

Identifying the Nature's Rationality with Galaxy NGC 3275

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Abstract Weakly interacted galaxies present very simple body structure. They are either three-dimensional objects resembling ellipsoids or flat-shaped disks showing spiral disturbance. Elliptical galaxies present little dust and gas but spiral galaxies demonstrate arms and rings which are characterized by containing a huge amount of dust and gas. Since arms and rings are linear-shaped, the body structure of spiral galaxies may be a textured one as earth-bound materials always do. This led to the concept of rational structure which is based on proportion curves. The proportion curves for normal spiral galaxies are all equiangular spirals which trace or cut through arms consistently. This paper demonstrates the spider-shaped proportion curves for barred spiral galaxies. It shows for the galaxy NGC 3275 that the curves do trace or cut through arms or rings consistently. More examples of barred galaxies will be studied for the testification of Nature's rationality.

keywords: Rational Structure, Spiral Galaxy, Proportion Curve

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1 Introduction to Rational Structure

On the internet are many images of galaxies. People should not be confused with color images. Color is essentially the different frequencies or wavelengths of light. In fact, the shape of an object or its image is the distribution of light arriving at one's eyes from the surface of the object. That is, it is the distribution of light frequency and density varying with the surface of the object. Light of longer wavelength that appears reddish has strong penetrating ability. In other words, reddish light resists the absorption by dust or gas. Elliptical galaxies are very clean, with little amount of gas and dust. Therefore, it does not matter to catch which color for one to take the images of elliptical galaxies. Images of the same elliptical galaxy of different colors are very similar and smooth. They are the good demonstration of star distribution in the galaxy. But elliptical galaxies are three-dimensional while their images are two-dimensional. The image of an elliptical galaxy is the cumulative density of stars in the observing direction.

Spiral galaxies are just the opposite. They have a large amount of gas and dust. Although their structures are two-dimensional, they have a certain degree of thickness. Therefore, if we take images of spiral galaxies at the shorter wavelength (i.e., bluish light) then the light from the stars that are behind gas and dust are basically absorbed by the gas and dust. As a result, the images are more or less the distribution of gas and dust. Because the distribution of gas and dust is not smooth, the images look rough. Internet images of spiral galaxies are usually short-wavelength ones, therefore, people are daunted by the mysterious look of gas and dust. Therefore, to get an image of spiral galaxy which is mainly stellar density

distribution, we take light of longer wavelength from the galaxy, e.g., infrared light. The resulting image is reddish. Although gas and dust have charming and bright colors, they generally have negligible mass.

All galaxies demonstrate the structure of uneven material distribution. The density distribution of a relatively independent galaxy is always simple and orderly. Thus, discovery of the fundamental law to galaxy mass distribution is the basic step toward the understanding of galaxies. Relatively independent galaxies fall into two categories. One is the three-dimensional elliptical galaxies, and the other is the flat-shaped spiral galaxies. The main structure of spiral galaxies is an axially symmetric disk with the stellar density decreasing exponentially along radial direction. It is the so-called exponential disk. In fact, the exponential disk of spiral galaxies can coexist with bar-shaped structure. There are two types of spiral galaxies. A barred spiral galaxy has additional bar structure. Its body is a combination of exponential disk and bar structure. A spiral galaxy without bar structure (i.e., its body is simply the exponential disk) is called a normal spiral galaxy.

This paper studies spiral galaxies only. Because spiral galaxies are flat-shaped, we use the function of two variables

$$\rho(x, y) \tag{1}$$

to represent the stellar density distribution of spiral galaxies. Modern galaxy images are generally digital ones. Thus, a longer-wavelength image of face-on spiral galaxy is essentially an array of positive numbers proportional to the stellar density $\rho(i, j)$. The size of the array is dependent on the resolution of the image. We use the array to draw galaxy image with the image brightness at each point being proportional to the corresponding value of the array.

People have not known which density distribution an independent natural material system should take. Since arms or rings are linear-shaped, the body structure of spiral galaxies may be a textured one decided by some net of curves. My suggestion for the texturing is proportion (see paper [1]). Proportion means that the distribution of matters in galaxies is orderly. For example, there are four giants standing in array. Their heights are A, B, C, D, respectively, and A, B stand in the first row from left to right, C, D in the second row from left to right. According to the common view, the four giants can have any heights and can stand at any position. That is why current scientific theories are incompetent and can not explain the origin of natural structure. They can not provide any basic principle to resolve the free motion of the most simple gravitational systems (such as interactional three bodies). However, the universe is orderly. The orderly force at the large scale requires that the distribution of heights is in proportion. In other words, A divided by B is equal to C divided by D. This means that A divided by C is equal to B divided by D. If there are nine giants standing in array then the ratios of heights from neighboring two rows are constant (proportion rows). Similarly, the ratios of heights from neighboring two columns are constant (proportion columns). In this way are galaxies created.

The above-said rows and columns are all straight (proportion lines). But each galaxy is a regional distribution of matters in the universe. Therefore, the proportion lines of each galaxy are curved but the rows and columns still cross at each other vertically and they form the orthogonal net of proportion curves on the galaxy plane. In general, a distribution of similar bodies is called the rational structure if its density varies proportionally along some particular net of orthogonal curves.

In fact, I never consider the density ratio of $\rho(x, y)$. Instead I consider the variance rate of the logarithmic density,

$$f(x, y) = \ln \rho(x, y). \quad (2)$$

The variance rate of $f(x, y)$ along the normal (i.e., perpendicular) direction to a curve is proportional to the density ratio of $\rho(x, y)$ as explained above. Therefore, considering the variance rate of the function $f(x, y)$, we get two functions $u(x, y)$ and $v(x, y)$ which record the variance rates in the normal direction to the row and column proportion curves, respectively (see paper [2]). Rational structure means that the proportion curves are simultaneously the level curves of the two functions $u(x, y)$ and $v(x, y)$, respectively.

Definition of rational structure (see papers [1-4]): The logarithmic density distribution $f(x, y)$ on a plane is not arbitrary. There exists a special net of orthogonal curves on the plane. The variance rate of $f(x, y)$ along the normal direction of a curve (from the net) is constant along the curve. The curve is called a proportion curve. The net of curves is called an orthogonal net of proportion curves. This kind of density distribution $\rho(x, y)$ is called a rational structure.

Now we prove that exponential disks are rational structure. The logarithmic density of an exponential disk is the circularly symmetric distribution of numerical values about the center point, which decreases linearly in the radial direction. Therefore, the magnitude of the directional derivative along radial direction (i.e., the magnitude of the gradient) is a global constant. An equiangular spiral is the curve which crosses all radial lines at a constant angle. Therefore, the directional derivative along the tangent direction of an equiangular spiral is constant along the spiral. Accordingly the directional derivative along the normal direction to the equiangular spiral is also constant along the spiral. However, this constant is different from the constant magnitude of the gradient. They differ by a factor of the cosine of the angle. Finally we have proved that exponential disk is a rational structure.

The fact that the body structure of normal spiral galaxies is a rational structure, is marvelous. But the most marvelous result is the explanation to the origin of gas and dust. Since we can not find any linear-shaped rational structure which fits to arm pattern, galaxy arms and rings must be the disturbance to rational body. Images of spiral galaxies show that the greater the disturbance to the body structure, the more dusts and gases resulted, and the greater the events of star birth. Only dusts and gases contain significant amount of elements that are heavier than hydrogen and helium which the living structures need. Elliptical galaxies do not show much evidence of life, and the disturbance to their bodies is hard to be observed. For example, elliptical galaxies present no significant arms.

The disturbing waves try to achieve the minimal disturbance and, as a result, trace the orthogonal or non-orthogonal proportion curves. Note that spiral arms are usually broken-shaped (not connecting end to end), composed of segments which trace orthogonal or non-orthogonal proportion curves. Take the normal spiral galaxy M51 as an example (see Figure 1). The proportion curves are all equiangular spirals. Coincidentally, astronomical observations show that the arms of any normal spiral galaxy are all equiangular. Therefore, the proportion curves of normal spiral galaxies trace or cut through the corresponding arms consistently.

Now that the origin of arms for normal spiral galaxies is explained, I go further to study barred spiral galaxies. Are barred spiral galaxies rational? If they are, what do the



Figure 1: The shorter-wavelength image of normal spiral galaxy M51 (image credit: NASA/ESA). The arms follow equiangular spirals which are the proportion curves of exponential disk.

corresponding proportion curves look like? In Section 2, I answer the first question. Section 3 is dedicated to the explanation of the second question. Section 4 is the conclusion.

2 Are Barred Spiral Galaxies Rational?

My PhD thesis showed that the density distribution of any barred spiral galaxy can be fitted with the superposition of several rational structures. However, I did not prove that the compound structure of rational components is still rational. Fortunately, the paper [2] completed the mathematical basis for the study of rational structure and provided a rational sufficient condition. Firstly let me introduce my model of barred spiral galaxies. The major part of any spiral galaxy is an axially symmetric disk with the stellar density decreasing exponentially along radial direction. It is the so-called exponential disk. A barred spiral galaxy has additional bar structure. Its body is the combination of exponential disk with bar structure (see Figure 2). We have already known that the exponential disk is a rational structure. Now we study the bar structure.

Without regard to the central bulge, barred spiral galaxies are the superposition of disks and bars. My research indicates that the bar of any barred spiral galaxy is a compound structure of two or three dual-handle structures. Each dual-handle structure is a rational structure whose logarithmic density is (see [4])

$$f(x, y) = (b_2/3) \left(p(x, y) + b_1^2 x^2 / p(x, y) \right)^{3/2} \quad (3)$$

where b_1, b_2 are constant parameters, r is the polar radius, and

$$p(x, y) = \frac{1}{2} \left(r^2 - b_1^2 + \sqrt{(r^2 - b_1^2)^2 + 4b_1^2 x^2} \right). \quad (4)$$

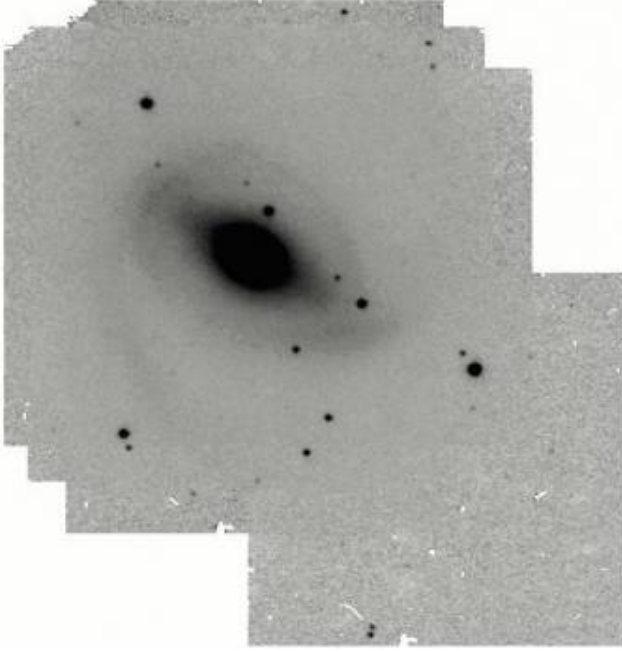


Figure 2: A longer-wavelength image of barred spiral galaxy NGC 3275 (see [5] for image credit). The galaxy presents arms as well as a ring.

The lower left panel of Figure 1 in paper [2] is the mass density distribution $\rho(x, y)$ of a dual-handle structure. The orthogonal net of proportion curves of dual-handle structure is composed of all confocal ellipses and hyperbolas. The distance between the two foci is $2b_1$, known as the length of the dual-handles. The variance rate of $f(x, y)$ along the normal direction to the confocal ellipses is also called the ‘radial’ variance rate, denoted by $u(x, y)$, and the one along the normal direction to the confocal hyperbolas is also called ‘axial’ variance rate, denoted by $v(x, y)$. In fact, the normal direction of the ellipses do not generally point to the galaxy center but point to the galaxy bar. For visual purpose, we can draw the image of the function $u(x, y)$ with the image brightness at each point corresponding to the value of $u(x, y)$ and the level curves of the brightness must be all confocal ellipses. The lower right panel of Figure 1 in paper [2] is such image of the function $u(x, y)$.

We study the barred spiral galaxy NGC 3275 as an example. Through visual simulation, the bar is found to be the superposition of two dual-handle structures. The upper right panel of Figure 3 in the current paper shows the simulated whole barred spiral galaxy (exponential disk plus galaxy bar). The fitting values of the parameters d_0, d_1 (exponential disk), b_0, b_1, b_2 (dual-handle structure) are presented in Table 1. We denote the density of the simulated shorter dual-handle structure by $\rho_1(x, y)$. The density of the longer dual-handle structure is denoted by $\rho_2(x, y)$, and the density of the simulated exponential disk is denoted by $\rho_0(x, y)$. Therefore, disregarding the central bulge, we have the visually simulated whole barred galaxy NGC 3275,

$$\rho(x, y) = \rho_0(x, y) + \rho_1(x, y) + \rho_2(x, y). \quad (5)$$

This is displayed in the upper-right panel of Figure 3. The gradient of the corresponding

logarithmic density is

$$\nabla f(x, y) = \frac{\rho_0}{\rho} \nabla f_0(x, y) + \frac{\rho_1}{\rho} \nabla f_1(x, y) + \frac{\rho_2}{\rho} \nabla f_2(x, y) \quad (6)$$

where $f_0(x, y)$, $f_1(x, y)$, $f_2(x, y)$ are the logarithmic densities of the simulated exponential disk and dual-handle structures respectively. It indicates that the gradient of the compound logarithmic density is the weighted average of the gradients of the components' logarithmic densities. The weight is the percentage of the component's density over the whole density (not the whole logarithmic density, see paper [2]). That is, the body structure is a compound one of rational structures.

In fact, the model of barred spiral galaxy structure, $\rho_{d_0, d_1, b_0, b_1, b_2, c_0, c_1, c_2}(x, y)$, is scale invariant (see formulas (3) and (4) for the definition of parameters). That is,

$$\rho_{d_0, d_1, b_0, b_1, b_2, \dots}(x, y) = \rho_{d_0, d_1 s^{-1}, b_0, b_1 s, b_2 s^{-3}, \dots}(xs, ys) \quad (7)$$

where s is the arbitrary scale. Therefore, we can assign an arbitrary positive number to the side length of a galaxy image, and the fitting galaxy structure is the same provided that the fitting parameter values vary according to the law given in formula (7). The evidence exists that the choice of a larger number for the side length can reduce the error of computer calculation.

Paper [7] is a breakthrough of rational-structure study because a rational necessary condition for a given density distribution was obtained. That is, if a given logarithmic density distribution $f(x, y)$ is rational then the slope of its orthogonal proportion curves must satisfy a cubic algebraic equation, called the instinct equation. The coefficients of the equation are known, which are combinations of the partial derivatives to the given logarithmic density $f(x, y)$. The slope is the tangent value of the azimuthal angle of the tangential line of the curves, i.e., $\tan \alpha$. Given a logarithmic density distribution $f(x, y)$, if we can prove that an orthogonal net of proportion curves exists which corresponds to the given density distribution then the density distribution must be rational. However, the existence of a smooth distribution of scalar angle

$$\alpha(x, y) \quad (8)$$

may not define a vectorial net of orthogonal curves. Therefore, a smooth root of the instinct equation can not guarantee that the given logarithmic density be a rational one. Fortunately, a sufficient condition is given in paper [2] which is composed of the boundary condition in addition to the so-called rational orthogonal condition,

$$\nabla f \cdot \nabla \alpha = 0 \quad (9)$$

(see the formula (14) in paper [2]) or the formula (10) in paper [1]). Since the instinct equation is an algebraic equation of the 'doubled' slope,

$$\gamma = \tan 2\alpha \quad (10)$$

and the logarithmic density $f(x, y)$ is determined by the formula (5), the slope as well as α are solved. Accordingly the rational orthogonal condition (9) can be expressed by the given density distribution $f(x, y)$.

Table 1: The Reference Values of Parameter Fitting

galaxy	real image side length	assigned length	d_0	d_1	b_0	b_1	b_2	c_0	c_1	c_2
3275	110 [arcsec]	100	8000	-0.528	134	8.0	-0.002	72	14.8	-0.001

Because the slope of proportion curves satisfies an algebraic equation, the requirement of rationality is extremely strict and the solutions of rational structure are very few. In fact, there are only three types of solutions which correspond to the sign of the discriminant of the cubic equation. The cubic equation can be factorized into a linear equation and a quadratic one. The slope of proportion curves must satisfy either the linear equation or the quadratic equation. The former type of rational structure is called the rational ground state, $|1\rangle$, and the latter type the rational excited state because the equations and their solutions have many similar aspects to the conventional discussion of quantum mechanics. The quadratic instinct equation is given by the formula (16) in paper [2],

$$A(x, y)\gamma^2 + B(x, y)\gamma + C(x, y) = 0 \quad (11)$$

where the coefficients A, B, C are the combinations of the partial derivatives to the given logarithmic density $f(x, y)$ (see the formulas (17) through (19) in the same paper). The solutions of the quadratic equation (11) are determined by its discriminant Δ and denoted as

$$\gamma_{\pm} = \frac{-B \pm \sqrt{\Delta}}{2A}. \quad (12)$$

If the ‘‘doubled’’ slope of the proportion curves of the excited state is γ_+ (or γ_-), the state is called the excited plus state, $|2+\rangle$ (or the excited minus state, $|2-\rangle$).

In summary, the rational orthogonal conditions for the excited plus state, the excited minus state, and the ground state are,

$$P(x, y) = (-B + \sqrt{\Delta})\nabla f \cdot (A\nabla B - B\nabla A) + 2A\nabla f \cdot (A\nabla C - C\nabla A) = 0, \quad (13)$$

$$M(x, y) = (-B - \sqrt{\Delta})\nabla f \cdot (A\nabla B - B\nabla A) + 2A\nabla f \cdot (A\nabla C - C\nabla A) = 0, \quad (14)$$

$$G(x, y) = (f''_{yy} - f''_{xx})f'_x f'_y - f''_{xy}(f'^2_y - f'^2_x) = 0 \quad (15)$$

respectively.

Because the body structure of barred galaxies is a compound one of rational structures (exponential disk plus dual-handle structure), the boundary condition is satisfied (see paper [2] for its discussion). Therefore, if one of the above rational orthogonal conditions is satisfied then the galaxy is rational. As shown in the following Section, if a barred galaxy is rational, its rational state can be one type at some area on the galaxy plane and the other state at other area.

3 The Proportion Curves of Barred Spiral Galaxies

Firstly let me introduce some potential origins of error for numerical calculation. Although the model of barred spiral galaxy is expressed with the simple elementary functions as

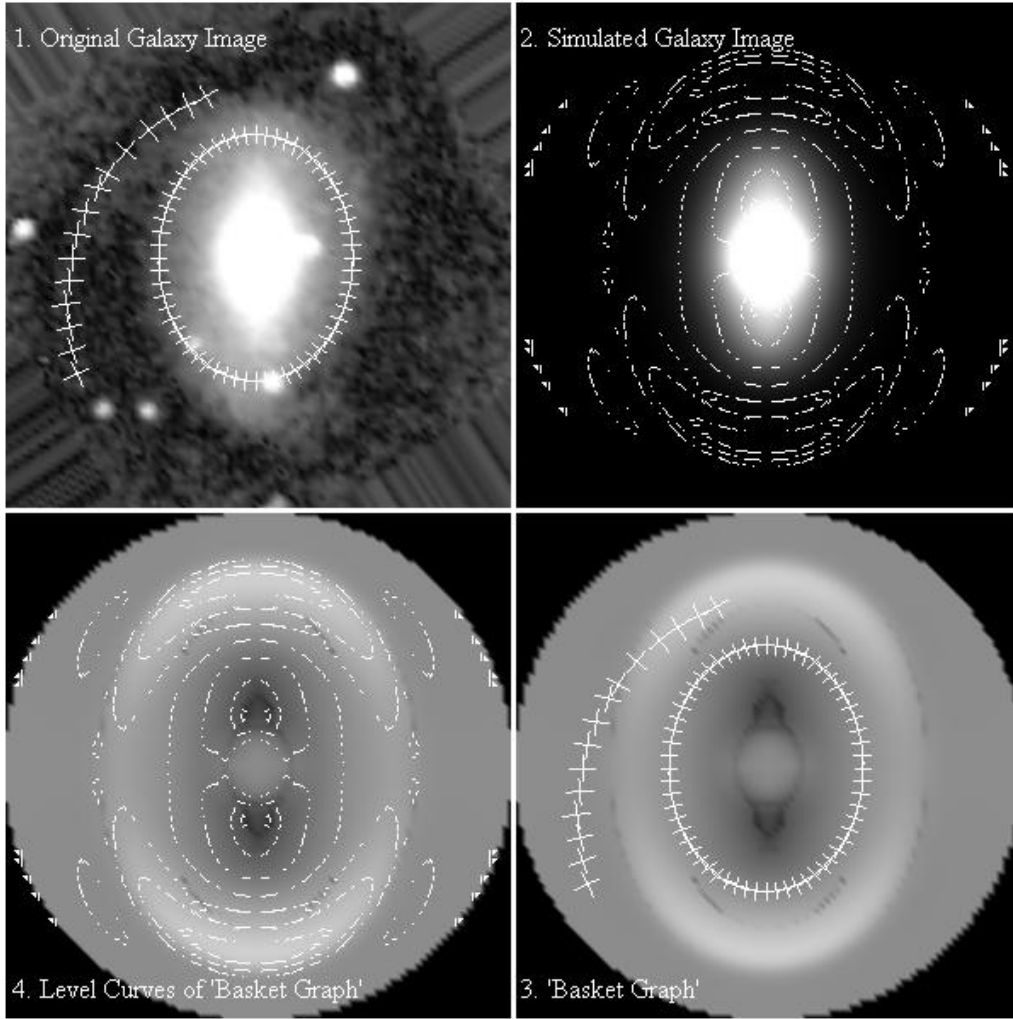


Figure 3: Upper left is the longer-wavelength image of barred spiral galaxy NGC 3275 (see [6] for image credit). Upper right is its simulation with an exponential disk and two dual-handle structures. Lower right is its 'basket graph' which is a demonstration of the function $u(x, y)$ with the image brightness at each point being proportional to the corresponding value of the function. The function is the variance rate of $f(x, y)$ along the direction of α that is solved from instinct equation. Lower left is the level curves of $u(x, y)$ which are simultaneously the proportion curves of barred galaxy. The curves are spider-shaped.



Figure 4: The longer-wavelength image of barred spiral galaxy NGC 3275 as shown on Figure 2. The spider-shaped proportion curves do trace or cut through arms or rings consistently as in the case of normal spiral galaxies (see Figure 1).

indicated by the formulas (3) and (4), it is unlikely that the proportion curves of barred spiral galaxies could be expressed with analytical formulas too. The calculation of the azimuthal angle α for the tangential lines of proportion curves involves the third derivatives of $f(x, y)$ while the calculation of the rational orthogonal conditions involves the fourth derivatives. Without further theoretical analysis and appropriate computing technique, the numerical calculation with common computers may present significant errors. Fortunately, the error can be greatly reduced by the choice of larger scale s in the formula (7). The choice of the scale in Table 1 (the third column) indicates that the left sides of the orthogonal conditions (13) and (15) (i.e., $P(x, y)$ and $G(x, y)$) are significantly small in the relevant areas for the galaxy NGC 3275. That is, the absolute values of these quantities are less than 10^{-16} . This suggests that the galaxy is rational. Detailed calculation indicates that there are a few lined areas where the discriminant of the quadratic equation (11) is negative. This means that the galaxy must be at the ground state in the relevant areas. In all other areas, the absolute values of $P(x, y)$ are much smaller than the ones of the quantities $M(x, y)$ and $G(x, y)$. It means that the galaxy is generally at the excited plus state.

Another potential error for the calculation is the simulation of exponential disk. The hard part of the simulation of barred galaxy images is not the bar fitting. Given some graphic software, it is easy for us to visually simulate bars with dual-handle structure. The criterion is that the galaxy image after bar subtraction must be an axi-symmetric disk. The hard part is galaxy disk simulation. Because of central bulges, disk simulation is done away from the bulges. Even though we have performed bar subtraction, the disks away from the galaxy centers are generally disturbed by arms or rings. Therefore, disk simulation is a sophisticated process, and Table 1 presents a preliminary result. Further more, it is not justified that the center part of any barred spiral galaxy is the superposition of a bulge and an *exponential*

disk.

In fact, the second partial derivatives to the dual-handle structure (3) do not exist at its two foci. In the vicinity of the foci, the partial derivatives exist but some of their values tend to infinity (keeping all important quantities, e.g. the slopes of confocal ellipses and hyperbolas, finite). Because the computer can not accurately handle large numbers, some results near the foci may present significant errors. The denominators generally contain the following term (see the formula (4)),

$$(r^2 - b_1^2)^2 + 4b_1^2x^2. \quad (16)$$

Fortunately the error can be greatly reduced as well by the choice of larger scale s in the formula (7). Note that the logarithmic density of the exponential disk has a singular point too, i.e., the center point, at which there is no partial derivatives to the logarithmic density though the structure $\rho(x, y)$ is continuous throughout. Away from the singular areas, the computer's calculation is normal and meets required precision.

Finally, I deal with the most important part of the paper, the drawing of the proportion curves of barred spiral galaxies. As shown in Figure 3, the first step is to find a longer-wavelength image of the barred galaxy NGC 3275 which represents the stellar distribution of the galaxy. The second step is the simulation of the image with exponential disk and dual-handle structure. This is generally done by visual simulation with graphic software. The result is the function $f(x, y)$ which is displayed in the part 2 of Figure 3. The third step is the demonstration of rational sufficient conditions. If the conditions are met with high confident level then the galaxy is rational. We calculate the solution α of the relevant instinct equations in relevant areas on the galaxy disk plane. The final result is the calculation of the 'radial' variance rate $u(x, y)$,

$$u(x, y) = f'_x \cos \alpha + f'_y \sin \alpha \quad (17)$$

(see the formula (7) in paper [2]). The demonstration of the function $u(x, y)$ is called the 'basket graph' (see the part 3 of Figure 3). Why is there 'basket graph'? The answer is the following. In the area near galaxy center and the four foci, the density of the exponential disk, i.e. the weight factor in the first term in (6), is the greatest and makes the overwhelming contribution to the gradient (the left-hand side of (6)). However, the absolute values of the logarithmic densities of the dual-handle structures increase with radius cubically whereas the absolute value of the logarithmic density of exponential disk increases linearly. Therefore, the gradient values of the logarithmic densities of the dual-handle structures, i.e. the second factors in the second and third terms in (6), are the greatest and make the overwhelming contribution to the gradient at around the area of the 'basket' rim. However, far away from the galaxy center, the first factor in the disk term, i.e. the weight factor, makes larger impact, and the density of the exponential disk is the greatest which makes the overwhelming contribution to the gradient (the left side of (6)).

The fourth step is the calculation of the level curves of the function $u(x, y)$. The result is the curves obtained by intersecting the surface of the function $z = u(x, y)$ with horizontal planes, $z = z_1, z_2, z_3, \dots$. The level curves are the proportion curves of the barred galaxy NGC 3275 which are displayed in the part 4 of Figure 3. The curves demonstrate a spider-shaped pattern which can be called the 'spider graph'. The result is very primitive because I use a very primitive computing algorithm.

The ‘spider graph’ shows that the ring follows the closed proportion curve while spiral arms cut through the orthogonal curves consistently. Note that the ellipse in the parts 1 and 3 of Figure 3 is artificial and shows the approximate position of the real ring. It is amazing that the closed proportion curve follows the ring more exactly. This suggests the rational model of galaxy structure approaches the nature’s truth.

Note that the ring is located inside the ‘basket’. This ring can be called the inner ring. There are other types of rings for barred galaxies: outer rings and nuclear rings [8]. The nuclear rings must be located in the central convex balls of the ‘basket graphs’, and they are approximately circular. Galaxy rings can not coincide with the bright ‘basket rims’. This is because the brightness of the rims is usually uneven as shown by the ‘spider graph’. Figure 3 shows that the ring is located immediately off the rims. However, arms can cross through the basket rims because the ‘non-radial’ variance rates of $f(x, y)$ are usually small. For example, the absolute value of the ‘axial’ variance rate $v(x, y)$ for the galaxy NGC 3275 is generally so small that it is buried in the numerical errors of ordinary computer calculation.

We can investigate ‘basket graphs’ deeply and find further evidences of rational galaxy structure. We take a preliminary overall look at ‘basket graphs’. Outside the ‘baskets’, the orthogonal proportion curves return to the ones of exponential disks, and the disks have infinite nets of proportion curves which are generally equiangular. Barred galaxy images do show that the arms are much richer outside of the bars. The area near galaxy bars is cleaner and has less gas and dust. Arms can cross the ‘baskets’ but their curves can not be parallel to the ‘basket’ rims for the same reason for galaxy rings. Galaxy images show that some arms originate from around the endpoints of bars and the arms make a sharp turn near the endpoints. The turn near the endpoints is so sharp that the arms are approximately perpendicular to the ‘basket’ rims.

More evidences of rational galaxy structure are presented in paper [4].

4 Conclusion and Future Work

Rational galaxy structure was suggested eleven years ago. The proportion curves for a barred spiral galaxy are finally obtained in the current paper. Proportion curves follow galaxy arms and rings naturally as shown in the Figures 1 and 4. I expect the ‘spider graphs’ for more examples of barred galaxies will be obtained and more people will join the study for the evaluation of Nature’s rationality.

“What exactly is a galaxy? Surprising as it may sound, astronomers don’t have an answer to this fundamental question.” These are the opening words of a piece of January news (2011) in Science magazine written by Jon Cartwright. The news introduces a scientific paper [9] and an online survey launched by the paper’s authors. Astronomers have long known that galaxy structure is very simple [10,11] but the two-body theories of gravity established by Newton and Einstein can not explain it. According to these theories, galaxy formation and evolution should be governed by six independent parameters. However, the astronomical observation shows that only one parameter is independent. My simple model of galaxy structure is based on galaxy images and the simple concept of rationality.

‘Basket graph’ will play an essential role in future study on barred galaxies. We humans live in a barred galaxy. What does the ‘basket graph’ of Milky Way look like? Do human

beings live inside or outside the ‘basket’? The mass distribution of galaxies, i.e., the stellar distribution, is represented by the longer-wavelength images. However, life phenomena are more involved in the distribution of gas and dust. Therefore, the study of the correlation between the ‘basket’ structure and the galaxy pattern shown on short-wavelength images may reveal many surprising results.

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5 Appendix

The Appendix is a free c++ computer program written by the author. Its main purpose is to provide with readers the program of proportion curve calculation for barred galaxies. All the functions are the same as the ones in Appendix of paper [1] and omitted. If your computer is equipped with c++ software, you can open your software and copy the following text as a whatever.cpp file. You run the file and get two data files: ‘endpointoffline’ and ‘oppositeendpoint’. Each line of a file contains two numbers that are the coordinates of an end point of a line. The corresponding line of the other file contains two numbers that are the coordinates of the other end point of the line. Connecting each pair of end points, you get the proportion curves as shown on Figures 3 and 4.

```
#include<stdio.h>
```

```
#include<math.h>
#include<iomanip.h>
#include<fstream.h>
long double pi=3.14159265358979323846,hpi=pi*0.5,tpi=pi*2,qpi=pi*0.25;
const int nb=4;
long double s0,d0,d1,b0[nb],b1[nb],b2[nb];
```

```
long double r0( long double x, long double y);
long double gi(int i, long double x, long double y);
long double ri(int i, long double x, long double y);
long double rh( long double x, long double y);
long double f0x( long double x, long double y);
long double f0y( long double x, long double y);
long double f0xx( long double x, long double y);
long double f0xy( long double x, long double y);
long double f0yy( long double x, long double y);
long double f0xxx( long double x, long double y);
long double f0xxy( long double x, long double y);
long double f0xyy( long double x, long double y);
long double f0yyy( long double x, long double y);
long double f0xxxx( long double x, long double y);
long double f0xxxy( long double x, long double y);
long double f0xxyy( long double x, long double y);
long double f0xyyy( long double x, long double y);
long double f0yyyy( long double x, long double y);
long double gix( int i, long double x, long double y);
long double giy( int i, long double x, long double y);
long double gixx( int i, long double x, long double y);
long double gixy( int i, long double x, long double y);
long double giyy( int i, long double x, long double y);
long double gixxx( int i, long double x, long double y);
long double gixxy( int i, long double x, long double y);
long double gixyy( int i, long double x, long double y);
long double giyyy( int i, long double x, long double y);
long double gixxxx( int i, long double x, long double y);
long double gixxxy( int i, long double x, long double y);
long double gixxyy( int i, long double x, long double y);
long double gixyyy( int i, long double x, long double y);
long double giyyyy( int i, long double x, long double y);
long double fix( int i, long double x, long double y);
long double fiy( int i, long double x, long double y);
long double fixx( int i, long double x, long double y);
long double fixy( int i, long double x, long double y);
long double fiyy( int i, long double x, long double y);
long double fixxx( int i, long double x, long double y);
```

```
long double fixxy( int i, long double x, long double y);
long double fixyy( int i, long double x, long double y);
long double fiyyy( int i, long double x, long double y);
long double fixxxx( int i, long double x, long double y);
long double fixxxy( int i, long double x, long double y);
long double fixxyy( int i, long double x, long double y);
long double fiyyyy( int i, long double x, long double y);
long double fx( long double x, long double y);
long double fy( long double x, long double y);
long double fxx( long double x, long double y);
long double fxy( long double x, long double y);
long double fyy( long double x, long double y);
long double fxxx( long double x, long double y);
long double fxxy( long double x, long double y);
long double fxyy( long double x, long double y);
long double fyyy( long double x, long double y);
long double fxxxx( long double x, long double y);
long double fxxxy( long double x, long double y);
long double fxxyy( long double x, long double y);
long double fxyyy( long double x, long double y);
long double fyyyy( long double x, long double y);
long double mu( long double p, long double q);
long double nu( long double p, long double q);
long double La( long double p, long double q);
long double Om( long double p, long double q);
long double mux( long double p, long double q);
long double muy( long double p, long double q);
long double muxx( long double p, long double q);
long double muxy( long double p, long double q);
long double muyy( long double p, long double q);
long double nux( long double p, long double q);
long double nuy( long double p, long double q);
long double nuxx( long double p, long double q);
long double nuxy( long double p, long double q);
long double nuyy( long double p, long double q);
long double Lax( long double p, long double q);
long double Lay( long double p, long double q);
long double Laxx( long double p, long double q);
long double Laxy( long double p, long double q);
long double Layy( long double p, long double q);
long double Omx( long double p, long double q);
long double Omy( long double p, long double q);
long double Omxx( long double p, long double q);
long double Omxy( long double p, long double q);
```

```

long double Omyy( long double p, long double q);
long double cA( long double x, long double y);
long double cB( long double x, long double y);
long double cC( long double x, long double y);
long double Ax( long double p, long double q);
long double Ay( long double p, long double q);
long double Bx( long double p, long double q);
long double By( long double p, long double q);
long double Cx( long double p, long double q);
long double Cy( long double p, long double q);
long double Orthm( long double p, long double q);
int main(){
ofstream of1("endpointoffline");
of1<<setw(16)<<setprecision (10)<< resetiosflags( ios::left )
<<setiosflags(ios::fixed — ios::showpoint); ofstream of1("oppositeendpoint");
of1<<setw(16)<<setprecision (10)<< resetiosflags( ios::left )
<<setiosflags(ios::fixed — ios::showpoint);

long double kx,ky,alp,A,a,b,c,x1,y1,x,y;

    const int M=60,N=60,hM=M/2,hN=N/2;
long double X,Y;
long double DI,DJ;
int m,n;

    int tz=3,tc,pn=0,ptr;
long double lev[M*N][10],qn,qm;
int ik[M*N][4];
    s0=11;
//2*s0 is the assigned galaxy length on a side of the square image
d0=8000;d1=-2.4; //fitted galaxy exponential disk
b0[1]=134;b1[1]=1.76;b2[1]=-0.2; //fitted dual-handle structure
b0[2]=72; b1[2]=3.25;b2[2]=-0.095;

    long double Az=1,rad,azi,R0;
d1=d1/Az;b1[1]=b1[1]*Az;b2[1]=b2[1]/(Az*Az*Az); // scale tranformation
b1[2]=b1[2]*Az;b2[2]=b2[2]/(Az*Az*Az); // scale tranformation
s0=s0*Az; // scale tranformation

    DI=2*s0/M;DJ=2*s0/N;

    for(n=hN+1;n<=N;n++){
Y=(n-hN-0.5)*DJ;
for(m=1;m<=hM;m++){
X=(m-hM-0.5)*DI;

```

```

    if( (m-hM-0.5)*(m-hM-0.5)+(n-hN-0.5)*(n-hN-0.5) <= hM*hM ){
a=cA(X,Y);
b=cB(X,Y);
c=cC(X,Y);

    A=b*b-4*a*c;
if(A>=0){
x1=( -b+sqrt(A) )/(2*a);
}else{
x1=-2*fx(X,Y)*fy(X,Y)/(fy(X,Y)*fy(X,Y)-fx(X,Y)*fx(X,Y));
}

    alp=0.5*atan(x1);

    kx=fx(X,Y);
ky=fy(X,Y);

    rad=sqrt(X*X+Y*Y);azi=atan(Y/X)+hpi;

    if( (alp<(azi-qpi)) && (alp<=(azi+qpi)) ){R0=alp;}
if( ((alp+hpi)<(azi-qpi)) && ((alp+hpi)<=(azi+qpi)) ){R0=alp+hpi;}
if( ((alp+pi)<(azi-qpi)) && ((alp+pi)<=(azi+qpi)) ){R0=alp+pi;}
if( ((alp+3*hpi)<(azi-qpi)) && ((alp+3*hpi)<=(azi+qpi)) ){R0=alp+3*hpi;}

    alp=R0;

    Rm[m][n]=kx*cos(alp-hpi)+ky*sin(alp-hpi);

    Rm[M-m+1][n]=Rm[m][n];
Rm[m][N-n+1]=Rm[m][n];
Rm[M-m+1][N-n+1]=Rm[m][n];

    }else{

    Rm[m][n]=0;

    Rm[M-m+1][n]=Rm[m][n];
Rm[m][N-n+1]=Rm[m][n];
Rm[M-m+1][N-n+1]=Rm[m][n];
}
} // n
} // m

//level =====

```



```

    for(n=hN+1;n<=N-1;n++){
y=(n-hN-0.5)*DJ;y1=((n+1)-hN-0.5)*DJ;
tc=0;
for(m=tz;m<=hM;m++){
x=(m-hM-0.5)*DI;x1=((m-1)-hM-0.5)*DJ;
if( (x1*x1+y1*y1>=hM*hN*DI*DJ)&&(x*x+y1*y1>=hM*hN*DI*DJ) ){
tc=tc+1;
}else if( (x1*x1+y1*y1>=hM*hN*DI*DJ)&&(x*x+y1*y1<hM*hN*DI*DJ) ){
ptr=2;

```

```

    if(Rm[m][n+1]>Rm[m-1][n]){
qn=Rm[m][n+1];qm=Rm[m-1][n];
}else{
qn=Rm[m-1][n];qm=Rm[m][n+1];
}
if( (Rm[m][n]>qn) ){
qn=Rm[m][n];
}else if( (Rm[m][n]>qm) ){
qm=Rm[m][n];
}else{;}

```

```

    if(qn>qm){
pn=pn+1;
ik[pn][1]=ptr; lev[pn][1]=x; lev[pn][2]=y1;lev[pn][3]=Rm[m][n+1];
lev[pn][4]=x1;lev[pn][5]=y; lev[pn][6]=Rm[m-1][n];
lev[pn][7]=x; lev[pn][8]=y; lev[pn][9]=Rm[m][n];
if(Rm[m][n+1]>Rm[m-1][n])
ik[pn][2]=1;
}else{
ik[pn][2]=2;
}
if( (Rm[m][n]>qn) ){
ik[pn][3]=1;
}else if( (Rm[m][n]>qm) ){
ik[pn][3]=2;
}else{ik[pn][3]=3;}
}else{
;
}
}else{
ptr=1;

```

```

    if(Rm[m-1][n+1]>Rm[m-1][n]){
qn=Rm[m-1][n+1];qm=Rm[m-1][n];

```

```

}else{
qn=Rm[m-1][n];qm=Rm[m-1][n+1];
}
if( (Rm[m][n]i;qm) ){
qn=Rm[m][n];
}else if( (Rm[m][n]i;qm) ){
qm=Rm[m][n];
}else{;}

    if(qn;qm){
pn=pn+1;
ik[pn][1]=ptr;
lev[pn][1]=x1; lev[pn][2]=y1;lev[pn][3]=Rm[m-1][n+1];
lev[pn][4]=x1;lev[pn][5]=y; lev[pn][6]=Rm[m-1][n];
lev[pn][7]=x; lev[pn][8]=y; lev[pn][9]=Rm[m][n];
if(Rm[m-1][n+1]i;Rm[m-1][n]){
ik[pn][2]=1;
}else{
ik[pn][2]=2;
}
if( (Rm[m][n]i;qm) ){
ik[pn][3]=1;
}else if( (Rm[m][n]i;qm) ){
ik[pn][3]=2;
}else{ik[pn][3]=3;}
}else{
;
}

```

```
ptr=2;
```

```

if(Rm[m][n+1]i;Rm[m-1][n]){
qn=Rm[m][n+1];qm=Rm[m-1][n];
}else{
qn=Rm[m-1][n];qm=Rm[m][n+1];
}
if( (Rm[m][n]i;qm) ){
qn=Rm[m][n];
}else if( (Rm[m][n]i;qm) ){
qm=Rm[m][n];
}else{;}

```

```

if(qn;qm){
pn=pn+1;
ik[pn][1]=ptr;

```

```

lev[pn][1]=x; lev[pn][2]=y1;lev[pn][3]=Rm[m][n+1];
lev[pn][4]=x1;lev[pn][5]=y; lev[pn][6]=Rm[m-1][n];
lev[pn][7]=x; lev[pn][8]=y; lev[pn][9]=Rm[m][n];
if(Rm[m][n+1];Rm[m-1][n]){
ik[pn][2]=1;
}else{
ik[pn][2]=2;
}
if( (Rm[m][n];qm) ){
ik[pn][3]=1;
}else if( (Rm[m][n];qm) ){
ik[pn][3]=2;
}else{ik[pn][3]=3;}
}else{
;
}
}
} // m
tz=tz+tc;
} // n

// result =====
double mnl=0.1, por=1.3, sca=0.6, mxl=25, zl, v3,w3,v6,w6,v9,w9;
double xx,yy,xx1,yy1;
int numpl=40;
for(m=1;m<=numpl;m++){
zl=mnl+sca*exp(por*log(m*1.))*(mxl-mnl)/numpl;
for(n=1;n<=pn;n++){

v3=lev[n][1];w3=lev[n][2];v6=lev[n][4];w6=lev[n][5];v9=lev[n][7];w9=lev[n][8];

if( (ik[n][2]==1)&&(ik[n][3]==1) ){
if( (zl<=lev[n][9])——(zl>=lev[n][6]) ){
}else{
xx=( v9+(v6-v9)*(zl-lev[n][9])/(lev[n][6]-lev[n][9]) );
yy=( w9+(w6-w9)*(zl-lev[n][9])/(lev[n][6]-lev[n][9]) );
of1<<" " <<xx;
of1<<" " <<yy<<endl;
if(zl>=lev[n][3]){
xx1=( v3+(v6-v3)*(zl-lev[n][3])/(lev[n][6]-lev[n][3]) );
yy1=( w3+(w6-w3)*(zl-lev[n][3])/(lev[n][6]-lev[n][3]) );
of2<<" " <<xx1;
of2<<" " <<yy1<<endl;
}else{
xx1=( v9+(v3-v9)*(zl-lev[n][9])/(lev[n][3]-lev[n][9]) );

```

```

yy1=( w9+(w3-w9)*(zl-lev[n][9])/(lev[n][3]-lev[n][9]) );
of2<<" "<<xx1;
of2<<" "<<yy1<<endl;
}
of1<<" "<<-xx;
of1<<" "<<yy<<endl;
of2<<" "<<-xx1;
of2<<" "<<yy1<<endl;
of1<<" "<<xx;
of1<<" "<<-yy<<endl;
of2<<" "<<xx1;
of2<<" "<<-yy1<<endl;
of1<<" "<<-xx;
of1<<" "<<-yy<<endl;
of2<<" "<<-xx1;
of2<<" "<<-yy1<<endl;

}
}else if( (ik[n][2]==2)&&(ik[n][3]==1) ){
if( (zl<=lev[n][9])——(zl>=lev[n][3]) ){;
}else{
xx=( v9+(v3-v9)*(zl-lev[n][9])/(lev[n][3]-lev[n][9]) );
yy=( w9+(w3-w9)*(zl-lev[n][9])/(lev[n][3]-lev[n][9]) );
of1<<" "<<xx;
of1<<" "<<yy<<endl;
if(zl>=lev[n][6]){
xx1=( v6+(v3-v6)*(zl-lev[n][6])/(lev[n][3]-lev[n][6]) );
yy1=( w6+(w3-w6)*(zl-lev[n][6])/(lev[n][3]-lev[n][6]) );
of2<<" "<<xx1;
of2<<" "<<yy1<<endl;
}else{
xx1=( v9+(v6-v9)*(zl-lev[n][9])/(lev[n][6]-lev[n][9]) );
yy1=( w9+(w6-w9)*(zl-lev[n][9])/(lev[n][6]-lev[n][9]) );
of2<<" "<<xx1;
of2<<" "<<yy1<<endl;
}
of1<<" "<<-xx;
of1<<" "<<yy<<endl;
of2<<" "<<-xx1;
of2<<" "<<yy1<<endl;
of1<<" "<<xx;
of1<<" "<<-yy<<endl;
of2<<" "<<xx1;
of2<<" "<<-yy1<<endl;
of1<<" "<<-xx;

```

```

of1<<" " <<-yy<<endl;
of2<<" " <<-xx1;
of2<<" " <<-yy1<<endl;

    }
}else if( (ik[n][2]==1)&&(ik[n][3]==2) ){
if( (zl<=lev[n][3])——(zl>=lev[n][9]) ){;
}else{
xx=( v3+(v9-v3)*(zl-lev[n][3])/(lev[n][9]-lev[n][3]) );
yy=( w3+(w9-w3)*(zl-lev[n][3])/(lev[n][9]-lev[n][3]) );
of1<<" " <<xx;
of1<<" " <<yy<<endl;
if(zl>=lev[n][6]){
xx1=( v6+(v9-v6)*(zl-lev[n][6])/(lev[n][9]-lev[n][6]) );
yy1=( w6+(w9-w6)*(zl-lev[n][6])/(lev[n][9]-lev[n][6]) );
of2<<" " <<xx1;
of2<<" " <<yy1<<endl;
}else{
xx1=( v3+(v6-v3)*(zl-lev[n][3])/(lev[n][6]-lev[n][3]) );
yy1=( w3+(w6-w3)*(zl-lev[n][3])/(lev[n][6]-lev[n][3]) );
of2<<" " <<xx1;
of2<<" " <<yy1<<endl;
}
of1<<" " <<-xx;
of1<<" " <<yy<<endl;
of2<<" " <<-xx1;
of2<<" " <<yy1<<endl;
of1<<" " <<xx;
of1<<" " <<-yy<<endl;
of2<<" " <<xx1;
of2<<" " <<-yy1<<endl;
of1<<" " <<-xx;
of1<<" " <<-yy<<endl;
of2<<" " <<-xx1;
of2<<" " <<-yy1<<endl;

    }
}else if( (ik[n][2]==2)&&(ik[n][3]==2) ){
if( (zl<=lev[n][6])——(zl>=lev[n][9]) ){;
}else{
xx=( v6+(v9-v6)*(zl-lev[n][6])/(lev[n][9]-lev[n][6]) );
yy=( w6+(w9-w6)*(zl-lev[n][6])/(lev[n][9]-lev[n][6]) );
of1<<" " <<xx;
of1<<" " <<yy<<endl;
if(zl>=lev[n][3]){

```

```

xx1=( v3+(v9-v3)*(zl-lev[n][3])/(lev[n][9]-lev[n][3]) );
yy1=( w3+(w9-w3)*(zl-lev[n][3])/(lev[n][9]-lev[n][3]) );
of2<<" "<<xx1;
of2<<" "<<yy1<<endl;
}else{
xx1=( v6+(v3-v6)*(zl-lev[n][6])/(lev[n][3]-lev[n][6]) );
yy1=( w6+(w3-w6)*(zl-lev[n][6])/(lev[n][3]-lev[n][6]) );
of2<<" "<<xx1;
of2<<" "<<yy1<<endl;
}
of1<<" "<<-xx;
of1<<" "<<yy<<endl;
of2<<" "<<-xx1;
of2<<" "<<yy1<<endl;
of1<<" "<<xx;
of1<<" "<<-yy<<endl;
of2<<" "<<xx1;
of2<<" "<<-yy1<<endl;
of1<<" "<<-xx;
of1<<" "<<-yy<<endl;
of2<<" "<<-xx1;
of2<<" "<<-yy1<<endl;

}
}else if( (ik[n][2]==1)&&(ik[n][3]==3) ){
if( (zl<=lev[n][3])——(zl>=lev[n][6]) ){
}else{
xx=( v3+(v6-v3)*(zl-lev[n][3])/(lev[n][6]-lev[n][3]) );
yy=( w3+(w6-w3)*(zl-lev[n][3])/(lev[n][6]-lev[n][3]) );
of1<<" "<<xx;
of1<<" "<<yy<<endl;
if(zl>=lev[n][9]){
xx1=( v9+(v6-v9)*(zl-lev[n][9])/(lev[n][6]-lev[n][9]) );
yy1=( w9+(w6-w9)*(zl-lev[n][9])/(lev[n][6]-lev[n][9]) );
of2<<" "<<xx1;
of2<<" "<<yy1<<endl;
}else{
xx1=( v3+(v9-v3)*(zl-lev[n][3])/(lev[n][9]-lev[n][3]) );
yy1=( w3+(w9-w3)*(zl-lev[n][3])/(lev[n][9]-lev[n][3]) );
of2<<" "<<xx1;
of2<<" "<<yy1<<endl;
}
of1<<" "<<-xx;
of1<<" "<<yy<<endl;
of2<<" "<<-xx1;

```

```

of2<<" "<<yy1<<endl;
of1<<" "<<xx;
of1<<" "<<-yy<<endl;
of2<<" "<<xx1;
of2<<" "<<-yy1<<endl;
of1<<" "<<-xx;
of1<<" "<<-yy<<endl;
of2<<" "<<-xx1;
of2<<" "<<-yy1<<endl;

```

```

}
}else{
if( (zl<=lev[n][6])——(zl>=lev[n][3]) );
}else{
xx=( v6+(v3-v6)*(zl-lev[n][6])/(lev[n][3]-lev[n][6]) );
yy=( w6+(w3-w6)*(zl-lev[n][6])/(lev[n][3]-lev[n][6]) );
of1<<" "<<xx;
of1<<" "<<yy<<endl;
if(zl>=lev[n][9]){
xx1=( v9+(v3-v9)*(zl-lev[n][9])/(lev[n][3]-lev[n][9]) );
yy1=( w9+(w3-w9)*(zl-lev[n][9])/(lev[n][3]-lev[n][9]) );
of2<<" "<<xx1;
of2<<" "<<yy1<<endl;
}else{
xx1=( v6+(v9-v6)*(zl-lev[n][6])/(lev[n][9]-lev[n][6]) );
yy1=( w6+(w9-w6)*(zl-lev[n][6])/(lev[n][9]-lev[n][6]) );
of2<<" "<<xx1;
of2<<" "<<yy1<<endl;
}
of1<<" "<<-xx;
of1<<" "<<yy<<endl;
of2<<" "<<-xx1;
of2<<" "<<yy1<<endl;
of1<<" "<<xx;
of1<<" "<<-yy<<endl;
of2<<" "<<xx1;
of2<<" "<<-yy1<<endl;
of1<<" "<<-xx;
of1<<" "<<-yy<<endl;
of2<<" "<<-xx1;
of2<<" "<<-yy1<<endl;
}
}
}

```

```
    } // n  
} // m  
return 0;  
}
```