A Note on the Dark Matter

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The constancy of the rotational velocity curves of the spiral galaxies from large distances from their galactic centers could be due to their geometries in form of arms.

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To explain why the rotational velocity curves of the spiral galaxies at large radii are constants; it was assumed the existence of dark matter [1, 2]. However, it could be due to their geometries in form of arms [3].

In effect, as in the gravitational rotation the centrifugal (repulsive) force is compensated by the gravitational (attractive) force, we would have that

$$m\frac{v^2}{r} = \frac{GMm}{r^2} \tag{1}$$

where G is the Newton's gravitational constant, M and r the mass and the radius of the spiral galaxy, respectively; m the mass of a star, and $v = \omega r$ and ω the linear and angular speeds of the star, respectively; v is the velocity of the orbit corresponding to the radius (or distance) r. We have put M instead of $M - m$ because $M - m \approx M$. From (1), it results:

$$v = \sqrt{\frac{GM}{r}} \tag{2}$$

Until a relatively small distance d_0 from the galactic center compared with r, the spiral galaxy would be a disc. For distances s between the galactic center and d_0 ($0 < s \le d_0$), the disc would have a thickness h_s, base area πs^2 and mass $M_s = \rho_s V_s = \rho_s \pi s^2 h_s$, where ρ_s and $V_s = \pi s^2 h_s$ are, respectively, the corresponding density and volume of the spiral galaxy until s. Then, from (2), it would be

$$v_s = \sqrt{\frac{GM_s}{s}} = \sqrt{\frac{G\rho_s \pi s^2 h_s}{s}} = \sqrt{G\rho_s \pi s h_s} = const. \times \sqrt{s} \tag{3}$$

and v_s varies proportionally to $\sqrt{s}$. And, the corresponding mass of the spiral galaxy until d_0 would be

$$M_{d_0} = \rho_s \pi d_0^{\,2} h_s = const. \tag{4}$$

and, from (2), the velocity of the orbit at d_0 would be

$$v_{d_0} = \sqrt{\frac{GM_{d_0}}{d_0}} = \sqrt{\frac{G\rho_s \pi d_0^{\,2} h_s}{d_0}} = \sqrt{G\rho_s \pi d_0 h_s} = const. \tag{5}$$

For distances *d* between d_0 and *r* ($d_0 < d \le r$), the spiral galaxy has a structure in form of arms. If we suppose that the arms are similar, then

$$M_q = nM_{qa} = n\rho_{qa} V_{qa} \tag{6}$$

where M_q is the corresponding mass of the spiral galaxy from d_0 until *d*, *n* the number of arms, and M_{qa}, ρ_{qa} and V_{qa} are, respectively, the corresponding mass, density and volume of a generic arm from d_0 until *d*. Although the arms are curved, any cross section of them can be considered circular; hence, the volume V_{qa} can be calculated as a sum of volumes of right cylinders:

$$V_{qa} = \sum_j \pi \frac{h_{qa}^{\,2}}{4} \ell_j = \pi \frac{h_{qa}^{\,2}}{4} \sum_j \ell_j = \pi \frac{h_{qa}^{\,2}}{4} \frac{q}{\alpha} \tag{7}$$

where h_{qa} is the thickness of the generic arm, ℓ_j the length of the right cylinder *j*, $q = d - d_0 = \alpha \sum_j \ell_j$ and α a real number, $0 < \alpha \le 1$. Substituting (7) into (6), it results

$$M_q = n\rho_{qa} \pi \frac{h_{qa}^{\,2}}{4} \frac{q}{\alpha} \tag{8}$$

If we substitute ρ_s and ρ_{qa} by ρ and h_s and h_{qa} by *h*, where ρ and *h* are, respectively, the density and the thickness of the spiral galaxy, then

$$M_d = M_{d_0} + M_q = \rho \pi d_0^{\,2} h + n\rho\pi \frac{h^2}{4} \frac{d - d_0}{\alpha} \tag{9}$$

where M_d is the corresponding mass of the spiral galaxy until *d*. For large values of *d* compared with d_0 (which implies large radii) and very small values of α (which implies very large arms), together with certain values of *h* and *n*; it results

$$M_d \approx n\rho\pi \frac{h^2}{4}\frac{d}{\alpha} \tag{10}$$

And, from (2), it would be

$$v_d = \sqrt{\frac{GM_d}{d}} \approx \sqrt{\frac{Gn\rho\pi\left(h^2/4\right)\left(d/\alpha\right)}{d}} = \sqrt{\frac{Gn\rho\pi h^2}{4\alpha}} = const. \tag{11}$$

Therefore, we conclude that the constancy of the rotational velocity curves of the spiral galaxies from large distances from their galactic centers could be due to their geometries in form of arms.

[1] F. Zwicky, "Die Rotverschiebung von extragalaktischen Nebeln", Helvetica Physica Acta 6: 110–127, 1933.

[2] F. Zwicky, Ap. J., **86**, 217, 1937.
http://adsabs.harvard.edu/abs/1937ApJ....86..217Z

[3] José Francisco García Juliá, A Note on the Dark Matter, May 24, 2011.
http://vixra.org/pdf/1105.0035v1.pdf