

Inconsistency of the Beckwith entropy formula

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Abstract

In my recent paper [1] published by Prespacetime Journal I discussed certain consequences of the entropy formula presented by A.W. Beckwith and his coauthors [2]. The main result of the deductions were bonons and the inflaton constant. However, I now consider the Beckwith entropy formula to be wrong, and deductions based on this relation can therefore be at most half-true. In this brief paper the right way to deduce the entropy formula is concisely discussed, the results obtained previously are revised, and certain new results are presented.

1 Introduction

Recently Beckwith et al (See e.g. the papers in the Ref. [2]) presented certain fruitful looking deductions for Higgs inflaton ϕ characterized by the potential $V(\phi) \sim \phi^2$. The key result is the entropy formula

$$[\Delta S] = [\hbar/T] \left[2k^2 - \frac{1}{\eta^2} \left[M_{Planck}^2 \cdot \left[\left[\frac{6}{4\pi} - \frac{12}{4\pi} \right] \cdot \left[\frac{1}{\phi} \right]^2 \right] - \frac{6}{4\pi} \cdot \left[\frac{1}{\phi^2} \right] \right] \right]^{1/2}. \quad (1)$$

Beckwith et al referred to the paper of J. Martin [3], in which the slow-roll inflation is applied, and its analogy to the Schwinger effect is discussed. Beckwith stated that the entropy formula (1) was obtained in such an approximation. I now claim that this statement is wrong, because the entropy formula (1) can not be deduced in a consistent way via the slow-roll approximation discussed by J. Martin.

In my recent paper [1] I have presented certain consequences of the Beckwith entropy formula (1). Despite that this paper connects quantum cosmology to inflationary cosmology by use of the Multiverse hypothesis, its main results can only be half-true, because the paper is based on the key result of Beckwith's deductions.

2 The Cosmological Perturbations

The entropy formula follows from the relation $\Delta S = \frac{\hbar\omega}{T}$, where

$$\omega = \sqrt{\omega_S^2(k, \eta) + \omega_T^2(k, \eta)}. \quad (2)$$

Here $\omega_S(k, \eta)$ and $\omega_T(k, \eta)$ are the frequencies of scalar and tensor perturbations

$$\omega_S^2(k, \eta) = k^2 c^2 - \frac{(a\sqrt{\gamma})''}{a\sqrt{\gamma}}, \quad (3)$$

$$\omega_T^2(k, \eta) = k^2 c^2 - \frac{a''}{a}, \quad (4)$$

which are the result of the slow-roll inflation [3]. Recall that a is the cosmic scale factor parameter, the prime means differentiation with respect to the conformal time η , $\gamma = 1 - \frac{\mathcal{H}'}{\mathcal{H}^2}$ and $\mathcal{H} = \frac{a'}{a}$ is the conformal Hubble parameter.

The slow-roll approximation uses the parameters δ and ε

$$\varepsilon = -\frac{\dot{H}}{H^2} = \varepsilon_1, \quad (5)$$

$$\delta = \varepsilon - \frac{1}{2H} \frac{\dot{\varepsilon}}{\varepsilon} = \varepsilon_1 - \frac{1}{2}\varepsilon_2, \quad (6)$$

$$\xi = \frac{\dot{\varepsilon} - \dot{\delta}}{H}, \quad (7)$$

obeying the following equations of motion

$$\frac{\dot{\varepsilon}}{H} = 2\varepsilon(\varepsilon - \delta), \quad (8)$$

$$\frac{\dot{\delta}}{H} = 2\varepsilon(\varepsilon - \delta) - \xi. \quad (9)$$

Here $H = \frac{\dot{a}}{a}$ is the cosmological/cosmic Hubble parameter (the dot means differentiation with respect to the cosmological/cosmic time t), and ε_1 and ε_2 are the horizon flow functions which are given by

$$\varepsilon_1 \simeq \frac{1}{4S_P} \left(\frac{V'}{V} \right)^2, \quad (10)$$

$$\varepsilon_2 \simeq \frac{1}{S_P} \left[\left(\frac{V'}{V} \right)^2 - \frac{V''}{V} \right], \quad (11)$$

where $S_P = 4\pi\ell_P^2$ is the area of the Planck sphere, and prime means differentiation with respect to the inflaton field ϕ . The frequencies of the cosmological perturbations are

$$\omega_S^2(k, \eta) = \omega_P^2 \left[\ell_P^2 k^2 - (2 + 3\delta) \frac{t_P^2}{\eta^2} \right], \quad (12)$$

$$\omega_T^2(k, \eta) = \omega_P^2 \left[\ell_P^2 k^2 - (2 + 3\varepsilon) \frac{t_P^2}{\eta^2} \right], \quad (13)$$

so that the frequency (2) becomes

$$\omega = \sqrt{\omega_P^2 \left[2\ell_P^2 k^2 - \left(4 + \frac{3}{2S_P} \frac{V''}{V} \right) \frac{t_P^2}{\eta^2} \right]} = \sqrt{\omega_P^2 \left[2\ell_P^2 k^2 - \left(4 + \frac{3}{S_P} \frac{1}{\phi^2} \right) \frac{t_P^2}{\eta^2} \right]}, \quad (14)$$

3 The Consistent Approach

The frequency $\omega(k, \eta)$ leading to the Beckwith entropy formula (1) can not be obtained via the composition (2). Beckwith evidently deduced his formula in this way, because the factor 2 in k^2 term can not be deduced in another way. After a closer examination I now feel that Beckwith has not justified the formula (1) so I no longer accept the results.

The recent draft of my book [4] contains the Chapter 5 entirely devoted to the consistent discussion of the cosmological Schwinger effect [3]. The results of this analysis have a much more physical nature, and remove many of the results obtained in my previous paper [1] based on the Beckwith entropy formula. The difference is what follows from the reasoning leading to the entropy formula.

The deductions of [4] differ from the ones [1] based on the inconsistent approach [2]. Bonons, the result of boson-phonon duality [1], can not be obtained in frames of the slow-roll approximation to the cosmological perturbations. Albeit,

it is not clear whether bonons exist beyond the slow-roll inflation. Another key result is the inflaton constant/N-atomic inflaton constant. Their values were established in [4]. N -atomic constant is independent of the number of identical atoms N , so its value is universal: $\Lambda_N = \Lambda_\infty = 4\pi$.

The Multiverse hypothesis can not be realized via bonons in frames of the slow-roll inflation. In this approximation, however, the inflationary quantum Universes are phonons, i.e. excitations in the solid-medium, which I call the phononic Hubble inflaton [4]. The solid-medium is the \mathcal{A} ether model, and realizes the idea of *Æthereal Multiverse*.

There are new results from the corrected reasoning. The Einstein–Hilbert action of the Friedmann–Lemaître–Robertson–Walker Universe, i.e. of the Hubble inflaton, describes a massless ϕ^4 -theory. The scalar field is the Higgs inflaton

$$\varphi(\eta) = \varphi_0 a(\eta) \quad , \quad \varphi_0 = \frac{2\sqrt{\pi}}{3\ell_P} \approx 7.3109596 \cdot 10^{34} \frac{1}{\text{m}}, \quad (15)$$

obeying the Hubble law

$$\varphi = \frac{\varphi_0}{1 \pm \frac{1}{\sqrt{2}} \frac{mc^2}{\hbar} (\eta - \eta_0)}. \quad (16)$$

where η_0 is the initial value of η , and m is the mass of the Higgs–Hubble inflaton

$$m = \frac{3}{2\pi} \sqrt{\frac{3V_P}{V}} M_P, \quad (17)$$

where $V_P = \frac{4}{3}\pi\ell_P^3$ is the volume of the Planck sphere. Consequently, the Callan–Symanzik beta function of the Hubble inflaton is

$$\beta(g) = g, \quad (18)$$

where g is the coupling constant

$$g = g(\ell_P) = 3 \cdot 3! \frac{m^2 c^3}{\hbar} \ell_P. \quad (19)$$

The scale of the system is the Planck length $\lambda = \ell_P$, and the renormalization group equation has the form

$$\left(\lambda \frac{\partial}{\partial \lambda} - \beta(g) \frac{\partial}{\partial g} + 3 \right) V(\varphi, g, \lambda) = 0, \quad (20)$$

where $V(\varphi, g, \lambda) = \ell_P^4 \frac{g(\ell_P)}{4!} \varphi^4$ is the potential of the Higgs–Hubble inflaton.

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