SOME STUDIES ON K-ESSENCE LAGRANGIAN

Somnath Mukherjee
Department of physics, Dharsa Mihirlal Khan Institution(H.S),New G.I.P Colony,Howrah-711112. *sompresi@gmail.com*

Abstract

It has by now established that the universe consists of roughly 25 percent dark matter and 70 percent dark energy.Parametric lagrangian from an exact k-essence lagrangian is studied of an unified dark matter and dark energy model.

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1.INTRODUCTION

It is more or less observationally correct that our universe is expanding.The main search is for the reason behind this expansion,with different proposed cosmological models of dark matter and dark energy.[1,2]it has been shown that it is possible to unify both these components into a single scalar field model with the scalar field ϕ having a non canonical kinetic term.these is known as k-essence fields.the lagrangian for these k-essence field is given by

$$L = V(\phi)F(X) \ \ ; \ \ X = \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi \qquad (1a)$$

Further

$$\rho = V(\phi)[F(X) - 2XF_X] \qquad (1b)$$

where ρ is an energy density.

Considering the Robertson- Walker metric of the form

$$ds^2 = c^2dt^2 - a^2(t)[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + sin^2\theta d\phi^2)] \qquad (2)$$

where symbols has an usual meaning.The zero zero component of Einstein's equation is given as

$$R_{00} - \frac{1}{2}g_{00}R = -\frac{8\pi G}{c^4}T_{00} \qquad (3)$$

This gives with the metric(2)

$$\frac{k}{a^2} + H^2 = \frac{8\pi G}{3}\rho \qquad (4)$$

Considering scaling relation[1]

$$XF_X^2 = Ca^{-6}$$

Considering $k = 0$ and using$(1a)$,$(1b)$,(2),(3),(4) an exact expression of the lagrangian is obtained as

$$L = 2\sqrt{C}\sqrt{X}a^{-3}V(\phi) + (\frac{3}{8\pi G})H^2 \qquad (5)$$

Homogeneity and isotropy of spacetime imply$\phi(t, x) = \phi(t)$ therefore (5) becomes

$$L = -c_1\dot{q}^2 - c_2V(\phi)\dot{\phi}e^{-3q} \qquad (6)$$

where $q(t) = \ln a(t)$,$c_1 = 3(8\pi G)^{-1}$ and $c_2 = 2\sqrt{C}$, Eqation (6)is further converted to a parametric equation[4]

1.1EquationofStateParameter

The parametric lagrangian is given by:

$$L = -\frac{M}{2}\dot{q}^2 + \frac{M}{2}\Omega^2(t)q^2$$

where

$$M = \frac{3}{8\pi G}$$

and

$$\Omega^2(t) = -12\pi Gg(t) \qquad (1)$$

the solution of euler lagrangian equation with this lagrangian is

$$q(t) = \ln a = A_0e^{ut}\cos\omega t + B_0e^{ut}\sin\omega t$$

The energy density is given by:

$$\rho = \frac{3}{8\pi G}H^2 = \frac{3}{8\pi G}\dot{q}^2 \qquad (2)$$

Pressure is given by:

$$P = L = -\frac{M}{2}\dot{q}^2 + \frac{M}{2}\Omega^2(t)q^2$$

or

$$P = -\frac{\rho}{2} + \frac{3}{16\pi g}\Omega^2(t)q^2$$

or

$$P = -\frac{\rho}{2} - \frac{3}{16\pi g}\Omega^2(t)q^2$$

Equation of state parameter:

$$W = \frac{P}{\rho} = -\frac{1}{2} + \frac{3}{16\pi g}\Omega^2(t)q^2\rho^{-1}$$

substituting (1)and (2):

$$W = -\frac{1}{2} - 6\pi G g(t)\frac{q^2}{\dot{q}^2} \qquad (3)$$

It shows that W is negative ,consistance with dark matter equation of state. For a particular very very small time scale given by the condition:

$$t = \frac{1}{\sqrt{12\pi G\sqrt{C}A_1\alpha(1+\beta)}}$$

$$W = -1$$

thus (3),for a particular choice of time scale leads to dark matter equation of state.

1.2Deacceleration Parameter

Deacceleration parameter is given by:

$$Q = -\frac{a}{\dot{a}^2}\ddot{a}$$

or

$$Q = -\frac{a^2}{\dot{a}^2}\frac{\ddot{a}}{a} = -\frac{\ddot{a}}{aH^2} \quad (4)$$

where $H = \frac{\dot{a}}{a}$ Hubble parameter

From Friedmann solution of Einstien's equation with $k = 0$ for observational flat universe:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho \quad (5)$$

or

$$H^2 = \frac{8\pi G}{3}\rho \quad (6)$$

From Friedmann's another solution of Einstien's equation with $k = 0$ for observational flat universe:

$$\frac{\dot{a}^2}{a^2} + 2\frac{\ddot{a}}{a} = -8\pi GP \quad (7)$$

Putting (5)in (7):

$$\frac{\ddot{a}}{a} = -4\pi G(P + \frac{\rho}{3}) \quad (8)$$

Putting (8)and(6)in (4):

$$Q = \frac{3}{2}(\frac{P}{\rho} + \frac{1}{3})$$

or

$$Q = \frac{1}{2} + \frac{3}{2}W$$

Since $P = W\rho$. Hence for $W = -1$,$Q = -1$, consistence with the accelerated expansion of the universe.

1.3Slow Roll Parameter

The K-essence potential chosen:

$$V(\phi) = \frac{A_1}{\phi + A_2} \tag{9}$$

where $A_1 and A_2$ are constants determined from observational cosmology The slow roll parameters are given as:

$$\epsilon(\phi) = \frac{1}{2}\frac{(V'(\phi))^2}{(V(\phi))^2} \tag{10}$$

$$\eta(\phi) = \frac{V''(\phi)}{V(\phi)} \tag{11}$$

$$\zeta(\phi) = \frac{V'(\phi)V'''(\phi)}{V^2(\phi)} \tag{12}$$

Considering(9) the slow roll parameters are:

$$\epsilon(\phi) = \frac{1}{2(\phi + A_2)^2} \tag{13}$$

$$\eta(\phi) = \frac{2}{(\phi + A_2)^2} \tag{14}$$

$$\zeta(\phi) = \frac{6}{(\phi + A_2)^4} \tag{15}$$

Observational constraints in slow roll parameter states that:

$$\epsilon(\phi) < 1; \eta(\phi) < 1$$

Hence considering(14) and imposing observational constraint,we get:

$$\frac{2}{(\phi + A_2)^2} < 1$$

or

$$\phi > (\sqrt{2} - A_2) \tag{16}$$

For this constraint

$$\epsilon(\phi) < 1$$

is also satisfied. From observational constraint on inflationary universe

$$\frac{V^{\frac{3}{2}}}{V'} \approx 5 \times 10^{-4}$$

putting (9)in this constraint we get:

$$\phi \approx \frac{25 \times 10^{-8}}{A_1} - A_2 \tag{17}$$

Thus by comparing(16)and(17),we obtain a constraint :

$$A_1 < 17.6778 \times 10^{-8} \tag{17.1}$$

The number of e-foldings that can be calculated in the slow roll approximation is by the relation:

$$N(\phi) \approx \int_{\phi}^{\phi_f} \frac{V(\phi)}{V'(\phi)} d\phi$$

Considering (9) we get:

$$N(\phi) \approx -\int_{\phi}^{\phi_f} (\phi + A_2) d\phi$$

$$N(\phi) \approx \frac{(\phi^2 - \phi_f^2)}{2} + A_2(\phi - \phi_f) \qquad (18)$$

The condition for inflation is given by$\epsilon(\phi) < 1$.In the chaotic inflationary scenario the scalar fieldϕ was initially sufficiently large($\phi \geq m_{pl}$),then it evolves to the minimum of the potential.Inflation ends at a field value ϕ_f,where $\epsilon(\phi_f) = 1$ With this condition ,we get from (13):

$$\phi_f = \frac{1}{\sqrt{2}} - A_2 \qquad (19)$$

Smoothness on scales comparable to the current horizon size requires $N \geq 60$,which places a lower limit on the initial field value $\phi_0 \geq \phi_{60}$,where$N(\phi_{60}) \equiv 60$ Hence from(18),(19) and $N(\phi_{60}) \equiv 60$,we obtain constraint on constant A_2 as:

$$A_2 = -\phi_{60} \pm 10.97724$$

A standard slow roll analysis gives observable quantities n_s in terms of the slow roll parameters to first order as:

$$n_s = 2 - 3\frac{(V'(\phi))^2}{(V(\phi))^2} + 2\frac{V''(\phi)}{V(\phi)}$$

Considering the form of k-essence potential(9)

$$n_s = 2 - \frac{1}{(\phi + A_2)^2} \tag{20}$$

Substituting (17)in(20) we get:

$$n_s = 2 - \frac{A_1^2}{625 \times 10^{-16}}$$

substituting (17.1)

$$n_s < 1.4999 \tag{20.1}$$

Experimental results with COBE shows $n_s \approx 1$, thus (20.1) is close to consistence.

1.4The effective potential

The chosen k-essence potential is of the form:

$$V(\phi) = \frac{A_1}{(\phi + A_2)} = \frac{A_1}{A_2}(1 + \frac{\phi}{A_2})^{-1}$$

Under binomial expansion with possible choice of different constants,this can be written in the form:

$$V(\phi) = V_0 + c_1\phi + \frac{C_2}{2}\phi^2 + \frac{C_3}{3}\phi^3 + \frac{C_4}{4}\phi^4 + ...$$

This is well consistent with the generic expression for the effective potential,

$$V(\phi) = V_0 + \alpha\phi + \frac{m^2}{2}\phi^2 + \frac{\beta}{3}\phi^3 + \frac{\lambda}{4}\phi^4 + ...$$

Thus the chosen K-essence potential of the form $\frac{A_1}{(\phi+A_2)}$,with the values of the constants A_1 and A_2 determined in this paper,satisfies all the conditions for inflationary scenario.

1.5 Conclusion

An exact form of k essence leads to a cosmological parametric equation,which satisfies all the cosmological parameters.Hence an exact lagrangian as well as parametric lagrangian is very important to study the dark matter and dark energy senario of modern cosmology.

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