 each, annihilate to two dark photons of spin 1 each which can annihilate to space-time curvature R (which is fundamentally a tensor field of rank two) of spin two. This is a graphical representation of equation (16).

2.3. Inflation

As for inflation, consider big bang cosmology in a universe dominated by these fields. At time equals on Planck time, the universe would have a very high curvature and these fields would have a very low energy density. As time begins and the universe expands initially the strength of the scalar and vector fields will increase as the curvature of space time decreases. The energy density of the fields will grow as the square of the field strength. This increase in the energy density of these fields will cause a pressure which will expand the universe even farther. In turn, the fields will grow in strength as the curvature of space time decreases. This process will continue until the universe is very nearly flat and it would happen very quickly. In this model inflation is driven as the energy bound up in the highly curved space-time near the big bang transforms into dark energy of the scalar and vector fields.

3. Conclusions

The proposed Lagrangian contains all the physics needed to represent the Lambda CDM model. There is a source of dark matter, dark energy, and inflation. The behavior of the fields is in agreement with our overall observations. This Lagrangian also provides a minimal explanation for why dark matter has been so hard to observe in experiments such as CDMS II and XENON100. The dark matter simply decays into dark energy when the curvature R is too high. Thus there are not "particles" to detect in a region of high space time curvature, like on Earth. This would provide an explanation for why it would be harder than expected to detect these particles in a ground based experiment.

This model also explains observations of a dark matter halo around galaxies at a characteristic distance in a simple and natural way.

There may well be other fermionic and super symmetric types of dark matter. Certainly numerous particles which will be discovered at accelerator laboratories in the future which may or may not be dark matter candidates exist. I have no hypothesis about such dark matter, or how the hypothesized particles could be produced via accelerator based experiments in this model at this time.

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