The finite element method (FEM)to Finding the Reverberation Times of Irregular Rooms

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Abstract

In this paper we applied a finite element method to finding the effects on the reverberation times of common irregularities like curved surfaces, non-parallel walls and large open-walled ante-rooms, found in auditoria. The number of modes having a reverberation time in a specified time interval is expressed as a function of the total allowed degrees of freedom and it is shown that even when the number of degrees of freedom of the model is large there is, in general, no one dominant group. Curved surfaces in particular lead to a situation where some modes have very long reverberation times, leading to bad acoustics. In such situations a knowledge of the offending mode shapes give an indication on where to position absorptive material for optimum effect.

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I. PRELIMINARY TOPICS

The reverberation time is one of the most important figures to characterize the acoustic behaviour in a room, which is relevant to the rate of the energy dissipation, and is defined by the time in which the pressure in a room decays into $\frac{1}{1000}$th after the sound source is terminated. The most well-known formula for this are the ones given by Sabine [1] and Eyring [6], which are valid under certain conditions when an ergodic state, i.e. a state that is aperiodic and (non-null) persistent, is possible and determined depending on the volume of the room, the average absorption coefficient and the surface area of the wall. In mathematically languages the fundamental theorem of Markov chains stated that the following properties hold for any finite, irreducible, aperiodic Markov chain[7]:

1. All states are ergodic
2. There is a unique stationary distribution. This distribution gives nonzero probability to each state.
3. Each state are persistent and the expected return time is the inverse of the probability given that state by the stationary distribution
4. If $N(i,t)$ is the number of visits to state i in t steps, then the limit of $\frac{N(i,t)}{t}$ as t goes to infinity is the probability given to state i by the stationary distribution

Traditionally the theory of room acoustics is divided into two parts: so-called *small room* acoustics which is treated in a similar way to any linear system having a finite number of degrees of freedom and, the acoustics of *large rooms*, usually analyzed with Sabine's geometric theory [1] which assumes a uniform diffuse sound field, the energy of which is controlled by the power supplied and by the rate of absorption at the bounding walls. The distinction between a small room and a large room is not so much dependent upon the physical dimensions, but rather on how these compare with the wavelength at a particular frequency. If the wavelength is larger or at least of the same order of magnitude as the room dimensions, then small room acoustics applies and an excellent model can be obtained by using a finite number of degrees of freedom governing the participating modes. Small room acoustics has been successfully applied to the understanding of how sound behaves in the passenger space of a vehicle in which the sources of sound are predominantly of a low frequency. On the other hand, when the wavelength is small compared with the dimensions of the room, then tens of thousands of modes can be involved. Geometric theory does not allow for any wave motion. It reduces the system with millions of degrees of freedom to a very simple equation giving the reverberation time in terms of the room volume, the surface area, the mean absorption coefficient and the speed of sound. The reverberation time is perhaps the single most important quantity relating to the acoustic quality of a room. For most purposes Sabine's equation will give a reliable estimate of its value, though its real importance is in identifying parameters which govern the sound quality of a room and hence it gives a guide on what is necessary to make corrective changes. Nevertheless, it is known that there are situations where poor acoustics due to persistent echoes and focusing of sound where the diffuse geometric theory of Sabine can't give an indication of how to solve the problem, although in the latter case ray theory gives a qualitative understanding. Problems such as flutter echoes and focusing have been discussed in the classical paper of Morse and Bolt [2], in the text by Morse and Ingard [3], and in the established work of Knudsen [4]. However, when these were prepared only simpler geometries could be considered in detail by the then purely analytical procedures. Predictions for the irregular room were then made using perturbations about a regular room and were intuitive, and qualitative at best. A numerical approach will more likely give better answers. Acoustic finite element procedures [5] are now well established for analyzing the small room acoustics of irregular enclosures. Firmly based on the approximation to the wave equation, the method can correctly model the dynamics, including resonance in the frequency domain, and wave reflections in the time domain. The only restrictions to the method appears to be in the size of computer available for modelling the room. However, dynamic models involving several thousands of degrees of freedom are currently common. These models can then be used to bridge the region between the two extremes of small and large rooms. Another paper [8] using the finite element models has given a simple procedure for calculating the decay constants for regular rooms where the wall effect was specified in terms of its normal impedance rather than the simpler Sabine absorption coefficient. It was shown that the decay constants, and hence the reverberation times, were dependent upon a *mode type factor* introduced in reference [2],[9–11]. In general there were as many decay constants as there were modes, but for regular rooms with uniform impedance on one or more walls then the reverberation times fall into several groups depending upon the mode type factor. The dominant group, i.e., one containing most number of modes with a common reverberation time was the oblique mode group. However, this was by no means a general conclusion: *for rectangular* *rooms with a small patch of absorption there was a large number of different reverberation times, cylindrical rooms and rooms with non-parallel walls had modes with extremely low decay constants which would lead to poor acoustics.* In the present paper the old works are extended to some common geometries that may occur in Auditoria. The information on the decay constants is expressed in the form of graphs giving the mode fraction having a group reverberation time. As the models have a large number of degrees of freedom they give accurate information on the room behavior in the transition region between small and large room acoustics.

II. PHYSICAL THEORY

In physics, the acoustic wave equation governs the propagation of acoustic waves through a material medium. The form of the equation is a second order partial differential equation. The equation describes the evolution of acoustic pressure p or particle velocity **u** as a function of position **r** and time t. A simplified form of the equation describes acoustic waves in only one spatial dimension (position x), while a more general form describes waves in three dimensions. For a small chunk of air there is a pressure differential. This leads to a net force that leads to a change in momentum of the air inside:

$$-\frac{\partial P}{\partial x} = \rho \frac{\partial u}{\partial t}$$

where P is pressure, u is the speed of the air molecules, and ρ is the average density of the air (think of the density before the wave begins). The left-hand-side of this equation is related to the difference in pressure and the right-hand-side is related to the change in momentum. A fluid can be compressed but it resists that compression. The parameter that describes this resistance is the Bulk Modulus, B.

$$\frac{\partial P}{\partial t} = -B \frac{\partial u}{\partial x}$$

Putting together the above equations through one gets the following equations:

$$\frac{\partial^2 P}{\partial t^2} = \frac{B}{\rho} \frac{\partial^2 P}{\partial x^2}$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{B}{\rho} \frac{\partial^2 u}{\partial x^2}$$

These are both in the form of wave equations. Thus both pressure and motion support waves and they both move at a speed of sound or propagation

$$v_{wave} = \sqrt{\frac{B}{\rho}}$$

In three dimensions Equation

$$\nabla^2 p - v_{wave}^{-2} \frac{\partial^2 p}{\partial t^2} = 0 \tag{1}$$

A. Solutions

The solutions are obtained by separation of variables in different coordinate systems. They are phasor solutions, that is they have an implicit time-dependence factor of $e^{i\omega t}$ where $\omega = 2\pi f$ is the angular frequency. The explicit time dependence is given by

$$p(r,t,k) = \text{Real}\left[p(r,k)e^{i\omega t}\right]$$

Here $k = \frac{\omega}{v_{wave}}$ is the wave number.

Cartesian coordinates

$$p(r,k) = Ae^{\pm ikr}.$$

Cylindrical coordinates

$$p(r,k) = AH_0^{(1)}(kr) + BH_0^{(2)}(kr).$$

where the asymptotic approximations to the Hankel functions, when $kr \to \infty$, are

$$H_0^{(1)}(kr) \simeq \sqrt{\frac{2}{\pi kr}} e^{i(kr-\pi/4)}, H_0^{(2)}(kr) \simeq \sqrt{\frac{2}{\pi kr}} e^{-i(kr-\pi/4)}.$$

Spherical coordinates

$$p(r,k) = \frac{A}{r} e^{\pm ikr}.$$

Depending on the chosen Fourier convention, one of these represents an outward travelling wave and the other an unphysical inward travelling wave

B. Finite element formulation of the acoustic wave equation

A finite element formulation of the acoustic wave equation which is covered in [1] leads to the equation of the form

$$A\ddot{\zeta} + B\dot{\zeta} + C\zeta = 0 \tag{2}$$

Here A, Band C are all symmetric matrices; A being found from the potential energy, C the kinetic energy and B from energy dissipated at the boundary walls ζ is the acoustic pressure response vector. It has been shown in [1] that if the system is lightly damped; i.e., if, Z the normal impedance, γ density and c the speed of sound, and ρ the ratio $\frac{|Z|}{\rho c} >> 1$, eq. (1) can be split into two equations: one governing the harmonic frequency ω given by

$$[C - \frac{1}{4}BA^{-1}B]\psi - \omega^2 A\psi = 0 \tag{3}$$

and the other, the decay constant m given by

$$B'\psi - 2mA\psi = 0 \tag{4}$$

where ψ is defined by the expression, $\zeta = \psi \exp(-imt \pm i\omega t)$ Here ω^2 is the eigenvalue corresponding to a given mode and ψ is the corresponding eigenvector. Matrix B' is given by

$$B' = (X^{-1})^T b X^{-1} \tag{5}$$

Where b is the modalized damping matrix, and X is the square matrix whose columns contain the eigenvectorsψ. From eq. (3) one can obtain

$$m_i = \frac{1}{2}\frac{\psi_i^T B'\psi_i}{\psi_i^T A\psi_i} \tag{6}$$

If the eigenvectors of the generalized eigenvalue problem defined by eq. (2) are normalized with respect to the matrix A then the term in the denominator of eq. (5), $\psi_i^T A\psi_i$ becomes unity. Thus the decay constant of a given mode depends on the square of the pressure distribution vector, ψ_i^2, along the absorptive surface; the greater this is, the greater the decay constant. This fact is exploited in this paper to demonstrate how the reverberation time (which is inversely proportional to the decay constant) can be controlled effectively.

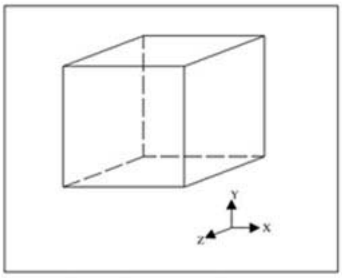

Fig. 1. Schematice of a rectangular room

FIG. 1. Schematic of a rectangular room

C. Effect of mesh size

The first step in any finite element application is the study of the effect of mesh dependency on the results. We used from a rectangular room of dimensions $4m \times 5m \times 6m$.

One of the walls parallel to x-direction is lined with absorptive material the absorptive properties of which were specified by the parameter, $\frac{|Z|}{\rho c} = 37$. The values on the abscissa of this figure are the mid-interval values of the reverberation time interval. One can see that modes having two different reverberation times are excited. The reverberation time for modes for which $n_x > 0$, where n_x is the mode number in the x-direction, coincides with the dominant peak, i.e., the first peak. Further, this peak also corresponds to Sabine's predictions. The reverberation time corresponding to the second peak in fact coincides with the reverberation time of modes for which $n_x = 0$. In obtaining the results only the first hundred modes were considered. The eigenvalues tend to be inaccurate beyond this value, because there are then less than five elements per wavelength. The latter being an empirical rule of thumb for good finite element modeling.

When more modes are included in modeling the system, the total number of modes for which $n_x > 0$ will be correspondingly higher [3]. The shifting of the peak occurs because, use of numerical methods, which are highly mesh dependent, lead to predicting higher decay constants. However, as the mesh is refined these decay constants tend to converge to a value lower than that for a coarse mesh. *The effect of refinement of mesh is to shift the reverberation times to higher values.* Also mesh refinement alters the mode fraction per group. If infinitely many modes are considered the number of oblique modes rises very sharply. In such a case, the mode fraction for the group which corresponds to the oblique modes, tends to unity.

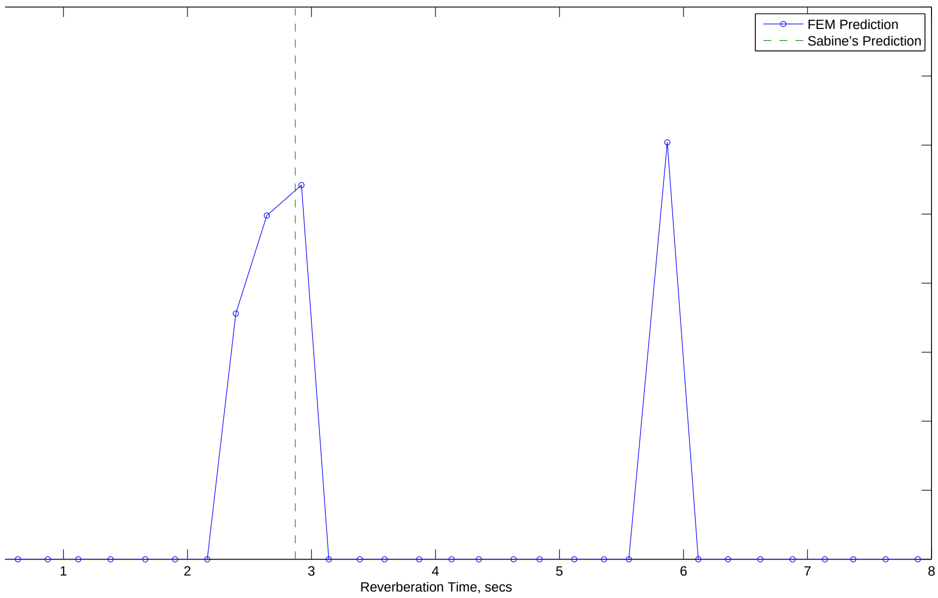

FIG. 2. Mode fraction distribution as a function of the reverberation time, for a rectangular room of dimensions $4m \times 5m \times 6m$ having lining distributed uniformly on one of the x-walls.

D. Applications

One would expect from Sabine's theory that a single reverberation time exists for an uncoupled room. However, we can observe from numerical analysis that this is not true.

Modes with two different and distinct reverberation times are excited. To further substantiate one can obtain the results for the rectangular room where the absorptive lining is on two orthogonal walls, instead of on one wall.

Fig3.Effect of the number of modes on the mode fraction distribution.

Fig. 6. Effect of using linear and quadratic elements on the mode fraction distribution.

1. *mesh's structure*

A mesh with linear elements and 1287 degrees of freedom was considered for this purpose. Four different peaks can be observed . The absorptive properties of the lining were specified by, $\frac{|Z_x|}{\rho c} = 37$ and $\frac{|Z_y|}{\rho c} = 20$, respectively, where Z_x and Z_y are the normal impedances in the directions x and y, respectively. The dominant peak again corresponds to the value predicted

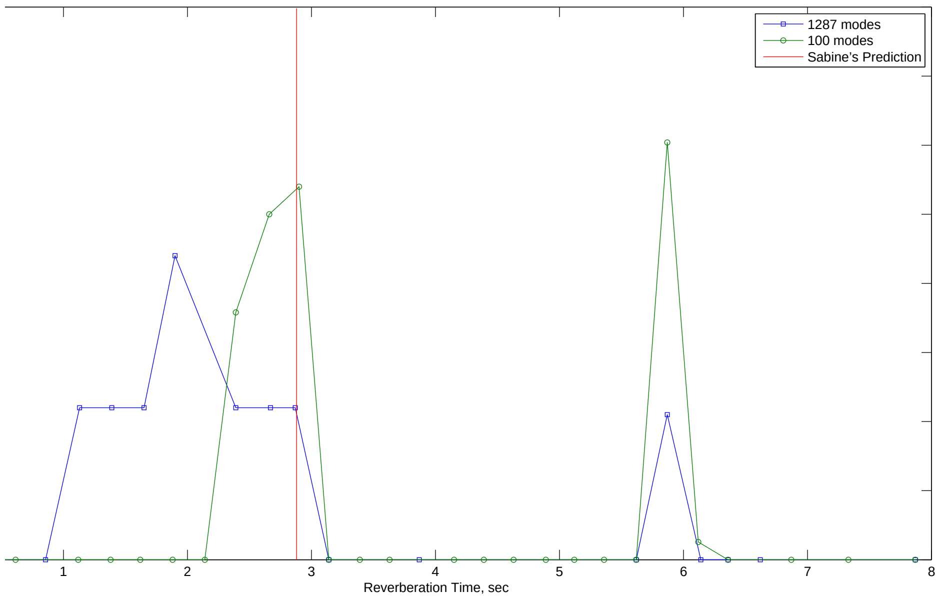

FIG. 3. Effect of the number of modes on the mode fraction distribution.

according to Sabine's theory. As in the previous case the dominant mode corresponds to the case for which $n_x > 0$ and $n_y > 0$. Here, n_y is the mode number in the y direction. The second peak corresponds to modes for which $n_x = 0$ and $n_y > 0$. The third peak corresponds to modes for which $n_y = 0$ and $n_x > 0$. Similarly, the fourth peak corresponds to modes for which $n_x = 0$ and $n_y = 0$.

To illustrate further the point that there is no single value for the decay constant, a rectangular room with a small absorptive patch applied to one of the corners was modeled.

A linear finite element mesh having 1287 degrees of freedom was used to model the system. Here again, it can be observed that there are numerous peaks. These peaks, however, cannot be easily classified as in rectangular rooms with uniformly distributed absorptive lining along the walls.

2. Numerical results

The centre of the radius of curvature of the barrelled ceiling lies just above the floor. It was decided to put the absorptive lining on the curved ceiling. The absorptive property of the material was specified, via. the parameter $\frac{|Z|}{\rho c} = 37$. A finite element mesh with

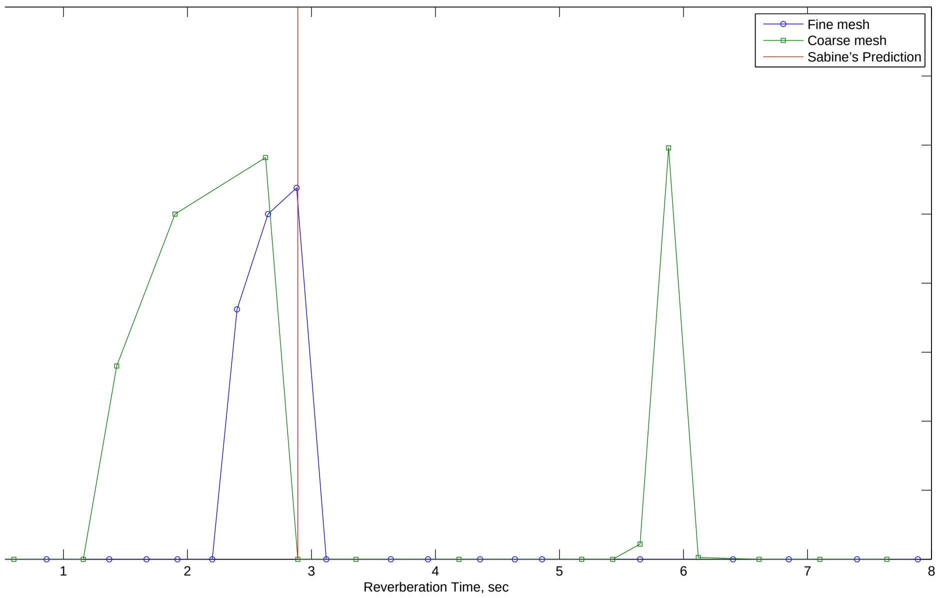

FIG. 4. Effect of refinement of the finite element mesh on the mode fraction distribution.

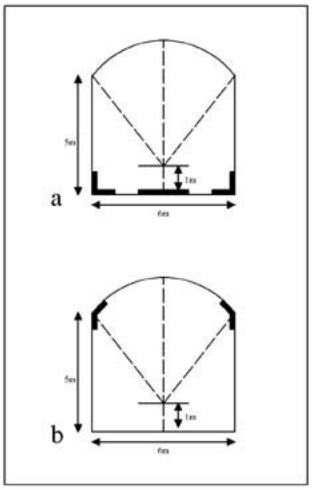

FIG. 5. Cross-sectional view of a room with a barreled ceiling. The ceiling is covered with absorptive material and the dark patches indicate effective positions for placing patches of absorptive material (a). Cross-secional view of a room with a barreled ceiling. The floor is covered with absorptive material and the dark patches indicate effective positions for placing patches of absorptive material (b).

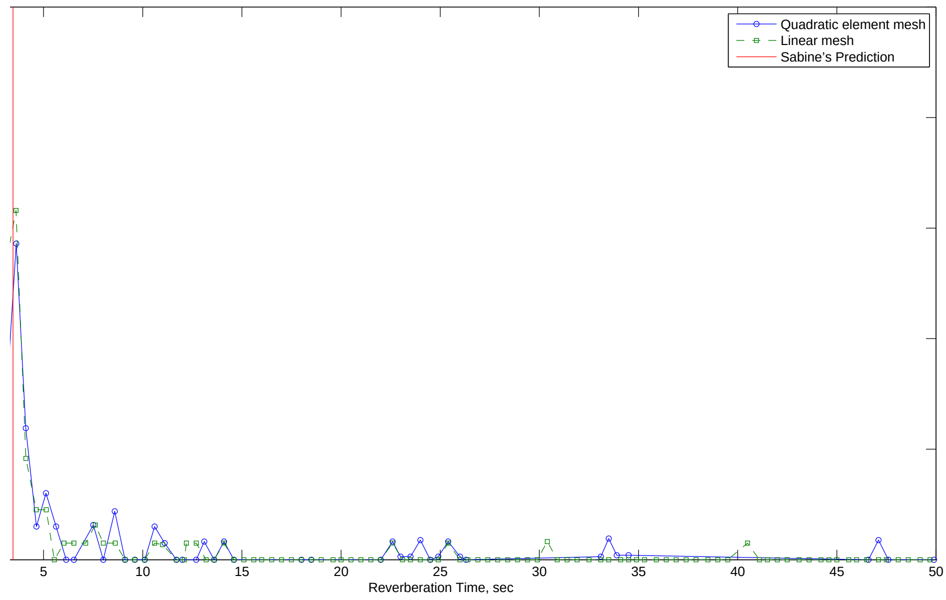

FIG. 6. Effect of using linear and quadratic elements on the mode fraction distribution.

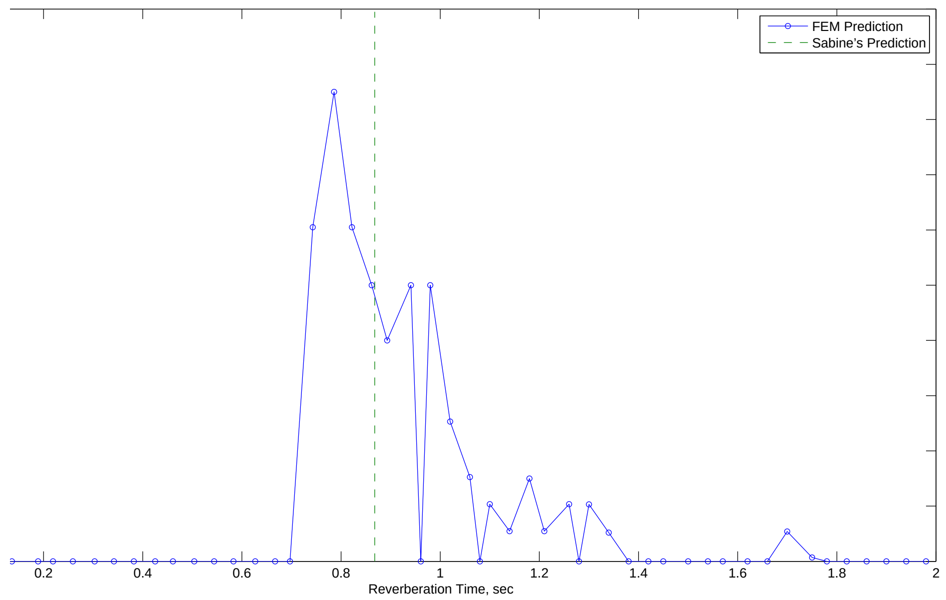

FIG. 7. Mode fraction distribution as a function of the reverberation time, for a rectangular room of dimensions 4 m x 5 m x 6 m having absorptive lining distributed uniformly on three orthogonal walls.

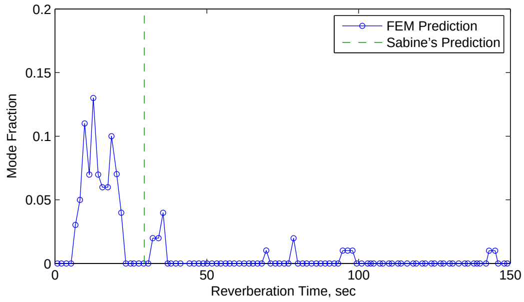

FIG. 8. Mode fraction distribution as a function of the reverberation time, for a rectangular room of dimensions 4 m x 5 m x 6 m having absorptive patch at one of its corners.

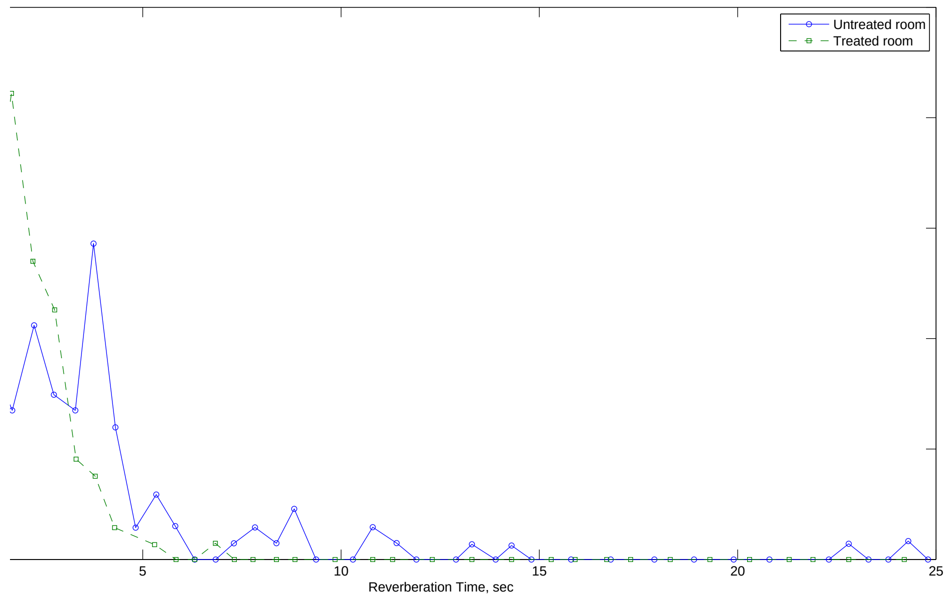

FIG. 9. Effect of treating the room of configuration of Fig. 5(a) with absorptive patches, on the reverberation time.

quadratic elements was used for modelling this room.

It can be noticed that more modes with high reverberation times appear here than those corresponding to the regular rectangular rooms. Also, the reverberation time for some of the modes can be seen to be high. These modes constitute what is known as the acoustic defect. If not suppressed, these modes tend to mask the sound excited at other frequencies, as they have longer reverberation times. This will result in certain frequency components being absent in such rooms which would make such rooms unsuitable as concert halls, unless

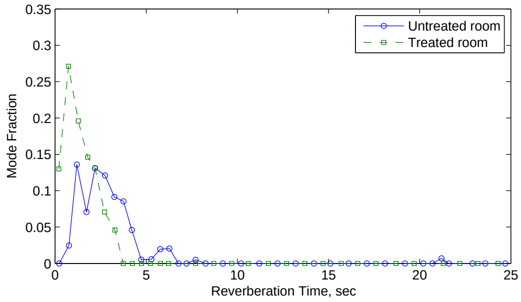

FIG. 10. Effect of treating the room of configuration of Fig. 17 with absorptive patches, on the reverberation time.

otherwise these defects are rectified. The high reverberation time modes are also primarily responsible for flutter echoes prevalent in rooms with parallel walls.

A study of the modal vectors of all the modes exhibiting very high reverberation time revealed that the pressure maximum occurred in most of these modes at the corner where the floor and the side walls intersect, and also near the middle of the floor. One can notice that most peaks, especially the longer reverberation time peaks, disappear. To illustrate this point further, the reverberation times for the untreated as well as the treated rooms are shown in Table I.

It appears that if one were to put the absorptive lining along the floor instead of along the ceiling, the modes with long reverberation times would cease to exist.

Even here, modes with longer reverberation times appear. The eigenvectors of these modes indicate that the modal pressure tends to be maximum at the corners where the side walls and the curved ceiling intersect. The dashed line in this figure corresponds to the mode fraction distribution of the room treated for the acoustic defects. Modes having long reverberation times disappear upon acoustical treatment. The reverberation times of such a room with and without acoustical treatment are illustrated in Table II.

It can be seen from the previous two tables that one could alter the reverberation times of the room quite substantially.

was also studied. For modelling this room, a finite element mesh having quadratic elements with a total of 156 degrees of freedom was used. This section is quite typical of concert halls and theaters. The corresponding results are shown in Table 2.

Here the absorptive lining was placed on the ceiling. The number of modes having long reverberation times is quite high in this case. This room can be treated acoustically to

TABLE I. Effect of damping treatment on the reverberation times of modes having longer reverberation times for the configuration of Fig. 1 with the absorptive lining on the curved ceiling..

Frequency $f = \omega/2\pi$	Reverberation time before damping treatment (sec) (Rad/s)	Reverberation time after damping t (Rad/s)
29.4	10.6	5.49
115.9	11.2	6.90
189.4	10.7	4.52
219.6	14.5	2.91
261.0	47.2	3.63
275.3	13.5	2.28
308.8	65.7	4.21
361.3	34.0	3.59
405.8	$1.39.10^5$	1.19
423.4	670.3	3.8
450.4	25.9	0.54
487.0	544.0	0.345
520.4	22.6	0.341

TABLE II. Effect of damping treatment on the reverberation times of modes having longer reverberation times for the configuration of Fig. 1 with the absorptive lining on the floor.

Frequency $f = \omega/2\pi$	Reverberation time before damping treatment (sec) (Rad/s)	Reverberation time after damping t (Rad/s)
517.3	14.0	0.144
517.3	51.8	0.145

reduce the reverberation times of such modes by properly placing the absorptive material at locations shown . To further demonstrate the usefulness of the finite element method in handling rooms of complex shapes, a room with domed ceiling as shown with absorptive lining along the curved wall, was modeled. .

For modelling this room, a finite element mesh with quadratic elements and 364 degrees of freedom was used. Results obtained for this room are shown . Here again more modes

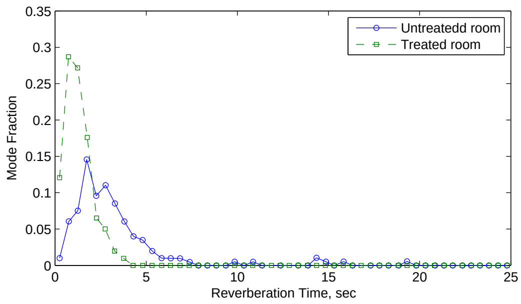

FIG. 11. Effect of treating the room of configuration with absorptive patches, on the reverberation time

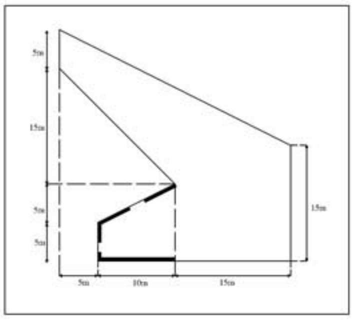

FIG. 12. Cross-sectional view showing the elevation of a simple Auditorium. The dark patches indicate locations where placing of absorptive lining would dampen the modes having long reverberation times.

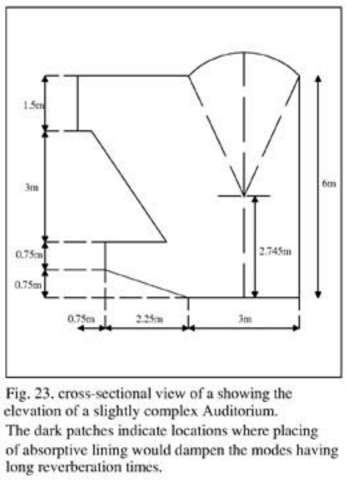

FIG. 13. Cross sectional view of a room showing the elevation of a slightly complex Auditorium. The dark patches indicate locations where placing of absorptive lining would dampen the modes having long reverberation times.

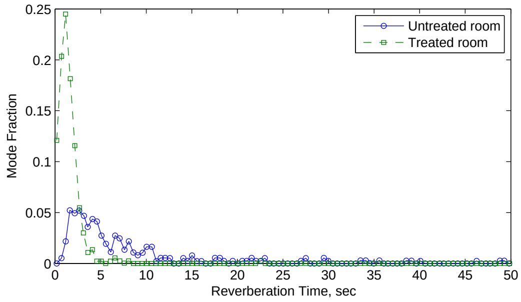

FIG. 14. Effect of treating the room of configuration with absorptive patches, on the reverberation time.

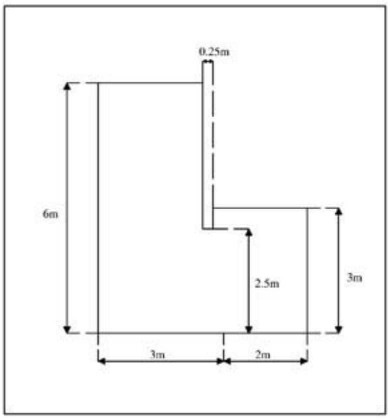

Fig. 25. Elevation view of a simple coupled room

FIG. 15. Elevation view of a simple coupled room.

with longer reverberation times appear. The effective treatment for these modes would be to put an absorptive patch at locations shown . The dashed line shows the effectiveness of such a damping treatment.

3. Coupled rooms

For coupled rooms, Sabine's theory predicts a single reverberation time which is a function of the volume as well as the lined surface of both the rooms.

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A finite element mesh with quadratic elements and having 213 degrees of freedom was used to model the situation. The corresponding results are depicted in next Fig.

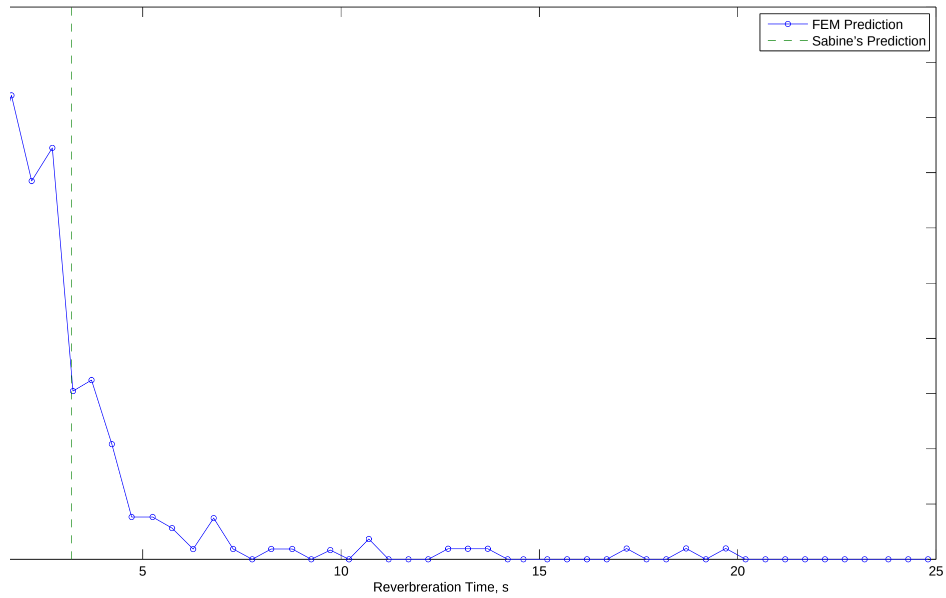

FIG. 16. Mode fraction distribution as a function of the reverberation time for the coupled room of configuration shown in Fig. 25.

Even with this many degrees of freedom there are at least ten different mode groups having a wide range of reverberation times from 0.5 to 10 seconds The use of Sabine's equation in this situation is then inappropriate.

III. DISCRETE HYYGENS MODEL APPROACH TO REVERBERATION IN A ROOM

In [12] the authors discussed the fundamental concept and the transmission line matrix approach as a discrete Huygens' modeling. The transmission-line matrix (TLM) modeling is an alternative to the Huygens' modeling, in which electrical impulse scattering are traced on a transmission-line network. Later the same authors discussed [13] the validity and capability of the modeling by presenting applied examples, the first being the simulation of the sound behavior in a room. The medeling was also valid when the direction of the time is reversed. Also the application was extended to the identification of the sound source location and intensity based on the measured data observed at the locations surrounding the sources, and also to the identification of an object shape from the response data observed at the locations surrounding it when a certain emanation is made. Further it has pointed out that the potential of this approach to acoustical problems [14, 15]. Transmission-line matrix modelling was originally developed by Johns and Beurle [16, 17] to solve electromagnetic wave problems. The method was then extensively developed for that purpose, which was well described in the literature [18, 19]. The explanation of the process of the discrete Huygens' modelling is possible without referring to the equivalent electrical circuit network by knowing the equivalent pressure and the volume velocity continuity at the node in acoustical network. Acousticians have preferred to use equivalent electrical circuit networks for the acoustical analysis so that the use of the transmission-lines is not foreign to them. One particular feature of the method is that the network is solved in discrete time domain to the impulse excitation, which provides the full wave analysis.

IV. SUMMARY AND CONCLUSIONS

Sabine's equation is only a reliable indicator of reverberation time, however, when the wavelength of the sound is much smaller than the dimensions of the absorbing patch and the sound field approaches a diffuse condition. At low frequencies in rectangular rooms and non-rectangular rooms other groups of modes exist having a significant mode fraction and different reverberation times which have a dramatic influence on the sound decay behaviour. This latter situation can be well simulated with a finite element model. The use of finite element method for predicting the reverberation times, and also for predicting the acoustical defects has been illustrated. Further usefulness of the method was demonstrated by showing how these defects can be corrected by the proper placement of absorptive patches. By using the finite element method one could study various possible room configurations an architect envisages, and choose the best possible shape which befits the objective for which the room is being designed. Modal pressure distribution also yields information on the locations of pressure minima. Such locations, if not avoided, can lead to poor acoustics for the audience. In this paper the dynamic characteristics of a room have been represented with a model having a finite number of degrees of freedom. If this number corresponds to the modes of vibration which a room has below a specified frequency, then the model accurately represents the essential characteristics of the room. When a patch of absorbing material is placed on one of the walls then each mode has a different decay rate and hence different reverberation time. If the absorption is made to cover completely one or more walls then there are groups of modes which have a similar reverberation time. For a rectangular sectioned room with uniformly distributed absorption the group with the largest mode fraction corresponds to the so-called oblique modes, where the reverberation time can be estimated using Sabine's equation. The mode fraction of this group becomes more dominant as the number of degrees of freedom is increased.

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