Electrodynamics and gravitation

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Abstract

In this manuscript, we examine the hypothesis of an electromagnetic origin of the gravitation. We suppose that the vacuum is polarized around a mass and we use the wave nature of matter in order to redefine gravity. We determine the relative permittivity and the relative permeability in order to find the expression of the energy and of the gravitational force.

1. Introduction

Einstein's theory of gravitation explains the deflection of the trajectory of matter and of light [1] in a gravitational field by the curvature of space-time. But this has not solved the mystery of the analogy between gravitation and electromagnetism. The use of the concept of curved space-time in order to explain the deviation of the trajectory of a body near a mass seems to be a mathematical abstraction which is difficult to reconcile with other natural phenomena, which should be linked by the existence of harmony between the infinitely small and the infinitely large. In this paper, we present a new theory of gravitation based on the fact that if we consider a particle with its electromagnetic field, as a wave in a polarizable vacuum near a mass, its trajectory will be refracted and this can help us redefine gravity and reconcile it with electromagnetism.

2. Trajectory of a particle in a gravitation field

Consider the vacuum surrounding a mass as a polarizable medium [2] which refract light and matter. We assume that matter affects the refractive index n of the surrounding environment, so that the trajectory of the particle depends on this index which varies with the distance r from the mass.

We try to generalize the Fermat's principle

$$S = \int n dl \qquad (1)$$

where dl is the shortest length of the stationary path S.

Assuming this principle is applicable not only to light but also to the trajectory of an incident particle. The refractive index is generally a tensor n_{ij} that may depend on many coordinates x^i and many parameters. *ndl* can be written as $(g_{ij}dx^i dx^j)^{1/2} = dS$

$$dS^2 = g_{ij}dx^i dx^j \tag{2}$$

$$\text{where } \sqrt{g_{\mathrm{ij}}} = n_{\mathrm{ij}} \tag{3}$$

3. Variation of the electric permittivity

The gravitation and the expansion of the universe are the two phenomena that dominate the space in its dynamic. If we assume that gravity is caused by the variation of the refractive index related to the variation of the relative permittivity ε_r around a mass, the expansion of the universe can also be linked to a variation of vacuum permittivity ε_0 over time. We suppose that when a ray of light with speed $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ travels a distance dr during the time dt in the medium, the permittivity ε_0 varies. Assuming that μ_0 is constant, we have:

$$\frac{d\varepsilon_0}{dr} = -2\varepsilon_0 \frac{1}{c}\frac{dc}{dr} \tag{4}$$

Due to the expansion of the universe, we observe an apparent recession velocity of galaxies $v=HL$ where L is the distance between galaxies and H is the Hubble constant [3]. We assume that the speed of light also varies depending on H. We suppose that

$$\frac{dc}{dr} = H \tag{5}$$

From equations (4) and (5) we have

$$\frac{d\varepsilon_0}{dr} = -2\varepsilon_0 \frac{H}{c} \tag{6}$$

4. Polarisation of the vacuum surrounding a spherically symmetric mass

Each electrical charge contained in a body, creates an electric field and a magnetic field in space and generates electromagnetic energy. The energies of all these electrical charges are added at any point of space.

If D is the electric displacement field of the particle and H its magnetic field strength, as ε_0 varies, the electromagnetic energy can be written as

$$W = W_{st} + W_{mg} = \frac{1}{2\varepsilon_0 \varepsilon_r} \iiint_{(v)} D^2 d\tau + \frac{\mu_0 \mu_r}{2} \iiint_{(v)} H^2 d\tau \tag{7}$$

$$W_{st} = \frac{1}{2\varepsilon_0 \varepsilon_r} \iiint_{(v)} D^2 d\tau \tag{8}$$

$$W_{mg} = \frac{\mu_0 \mu_r}{2} \iiint_{(v)} H^2 d\tau \tag{9}$$

where W_{st} , W_{mg} and $d\tau$ are respectively the electrostatic energy, the magnetic energy and the volume element. ε_r and μ_r are the relative permittivity and the relative permeability of the medium.

We assume an equivalence between electromagnetic energy and mass energy, so that electromagnetic energies from all elementary sources that make up a particle add-up to give mass energy, although the resulting total field can be null.

Consider a particle of charge q and mass M in the vacuum where $\varepsilon_r = \mu_r = 1$. With the equivalence of these two energies, we have

$$Mc^2 = W_{st} + W_{mg} \tag{10}$$

The electrostatic energy of this particle is

$$W_{st} = \frac{1}{2\varepsilon_0} \int_R^\infty D^2 4\pi r^2 dr = \frac{1}{2\varepsilon_0} \int_R^\infty (\frac{q}{4\pi r^2})^2 4\pi r^2 dr = \frac{1}{2} \frac{q^2}{4\pi\varepsilon_0 R} = \frac{1}{2} Mc^2 \tag{11}$$

$$\text{where} \quad M = \frac{\mu_0 q^2}{4\pi R} \tag{12}$$

and R is the classical radius of the particle.

from equations (10) and (11), we have $W_{mg} = Mc^2 - W_{st} = \frac{1}{2} Mc^2$, the magnetic energy of this particle is

$$W_{mg} = \frac{\mu_0}{2}\int_R^\infty H^2 4\pi r^2 dr = \frac{1}{2}Mc^2 = \frac{1}{2}\frac{q^2}{4\pi\varepsilon_0 R} \tag{13}$$

The electrostatic energy at a distance r from the particle is obtained from equation (11) by integrating from r up to ∞ :

$$W_{st}(r) = \frac{1}{2}\frac{q^2}{4\pi\varepsilon_0 r} \tag{14}$$

By analogy with equations (11) and (13) and replacing R by r, electromagnetic energy at a distance r will be

$$W(r) = W_{st}(r) + W_{mg}(r) = 2W_{st}(r) = \frac{q^2}{4\pi\varepsilon_0 r} \tag{15}$$

We assume that electromagnetic energy at a distance r creates pairs of virtual electrons e^- and positrons e^+, which form a polarisation density $\vec{P}$ in the presence of an electric field $\vec{E}_i$ of an incident particle. This polarisation density will be added to this electric field to act again in the medium.

We need to find the relationship between P and $E_i + \frac{P}{\varepsilon_0}$.

We expect this relationship depends on the variation of ε_0 (6) and of a quantity w_p which we call electromagnetic potential energy of polarization of the charge q at a distance r deduced from the equation (15):

$$w_p = -\frac{q^2}{4\pi\varepsilon_0 r} \tag{16}$$

The relationship will be

$$P = Kw_p \frac{d\varepsilon_0}{dr}(E_i + \frac{P}{\varepsilon_0}) \tag{17}$$

where K is a proportionality factor.

From equations (6), (16) and (17) we obtain

$$P = K(-\frac{q^2}{4\pi\varepsilon_0 r})(-2\varepsilon_0 \frac{H}{c})(E_i + \frac{P}{\varepsilon_0}) \tag{18}$$

$$\Rightarrow P = \varepsilon_0 \left(\frac{\frac{Hq^2K}{2\pi\varepsilon_0 cr}}{1 - \frac{Hq^2K}{2\pi\varepsilon_0 cr}} \right) E_i = \varepsilon_0 \chi_e E_i \tag{19}$$

$$\Rightarrow \varepsilon_r = 1 + \chi_e = \frac{1}{1 - \frac{Hq^2K}{2\pi\varepsilon_0 cr}} \tag{20}$$

where χ_e is the electric susceptibility of the medium

Dimensional analysis shows that

$$K = \frac{\alpha}{Rm_0\omega_0^{\ 2}} \tag{21}$$

where R, m_0, ω_0 are respectively a length, a mass and a frequency and α is a dimensionless constant.

$$\Rightarrow \varepsilon_r = \frac{1}{1 - \frac{H\alpha q^2}{2\pi\varepsilon_0 cRm_0\omega_0^{\ 2} r}} \tag{22}$$

We can rearrange the formula (22) to bring up the mass of the particle $M = \frac{\mu_0 q^2}{4\pi R}$, where R is the classical radius of the particle.

$$\Rightarrow \varepsilon_r = \frac{1}{1 - \frac{2(cH)\alpha M}{m_0\omega_0^{\ 2} r}} \tag{23}$$

We can rewrite ε_r as

$$\varepsilon_r = \frac{1}{1 - \frac{Nq^2}{\varepsilon_0 m_0\omega_0^{\ 2}}} \tag{24}$$

where N is the volume charge density of the vacuum at a distance r from the mass M :

$$N = \frac{2\varepsilon_0 (cH)\alpha M}{rq^2} \tag{25}$$

5. Particle in a gravitation field

If a particle with a mass m is in a gravitational field $(\varepsilon_r \neq 1)$, from equations (8) and (11), we can deduce its electrostatic energy which is the potential energy:

$$W_{st} = \frac{1}{2}\frac{mc^2}{\varepsilon_r} \tag{26}$$

For a spherical symmetry where we consider the refractive index depending on r and considering equations (2) and (3), we can write dS^2 with the use of the signature $(+,-,-,-)$ as:

$$dS^2 = g_{ij}dx^i dx^j = n_{ij}{}^2 dx^i dx^j = n_t{}^2 c^2 dt^2 - n_r{}^2 dr^2 - n_\theta{}^2 r^2 d\theta^2 - n_\varphi{}^2 r^2 \sin^2\theta d\varphi^2 \tag{27}$$

where n_t, n_r, n_θ and n_φ are the respective refractive indices of the corresponding coordinates t, r, θ and φ . For a local observer all these refractive indices are equal to 1 but for an external observer $n_r = \sqrt{\varepsilon_r} \Rightarrow n_t = \frac{1}{\sqrt{\varepsilon_r}}$ and due to the spherical symmetry we deduce that $n_\theta = n_\varphi = 1$.

The equation (27) becomes

$$dS^2 = \frac{c^2}{\varepsilon_r}dt^2 - \varepsilon_r dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{28}$$

by comparing the equation (28) with the spherically symmetric solution (Schwarzschild solution [4]) of the Einstein's equations:

$$dS^2 = (1 - \frac{2GM}{c^2 r})c^2 dt^2 - \frac{1}{(1 - \frac{2GM}{c^2 r})}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{29}$$

where G is the gravitational constant, and taking account of equation (23), we have

$$\varepsilon_r = \frac{1}{1 - \frac{2(cH)\alpha M}{m_0 \omega_0{}^2 r}} = \frac{1}{1 - \frac{2GM}{c^2 r}} \tag{30}$$

$$\Rightarrow \omega_0 = \sqrt{\frac{\alpha H c^3}{G m_0}} \tag{31}$$

where the electron of mass m_0 is a harmonic oscillator whose frequency is equal to ω_0.

In a gravitational field, the particle moves in the direction which minimizes its potential energy. The gravitational force derived from this energy and directed to M is

$$\overrightarrow{F_g} = -\overrightarrow{grad}(W_{st}) = -\overrightarrow{grad}(\frac{1}{2}\frac{mc^2}{\varepsilon_r}) \tag{32}$$

Substituting ε_r with its classical value $\varepsilon_r = \frac{1}{1 - \frac{2GM}{c^2 r}}$ from equation (30), this force is

$$\overrightarrow{F_g} = -\overrightarrow{grad}\{\frac{1}{2}mc^2(1 - \frac{2GM}{c^2 r})\} = -mc\frac{dc}{dr}\vec{u}_r + \overrightarrow{grad}(m\frac{GM}{r}) \tag{33}$$

Taking account of equation (5), we have

$$\overrightarrow{F_g} = -m(cH + \frac{GM}{r^2})\vec{u}_r \tag{34}$$

We note the acceleration (cH) due the variation of the vacuum permittivity ε_0.

6. Particle moving in the vacuum

The magnetic energy of a particle with mass m is deduced from equations (9) and (13)

$$W_{mg} = \frac{\mu_r mc^2}{2} \tag{35}$$

When a particle with an electric charge q is moving in the vacuum with the speed v, we assume that it creates a magnetic field $\overrightarrow{H}$, which in turn creates a magnetization $\overrightarrow{M} = \chi_m \overrightarrow{H}$, where χ_m is the magnetic susceptibility of the medium. The magnetic field will be added to this magnetization to act again in the medium.

We want to find the relationship between $\overrightarrow{M}$ and $\overrightarrow{H} + \overrightarrow{M}$.

It is evident that the magnetization is proportional to the speed v of the particle relative to the medium $(v < c)$; this relationship can be written as

$$\overrightarrow{M} = Yv^{\beta}(\overrightarrow{H} + \overrightarrow{M}) \tag{36}$$

where β is a constant and Y is a proportionality factor. (Yv^{β}) is a dimensionless term $\Rightarrow Y = \dfrac{b}{c^{\beta}}$, where c is the speed of light and b is a dimensionless constant. We have

$$M = \frac{bv^{\beta}}{c^{\beta}}(H + M) \tag{37}$$

$$\Rightarrow M = \frac{\dfrac{bv^{\beta}}{c^{\beta}}}{1 - \dfrac{bv^{\beta}}{c^{\beta}}} H = \chi_m H \tag{38}$$

A choice of $b = 1$ and $\beta = 2$ allows us obtain the expression of magnetic energy and kinetic energy. The magnetic permeability is

$$\mu_r = 1 + \chi_m = \frac{1}{1 - \dfrac{v^2}{c^2}} \tag{39}$$

The total energy of the particle according to (7), (26) and (35) is

$$W = \frac{mc^2}{2}\left(\frac{1}{\varepsilon_r} + \mu_r\right) \tag{40}$$

The expressions of the refractive indices are $n_r = \sqrt{\varepsilon_r \mu_r}$ and $n_t = \dfrac{1}{\sqrt{\varepsilon_r \mu_r}}$

Substituting μ_r from equations (39) and the classical value $\varepsilon_r = \dfrac{1}{1 - \dfrac{2GM}{c^2 r}}$ from equations (30), the total energy of the particle is

$$W = \frac{1}{2}mc^2\left(1 - \frac{2GM}{c^2 r}\right) + \frac{1}{2}\frac{mc^2}{1 - \dfrac{v^2}{c^2}} \tag{41}$$

In case $v << c \Rightarrow W_{mg} = \frac{1}{2}\frac{mc^2}{1-\frac{v^2}{c^2}} \approx \frac{1}{2}mc^2 + \frac{1}{2}mv^2$, the kinetic energy $\frac{1}{2}mv^2$ appears as an extension of the magnetic energy, there is a correspondence between these two energies. In equation (41), the term $\frac{1}{2}mc^2(1-\frac{2GM}{c^2 r})$ represents the potential energy of the particle and $\frac{1}{2}\frac{mc^2}{1-\frac{v^2}{c^2}}$ represents its kinetic energy which equal to $\frac{1}{2}mc^2$ when $v = 0$.

7. Conclusion

We have demonstrated that we can link the gravitation and the expansion of the universe to the variation of the vacuum permittivity ε_0 and to the variation of the relative permittivity ε_r, taking into account the vacuum polarization and the wave nature of matter whose trajectory is deflected near the mass. We also deduced the expression of the energy of the particle by determining the corresponding ε_r and μ_r. In addition to redefining gravity from electromagnetism, Schwarzschild solution is gotten from this new theory which may predict a dispersion of particles in a gravitational field depending on their energies.

References

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