The First Moment of $\delta g(x)$ – a Comparative Study

Bodo Lampe

Department of Physics, University of Munich

Theresienstrasse 37, D–80333 Munich

Andreas Ruffing

Max Planck Institute for Physics

Föhringer Ring 6, D–80805 Munich

Abstract

The sensititivity of various future polarization experiments to the first moment Δg of the polarized gluon density is elucidated in detail. It is shown to what extent the first moment can be extracted from the future data as compared to the higher moments. We concentrate on two processes which in the near future will become an important source of information on the polarized gluon density, namely the photoproduction of open charm to be studied at CERN (COMPASS) and SLAC and the production of direct photons at RHIC.

1. Introduction

One of the main issues in polarized DIS experiments is the question of how the proton spin at high energies is composed out of the spins of its constituents, possibly

$$+\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_z \tag{1}$$

where $\Delta\Sigma = \Delta(u+\bar{u}) + \Delta(d+\bar{d}) + \Delta(s+\bar{s})$ is the contribution from the quark spins. In the (static) constituent models, like SU(6), one has $\Delta\Sigma(SU_6) = 1$ but experimentally [1] it seems that this rule is violated by a large amount ($\Delta\Sigma_{exp} \approx 0.25$). This experimental fact is also in disagreement with the Ellis-Jaffe sum rule [2] which predicts $\Delta\Sigma_{EJ} \approx 0.65$ on the basis of the approximation $\Delta s = \Delta\bar{q} = \Delta g = 0$. It will probably turn out that both the gluon and the strange quark are needed to understand the proton spin structure [3].

In recent studies of polarized DIS phenomena [4, 5] these questions have been superimposed by attempts to guess the full x–dependence of the polarized gluon density, but the question of the first moment is still particularly interesting because it is related to the anomaly. It is true that the first moment is only one among an infinite set of moments. However, the first moment Δg certainly has its significance, because it enters the fundamental spin sum rule (1) and because it gives the contribution within the proton to the γ_5 anomaly.

In lepton–nucleon DIS the gluon arises only as a higher order effect and consequently one has difficulties to extract the polarized gluon distribution and in particular its first moment from inclusive deep inelastic data. These problems have been anticipated several years ago by theoretical studies [7, 8, 6], and they are in fact not surprising in view of the subtleties in determining

the unpolarized gluon density in unpolarized DIS experiments [9].

A popular way out of this dilemma is the study of semi–inclusive cross sections, and in particular of charm production, because the production of heavy quark hadrons is triggered in leading order by the photon–gluon fusion mechanism and is therefore sensitive to the gluon density inside the proton, whereas the heavy quark content of the proton is usually negligible at presently available Q^2–values. Another possibility is to look at hard prompt photons produced in proton–proton scattering, a process which is well known to lead to good results for $g(x)$ in unpolarized scattering. In this article we shall study these two processes in some detail and examine the question to what extent they are sensitive to the first moment Δg as compared to the higher moments.

Throughout the paper we are using polarized quark densities as suggested by [4]. Likewise, we could have used densities parametrized in ref. [5]. We stress that we make no assumptions on the form of $\delta g(x)$ and that our results depend only marginally on the quark density parametrization choosen.

2. Polarized Open Charm Photoproduction

Due to its prominent decay mechanism, J/ψ–events are the most prominent within charm production, and this fact has led to attempts to determine the unpolarized gluon density from the J/ψ–production cross section [10, 11, 12]. Similar ideas hold in the case of polarized J/ψ–production [13, 14]. However, these suggestions are model dependent and depend on assumptions which go beyond the QCD improved parton model. For example, according to the suggestion of [11] elastic J/ψ–production should measure the square of $\delta g(x)$ and therefore be very sensitive to its magnitude. However, it is not clear whether in the cross section formula there is a factor $g(x_1)g(x_2)$ or

whether some independent 2–gluon correlation function $K(x_1, x_2)$ appears. Furthermore, it has recently been stressed [15], that color octet contributions may appear in addition to the color singlet pieces in inelastic J/ψ–production. If true, this would upset the inelastic J/ψ–analysis because several new free parameters, the color octet matrix elements, would enter the game.

Therefore, from the theoretical point of view the cleanest signal for the gluon in heavy quark production is probably open charm production, although experimentally it has worse statistics due to the difficulties in identifying D–mesons. Instead of the deep inelastic process one may as well look at photoproduction, because the mass of the charm quark forces the process to take place in the perturbative regime. The advantage of photoproduction over DIS is its larger cross section. Two fixed target experiments, one at SLAC and COMPASS at CERN [16] are being developed to make use of this advantage and measure the polarized gluon distribution via photoproduction.

In leading order the inclusive polarized *deep inelastic* open charm production cross section is given by [17, 18]

$$\frac{d\sigma_c}{dQ^2 dy} = \frac{4\pi\alpha^2}{Q^2} \frac{2-y}{yS} g_1^c(\frac{Q^2}{yS}, Q^2) \tag{2}$$

where S is the Mandelstam–S for the lepton–nucleon scattering process and

$$g_1^c(x, Q^2) = \frac{\alpha_s}{9\pi} \int_{(1+\frac{4m_c^2}{Q^2})x}^1 \frac{dw}{w} \delta g(w, Q^2) h_{ec}(\frac{x}{w}) \tag{3}$$

is the charm contribution to the polarized structure function g_1 and where

$$h_{ec}(z) = (2z - 1)\ln\frac{1+\beta}{1-\beta} + (3 - 4z)\beta \tag{4}$$

is the parton level matrix element. One has $\beta = \sqrt{1 - \frac{4m_c^2}{\hat{s}}}$ where the Mandelstam variable $\hat{s}$ is defined by $\hat{s} = (p+q)^2 = Q^2\frac{1-z}{z}$. By combining these

formulae with the unpolarized cross section one can obtain the polarization asymmetries. If one plugs in the relatively large gluon contribution of references [7, 4, 5], one gets asymmetries of the order 0.1 in a fixed target experiment which would operate well above charm threshold.

It is straightforward to obtain from the above expressions (2) – (4) the inclusive open charm *photoproduction* cross section by taking the simultaneous limits $Q^2 \to 0$ and $z \to 0$ while keeping $\frac{Q^2}{z} \approx \hat{s}$ fixed [19, 20]:

$$\sigma_{\gamma p}^c(S_\gamma) = \frac{8\pi\alpha\alpha_s}{9S_\gamma} \int_{\frac{4m_c^2}{S_\gamma}}^1 \frac{dw}{w} \delta g(w, S_\gamma)(3v - \ln\frac{1+v}{1-v}) \tag{5}$$

where $v = \sqrt{1 - \frac{4m_c^2}{\hat{s}}}$ and $\hat{s} = wS_\gamma$. This integrated cross section depends only on the total proton–photon energy $S_\gamma = (P + q)^2$ which for a fixed target experiment is given by $S_\gamma = 2ME_\gamma$ where E_γ is the photon energy. By varying the photon energy it is in principle possible to explore the x–dependence of δg. Very high photon energies correspond to small values of x. However, as we shall see later, it is not trivial to obtain the first moment of $\delta g(x)$ from the cross section Eq. (5) even if the full energy dependence is known.

It should be noted that the second argument of δg in Eqs. (3) and (5) is not certain. It might as well be $4m_c^2$ or any number in between. This uncertainty reflects our ignorance about the magnitude of the higher order correction and could be resolved if a higher order calculation of these cross sections would be performed. The same statement holds true for the argument of α_s. Therefore, in the equations presented below the energy arguments of δg and α_s will be chosen to be more general, μ_S and μ_R respectively.

Eq. (5) was obtained after integration over the charm quark production angle ($\hat{\theta}$ in the gluon photon cms). If one is interested in the p_T distribution or

wants to introduce a p_T–cut, it is appropriate to keep the $\hat{\theta}$ dependence in the matrix element

$$ME = \frac{\hat{t}^2 + \hat{u}^2 - 2m_c^2\hat{s}}{\hat{t}\hat{u}} + 2m_c^2\frac{\hat{t}^3 + \hat{u}^3}{\hat{t}^2\hat{u}^2} \tag{6}$$

where $\hat{s} = wS_\gamma$, $\hat{t} = -\frac{\hat{s}}{2}(1 - v\cos\hat{\theta})$ and $\hat{u} = -\frac{\hat{s}}{2}(1 + v\cos\hat{\theta})$. It is possible to make a transformation to the transverse charm quark momentum by using $p_T^2 = (\frac{\hat{s}}{4} - m_c^2)\sin^2\hat{\theta}$ and to obtain the cross section for all processes with p_T greater than a given p_{Tcut} :

$$\sigma(p_{Tcut}) = \frac{\pi\alpha\alpha_s(\mu_R^2)}{9S_\gamma} \int_{\frac{4(m_c^2+p_{Tcut}^2)}{S_\gamma}}^1 \frac{dw}{w}\delta g(w,\mu_S^2)\frac{v}{\frac{\hat{s}}{4} - m_c^2} \int_{p_{Tcut}^2}^{\frac{\hat{s}}{4}-m_c^2} \frac{dp_T^2}{\sqrt{1 - \frac{p_T^2}{\frac{\hat{s}}{4}-m_c^2}}}$$

$$\left\{2m_c^2\frac{\hat{s}}{\hat{t}\hat{u}} - \frac{\hat{t}}{\hat{u}} - \frac{\hat{u}}{\hat{t}} - 2m_c^2(\frac{\hat{t}}{\hat{u}^2} + \frac{\hat{u}}{\hat{t}^2})\right\} \tag{7}$$

There are several good reasons to study the p_T distribution. First of all and in general, it gives more information than the inclusive cross section. Secondly and in particular, it can be shown that the integrated photoproduction cross section Eq. (5) as well as the corresponding DIS charm production cross section are not sensitive to the first moment of $\delta g(x)$. The sensitivity is increased, however, if a p_T–cut of the order of $p_T \approx 1$ GeV is introduced (see below). Last but not least, it is experimentally reasonable to introduce a p_T–cut.

Now we want to follow the question what the contribution of the first moment Δg to the cross sections (3), (5) and (7) is. In *massless* DIS this question is easy to answer. One can apply the convolution theorem on Eq. (3) (with $m_c = 0$) to see that the contribution of Δg is given by the first moment of the parton matrix element. If masses are involved, like m_c, the answer to this question is somewhat more subtle. Since the cross section is not any more a convolution of the standard form, one can not directly apply the convolution

theorem, but has to write it artificially as

$$\sigma(a) = \int_a^1 \frac{dw}{w} \delta g(w) H(\frac{a}{w}) \tag{8}$$

where H is some function (to be given below) and $a = a_{\gamma c} = \frac{4m_c^2}{S_\gamma}$ for photopro-
duction and $a = a_{ec} = (1 + \frac{4m_c^2}{Q^2})x$ for DIS charm production. Now one can ap-
ply the convolution theorem to prove that the first moment $H^{(1)} = \int_0^1 dz H(z)$
gives the contribution from Δg to the cross section [and in general for the
n–th moment : $\sigma^{(n)} = H^{(n)} \delta g^{(n)}$]. Note that the moments $\sigma^{(n)}$ are taken
with respect to a. In DIS charm production, the function H is given by
$H_{ec}(z) \sim h_{ec}(\frac{z}{1 + \frac{4m_c^2}{Q^2}})$, Eq. (4), and in open charm photoproduction without
cuts it is given by $H_{\gamma c}(z) = \frac{8\pi\alpha\alpha_s(\mu_R)}{9S_\gamma} h_{\gamma c}(z)$ with

$$h_{\gamma c}(z) = 3\sqrt{1-z} - \ln\frac{1 + \sqrt{1-z}}{1 - \sqrt{1-z}} \tag{9}$$

where $z = \frac{a_{\gamma c}}{w}$. After some algebra one can see that both for the inclusive
charm photoproduction and DIS the relevant quantities $\int_0^1 dz H(z)$ identically
vanish. For example, for the photoproduction case the moment function $h_{\gamma c}^{(n)}$
is given by

$$h_{\gamma c}^{(n)} = (n^{-1} - n^{-2}) \int_0^1 (1 - t^2)^n dt \tag{10}$$

On physical grounds the result $H^{(1)} = 0$ can be traced back to the small–p_T
behaviour of the matrixelement for $\gamma g \to c\bar{c}$ which cancels the contribution
of the large–p_T region in $\int_0^1 dz H(z)$ [21]. It is not really a surprise in view of
the structure of the anomaly in massive QCD (cf. the appendix of ref. [22]).

Eq. (10) allows to calculate $h_{\gamma c}^{(n)}$ for arbitrary complex n. For example, there
is an expansion $h_{\gamma c}^{(1+\epsilon)} = \frac{2}{3}\epsilon + O(\epsilon^2)$ (and similarly for $H_{ec}^{(1+\epsilon)}$) which shows
that the $H^{(n)}$ keep being small in the neighbourhood of n=1. From this one

may conclude that the cross sections are not suited for determining the first moment of δg.

Fortunately, the situation changes if one includes a p_T–cut of greater than 1 GeV. In that case, some sensitivity to Δg is re–established because the small–p_T behaviour of the matrix element for $\gamma g \rightarrow c\bar{c}$ does not cancel the contribution of the large–p_T region any more. Let us discuss this issue in some more detail for the photoproduction case. One can put the cross section (7) in the form (8) with $H_{\gamma c}(z, p_{Tcut}) = \frac{8\pi a \alpha_s(\mu_R)}{9S_\gamma} h_{\gamma c}(z, p_{Tcut})$ and

$$h_{\gamma c}(z, p_{Tcut}) = \frac{v}{8} \frac{1}{\frac{\hat{s}}{4} - m_c^2} \int_{p_{Tcut}^2}^{\frac{\hat{s}}{4} - m_c^2} \frac{dp_T^2}{\sqrt{1 - \frac{p_T^2}{\frac{\hat{s}}{4} - m_c^2}}} \left\{ 2m_c^2 \frac{\hat{s}}{\hat{t}\hat{u}} - \frac{\hat{t}}{\hat{u}} \right.$$
$$\left. - \frac{\hat{u}}{\hat{t}} - 2m_c^2 (\frac{\hat{t}}{\hat{u}^2} + \frac{\hat{u}}{\hat{t}^2}) \right\} \tag{11}$$

z is defined by $z = \frac{4m_c^2}{\hat{s}}$. All other relevant quantities have been defined before and after (7). Note that for $p_{Tcut} \rightarrow 0$ one recovers $h_{\gamma c}(z, p_{Tcut}) = h_{\gamma c}(z)$, Eq. (9). The integral Eq. (11) will be the starting point for several important observations, cf. Figs. 1–6 below. The point is, that one can use the moments $H_{\gamma c}^{(n)}(p_{Tcut})$ of $H_{\gamma c}(z, p_{Tcut})$ to reconstruct the cross section according to the Mellin formula

$$\sigma(p_{Tcut}) = \frac{1}{2\pi i} \int_{k-i\infty}^{k+i\infty} dn a_{\gamma c}^{-n} H_{\gamma c}^{(n)}(p_{Tcut}) \delta g^{(n)} \tag{12}$$

This is because the cross section has the form Eq. (8) with $H(z) = H_{\gamma c}(z, p_{Tcut})$. Since the integral in Eq. (12) extends along the imaginary axis, one has to consider complex values of n. We have studied the behavior of $H_{\gamma c}^{(n)}(p_{Tcut})$ as a function of $n = 1 + iy$, $n = 2 + iy$, $n = 3 + iy$ etc. for real values of y and several values of p_{Tcut}. In this way we are able to determine the circumstances under which the first moment $H_{\gamma c}^{(1)}(p_{Tcut})$ dominates in the in-
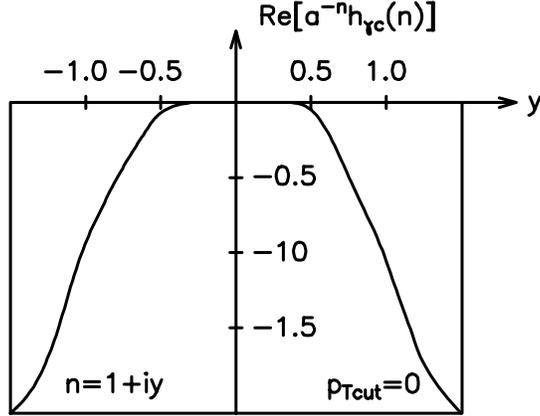
Figure 1: The behavior of the n–th moment of $h_{\gamma c}$ in the imaginary neighbourhood of $n = 1$ at $p_{Tcut} = 0$. It is shown that there is no sensitivity to the first moment at all.

tegral Eq. (12) over the higher moments. For definiteness, we have choosen a photon energy of 30 GeV ($S_\gamma = 60$ GeV^2) and $m_c = 1.3$ GeV.

More precisely, in Figs. 1–6 the quantity $\mathrm{Re}[a^{-k-iy} h_{\gamma c}^{(k+iy)}(p_{Tcut})]$ is presented for $-1.5 \leq y \leq +1.5$, $k = 1, 2$ and $p_{Tcut} = 0., 0.75, 1.5$ GeV. The curves for $p_{Tcut} = 0$ (Figs. 1 and 4) can be obtained directly from Eq. (10). The insensitivity at $p_{Tcut} = 0$ as to the first moment is displayed by the zero of the curve $k = 1$ (Fig. 1). If one compares Figs. 1 and 4, one sees that the sensitivity to the second moment is much larger for $p_{Tcut} = 0$. The situation changes as the p_{Tcut} is increased. This can be seen in Figs. 2, 3, 5 and 6, but also in Fig. 7 where $h_{\gamma c}^{(1)}(p_{Tcut})$ and $h_{\gamma c}^{(2)}(p_{Tcut})$ are given as a function of p_{Tcut}. $h_{\gamma c}^{(1)}(p_{Tcut})$ has an extremum at some point $p_{Tcut} \approx 1$GeV which should be suspected to be the most optimal value for a measurement of Δg. That this is really the case, can be deduced from a closer study of Figs. 2 and
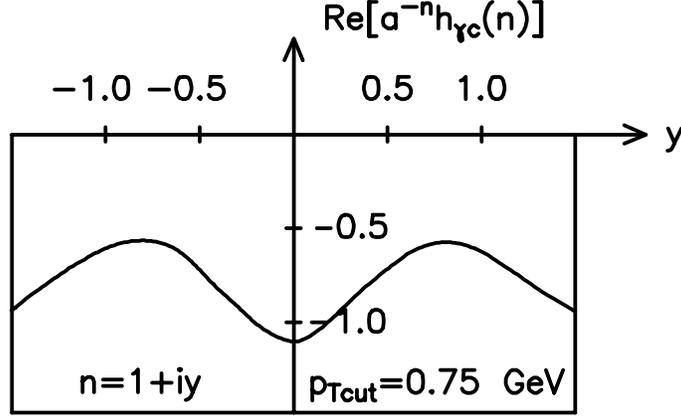
Figure 2: The behavior of the n–th moment of $h_{\gamma c}$ in the imaginary neighbourhood of $n = 1$ at $p_{Tcut} = 0.75$ GeV. An increasing sensitivity as to the first moment is visible.

3. We suggest to associate an "observational significance" R_1 to any cross section sensitive to Δg by means of the following procedure : It should be defined as the ratio of the "signal" over the squareroot of the "background", $R_1 = \frac{N_{signal}}{\sqrt{N_{background}}}$ which in the ideal situation of a Gaussian corresponds to $R_1 = \sqrt{\frac{height}{width}}$. In the case at hand one has $R_1 \approx \frac{|H^{(1)}|}{\sqrt{\int_{\approx -1}^{\approx +1} dy a^{-1-iy}|H^{(1+iy)}|}}$. Similarly, one can define a significance R_2 for the determination of the second moment and so on. For simplicity, R_3, R_4 etc. are not considered in this paper because qualitatively they behave not much different than R_2. From Figs. 1–6, the "observational significances" R_1 and R_2 can be deduced as a function of p_{Tcut}. If one makes a more thorough analysis by taking other values of p_{Tcut} into account, one finds that R_1 has a maximum near $p_{Tcut} \approx m_c$ whereas R_2 goes down for increasing values of p_{Tcut}. Unfortunately, R_3, R_4 etc do not decrease as strongly as R_2 at higher values of p_{Tcut}. One may also
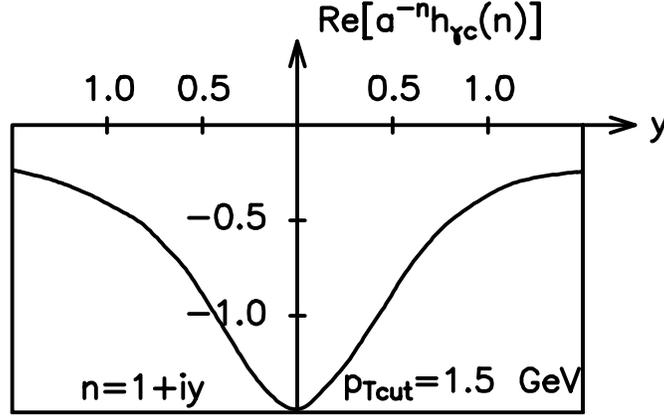
Figure 3: The behavior of the n–th moment of $h_{\gamma c}$ in the imaginary neighbourhood of $n = 1$ at $p_{Tcut} = 1.5$ GeV. The first moment dominates the higher moment contributions which are comprised at values $y \neq 0$.

determine the "12–significance" $R_{12} = \frac{R_1}{R_2}$. This ratio is important, because it does not depend on the overall cross section. For example, at $p_{Tcut} = 0.75$ GeV one has $R_{12} = 0.67$. It will be shown later that the "12–significance" is much larger for the RHIC process than this value obtained for charm photoproduction!

4. Polarized Production of Direct Photons in PP Collisions

In unpolarized hadron scattering hard photons are known to be a clean probe of the gluon distribution, because they can be directly detected, without undergoing fragmentation. Similarly, the most interesting prospect for polarized high energetic proton experiments is the possibility to determine $\delta g(x)$ from the process $pp \to \gamma X$ with a high energetic photon in the final state. Consequently several groups have studied this process theoretically
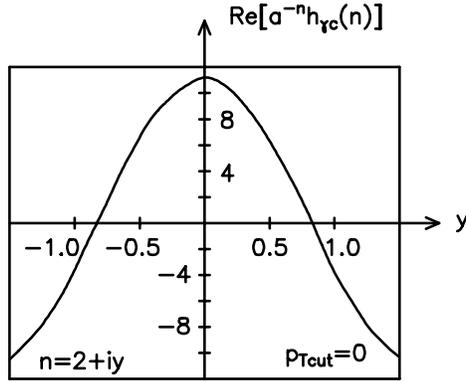
Figure 4: The behavior of the n–th moment of $h_{\gamma c}$ in the imaginary neighbourhood of $n = 2$ at $p_{Tcut} = 0$. A sensitivity of the second moment $(y = 0)$ as compared to the other moments $(y \neq 0)$ is visible. The decrease at large values of $y > 1.5$ is due to moments $n \geq 3$.
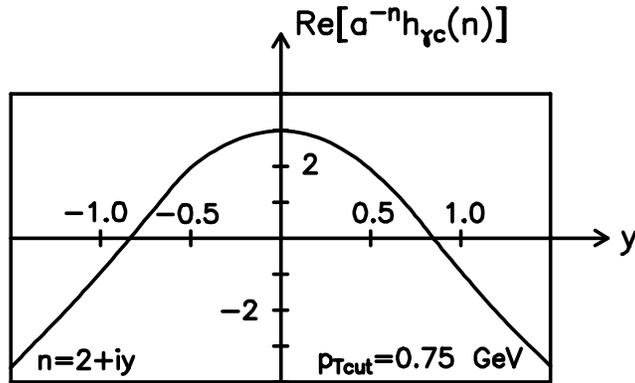

Figure 5: The behavior of the n–th moment of $h_{\gamma c}$ in the imaginary neighbourhood of $n = 2$ at $p_{Tcut} = 0.75$ GeV. The sensitivity of the cross section as regards the second moment is going down.
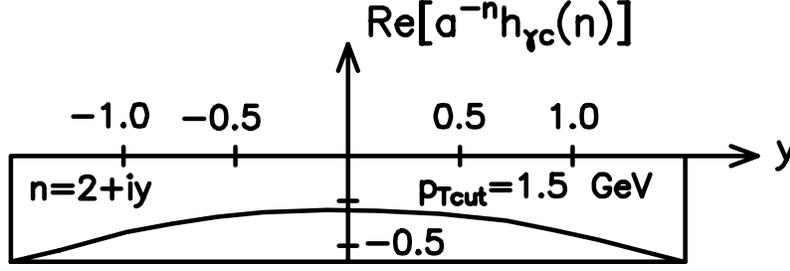
Figure 6: The behavior of the n–th moment of $h_{\gamma c}$ in the imaginary neighbourhood of $n = 2$ at $p_{Tcut} = 1.5$ GeV. The sensitivity of the cross section as regards the second moment has gone down further.

[23, 24, 25, 26] and even higher order QCD corrections are completely known [27, 28]. On the experimental side, there is the upcoming very promising experimental spin program of the Relativistic Heavy Ion Collider (RHIC) Spin Collaboration [29] at the Brookhaven National Laboratory. At RHIC both proton beams, with an average energy of 250 GeV each, will be polarized, using 'Siberian snakes' , to an expected polarization of about 70%. Due to the high luminosity of the order of $\sim 10^{32}$cm^{-2}s^{-2} (corresponding to an integrated luminosity of about 800 pb^{-1}) the polarized RHIC pp collisions will play a decisive role for measuring the polarized gluon density.

On the parton level, direct photon production is induced in lowest order by the annihilation of light quarks $q\bar{q} \rightarrow \gamma g$ and by the Compton scattering $qg \rightarrow \gamma q$. Among the two, the contribution from the annihilation process is small because it is a valence–sea scattering process. Since the Compton scattering dominates, the cross section is strongly dependent on the magnitude of $\delta g(x)$. Assuming the above luminosity at RHIC, a sensitivity of about 5% is expected for $\frac{\delta g(x)}{g(x)}$ [30]. If $\delta g(x)$ is large, one encounters large
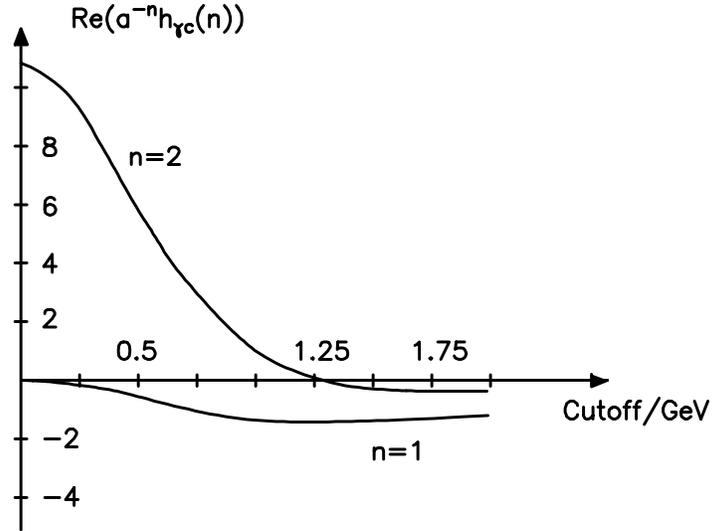
Figure 7: The p_{Tcut}–dependence of of $h_{\gamma c}^{(n)}$ for $n = 1$ and $n = 2$. The sensitivity as regards the second moment is strongly reduced as the p_{Tcut} is switched on. Unfortunately, the decrease of $h_{\gamma c}^{(1)}$ is not as strong as one might hope.

positive values of the spin asymmetry up to 50 percent. The form of $\delta g(x)$ can largely be reconstructed from the k_T–distribution of the direct photons which is given by the simple formula

$$\frac{d\sigma(PP \to \gamma X)}{da} = \sum_{qg,gq} \int_a^1 dw \delta g(w) \int_{\frac{a}{w}}^1 dx \frac{d\hat{\sigma}(qg/gq \to \gamma q)}{da} \sum_q Q_q^2 \delta q(x) \tag{13}$$

where $a = a_{pp} = \frac{4k_T^2}{S}$ is the rescaled transverse momentum and the parton level k_T–distribution $\frac{d\hat{\sigma}}{da} = \frac{1}{2|c|xw} \frac{d\hat{\sigma}}{d|c|}$ is given by

$$\frac{d\hat{\sigma}(qg(gq) \to \gamma q)}{dc} = \frac{\pi\alpha\alpha_s}{6xwS} [\frac{1}{\frac{1}{2}(1 \pm c)} - \frac{1}{2}(1 \pm c)] \tag{14}$$

The upper and lower sign stand for the processes $qg \to \gamma q$ and $gq \to \gamma q$, respectively. c is the cosine of the parton–parton scattering angle in the parton cms. Note that the photon transverse momentum $k_T^2 = \frac{1}{4}\hat{s}(1 - c^2) = \frac{1}{4}xwS(1 - c^2)$ is identical for parton and proton level.

The question then arises, how sensitive this measurement is to the first moment of $\delta g(x)$ as compared to the higher moments. This question can be answered in a similar fashion than for charm photoproduction. The main ingredient is again the Mellin theorem which can be applied to the photon k_T–distribution Eq. (13) because $\frac{d\sigma(PP \to \gamma X)}{da_{pp}}$ is of the form $\int_{a_{pp}}^1 \frac{dw}{w} \delta g(w) H_{pp}(\frac{a_{pp}}{w})$ where $H_{pp}(z) = H_{pp}^+(z) + H_{pp}^-(z)$ is the sum of contributions from the processes $qg \to \gamma q$ and $gq \to \gamma q$.

$$H_{pp}^\pm(z) = \frac{\pi\alpha\alpha_s}{12a_{pp}S} \int_z^1 \frac{dx}{x} [Q_u^2 \delta u(x) + Q_d^2 \delta d(x)] \frac{z}{x|c|} [\frac{1}{\frac{1}{2}(1 \pm c)} - \frac{1}{2}(1 \pm c)] \tag{15}$$

where $c = \pm\sqrt{1 - \frac{z}{x}}$. Therefore, according to the convolution theorem, the contribution of Δg to the cross section is given by the first moment of $H_{pp}(z)$.
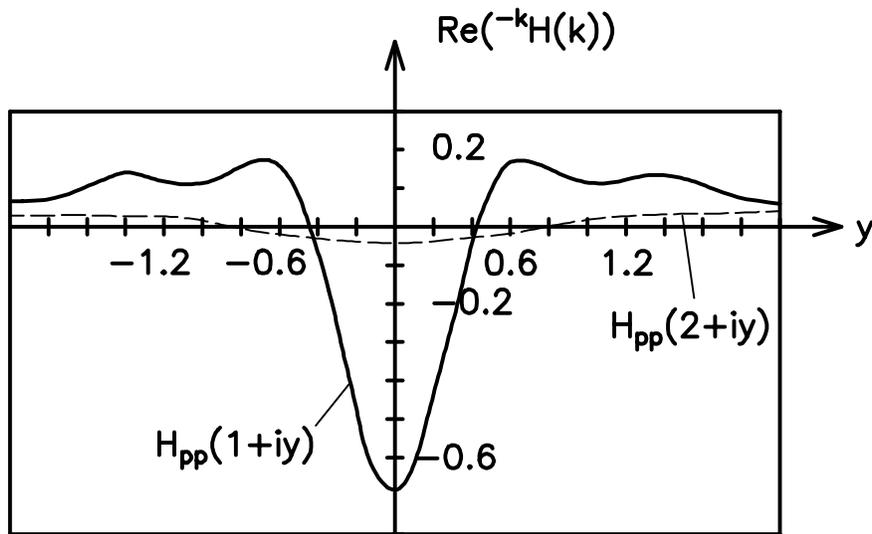
Figure 8: The behavior of the n–th moment of H_{pp} in the imaginary neighbourhood of $n = 1$ and $n = 2$. The dominance of the first moment over the higher moment is clearly visible. It is much more pronounced than in any of the photoproduction cases considered before. Numbers for H_{pp} are given in units of $\frac{\pi \alpha \alpha_s}{6 a_{pp} S}$.

Introducing $u = \frac{z}{x}$, it can be seen that the moments of $H_{pp}(z)$ factorize in a very simple way

$$H_{pp}^{(n)} = \frac{\pi\alpha\alpha_s}{6a_{pp}S} \int_0^1 du\, u^{n-1} (\frac{u}{2} + 2) \int_0^1 dx\, x^{n-1} [Q_u^2 \delta u(x) + Q_d^2 \delta d(x)]$$

(16)

Just as in the case of charm photoproduction we can now study the behavior of $H_{pp}^{(n)}$ in the imaginary neighbourhood of $n = 1$ and $n = 2$, and in principle also for higher moments. The results are shown in Fig. 8 and exhibit a distinct peak at $n = 1$ whereas the sensitivity to the second moment is much weaker (almost flat). The peak is in fact much more pronounced than any of the peaks which were found in the last section for charm photoproduction. From the figure we calculate the 12–sensitivity defined in the last section to be $R_{12} = \frac{0.175}{0.042} = 4.2$. Similar large values arise for $R_{13} = \frac{R_1}{R_3}$, $R_{14} = \frac{R_1}{R_4}$ etc. Therefore one concludes that this process is much more sensitive to Δg than polarized charm photoproduction.

Higher order QCD corrections to the process $pp \to \gamma X$ involving polarized proton beams have been calculated in refs. [27, 28]. It is possible to extend our method to include the higher order effects because the results of the calculation are given in the form of a K–factor to the k_T distribution. The higher order corrections turn out to be positive and quite large. Nevertheless, the ”12–significance” is hardly modified because the higher order effects cancel between the numerator R_1 and denominator R_2 (cf. the end of section 3).

There is another RHIC process which will serve to give information about $\delta g(x)$, namely the production of heavy quarks in the collision of polarized protons [29]. Using our method it is possible to determine its sensitivity to Δg. At low and intermediate values of the heavy quark p_T, the cross section is dominated by the subprocess $gg \to Q\bar{Q}$. Correspondingly, the

moments of the cross section are generically of the form $\sigma^{(n)} = [\delta g^{(n)}]^2 \hat{\sigma}^{(n)}$. Due to the quadratic dependence on $\delta g^{(n)}$ one concludes that this process is very sensitive to Δg if Δg is large and not very sensitive to Δg if Δg is small. The 'weight' $\sigma^{(1)}$ can be calculated to be relatively large. However, a quantitative comparison to the results obtained above is not possible, because direct photon production is linear in $\delta g(x)$ and heavy quark production is quadratic in $\delta g(x)$ and because the magnitude of Δg is not known.

5. Summary

Present experimental data do not really give good information about the polarized gluon density $\delta g(x)$, [1] because in DIS the gluon is a higher order effect. However, in the near future several experiments at BNL, CERN and SLAC will directly test for $\delta g(x)$. In this paper we have examined the question of how sensitive these experiments will be to Δg as compared to the higher moments. We have found that for the determination of Δg the RHIC experiment is suited much better than the charm photoproduction process. This arises due to the property of the matrix elements and is not just because RHIC allows to study smaller values of x, but because the full cross section gets a relatively larger contribution from the first (as compared to the higher) moments. Using our method it is in principle possible to separately analyze the sensitivity to any moment, but we have restricted ourselves to the first moment because of its outstanding importance. For comparative reasons we have included in our analysis the second moment as well.

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[1]It has been attempted to determine the magnitude of the first moment Δg [31] from the DIS data. As an essential input the presently known Q^2–dependence of the data (both theoretically and experimentally) was used, in which the polarized gluon plays some role. On the basis of this, a value $\Delta g \approx 1.3 \pm 0.5$ was quoted. Our opinion is that the error here is underestimated but the order of magnitude of Δg looks reasonable. Note there are also attempts to determine Δg from meson properties, e.g. recently ref. [32].

In principle, to determine the first moment Δg precisely it is necessary to know the small–x behaviour $\delta g(x)$. In reality, all processes allow to determine $\delta g(x)$ only down to some lower limit $x \geq a$. For example, in charm photoproduction one has $x \geq \frac{4m_c^2}{S_\gamma}$ for kinematical reasons. Thus, increasing the photon energy from 30 ($x \geq 0.12$) to 300 GeV ($x \geq 0.012$), one can penetrate deeper into the small–x region, and so on. Clearly, working at a fixed photon energy (fixed a), one will not get any information on $\delta g(x \leq a)$. As a consequence, the moments determined from $\delta g(x)$ will be "cut moments". For example, the first cut moment will be $\Delta_a g := \int_a^1 dx \delta g(x)$. We want to make clear that the whole analysis presented in this article refers to the first cut moment. One has $a = \frac{4m_c^2}{S_\gamma}$ and $a = 4\frac{m_c^2 + p_{Tcut}^2}{S_\gamma}$ for charm photoproduction with and without p_{Tcut}, respectively. Thus, application of the p_T–cut effectively increases a and shrinks the x–region which can be studied. However, we have seen in Sect. 3 that one has to apply a p_T–cut because otherwise there would be no sensitivity to the first moment at all. On the other hand, our results apply to cut moments only up to terms of order $O(a)$ because the convolution formula is true strictly speaking for moments and not for cut moments. In charm photoproduction we studied the case $a = 0.12$ (cf. Figs. 1–6), so that we expect corrections of about 10% to our results.

The situation is much better for the production of direct photons at RHIC. First of all, as shown in Sect. 4, there is a stronger sensitivity to the first moment as in charm photoproduction, and secondly, it will be possible to penetrate deeper into the small–x region, because in this case $a = \frac{4k_T^2}{S}$. It is true that hard photons with transverse energies less than a few GeV are difficult to identify, but even if one considers only photons with $k_T \geq 10$ GeV, one still has $a \geq 0.0016$, and the first cut moment $\Delta_{0.0016} g$ will contain a lot of contributions from the small–x regime.

Recently, the small–x behavior of the polarized parton densities have attracted some attention. We think that it can safely be stated that the polarized gluon density behaves much more moderate than the unpolarized one. It can be shown on rather general grounds that $\delta g(x)$ is one power of x less singular than $g(x)$. Some results in the literature [33] indicate that logarithms of x may be present and consequently predict a relatively strong rise of $\delta g(x)$ as $x \to 0$. These results are not really indicative for the first moment, because they neglect certain contributions at very small x which are assumed to compensate those logarithms [34]. In fact, the first moment of g_1 would not exist if one would take the results of [33] literally. They are, however, a good starting point for measurements of $g_1(x)$ in the small–x regime feasibale at HERA. Maybe, they will already be seen at RHIC.

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