COMPLETE DESCRIPTION OF POLARIZATION EFFECTS IN TOP QUARK DECAYS INCLUDING HIGHER ORDER QCD CORRECTIONS [a]

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The complete set of matrix elements for all polarization configurations in top quark decays is presented including higher order QCD corrections. The analysis is done in the framework of the helicity formalism. The results can be used in a variety of circumstances, e.g. in the experimental analysis of top quark production and decay at Tevatron, LHC and NLC. Relations to LEP1 and LEP2 physics are pointed out.

1 Introduction

Since its discovery in 1995 the top quark has been an object of increasing interest. The production process for top quarks has been analyzed in various theoretical studies both for proton and e^+e^- collisions. Early references on the lowest order cross section are [1] for pp collsions and [2] for e^+e^- annihilation. Higher order corrections to the cross section (total cross section, p_T distribution etc.) have been calculated by several groups, [3,4] for pp and [5,6] for e^+e^-. These total cross sections do not involve information on the top quark polarization. Such spin effects only come in, if one studies distributions of top quark decay products. In some cases, spin effects have been studied, i.e. the distribution of the top quark spin vector, in ref. [7] for pp and in ref. [8] for e^+e^- collisions. These latter studies have not been extended to higher orders yet. However, an interesting step in this direction has been taken in ref. [6], where a Monte Carlo progam including final state spin terms has been written. Unfortunately, that paper is in fact concerned with higher order corrections to $\tau^+\tau^-$ production in e^+e^- annihilation at lower energies and does not take into account axial vector couplings. More general results, including axial vector couplings, can be found in [9], but not in the form of a Monte Carlo program.

Nothing is known about higher order spin corrections to the processes which induce top quark production in proton collisions (light quark annihilation $q\bar{q} \to t\bar{t}$ and gluon–gluon fusion $q\bar{q} \to t\bar{t}$). In contrast to e^+e^- annihilation, there are many higher order diagrams involved in these processes, so that the calculations are very difficult.

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Of course, in studying distributions of top quark decay products one has to take into account the higher order top quark decay matrix elements, too. The spin averaged matrix elements have been calculated including higher order corrections. For example, oneloop QCD corrections are known to decrease the total width of the top quark by about 8%. If one studies distributions of top quark decay products with reference to the production process, one needs in addition the spin correlations in the top quark decay matrix elements. In lowest order, these spin correlations are known, but in higher order only rudimentary results exist [10,11]. In those calculations, higher order corrections to special decay product distributions were calculated. This corresponds to certain linear combinations of the polarized matrix elements. The present article describes how *all* the polarized matrix elements can be obtained in a complete and systematic way in the framework of the so–called helicity formalism.

Within the Standard Model, all couplings of the top quark to other particles are completely fixed by its mass and by a few quantum numbers. For example, the coupling of the top quark to gluons is a pure vector coupling with strength g_s, the coupling to the W–boson is a V–A coupling etc. The coupling to the W–boson is particularly interesting for this article, because it induces the decay $t \to W^+ b$. b–quark mass terms $O(m_b/m_t)$ can very probably be neglected in higher order corrections to the decay process, because they are known to be small ($\sim 1\%$) in leading order. In this approximation it can be shown that the b–quark is always left–handed in the top quark decay process, both in lowest order *and in higher order QCD*. As will be seen, this strongly reduces the number of independent helicity amplitudes and makes the results quite intuitive.

2 Helicity vs. Spin Vector Description

Helicity amplitudes have been considered in many applications of phenomenological importance in high energy physics, like jet production [12], nonstandard effects in top quark processes [13], and many others. The idea is, first to separate a given process into simpler subprocesses and then to explicitly evaluate all the possible spin amplitudes for the subprocesses in special Lorentz and Dirac frames. The results can afterwards be put together with the help of a master formula (to be given below). For example, consider the lowest order helicity amplitudes for top quark decay in the Standard Model. They are given by

$$A(h_t, h_W) = \frac{g}{\sqrt{2}} \bar{u}_{-1/2}(p_b) \gamma_\mu \frac{1}{2}(1-\gamma_5) u_{h_t}(p_t) \epsilon^\mu_{h_W} \tag{1}$$

where $g = \frac{e}{s_W}$ and h_t and h_W label the spins for the top quark and the W–boson, $h_t = \pm\frac{1}{2}$ and $h_W = 0, \pm 1$. The helicity of the (massless) bottom quark is fixed to be $-1/2$ by the V–A nature of the interaction. Thus, in the Standard Model there are six amplitudes to be considered, and this number remains the same in higher order QCD (after integrating over the gluon degrees of freedom). The six amplitudes can be used to define the 36 elements of the density matrix

$$\rho(h_t, h_W, h_t', h_W') := A(h_t, h_W) A(h_t', h_W')^* \,. \tag{2}$$

Note that the amplitudes are only determined up to an overall phase, and that this arbitrariness goes away when forming the density matrix elements. Unfortunately, higher order QCD corrections cannot be fully calculated on the level of amplitudes, but only on the level of the density matrix which contains in principle 36 degrees of freedom. As will be discussed below, hermiticity and CP invariance reduce this number, but still leave an appreciable set of matrix elements.

The full density matrix is in fact needed in the 'master formula', if one considers some combined production and decay process for the top quark. Namely, assume that top quarks are produced in some process $ab \to t\bar{t}$ and then decay according to $t \to W^+ b$ and $\bar{t} \to W^- \bar{b}$, where the W's further decay to light (massless) fermions, $W^+ \to f_1 \bar{f}_2$ and $W^- \to f_3 \bar{f}_4$. The cross section is then given by the 'master formula'

$$\sigma = \sum_{EXT} | \sum_{INT} A(h_a, h_b, h_t, h_{\bar{t}}) A(h_t, h_{W^+}) A(h_{\bar{t}}, h_{W^-}) A(h_{W^+}) A(h_{W^-}) |^2 \tag{3}$$

where $EXT = h_a, h_b$ denotes the spins of the external particles and $INT = h_t, h_{\bar{t}}, h_{W^+}, h_{W^-}$ the spins of the internal particles of the process. $A(h_a, h_b, h_t, h_{\bar{t}})$ are the helicity amplitudes for the production process, $A(h_{\bar{t}}, h_{W^-})$ for the decay of the antitop quark, and $A(h_{W^+})$ and $A(h_{W^-})$ are the amplitudes for the decay of the W^+ and W^-, respectively. Note that before Eq. (3) the W^+ spin has been denoted by h_W instead of h_{W^+}, and for simplicity it will again be denoted by h_W below. The fact that the formula makes no explicit reference to the spins of the massless fermions f_i, i=1,2,3,4, has the same reason as the non–appearance of the b–quark spins. Namely, the h_{f_i} are fixed by the V–A nature of the W decays. Furthermore, if one is not interested in the decay of the antitop or of the W's, the corresponding amplitudes and helicities will not appear in the above formulas. In that case, the $\bar{t}$ and/or the W's will be one of the *external* (EXT) particles, whose spins have to be summed over after taking the square in Eq. (3).

Besides neglecting the b–quark mass, it is also a good approximation in Eq. (3) to take the internal particles on–shell, because off-shell contributions are suppressed by powers of the width Γ_t and Γ_W. More precisely, one has

$$\frac{1}{(P^2-M^2)^2+M^2\Gamma^2}=\frac{\pi}{M\Gamma}\delta(P^2-M^2)+O(\frac{\Gamma}{M}) \tag{4}$$

for a particle of mass M and 4–momentum P. For a top quark of mass 175 GeV, the width is about 1.5 GeV, so that terms of order $\frac{\Gamma}{M}$ can be neglected, in particular in higher orders where $O(\alpha_s\frac{\Gamma}{M})$ is of the order of a permille. Similar considerations apply to the W–boson.

In higher order the narrow width approximation Eq. (4) is of particular use in reducing the number of diagrams to be calculated. The reason is that all diagrams where a gluon runs from the production part of the process to the decay part, and also the corresponding interference diagrams, give contributions which are suppressed by powers of $\frac{\Gamma}{M}$. Therefore, in this approximation the process can be really decomposed into a number of building blocks, the production block, the t–decay blocks and the W–decay blocks. In the narrow width approximation, these blocks are interrelated by spin–indices, but not by gluon exchange.

When one carries out the modulus squared in Eq. (3), it becomes apparent that in general the full density matrix $A(h_t,h_W)A(h'_t,h'_W)^*$ is needed to calculate the cross section of the decay products.

I have calculated the six amplitudes $A(h_t,h_W)$ using the 'chiral representation' of γ matrices, in which γ_5 =diag$(-1,-1,1,1)$ etc. This representation makes calculations with massless fermions (the b–quark in the case at hand) quite transparent. Furthermore, I shall present results in two different Lorentz frames, in the rest frame of the top quark and the rest frame of the W–boson. Both frames have special virtues, so it is worthwhile to study them both. The top quark rest frame is of course the natural frame to study top decays, and to look at distributions in the energies of the decay products etc. The W rest frame has the particular virtue that the amount of longitudinal W's can be read off most easily in this frame. I am quite sure there is a (complicated) transformation between the amplitudes in both frames. However, I was not able to derive it and, furthermore, found it reasonably convenient to do the calculations in both frames separately.

There is a popular alternative to the helicity formalism, where use is made by the fact that the spin of a fermion with 4–momentum P can be described by a 'spin vector' S, a pseudo 4–vector which fulfills $S^2=-1$ and $S\cdot P=0$. In this formalism one does not calculate amplitudes but (squared) matrixelements. The matrix element for the production of a $t\bar{t}$ pair with spin vectors s_t and $s_{\bar{t}}$ has the generic form

$$|M|^2_P \sim tr(\not{p}_t+m_t)(1+\gamma_5\not{s}_t)\ldots\ldots(\not{p}_{\bar{t}}+m_t)(1+\gamma_5\not{s}_{\bar{t}})\ldots\ldots$$

$$= a + bs_t + cs_{\bar{t}} + ds_t s_{\bar{t}} \tag{5}$$

where, for example, d is a tensor with two Lorentz indices and b and d are 4–vectors. Similarly, the matrix elements for the decay of the top quarks have the form

$$|M|^2_D = e + fs_t \qquad |M|^2_{\bar{D}} = \bar{e} + \bar{f}s_{\bar{t}} \tag{6}$$

and the full matrix element for production and decay is then given by

$$|M| = ae\bar{e} - bf\bar{e} - ce\bar{f} + df\bar{f}\,. \tag{7}$$

According to this formula, the cross section will not be just a product of a production and of a decay piece, but is given by a sum of such products. In fact, Eq. (7) can be shown to be equivalent to Eq. (3) [14].

3 The Method

The results to be presented were obtained in the helicity formalism. Furthermore, they concern the oneloop QCD corrections to the lowest order matrix elements. The lowest order expressions are usually simple, whereas the oneloop expressions, in particular for the case of hard gluons are quite lenghty for arbitrary spin orientations. Furthermore, the hard gluon contributions cannot be treated on the level of amplitudes, because they have to be integrated over the gluon's energy and angles. One really has to go to the level of the (spin) density matrix, Eq. (2), to do the phase space integrations. There is, however, one circumstance which simplifies the task. This is related to the fact, that the oneloop QCD corrections to the total (spin–averaged) width of the top quark are known [15]. This allows to get rid of the infrared and collinear singularities present in the matrix elements, by forming suitable singularity–free combinations of the spin–dependent and the spin–averaged expressions. The point is that the infrared and collinear singularities are 'universal', i.e. independent of the spin direction, so that they will drop out in suitable differences. We shall discuss our procedure in more detail in the next section, where QCD corrections to $W \to q\bar{q}'$ are considered as a rather simple warming up exercise.

4 QCD Corrections to $W \to q\bar{q}'$

This process is simpler because both outgoing quarks can be considered to be massless particles, so that their helicities are fixed by the $V - A$ nature of the decay. Therefore, the decay amplitudes depend only on the W–spin and in lowest order are given by

$$A(0) = \sin\theta \qquad A(\pm 1) = \frac{1 \pm \cos\theta}{\sqrt{2}} e^{\pm i\phi} \tag{8}$$

where θ and ϕ are the (polar and azimuthal) angles between the z–direction (defined as the direction of the outgoing quark q) and the direction of the W as given in some LAB frame (it may also be considered as the direction to which the W–spin points). These formulae hold in the rest frame of the W–boson. An overall factor $\frac{em_W}{2\sqrt{2}s_W}$ has been left out in the amplitudes. Note that the integrated W–width $\Gamma_W = \frac{e^2 m_W^2}{48\pi s_W^2}$ can be obtained from the trace of the density matrix $\sum_{h_W} A(h_W)A(h_W)^*$ by multiplying with the square of the factor $\frac{em_W}{2\sqrt{2}s_W}$ and by dividing by the well–known $16\pi m_W$ [16].

Using Eq. (8), the corresponding density matrix $D_{h_W,h'_W} \equiv A(h_W)A(h'_W)^*$ can easily be calculated in lowest order to yield

$$\begin{aligned} D^{lo}(0,0) &= \sin^2\theta \\ D^{lo}(\pm 1,\pm 1) &= \frac{1}{2}(1+\cos^2\theta)\underline{\pm\cos\theta} \\ D^{lo}(+1,-1) &= \frac{1}{2}\sin^2\theta e^{2i\phi} \\ D^{lo}(\pm 1,0) &= (\pm\cos\theta\sin\theta\underline{-\sin\theta})\frac{e^{\pm i\phi}}{\sqrt{2}}\,. \end{aligned} \tag{9}$$

The underlined terms refer to parity violating effects and cannot be measured in hadronic W decays, because the outgoing quark is detected in the form of a jet and its flavor and charge cannot be identified. The functions $D(h_W,h'_W)$ are sometimes called the decay functions of the W. An upper index lo has been introduced in Eq. (9) in order to make clear that these are the Born level contributions to the density matrix. The aim is then to calculate higher order corrections to the decay functions/density matrix, in the form

$$D(h_W,h'_W) = D^{lo}(h_W,h'_W) + \frac{\alpha_s}{\pi}D^{ho}(h_W,h'_W) \tag{10}$$

To accomplish this calculation, use was made of the well–known higher order result for the spin averaged decay function, the 'trace of the spin density matrix', which in our normalization is given by

$$D_{total} = D^{lo}_{total} + \frac{\alpha_s}{\pi}D^{ho}_{total} = 2(1+\frac{\alpha_s}{\pi})\,. \tag{11}$$

The point to notice is that one has

$$\frac{D(h_W,h'_W)}{D_{total}} = \frac{D^{lo}(h_W,h'_W)}{D^{lo}_{total}} + \frac{\alpha_s}{\pi}\frac{D^{ho}(h_W,h'_W)D^{lo}_{total} - D^{lo}(h_W,h'_W)D^{ho}_{total}}{(D^{lo}_{total})^2} \tag{12}$$

where the difference $D^{ho}(h_W, h'_W) D^{lo}_{total} - D^{lo}(h_W, h'_W) D^{ho}_{total}$ is completely free of singularities separately for hard and virtual gluons. In fact, the diagrams with virtual gluon exchange contribute nothing to the ratio $\frac{D(h_W, h'_W)}{D_{total}}$. (In top decay, where we shall proceed similarly, the virtual diagrams will contribute, but only a finite amount.) The real gluon processes $W \to q\bar{q}' g$ can be explicitly seen to give a finite contribution to the above difference, i.e. the result is finite for $E_g \to 0$ and $\theta_g \to 0$, where E_g and θ_g are the gluon energy and angle with respect to the quark direction [17]. The integration over E_g and θ_g is therefore straightforward and one obtains the corrections to the W decay functions in a very compact form [17]

$$D(h_W, h'_W) = (1 + \frac{\alpha_s}{\pi})(1 - 0.975\frac{\alpha_s}{\pi})[D^{lo}(h_W, h'_W) + 0.653\frac{\alpha_s}{\pi}\delta_{h_W, h'_W}] \,. \quad (13)$$

This representation can only be obtained after neglecting the irrelevant parity violating terms. Furthermore, in higher orders the angles θ and ϕ have been defined as to refer to the thrust– instead of the quark–momentum direction.

These results have been applied to W–pair production at LEP2 [17] and NLC [18]. According to a master formula similar to (3), the cross section for W–pair production and decay in e^+e^- annihilation was calculated including the higher order corrections Eq. (13) and nonstandard contributions. Our main motivation in studying this cross section was twofold:

- first of all we wanted to know how QCD corrections to angular correlations of W decay products differ from the naive expection of a constant K–factor $\sim 1 + \frac{\alpha_s}{\pi}$. We found that depending on the kinematic point, the deviations from a constant K–factor can be appreciable (of the order of a few percent). Unfortunately, at LEP2 with its few thousand W events these effects are just at the edge to become visible. The situation is different at NLC with its larger statistics, where the QCD corrections really become relevant.

- secondly we have proven that QCD correction can mimic the presence of nonstandard physics. As has been shown by Monte Carlo studies [19], NLC is sensitive to nonstandard couplings as small as 10^{-3}. This is well below the magnitude of the QCD effects induced by Eq. (13) [18].

In addition to the QCD corrections presented above, there are off-shell W corrections (with and without QCD [20]) which are particularly important at LEP2, i.e. near threshold. Unfortunately, there is no space here to discuss them in detail.

5 Complete Lowest Order Analysis of the Spin Density Matrix for $t \to bW$

The decay of the top quark has some extremely interesting physics features, which are related to the spin decomposition of the matrix element. In particular, it is well known that, due to the large top quark mass value, t–decay is dominated by longitudinal W's. In fact, the total width of the W is given by

$$\Gamma_t = \Gamma_L + \Gamma_T = \frac{G_F m_t^3}{8\sqrt{2}\pi}(1 - \frac{m_W^2}{m_t^2})^2(1 + 2\frac{m_W^2}{m_t^2}) \tag{14}$$

and the ratio of the number of longitudinal over transverse W's is given by $\Gamma_L/\Gamma_T = \frac{m_t^2}{2m_W^2}$. Note that the QCD corrections to Γ_t, Γ_L and Γ_T are known [15,11] to be about -9% resp. 5%, and that all other correction effects like electroweak, b–mass and finite width effects contribute roughly 1%. Note further that Γ_L is the most interesting source of loop corrections to the famous R_b value [the partial width $\Gamma(Z \to b\bar{b})$ measured at LEP1]. The point is that the exchange of longitudinal W's between the b-quarks gives rise to the celebrated corrections of order $O(G_F m_t^2) \sim \frac{m_t^2}{m_W^2}$. Unfortunately, at LEP1 this is only a small loop effect which is of the order of 1%, because it is suppressed by a factor $\frac{1}{16\pi^2}$. In contrast, in t–decays the longitudinal W's enter as the most dominant leading order effect, so that they can be studied much clearer.

Top quark decay may be looked at in different frames; particularly interesting are the rest frames of the t–quark or that of the W. In the W rest frame, for example, there is a very simple way to experimentally determine the ratio Γ_L/Γ_T and other spin–dependent observables [11]. Of course, the two systems are related just by a simple boost. However, the transformation formula between the spin–amplitudes in the two systems is quite complicated, so that I prefer to give results in the two systems separately. Let's start with the t rest system. In order to calculate the amplitudes (1), I have choosen the following parametrization of momenta, polarization vectors and spinors. First, the momenta:

$$p_t = (m_t, \vec{0}) \qquad p_W = \frac{m_t}{2}(f_+, 0, 0, f_-) \qquad p_b = p_t - p_W \tag{15}$$

where $f_\pm = 1 \pm f$ and $f = \frac{m_W^2}{m_t^2}$ and the W–boson has been chosen to define the z–direction. The most general top quark spinor is given by

$$\begin{aligned} u_{+1/2}(p_t) &= \sqrt{m_t}(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}e^{i\phi}, \cos\frac{\theta}{2}, \sin\frac{\theta}{2}e^{i\phi}) \\ u_{-1/2}(p_t) &= \sqrt{m_t}(-\sin\frac{\theta}{2}e^{-i\phi}, \cos\frac{\theta}{2}, -\sin\frac{\theta}{2}e^{-i\phi}, \cos\frac{\theta}{2}) \end{aligned} \tag{16}$$

where θ and ϕ refer to some direction, e.g. to the direction of the top quark in some lab–system. One may put $\phi = 0$ without restriction, because this corresponds to defining the y–direction. The possible b– and W–polarizations are fixed to be

$$\bar{u}_{-1/2}(p_b) = \sqrt{m_t}(0, 0, \sqrt{f_-}, 0) \tag{17}$$

and

$$\epsilon_{\mp 1} = -\frac{1}{\sqrt{2}}(0, \pm 1, i, 0) \qquad \epsilon_0 = -\frac{1}{2\sqrt{f}}(f_-, 0, 0, f_+) \,. \tag{18}$$

This leads to the following amplitudes for t–decays in the top quark rest frame:

$$\begin{aligned} A_t^{lo}(-\tfrac{1}{2}, 0) &= \tfrac{1}{\sqrt{f}} \sin\tfrac{\theta}{2} e^{-i\phi} \qquad & A_t^{lo}(+\frac{1}{2}, 0) &= -\frac{1}{\sqrt{f}}\cos\frac{\theta}{2} \\ A_t^{lo}(-\tfrac{1}{2}, +1) &= 0 & A_t^{lo}(+\frac{1}{2}, +1) &= 0 \\ A_t^{lo}(-\tfrac{1}{2}, -1) &= -\sqrt{2}\cos\tfrac{\theta}{2} & A_t^{lo}(+\frac{1}{2}, -1) &= -\sqrt{2}\sin\frac{\theta}{2} e^{i\phi} \end{aligned} \tag{19}$$

where the upper index refers to 'lowest order' and the lower index to top quark decay in the t rest frame. A universal spin independent coefficient $c_0 = \frac{em_t}{2s_W}\sqrt{f_-}$ has been left out in all the amplitudes. Note that the amplitudes are only determined up to an overall phase, and that this arbitrariness goes away when forming the density matrix elements. Since the amplitudes are explicitly given, it is straightforward to obtain the density matrix in lowest order. Its trace is easily obtained from the above expressions to be

$$\sum_{h_t, h_W} A_t^{lo}(h_t, h_W) A_t^{lo}(h_t, h_W)^* = c_0^2 \frac{1 + 2f}{f} \tag{20}$$

and one can reproduce from this the total width of the top quark, Eq. (14) by dividing by the phase space factor $\frac{f_-}{16\pi m_t}$ [16].

There are lots of other combinations of density matrix elements to describe interesting physics. For example, the above mentioned ratio $\frac{\Gamma_L}{\Gamma_T}$ is obtained as

$$\frac{\Gamma_L}{\Gamma_T} = \frac{\sum_{h_t = \pm\frac{1}{2}, h_W = 0} |A_t^{lo}(h_t, h_W)|^2}{\sum_{h_t = \pm\frac{1}{2}, h_W = \pm 1} |A_t^{lo}(h_t, h_W)|^2} = \frac{1}{2f} \,. \tag{21}$$

Let us now repeat the same analysis in the W rest frame. This time I chose to define the z–direction by the direction of the top quark, i.e.

$$p_W = (m_W, \vec{0}) \qquad p_t = \frac{m_t}{2\sqrt{f}}(f_+, 0, 0, f_-) \qquad p_b = p_t - p_W \,. \tag{22}$$

The top and bottom quark spinors are then fixed as

$$u_{+1/2}(p_t) = \sqrt{m_t}(f^{1/4}, 0, f^{-1/4}, 0) \qquad u_{-1/2}(p_t) = \sqrt{m_t}(0, f^{-1/4}, 0, f^{1/4}) \tag{23}$$

and

$$\bar{u}_{-1/2}(p_b) = \sqrt{m_t}(0, 0, 0, \sqrt{\frac{f_-}{\sqrt{f}}}) \tag{24}$$

whereas the W polarization direction is arbitrary:

$$\begin{aligned} \epsilon_{-1} &= -\frac{e^{i\phi}}{\sqrt{2}}(0, \cos\phi\cos\theta - i\sin\phi, \sin\phi\cos\theta + i\cos\phi, -\sin\theta) \\ \epsilon_{+1} &= -\epsilon^*_{-1} \\ \epsilon_0 &= -(0, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)\,. \end{aligned} \tag{25}$$

The angles θ and ϕ refer to some arbitrary direction, e.g. to the direction of the W–boson in some lab–system. Note that although I am using the same symbols θ and ϕ as in Eqs. (19) and (8), the meaning of these angles is completely different in the three cases. As before, one may in principle put $\phi = 0$ without restriction, because this corresponds to defining the y–direction. One is lead to the following amplitudes for t–decays in the W rest frame:

$$\begin{aligned} A^{lo}_W(-\frac{1}{2}, 0) &= \frac{1}{\sqrt{f}}\cos\theta \qquad & A^{lo}_W(+\frac{1}{2}, 0) &= -\sin\theta e^{i\phi} \\ A^{lo}_W(-\frac{1}{2}, +1) &= \frac{1}{\sqrt{2f}}\sin\theta e^{-i\phi} \qquad & A^{lo}_W(+\frac{1}{2}, +1) &= \frac{1}{\sqrt{2}}(1+\cos\theta) \\ A^{lo}_W(-\frac{1}{2}, -1) &= -\frac{1}{\sqrt{2f}}\sin\theta e^{i\phi} \qquad & A^{lo}_W(+\frac{1}{2}, -1) &= \frac{1}{\sqrt{2}}(1-\cos\theta)e^{2i\phi} \end{aligned} \tag{26}$$

Again, the universal coefficient $c_0 = \frac{em_t}{2s_W}\sqrt{f_-}$ has been left out in all the amplitudes. From the trace of the corresponding density matrix one can reconstruct the total width of the top quark, just as in (20). However, the ratio (20) can only be obtained for $\theta = 0$, because otherwise the notion of 'longitudinal' does not refer to the heavy quark direction.

6 QCD Corrections to the Spin Density Matrix of $t \to bW$ from the Exchange of Virtual Gluons

The oneloop QCD Corrections to t–decay are somewhat more complicated than to W–decay because even neglecting the b–mass there is one more mass parameter involved. This is despite the fact that the Feynman diagrams needed are exactly the same as for W–decay (with the directions of the W and one of the quarks interchanged). Namely, there are the 'virtual gluon' vertex and self–energy diagrams and two 'real' diagrams with gluon emission from one of the quark legs. As discussed in the introduction, there are 36 density matrix elements (2) which will be considered in the normalized form

$$\rho_{norm}(h_t, h_W, h_t', h_W') = \frac{\rho(h_t, h_W, h_t', h_W')}{\rho_{total}} \tag{27}$$

because this helps to cancel universal contributions, in analogy to the case of W decay, Eq. (12). $\rho_{total} \equiv \sum_{h_t, h_W} |\rho(h_t, h_W, h_t, h_W)|$ is defined to be the trace of the density matrix and related to the total width Γ_t of the top quark as discussed above.

Let us start the discussion with the virtual contributions, because they can easily be obtained from the corrections to the V–A vertex calculated a long time ago [21]

$$\Gamma_\mu(t \to bW) = -\frac{ie}{\sqrt{2}s_W}\left\{H\gamma_\mu \frac{1}{2}(1-\gamma_5) + \alpha_d H_+ \frac{i\sigma_{\mu\nu}p_W^\nu}{2m_t}\frac{1}{2}(1+\gamma_5)\right\} \tag{28}$$

where the known function $H = 1 + O(\alpha_s)$ 'renormalizes' the V–A structure, and contains all the ultraviolet, infrared and collinear singularities, and

$$H_+ = -\frac{\ln f_-}{f} \tag{29}$$

is a regular function of $f = m_W^2/m_t^2$. Note that the appropriate expansion parameter in the case of top quark decay is $\alpha_d = -C_F \frac{\alpha_s}{2\pi}$. It should further be noted that the contribution of H to the normalized density matrix ρ_{norm} vanishes because it cancels between the numerator and denominator in Eq. (27). The argument works in the same way as was discussed in the case of the W–boson. The only difference is that now a finite contribution $\sim H_+$ from the $\sigma_{\mu\nu}$ term survives due the nonvanishing (top–)quark mass. In turn, one may conclude that the contribution from real gluon emission to the normalized density matrix is finite, too, because of the Lee Nauenberg theorem, which says that any such singularity cancels between real and virtual corrections. One may write down the contribution $\sim H_+$ to the amplitudes in the form

$$A_t(h_t, h_W) = A_t^{lo}(h_t, h_W) + \alpha_d A_t^+(h_t, h_W) H_+ \tag{30}$$

where the lowest order amplitudes A_t^{lo} were given in (19) and the higher order amplitudes A_t^+ originating from the $\sigma_{\mu\nu}$ term are given by

$$A_t^+(-\tfrac{1}{2}, 0) = -\tfrac{\sqrt{f}}{2}\sin\tfrac{\theta}{2}e^{-i\phi} \qquad A_t^+(+\frac{1}{2}, 0) = \frac{\sqrt{f}}{2}\cos\frac{\theta}{2}$$

$$A_t^+(-\tfrac{1}{2},+1)=0 \qquad A_t^+(+\frac{1}{2},+1)=0$$

$$A_t^+(-\tfrac{1}{2},-1)=\tfrac{1}{\sqrt{2}}\cos\tfrac{\theta}{2} \qquad A_t^+(+\frac{1}{2},-1)=\frac{1}{\sqrt{2}}\sin\frac{\theta}{2}e^{i\phi} \tag{31}$$

Again, the universal coefficient $c_0=\frac{em_t}{2s_W}\sqrt{f_-}$ has been left out in all the amplitudes. These results imply a higher order contribution due to the $\sigma_{\mu\nu}$ term on the level of the density matrix $\rho(h_t,h_W,h'_t,h'_W)$. Including the lowest order piece it will be of the form

$$\begin{aligned} A_t^{lo}(h_t,h_W)A_t^{lo}(h'_t,h'_W)^* + \alpha_d H_+\big[A_t^{lo}(h_t,h_W)A_t^+(h'_t,h'_W)^* \\ +A_t^+(h_t,h_W)A_t^{lo}(h'_t,h'_W)^*\big] \end{aligned} \tag{32}$$

Note that to calculate ρ_{norm}, one has to divide afterwards by ρ_{total}, whose contribution from the $\sigma_{\mu\nu}$ term (including lo) can be calculated to be

$$\begin{aligned} \sum_{h_t,h_W}|A_t^{lo}(h_t,h_W)|^2+\frac{\alpha_s}{2\pi}H_+\sum_{h_t,h_W}\big[A_t^{lo}(h_t,h_W)A_t^+(h_t,h_W)^* \\ +A_t^+(h_t,h_W)A_t^{lo}(h_t,h_W)^*\big]=c_0^2\big\{\frac{1+2f}{f}-3\alpha_d H_+\big\} \end{aligned} \tag{33}$$

This result was obtained by summing over all spin configurations and using the explicit representation of the amplitudes given above. It shows explicitly that the contribution from virtual gluon exchange to ρ_{norm} is completely under control.

The amplitudes corresponding to the $\sigma_{\mu\nu}$ term in the vertex may also be evaluated in the W–rest frame

$$\begin{aligned} A_W^+(-\tfrac{1}{2},0)&=-\tfrac{\sqrt{f}}{2}\cos\theta & A_W^+(+\frac{1}{2},0)&=\frac{1}{2}\sin\theta e^{i\phi} \\ A_W^+(-\tfrac{1}{2},+1)&=-\tfrac{\sqrt{f}}{2\sqrt{2}}\sin\theta e^{-i\phi} & A_W^+(+\frac{1}{2},+1)&=-\frac{1}{2\sqrt{2}}(1+\cos\theta) \\ A_W^+(-\tfrac{1}{2},-1)&=\tfrac{\sqrt{f}}{2\sqrt{2}}\sin\theta e^{i\phi} & A_W^+(+\frac{1}{2},-1)&=-\frac{1}{2\sqrt{2}}(1-\cos\theta)e^{2i\phi} \end{aligned} \tag{34}$$

and analgous relations as (32) and (33) apply. As discussed before, the angles θ and ϕ have completely different meanings in the t and W rest frames.

7 QCD Corrections to the Spin Density Matrix of $t\to bW$ from Real Gluon Emission

The amplitudes and density matrix for real gluon emission are the most difficult part of the higher order calculation and are also the most difficult to document.

The point is not only that the amplitudes are quite complicated expressions, but also that one has to deal with the phase space integration over the real gluon's degrees of freedom on the level of the spin density matrix. Furthermore, the cancellations of the singularities in ρ_{norm} in Eq. (27) require a subtle understanding of the interplay between lowest order and first order QCD. All this enforces the use of an algebraic computer program like FORM, REDUCE or MATHEMATICA to handle the long and complicated expressions. After calculating the spin density matrix, the real gluon's degrees of freedom have to be integrated over. This integration can be done without regularization, because according to the last section the correction to ρ_{norm} from real gluons is finite and of the generic form, cf. Eq. (12),

$$\rho_{norm}(h_t, h_W, h'_t, h'_W) = \rho^{lo}_{norm} + \alpha_d \frac{\rho^{ho}\rho^{lo}_{total} - \rho^{lo}\rho^{ho}_{total}}{(\rho^{lo}_{total})^2} \tag{35}$$

where $\alpha_d = -C_F \frac{\alpha_s}{2\pi}$. The dependence on the gluon spin is not considered, because it cannot be determined experimentally. Accordingly, the gluon polarizations have been summed in the standard fashion.

To be more explicit, let us write the 4–momenta relevant for the process $t \to Wbg$ in the rest system of the top quark as

$$\begin{aligned} p_t &= (m_t, \vec{0}) \qquad p_W = \frac{m_t}{2} f_+ x_W (1,0,0,\beta_W) \qquad p_b = p_t - p_W - p_g \\ p_g &= \frac{m_t}{2} f_- x_g (1, \sin\theta_g \cos\phi_g, \sin\theta_g \sin\phi_g, \cos\theta_g) \end{aligned} \tag{36}$$

where

$$\beta_W^2 = 1 - \frac{4f}{f_+^2 x_W^2} \tag{37}$$

and where θ_g is given in terms of the other variables according to

$$f_- x_g - f_+(1 - x_W) = \frac{1}{2} f_+ x_W f_- x_g (1 - \beta_W \cos\theta_g) \tag{38}$$

Note that the meaning of f, f_-, f_+ etc is as in lowest order. The integrations over the gluon's degrees of freedom then have to be done with the phase space

$$dPS_3(t \to Wbg) = \frac{m_t^2 \pi^2}{4} f_+ f_- \int_0^1 dx_g \int_0^{\frac{f_-^2 x_g (1-x_g)}{f_+(1-f_- x_g)}} d(1 - x_W) \int_0^{2\pi} \frac{d\phi_g}{2\pi} \tag{39}$$

The top quark spinors may be taken as in Eq. (16), with $\phi = 0$ because the y–z plane has not yet been specified. However, the W polarization vectors are different from the lowest order expressions, Eq. (18), because the W–momentum has changed. More precisely, the transverse polarization vectors are left unchanged, but the longitudinal polarization vector now reads

$$\epsilon_0 = -\frac{1}{2\sqrt{f}}(f_+ x_W \beta_W, 0, 0, f_+ x_W) \tag{40}$$

The $h_b = -1/2$ spinor for the b–quark changes, too, but for the density matrix one needs only the combination $u_{-1/2}\bar{u}_{-1/2} = \frac{1}{2}(1-\gamma_5)\not{p}$.

I have calculated all the $6 \times 6 = 36$ spin density matrix elements using these parametrizations

$$(1+2f)\rho_{norm}(-\frac{1}{2},0,-\frac{1}{2},0) = c_-\,(1+0.188\,\alpha_d) - 0.0246\,c_+\,\alpha_d \tag{41}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},0,+\frac{1}{2},0) = c_+\,(1+0.188\,\alpha_d) - 0.0246\,c_-\,\alpha_d \tag{42}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},0,+\frac{1}{2},0) = -s_0\,(1+0.212\,\alpha_d) \tag{43}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},0,-\frac{1}{2},0) = -s_0\,(1+0.212\,\alpha_d) \tag{44}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},-1,-\frac{1}{2},-1) = 2fc_+\,(1-0.236\,\alpha_d) - 0.00751\,c_-\,\alpha_d \tag{45}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},-1,+\frac{1}{2},-1) = 2fc_-\,(1-0.236\,\alpha_d) - 0.00751\,c_+\,\alpha_d \tag{46}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},-1,+\frac{1}{2},-1) = 2f\,s_0\,(1-0.216\,\alpha_d) \tag{47}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},-1,-\frac{1}{2},-1) = 2f\,s_0\,(1-0.216\,\alpha_d) \tag{48}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},+1,-\frac{1}{2},+1) = -0.00587\,c_+\,\alpha_d - 0.0518\,c_-\,\alpha_d \tag{49}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},+1,+\frac{1}{2},+1) = -0.00587\,c_-\,\alpha_d - 0.0518\,c_+\,\alpha_d \tag{50}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},+1,+\frac{1}{2},+1) = 0.0460\,s_0\,\alpha_d \tag{51}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},+1,-\frac{1}{2},+1) = 0.0460\,s_0\,\alpha_d \tag{52}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},0,-\frac{1}{2},+1) = -0.0150\,s_0\,\alpha_d \tag{53}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},+1,-\frac{1}{2},0) = -0.0150\,s_0\,\alpha_d \tag{54}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},0,+\frac{1}{2},+1) = +0.0150\,s_0\,\alpha_d \tag{55}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},+1,+\frac{1}{2},0) = +0.0150\, s_0\, \alpha_d \tag{56}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},0,+\frac{1}{2},+1) = 0.0150\, c_+\, \alpha_d \tag{57}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},+1,-\frac{1}{2},0) = 0.0150\, c_+\, \alpha_d \tag{58}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},0,-\frac{1}{2},+1) = -0.0150\, c_-\, \alpha_d \tag{59}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},+1,+\frac{1}{2},0) = -0.0150\, c_-\, \alpha_d \tag{60}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},0,-\frac{1}{2},-1) = -s_0\, \sqrt{2f}\, (1+0.00600\, \alpha_d) \tag{61}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},-1,-\frac{1}{2},0) = -s_0\, \sqrt{2f}\, (1+0.00600\, \alpha_d) \tag{62}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},0,+\frac{1}{2},-1) = +s_0\, \sqrt{2f}\, (1+0.00600\, \alpha_d) \tag{63}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},-1,+\frac{1}{2},0) = +s_0\, \sqrt{2f}\, (1+0.00600\, \alpha_d) \tag{64}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},0,+\frac{1}{2},-1) = -c_-\, \sqrt{2f}\, (1+0.00600\, \alpha_d) \tag{65}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},-1,-\frac{1}{2},0) = -c_-\, \sqrt{2f}\, (1+0.00600\, \alpha_d) \tag{66}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},0,-\frac{1}{2},-1) = +c_+\, \sqrt{2f}\, (1+0.00600\, \alpha_d) \tag{67}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},-1,+\frac{1}{2},0) = +c_+\, \sqrt{2f}\, (1+0.00600\, \alpha_d) \tag{68}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},+1,-\frac{1}{2},-1) \equiv 0 \tag{69}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},-1,-\frac{1}{2},+1) \equiv 0 \tag{70}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},+1,+\frac{1}{2},-1) \equiv 0 \tag{71}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},-1,+\frac{1}{2},+1) \equiv 0 \tag{72}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},+1,-\frac{1}{2},-1) \equiv 0 \tag{73}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},-1,+\frac{1}{2},+1) \equiv 0 \tag{74}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2},+1,+\frac{1}{2},-1) \equiv 0 \tag{75}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},-1,-\frac{1}{2},+1) \equiv 0 \tag{76}$$

where $c_\pm = \frac{1}{2}(1 \pm \cos\theta)$ and $s_0 = \frac{1}{2}\sin\theta$ and where the numerical coefficients in order $\alpha_d = -C_F \frac{\alpha_s}{2\pi}$ have been obtained with $m_t = 175$ GeV, for which $f = 0.21$. With the help of my intergation programs I have shown that the m_t dependence of these coefficients is in all cases moderate. For example, the coefficient 0.188 in Eq. (41) depends on $\frac{m_t}{m_W} = \frac{1}{\sqrt{f}}$ in the way depicted in Figure 1. The m_t dependence of the other independent QCD coefficients (denoted by -0.0246, 0.0212, -0.0150, 0.00600, -0.0518, -0.00587, 0.0460, -0.216, -0.236 and -0.00751, i.e. by their values at $m_t = 175$ GeV) are given in the figures that follow. Note that only lowest order and real gluons are incorporated in Eqs. (41)–(76). The virtual corrections Eqs. (32) and (31) from the $\sigma_{\mu\nu}$ term have to be added. The factors $1+2f$ appearing on the left hand side are a relic of the fact that I am presenting the density matrix 'normalized' to the total width/trace. There are several possibilities to make checks on this list. For example, I have checked that

$$\sum_{h_t,h_W} \rho_{norm}(h_t,h_W,h_t,h_W) \equiv 1 \tag{77}$$

is true including the oneloop QCD corrections. Furthermore, I have also checked that the ratio

$$\frac{\sum_{h_t}\rho_{norm}(h_t,0,h_t,0)}{\sum_{h_t}[\rho_{norm}(h_t,+1,h_t,+1)+\rho_{norm}(h_t,-1,h_t,-1)]} = \frac{\Gamma_L}{\Gamma_T} = \frac{1}{2f}(1+\alpha_s\cdots) \tag{78}$$

reproduces the ratio of longitudinal over transverse W's as calculated in [11] *including* higher order QCD corrections. Finally, there is the check as to the hermiticity of the density matrix, $\rho_{norm}(h_t,h_W,h'_t,h'_W) = \rho_{norm}(h'_t,h'_W,h_t,h_W)^*$. Note that the density matrix is real in the present case, because in the considered frame there is not azimuthal dependence.

I have carried through a second analogous calculation in the rest system of the W and obtained the real gluon QCD corrections to the density matrix $\rho_{norm}(h_t,h_W,h'_t,h'_W)$ also in that system. The momenta are now parametrized as follows

$$\begin{aligned} &p_W = (m_t,\vec{0}) \qquad p_t = \frac{m_t}{2\sqrt{f}}f_+x_t(1,0,0,\beta_t) \qquad p_g = p_t - p_W - p_b \\ &p_b = \frac{m_t}{2\sqrt{f}}f_-x_b(1,\sin\theta_b\cos\phi_b,\sin\theta_b\sin\phi_b,\cos\theta_b) \end{aligned} \tag{79}$$

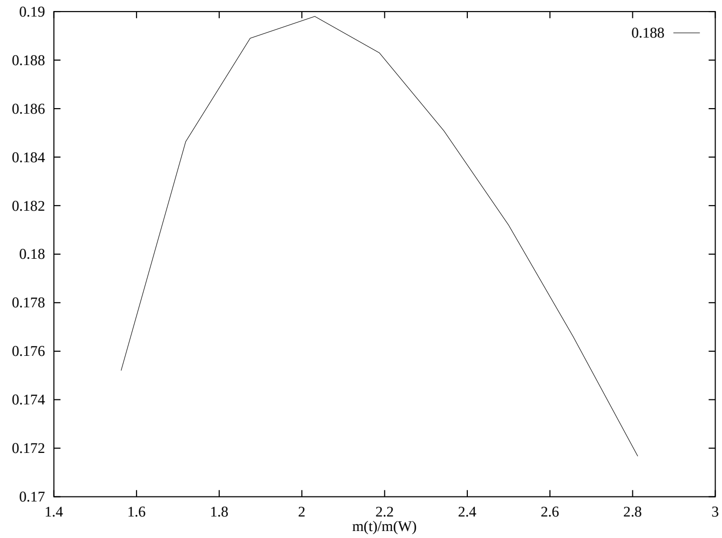


Figure 1:

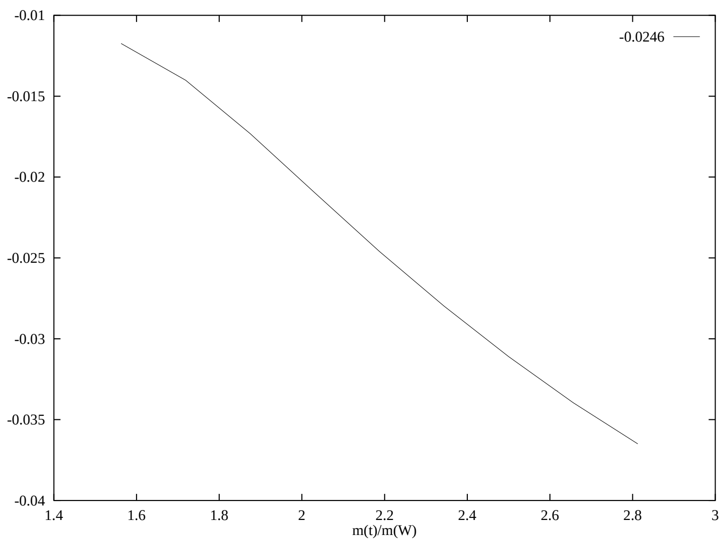


Figure 2:

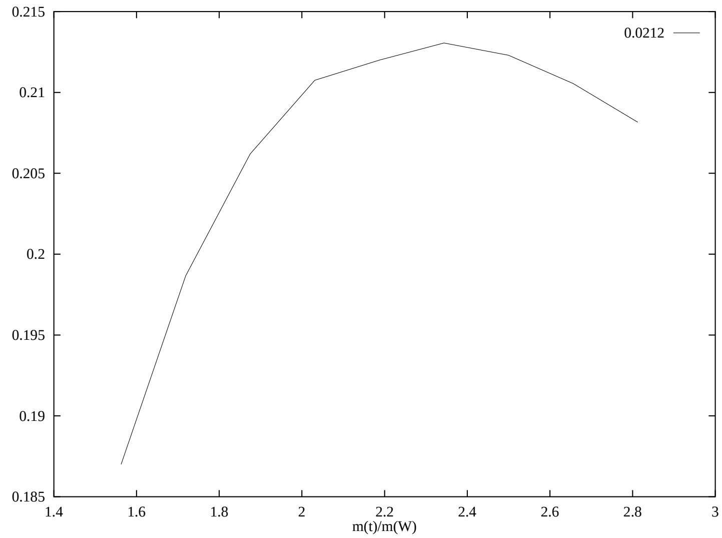
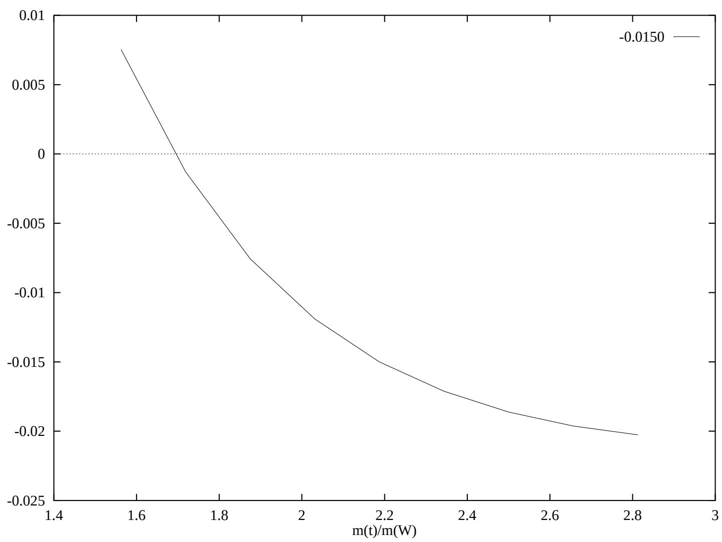


Figure 3:



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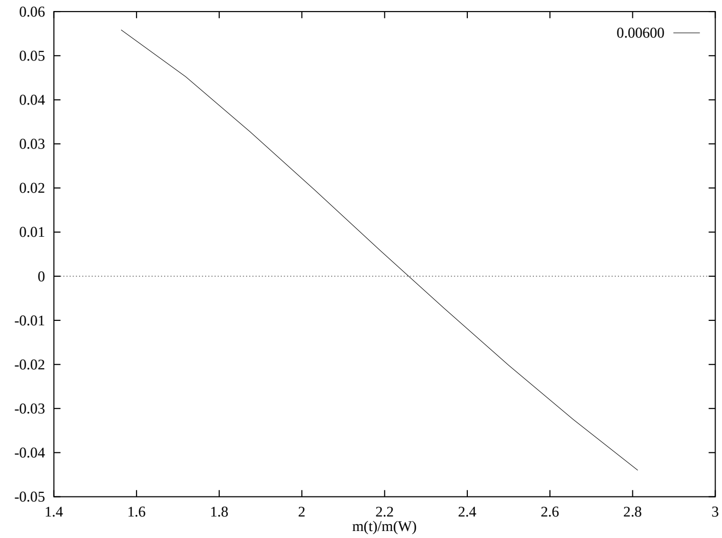

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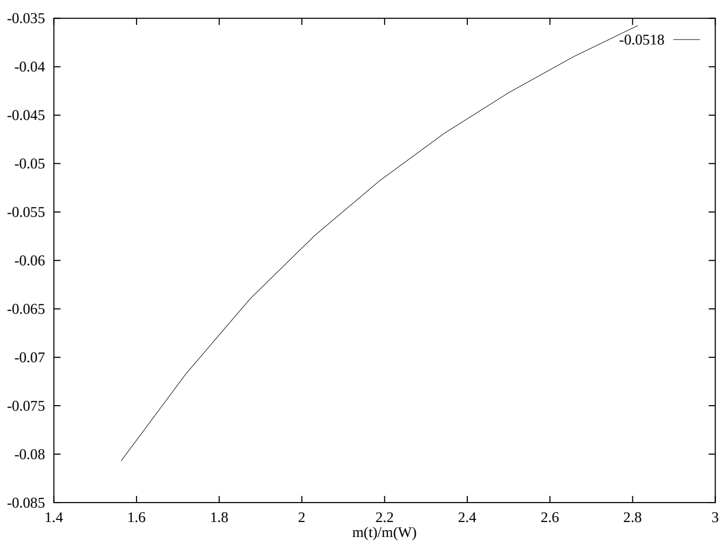

Figure 6:

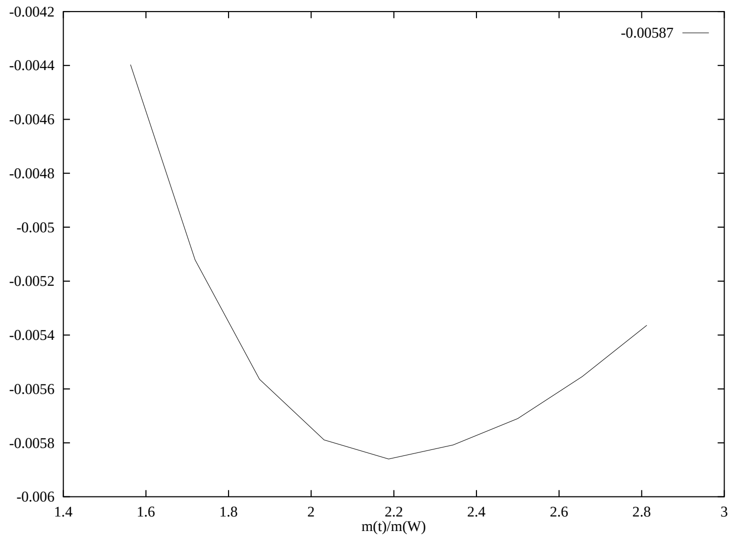

Figure 7:

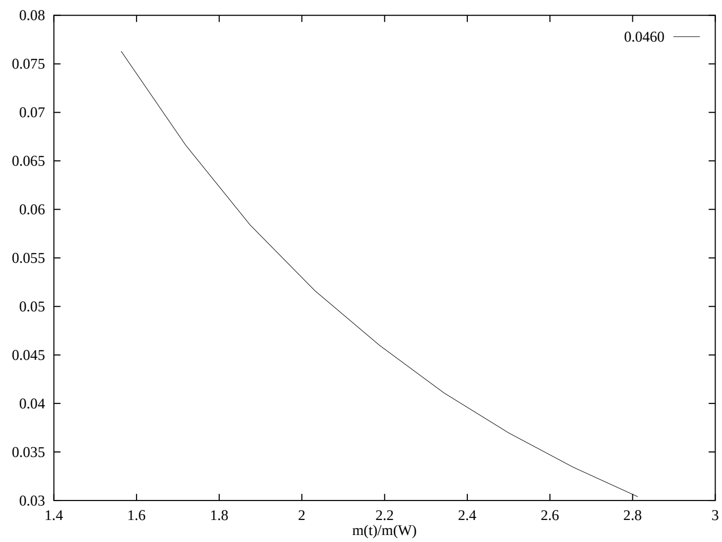

Figure 8:

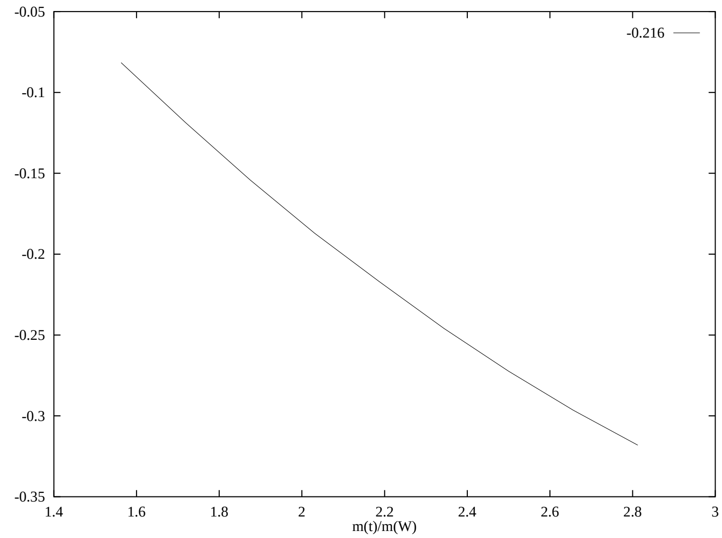

Figure 9:

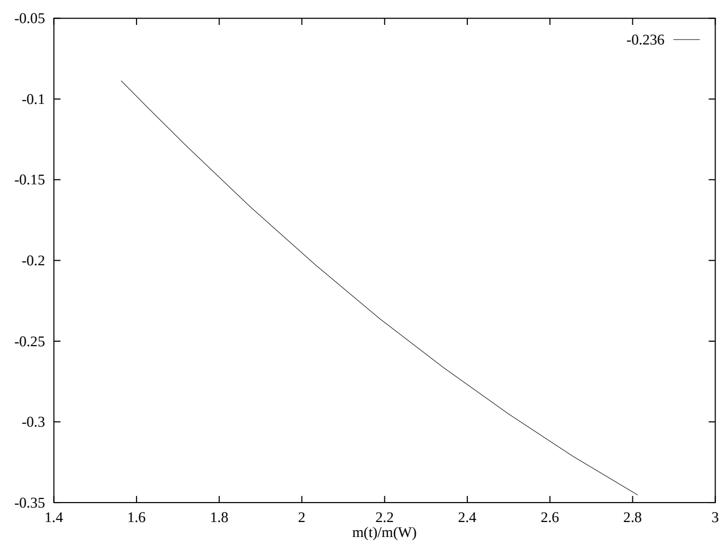

Figure 10:

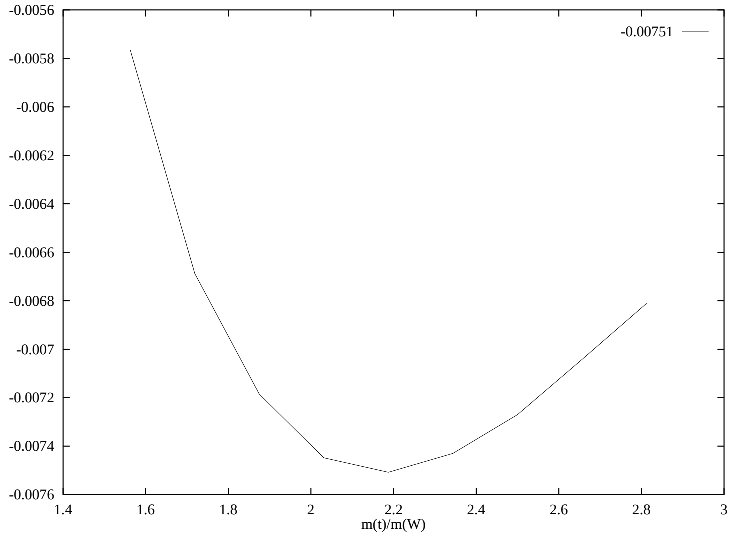

Figure 11:

where

$$\beta_t^2 = 1 - \frac{4f}{f_+^2 x_t^2} \tag{80}$$

and where θ_b is given in terms of the other variables according to

$$f_- x_b + f_+(1 - x_t) = \frac{1}{2f} f_+ x_t f_- x_b (1 - \beta_t \cos\theta_b)\,. \tag{81}$$

The integrations over the gluon's degrees of freedom are encoded as integrations over x_t, x_b and ϕ_b and have to be done with the phase space

$$dPS_3(t \to Wbg) = \frac{m_t^2 \pi^2}{32} f_+ f_- \int_0^1 d(1-x_b) \int_0^{\frac{f_-^2 x_b(1-x_b)}{f_+(1-f_-(1-x_g))}} d(1-x_t) \int_0^{2\pi} \frac{d\phi_b}{2\pi}\,. \tag{82}$$

About the effects of gluon emission on polarization: Since the W–momentum is unchanged as compared to lowest order, the form of the W polarization vectors remains as in (25). However, the top and bottom spinors are modified. They now read

$$u_{+1/2}(p_t) = \sqrt{m_t}(a_-, 0, a_+, 0) \qquad u_{-1/2}(p_t) = \sqrt{m_t}(0, a_-, 0, a_+) \tag{83}$$

with

$$a_{\pm} = \sqrt{\frac{x_t f_+}{2\sqrt{f}}}\sqrt{1 \pm \beta_t} \tag{84}$$

and

$$\bar{u}_{-1/2}(p_b) = \sqrt{m_t}(0, 0, b_1^*, b_2^*) \tag{85}$$

where b_1^* and b_2^* are given indirectly by $u_{-1/2}(p_b)\bar{u}_{-1/2}(p_b) = \frac{1}{2}(1-\gamma_5)\not{p}$. The 36 elements of the normalized density matrix obtained in this frame are given by

$$(1+2f)\rho_{norm}(-\frac{1}{2}, 0, -\frac{1}{2}, 0) = \cos^2\theta + (-0.248\, z_- + 0.188\, z_+)\,\alpha_d \tag{86}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2}, 0, +\frac{1}{2}, 0) = f\sin^2\theta + (-0.080\, z_- - 0.0246\, z_+)\,\alpha_d \tag{87}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2}, 0, +\frac{1}{2}, 0) = -\sqrt{f}\,\sin\theta\cos\theta\,(1 - 0.0171\,\alpha_d) \tag{88}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2}, 0, -\frac{1}{2}, 0) = -\sqrt{f}\,\sin\theta\cos\theta\,(1 - 0.0171\,\alpha_d) \tag{89}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2}, -1, -\frac{1}{2}, -1) = \frac{1}{2}\sin^2\theta \tag{90}$$

$$+(0.188\, z_- - 0.0296\, z_+ - 0.0222\,\cos\theta)\,\alpha_d \tag{91}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2}, -1, +\frac{1}{2}, -1) = \frac{f}{2}(1-\cos\theta)^2 \tag{92}$$

$$+(-0.0246\, z_- - 0.0522\, z_+ + 0.0464\,\cos\theta)\,\alpha_d \tag{93}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2}, -1, -\frac{1}{2}, -1) = (1+2f)\rho_{norm}(-\frac{1}{2}, -1, +\frac{1}{2}, -1) \tag{94}$$

$$= \frac{\sqrt{f}}{2}\left[-\sin\theta\,(1 + 0.0640\,\alpha_d) + \sin\theta\cos\theta\,(1 - 0.0374\,\alpha_d)\right] \tag{95}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2}, +1, -\frac{1}{2}, +1) = \frac{1}{2}\sin^2\theta \tag{96}$$

$$+(0.188\, z_- - 0.0296\, z_+ + 0.0222\,\cos\theta)\,\alpha_d \tag{97}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2}, +1, +\frac{1}{2}, +1) = \frac{f}{2}(1+\cos\theta)^2 \tag{98}$$

$$+(-0.0246\, z_- - 0.0522\, z_+ - 0.0464\,\cos\theta)\,\alpha_d \tag{99}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2}, +1, -\frac{1}{2}, +1) = (1+2f)\rho_{norm}(-\frac{1}{2}, +1, +\frac{1}{2}, +1) \tag{100}$$

$$= \frac{\sqrt{f}}{2}\left[\sin\theta\,(1 + 0.0293\,\alpha_d) + \sin\theta\cos\theta\,(1 - 0.0171\,\alpha_d)\right] \tag{101}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2}, 0, -\frac{1}{2}, +1) = (1+2f)\rho_{norm}(-\frac{1}{2}, +1, -\frac{1}{2}, 0) \tag{102}$$

$$= \frac{1}{\sqrt{2}} \sin\theta \cos\theta (1 + 0.218\,\alpha_d) - 0.0157\,\alpha_d\,\sin\theta \tag{103}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2}, 0, +\frac{1}{2}, +1) = (1+2f)\rho_{norm}(+\frac{1}{2}, +1, +\frac{1}{2}, 0) \tag{104}$$

$$= -\frac{f}{\sqrt{2}} \left[\sin\theta\,(1 - 0.222\,\alpha_d) + \sin\theta\cos\theta\,(1 - 0.1325\,\alpha_d) \right] \tag{105}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2}, 0, +\frac{1}{2}, +1) = (1+2f)\rho_{norm}(+\frac{1}{2}, +1, -\frac{1}{2}, 0) \tag{106}$$

$$= \sqrt{\frac{f}{2}} \cos\theta\,(1 + \cos\theta) + (0.0130\,z_- + 0.00196\,z_+ - 0.00196\,\cos\theta)\,\alpha_d \tag{107}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2}, 0, -\frac{1}{2}, +1) = (1+2f)\rho_{norm}(-\frac{1}{2}, +1, +\frac{1}{2}, 0) \tag{108}$$

$$= -\sqrt{\frac{f}{2}} \sin^2\theta + (0.00358\,z_- - 0.00750\,z_+ + 0.00750\,\cos\theta)\,\alpha_d \tag{109}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2}, 0, -\frac{1}{2}, -1) = (1+2f)\rho_{norm}(-\frac{1}{2}, -1, -\frac{1}{2}, 0) \tag{110}$$

$$= -\frac{1}{\sqrt{2}} \sin\theta \cos\theta\,(1 + 0.218\,\alpha_d) - 0.0157\,\alpha_d\,\sin\theta \tag{111}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2}, 0, +\frac{1}{2}, -1) = (1+2f)\rho_{norm}(+\frac{1}{2}, -1, +\frac{1}{2}, 0) \tag{112}$$

$$= -\frac{f}{\sqrt{2}} \left[\sin\theta\,(1 - 0.222\,\alpha_d) - \sin\theta\cos\theta\,(1 - 0.1325\,\alpha_d) \right] \tag{113}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2}, 0, +\frac{1}{2}, -1) = (1+2f)\rho_{norm}(+\frac{1}{2}, -1, -\frac{1}{2}, 0) \tag{114}$$

$$= \sqrt{\frac{f}{2}} \cos\theta\,(1 - \cos\theta) + (-0.0130\,z_- - 0.00196\,z_+ + 0.00196\,\cos\theta)\,\alpha_d \tag{115}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2}, 0, -\frac{1}{2}, -1) = (1+2f)\rho_{norm}(-\frac{1}{2}, -1, +\frac{1}{2}, 0) \tag{116}$$

$$= \sqrt{\frac{f}{2}} \sin^2\theta + (-0.00358\,z_- + 0.00750\,z_+ + 0.00750\,\cos\theta)\,\alpha_d \tag{117}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2}, +1, -\frac{1}{2}, -1) = (1+2f)\rho_{norm}(-\frac{1}{2}, -1, -\frac{1}{2}, +1) \tag{118}$$

$$= -z_-\,(1 + 0.218\,\alpha_d) \tag{119}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2}, +1, +\frac{1}{2}, -1) = (1+2f)\rho_{norm}(+\frac{1}{2}, -1, +\frac{1}{2}, +1) \tag{120}$$

$$= f\,z_-\,(1 - 0.1325\,\alpha_d) \tag{121}$$

$$(1+2f)\rho_{norm}(-\frac{1}{2}, +1, +\frac{1}{2}, -1) = (1+2f)\rho_{norm}(+\frac{1}{2}, -1, -\frac{1}{2}, +1) \tag{122}$$

$$= \frac{\sqrt{f}}{2}\,\sin\theta\,(\,1-\cos\theta\,)\,(1-0.0171\,\alpha_d) \tag{123}$$

$$(1+2f)\rho_{norm}(+\frac{1}{2},+1,-\frac{1}{2},-1) = (1+2f)\rho_{norm}(-\frac{1}{2},-1,+\frac{1}{2},+1) \tag{124}$$

$$= -\frac{\sqrt{f}}{2}\,\sin\theta\,(\,1+\cos\theta\,)\,(1-0.0171\,\alpha_d) \tag{125}$$

where $z_{\pm} = \frac{1}{2}(1 \pm \cos^2\theta)$. The QCD coefficients have again been obtained by numerical integration with $m_t = 175$ GeV. The m_t dependence of these coefficients is again moderate. In fact, it can be shown that the coefficients in the rest frame of the top quark, Eqs. (41)–(76), and of the W boson, Eqs. (86)–(125), are related. I was not able to derive a general formula, but I have found, for example, that -0.0518[Eq.(49)]$= -0.0296$[Eq.(91)]-0.0222 [Eq.(91)] and -0.00587[Eq.(49)]$= -0.0522$[Eq.(93)]$+0.0464$ [Eq.(93)]. Corresponding equalities are true for all other values of m_t, i.e. they hold for the coefficients in general. There are some other relations which I do not want to quote here.

8 Summary

In this report I have summarized a recent new calculation of a complete spin analysis of the Standard Model top quark decay including higher order QCD corrections. The QCD corrections to the 'normalized' density matrix are in general quite small, of the order of 1%, in particular for the real gluon contribution in the rest frame of the top quark, cf. Eqs. (41)–(76) and of the W, cf. Eqs. (86)–(125). The contribution from virtual gluons is somewhat larger, cf. Eq. (29). Note that the *relative* magnitude of the QCD corrections can be very large – in all cases, where the lowest order contribution vanishes, like Eqs. (49)–(60). In other cases, the symmetry requirement of CP gives vanishing matrix elements beyond the leading order, cf. Eqs. (69)–(76).

Complete results including azimuthal dependence, numerical analysis and physical applications have not been included here. However, I plan to write a long article with J. Körner and his group [22], in which not only this, but also analytical formulae for all the QCD coefficients will be given.

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