

Continuum and discrete features of Nature from a canonical science perspective

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From the ancient Greek atoms to quantum mechanics

Is Reality digital or analog? This is an old question, which was already formulated by the early Greek philosophers. These concluded that matter could not be infinitely divisible and introduced the idea of a minimal amount of matter: the atom. Indeed, the term «atom» comes from the Greek «ἄτομος», which means something that cannot be divided further.

In «*The Sceptical Chymist*» ROBERT BOYLE argued that matter was composed of various combinations of different atoms.¹ In the 17th and 18th centuries, chemists provided a scientific basis for the concept of atom—as discrete component of matter—by showing that certain substances could not be further broken down by chemical methods. The modern atomic theory, initiated by JOHN DALTON, elegantly explained new discoveries in the field of chemistry. However, during those centuries and in the following, physicists developed the physics of mechanics, electrodynamics, relativity, and thermodynamics according to the ideas of the continuum.

Of course, we now know that atoms can be divided further but, together with molecules, they continue being the discrete units that characterize the properties of chemical substances. Currently, the elementary particles of the Standard Model of physicists play the role of indivisible components of Nature.

The discovering of the divisibility of the atom introduced several revolutions in the physics. The first is evident; this discovering was against a doctrine that considered the atoms to be the smallest possible division of matter during nearly fifteen centuries—e.g., JAMES MAXWELL started his article «*Molecules*», published in 1873, with «*An atom is a body which cannot be cut in two*»—.² However, the main revolutions became from the new properties of the atomic models. One of the revolutions introduced by NIELS BOHR in 1913 was that the energy of an atom was discrete rather than continuous—the idea that light energy is emitted or absorbed in discrete amounts in black body radiation had been postulated by MAX PLANCK and ALBERT EINSTEIN few years before, although none of them addressed the incompatibility between a quantum of radiation and the continuum picture associated to classical mechanics and electrodynamics—.³

WERNER HEISENBERG, MAX BORN, & PASCUAL JORDAN extended the BOHR's idea of quantum jumps and developed the first formulation of quantum mechanics in 1925.³ This revolutionary theory considered that physical quantities others than energy could take discrete rather than continuous values. The principles of quantum mechanics were used to successfully model the atom.

From quantum mechanics to the quantum gravity fiasco

The success of quantum mechanics is not limited to atomic theory. The new mechanics was subsequently applied to small molecules and later to larger objects as clusters and solids. Its principles were adapted to fields, giving us a quantum field theory. Physicists like to say to broad audiences that quantum electrodynamics is the most stringently tested physical theory, with its predictions for the anomalous magnetic dipole moment of the electron agreeing with experiments to within one part in 10^{10} . That is an amazing achievement of quantum field theory, but the theory also meets with formidable difficulties. Already in 1978 PAUL DIRAC expressed his dissatisfaction for the limitations of quantum electrodynamics and for its inconsistency with quantum mechanics:⁴

«Most physicists are very satisfied with this situation. They argue that if one has rules for doing calculations and the results agree with observation, that is all that one requires. But it is not all that one requires. One requires a single comprehensive theory applying to all physical phenomena. Not one theory for dealing with non-relativistic effects and a separate disjoint theory for dealing with certain relativistic effects. [...] For these reasons I find the present quantum electrodynamics quite unsatisfactory.»

Modern textbooks on quantum field theory notice part of this inconsistency.⁵ A detailed and rigorous discussion of the inconsistency with quantum mechanics is given in a recent report.⁶

These formidable difficulties of quantum field theory are the outcome of certain incompatibilities between special relativity and quantum mechanics. However, modern physicists have ignored them and tried to quantize general relativity as well. Since the incompatibilities between general relativity and quantum mechanics are of a more fundamental kind, it is unsurprising that they have failed to quantize it, despite of five decades of efforts.

Contrary to statements done in both specialized literature and popular books, there is nothing exceptional in the gravity impeding its quantization. What happens is that most physicists and mathematicians have taken «*quantum gravity*» as synonym for «*quantum general relativity*» and relied on the fiasco to quantize general relativity as a sign of the existence of something radically new associated to gravity. For instance, since that general relativity is a geometric theory that relates gravitational effects to the curvature of spacetime, many workers have incorrectly considered that any theory of quantum gravity would be developed over a hypothetical quantum spacetime. However, a naive quantum approach on a discretized spacetime will meet with difficulties related to causality, and this was often interpreted as one of the symptoms.

These difficulties, and others that we have not mentioned, are traced to the current emphasis by many physicists on that everything in Nature would be quantized; time would be discrete and 'fuzzy' at the PLANCK scale, as EDWARD WITTEN affirms in an interview,⁷

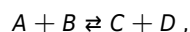
«Quantum mechanics brought an unexpected fuzziness into physics because of quantum uncertainty, the Heisenberg uncertainty principle. String theory does so again [...] when we study it more deeply, we find that in string theory, spacetime becomes fuzzy.»

They are so misguided as the classical physicists believing that a final theory would be based in the continuum. *Nature is neither completely digital nor analog but a wise mixture of both.*

Basic lessons from early canonical science

Canonical science is the elegant discipline based in the canonical theory initially developed by JOEL E. KEIZER, a recognized theoretical physical chemist who did relevant contributions to the theory of bimolecular reactions in three and two dimensions and to nonequilibrium thermodynamics, and who became later known as mathematical biologist for his pioneering work in computational biology. A list of his foundational papers is given in «*Canonical science: its history, goals, and future*». ⁸

KEIZER considered elementary chemical reactions as



where one molecule of A and one of B react for producing one molecule of C plus one of D , or vice versa, and rewrote it as

$$(n_A^+, n_B^+, n_C^+, n_D^+) \rightleftharpoons (n_A^-, n_B^-, n_C^-, n_D^-),$$

where n_i^+ describe the numbers of each molecule of the kind i involved in the forward rate and n_i^- describe the fate of those molecules after reaction.

KEIZER found that the rate for the forward and reverse process can be written as

$$V^\pm = \Omega \exp\left(-\sum_i \frac{n_i^\pm F_i}{k}\right),$$

where the Ω are transport coefficients, F_i thermochemical forces, and k the BOLTZMANN constant.

Those n_i^\pm are the 'quanta' associated to the elementary process that describes the change in the macroscopic quantities n_i . For instance, each elementary reaction adds or subtracts one molecule of D , but the overall change in the macroscopic concentration of the chemical substance D in the system is due to trillions and trillions of elementary reactions that happen in it. RICHARD FEYNMAN begins «*The Feynman Lectures on Physics*», emphasizing that the atomic-molecular picture of matter is one of the most powerful, compacts, and elegant achievements of science. ⁹

KEIZER showed this form for V^\pm to be canonical and successfully describing a wide range of physical, chemical, and biological processes. ⁸ By the first time in the history of science, we could study disparate topics as eggs metabolism, chemical reactions, and heat transport; all using an unified formalism!

KEIZER actually went beyond the mere unification and used the canonical theory to yield generalizations of available physical theories, giving us a nonequilibrium thermodynamics valid also for nonlinear regimes and a fluctuating hydrodynamics, for instance.

A fundamental lesson from canonical science is that many macroscopic processes studied in physics, chemistry, and biology are directly related to the microscopic structure of matter described by the n_i^\pm . Effectively, macroscopic observations of variations in energy, chemical composition, momentum, volume, electric current, and others described by the V^\pm could not happen in an Universe where the n_i^\pm do not take the required discrete values.

The canonical form explains, in a natural way, why the changes in chemical composition observed when two grams of reactant are introduced in a liter of water or why the variations in electric currents observed in Ca^{2+} -activated potassium channels from rat skeletal muscle *cannot be explained by theories based in the continuum* as classical mechanics or classical electrodynamics, respectively.

Further lessons from post-KEIZER canonical science

The first canonical theory worked well for a wide range of macroscopic processes. However, it was based in KEIZER macroscopic postulates and even in this domain the original theory was too restrictive; for example, it ignored relativistic gravitational effects.

A first extension of his theory considered higher order corrections in the discrete n_i^\pm .¹⁰ KEIZER showed that his canonical theory contained the classical thermodynamics of equilibrium as a special case when the forward and inverse rates of thermal processes satisfy the equation $V^+ = V^-$. What happens beyond the continuum? Consider an elementary process

$$(\varepsilon_A^+, 0) \rightleftharpoons (0, \varepsilon_B^-)$$

describing the transport of energy between systems A and B . By conservation of energy $\varepsilon_A^+ = \varepsilon_B^- = \varepsilon$.

When second order terms due to a discrete ε are retained in the canonical form at equilibrium, one obtains a generalization of the zeroth law of classical thermodynamics

$$\frac{\varepsilon}{T_B} - \frac{\varepsilon}{T_A} = \frac{\varepsilon^2}{2} \left(\frac{1}{T_A^2 N_A c_{V,A}} - \frac{1}{T_B^2 N_B c_{V,B}} \right),$$

where T_j , N_j , and $c_{V,j}$ denote the absolute temperature, number of molecules and heat capacity, respectively, of system j .

This generalization agrees with the results obtained by other methods.¹⁰ The main difference being that other authors have interpreted the inequality of temperatures as a sign of the violation of thermodynamics in the quantum realm, whereas we interpret them as the result of the canonical extension of thermodynamics to small systems. In any case, the traditional zeroth law is recovered in the limit when $\varepsilon^2 / (T_j^2 N_j c_{V,j}) \rightarrow 0$.

We can obtain similar corrections for other physical quantities. For instance, if we consider an elementary process with discrete volume $v_A^+ = v_B^- = v = 2kT/P_B$, then we can obtain the relation

$$P_A - P_B = -\frac{kT}{P_B \kappa_T V_B},$$

for the mechanical equilibrium between a macroscopic system A and a nanoscopic system B with isothermal compressibility κ_T and volume V_B , both at the same temperature T . The same relation was obtained by GER KOPER & HOWARD REISS by other methods.¹¹ Using the above relation, the pair of authors computed that the pressures differ about a 1% for an ideal gas with only 100 molecules.

A fundamental lesson from modern canonical science is that the «exotic» processes observed at the nanoscale, and needed for a fundamental understanding of the basic mechanisms of life and for the development of the nanotechnology, could not exist in an Universe where the n_i^\pm do not take the required discrete values. The continuum picture associated to classical thermodynamics and classical mechanics is recovered when ε and v are infinitesimal and the corrections of order ε^2 and v^2 vanish.

The modern canonical theory allows us to understand nanophenomena that could not be understood by the physics and mathematics associated to the continuum. For instance, the recently discovered flow of heat from cold to hot regions^{12,13} can be explained when higher order terms due to discrete ε are considered in the canonical rate equation for the heat flow.¹⁰

A fundamental concept of time beyond any hypothetical discrete spacetime

As explained in the two previous sections, a model of Nature where the quantities n_i^\pm are discrete allows us to understand phenomena could not be described otherwise. The analog models can be recovered from the digital model as an limiting case. In this perspective, the continuum is an approximation but *this does not imply that anything must be discrete.*

The state of any system is described in canonical theory by a generalized vector $(n_1(t), n_2(t), \dots, n_i(t), \dots)$. The physical, chemical, and biological quantities $n_i(t)$ have associated 'quanta' n_i^\pm , but time t enters here as a *continuous and implicit evolution parameter*. There is not 'quanta' of time. Now, the literature in quantum gravity very often states that time would be also quantized. At first, this seems a serious deficiency of the canonical theory, but a rigorous analysis shows that the deficiency is in that literature.

It is illuminating to trace the origin of such deficiency. During the development of special relativity, EINSTEIN misunderstood the concept of time used in the Newtonian theory and substituted it by the Minkowskian concept of time found in the MAXWELL equations¹⁴—this old confusion is perpetuated in that modern literature still denoting both times as t —. Years after, the special theory was generalized to curved spacetimes. Luckily his rejection of the Newtonian concept of time is almost unnoticeable for the broad kind of simple phenomena associated to both special and general relativity. The problems arise when these theories are applied to complex systems, then they either give wrong answers or no answer at all.

Worried by this fundamental deficiency of the usual relativistic theory, authors as FEYNMAN, SCHWINGER, STUECKELBERG, FANCHI, COLLINS, HORWITZ, PIRON, and others, studied extensions of it. In their extension to complex systems, STUECKELBERG, HORWITZ, & PIRON (SHP) retained the relativistic concept of time as dimension, but added a new concept of time—as implicit evolution parameter—with the mathematical and physical properties missed in special and general relativity, and missed also in further developments as relativistic quantum field theory, superstring theory, and others based in the concept of dimensional time.¹⁴

For instance, the gravitational potential for a two-body system in the new SHP theory has the form $\Phi_{SHP} = \Phi_{SHP}(\rho(\tau))$ where ρ is a generalized distance and τ the new concept of time. It has been rigorously showed that this potential reduces to the Newtonian potential $\Phi = \Phi(R(t))$ —where t is Newtonian time—in the non-relativistic limit; whereas that general relativity (GR) fails, because only can give a metric potential $\Phi_{GR} = \Phi_{GR}(\tilde{x}, \tilde{t}) \neq \Phi(R(t))$ in the limit.¹⁸ Indeed, general relativity cannot even give the correct dimensions, because Φ_{GR} is a four-dimensional function whereas the Newtonian Φ is six-dimensional; evidently, this is not a problem for the eight-dimensional Φ_{SHP} .

It is important to emphasize that proclaimed «*theories of everything*» as the *superstring theory* are built over limited concepts of dimensional time, causality, topology of interactions, and others beyond the scope of this essay, although these theories consider «*exotic*» spacetimes of ten or eleven dimensions.

An introduction to the SHP theory for free particles and its application to correct some of the limitations of the conventional string and p -brane theories is given in a recent book by the \mathcal{M} -theorist MATEJ PAVSIC.¹⁵ A presentation of the classical SHP theory is given in the monograph by the relativistic chaos expert WILLIAM SCHIEVE.¹⁶ A detailed discussion of both the classical and quantum versions, together with some of their advantages over classical electrodynamics, quantum field theory, and special and general relativity is given in other works.^{6, 17, 18}

Time is, at the most fundamental level achievable today, the universal implicit evolution parameter in a generalized canonical theory with direct particle interactions. At the quantum level of description, dynamical states for a N -particle system are given by a LIOUVILLE space state operator $\hat{\sigma} = \hat{\sigma}(\{\hat{x}\}_N; t)$. This $\hat{\sigma}$ is a generalization of the DIRAC bra-ket formalism in HILBERT space. It can be shown that the other concepts of time arise as approximations from this fundamental time t . Consider the concepts of time used in general relativity and quantum field theory as illustration.

Detailed derivations of general relativity and quantum field theory and of their respective concepts of time, from the more fundamental canonical theory, are rather complex and beyond the scope of this essay; however, lists of the approximations involved in each one of the derivations will allow us to understand all the limitations of general relativity and of quantum field theory as direct consequences of these approximations. These lists are given as two technical notes at the end.^{1,2}

The main lesson here is that the concepts of time \tilde{t} used in general relativity and of time $\overset{\circ}{t}$ used in quantum field theory have only approximated validity and, contrary to a well-known confusion initiated by EINSTEIN,¹⁴ none of them can reproduce the mathematics and physics associated to the implicit evolution parameter t .

Another important lesson here is that the sequences of approximations involved in the derivation of the, generally curved, classical spacetime of general relativity

$$(\{\hat{x}\}_N; t) \rightarrow (\tilde{x}, \tilde{t})$$

and of the «dummy» spacetime of quantum field theory

$$(\{\hat{x}\}_N; t) \rightarrow (\overset{\circ}{x}, \overset{\circ}{t})$$

are mutually incompatible

$$(\tilde{x}, \tilde{t}) \neq (\overset{\circ}{x}, \overset{\circ}{t}),$$

which implies that the so-named «modern» approaches to quantum general relativity—where the classical spacetime (\tilde{x}, \tilde{t}) is discretized—do not give us an ordinary quantum field theory of gravity on $(\overset{\circ}{x}, \overset{\circ}{t})$ —which is obtained from quantizing the classical field theory directly—¹⁹ and less still will give us a fundamental quantum theory of gravity on $(\{\hat{x}\}_N; t)$ —where the fundamental time t is continuous—.

Final remarks

We have offered answers to several foundational questions asked in this «FQXi ESSAY CONTEST» about the continuum and discrete aspects of Nature. We will offer answers to other questions in this section.

«How would a discrete universe expand without the discreteness becoming evident? Or, does it become evident?» Universe spatial expansion follows from solving the HILBERT & EINSTEIN equations, associated to (\tilde{x}, \tilde{t}) , at cosmological scales. There is not expansion of space in the classical field theory of gravity and its (\check{x}, \check{t}) ,¹⁹ nor in a fundamental quantum theory of gravity on $(\{\hat{x}\}_N; t)$. In them the physical phenomena usually interpreted in terms of the expansion receive a different explanation, one without the well-known observational difficulties introduced by inhomogeneities associated to cosmological discreteness and fractal structures.

«What are the implications of a minimal length, time, or energy, and how could we observe them now? Or, is this the wrong way to view fundamental discreteness?» The correct way is to consider expressions such as $(n_i^\pm F_i/k)$ and others coming into in the fundamental equations of motion, rather than just consider the n_i^\pm outside of any context. As emphasized in the previous section, fundamental time t enters in a different way; this leaves out discrete length λ_i^\pm , energy ε_i^\pm and the possible 'quanta' associated to dimensional times as \check{t} , \tilde{t} and \hat{t} . Corrections to usual expressions by considering a discrete volume or a discrete energy were presented in this essay. Corrections due to discrete lengths or surfaces follow in a similar way from the canonical form. We have not studied the implications for the dimensional times.

«Is a universe that is infinite in various ways incompatible with a digital description?» Consider the deviation from classical mechanics due to a discrete volume v discussed in a previous section. In an infinite volume $V_B \rightarrow \infty$ and $P_A = P_B$, as in classical mechanics, for any discrete small v . There is not incompatibility; at contrary, the digital description is equivalent to the analog description $v \rightarrow 0$ for a finite, albeit large, volume.

«How is a digital description consistent with a 'flow' of time? How does causality work?» The first question is ambiguous. If by «'flow' of time» we refer to the so-named «arrow of time», then the digital description associated to the canonical theory is perfectly consistent with irreversibility, because gives a generalization of the usual H-theorems. Regarding the second question, precisely the fact that the canonical theory uses fundamental time t instead of dimensional times as \check{t} , \tilde{t} and \hat{t} , implies that causality continues to work in despite of the discreteness associated to the n_i^\pm . *It must be emphasized that there is not «problem of time» in the canonical theory because the proper quantum generator of time translations is not the constrained 'Hamiltonian' proposed by the naive quantum general relativity approach.* The «problem of time» is also absent in the SHP theory by the same reason.¹⁵

«Are simple discrete models like cellular automata, etc., effective approaches to physics?» So far as we know, all the cellular automata with application to physics, chemistry, or biology arise from approximations to more fundamental models developed in these disciplines. The speculations by authors as FREDKIN & WOLFRAM^{22,23} about the possibility to describe all of physics as a cellular automata do not hold up on close inspection.²⁴

References

Scope: Literature survey is not exhaustive. The large volume of the literature makes it impossible to read or quote all relevant works.

- 1 The Sceptical Chymist: or Chymico-Physical Doubts & Paradoxes **1661:** *J. Cadwell (Publisher); London.* BOYLE, ROBERT.
- 2 Molecules **1873:** *Nature, September, 437–441.* MAXWELL, JAMES CLERK.
- 3 Sources of Quantum Mechanics **1968:** *In Classics of science, volume five; Courier Dover Publications; Holton, Gerald (General Editor).* SEVERAL AUTHORS; VAN DER WAERDEN, BARTEL LEENDERT (EDITOR AND AUTHOR).
- 4 Mathematical Foundations of Quantum Theory **1978:** *In Mathematical Foundations of Quantum Theory; Academic Press, Inc.; Marlow, A.R. (Editor).* DIRAC, PAUL ANDREW MAURICE.
- 5 Quantum field theory **1999:** *John Wiley & Sons Ltd.* MANDL, F.; SHAW, G.
- 6 Theories reducing to quantum mechanics, quantum electrodynamics doesn't **2010:** *Can. Sci. Rep. 20101v1.* GONZÁLEZ-ÁLVAREZ, JUAN R.
- 7 NOVA | The Elegant Universe | Edward Witten | PBS **27 January 2011 (Access):** <http://www.pbs.org/wgbh/nova/elegant/view-witten.html>. AUTHOR NOT AVAILABLE.
- 8 Canonical science: its history, goals, and future **2008:** *Can. Sci. Rep. 20083v1.* GONZÁLEZ-ÁLVAREZ, JUAN R.
- 9 The Feynman Lectures on Physics **1963:** *Addison-Wesley.* FEYNMAN, R. P.; LEIGHTON, R. B.; SANDS, M.
- 10 Has thermodynamics been violated? **2003:** *CPS: physchem/0309002.* GONZÁLEZ-ÁLVAREZ, JUAN R.
- 11 Length Scale for the Constant Pressure Ensemble: Application to Small Systems and Relation to Einstein Fluctuation Theory **1996:** *J. Phys. Chem., 100, 422–432.* KOPER, GER J. M.; REISS, HOWARD.
- 12 Zeroth and second laws of thermodynamics simultaneously questioned in the quantum microworld **2002:** *Eur. Phys. J. B, 25(1), 101–113.* ČÁPEK, V.
- 13 Heat can flow from cold to hot in Microcanonical Thermodynamics of finite systems and the microscopic origin of phase transitions **2003:** <http://arxiv.org/abs/cond-mat/0307054>. GROSS, D. H. E.
- 14 Relativistic dynamics **23 January 2011 (Access):** http://en.wikipedia.org/wiki/Relativistic_dynamics. AUTHOR NOT AVAILABLE.
- 15 The Landscape of Theoretical Physics: A Global View; From Point Particles to the Brane World and Beyond, in Search of a Unifying Principle **2006:** *arXiv:gr-qc/0610061v2.* PAVSIC, MATEJ.
- 16 Classical Relativistic Many-Body Dynamics **1999:** *Springer.* TRUMP, MATTHEW A.; SCHIEVE, WILLIAM C.
- 17 Theories reducing to Coulombian electrodynamics, relativistic electrodynamics doesn't **2009:** *Can. Sci. Rep. 20093v1.* GONZÁLEZ-ÁLVAREZ, JUAN R.
- 18 Theories reducing to Newtonian gravity, general relativity doesn't **2009:** *Can. Sci. Rep. 20092v1.* GONZÁLEZ-ÁLVAREZ, JUAN R.
- 19 Field Theory of Gravitation: Desire and Reality **1999:** *arXiv:GR-qc/9912003v1.* BARYSHEV, YURIJ, V.
- 20 The notions of localizability and space: from Eugene Wigner to Alain Connes **1989:** *Nucl. Phys. Proc. Suppl. 6, 222.* BACRY, H.
- 21 Classical electrodynamics **1962:** *John Wiley & Sons, Inc.; New York.* JACKSON, JOHN DAVID.
- 22 Digital mechanics: An informational process based on reversible universal cellular automata **1990:** *Physica D: Nonlinear Phenomena 45(1–3), 254–270.* FREDKIN, EDWARD.
- 23 A New Kind of Science **2002:** *Wolfram Media, Inc.* WOLFRAM, STEPHEN.
- 24 Is the Universe a Computer? **24 October 2002:** <http://www.nybooks.com/articles/15762>. WEINBERG, STEVEN.

Notes

- 1 We start with the basic canonical equation —see equation 3 in «*Canonical science: its history, goals, and future*»—⁸ and first ignore the random component f associated to non-deterministic chaos. The following approximation involves ignoring quadratic and higher order terms in the interaction Liouvillian, which implies that $\omega V = 0$. At this stage of the derivation, we have a quantum theory with a kinetic Liouvillian plus a first order interaction Liouvillian —see equation 4—.⁸ Next, we apply a classical limit $\hbar \rightarrow 0$. The generalized quantum state $\hat{\sigma}$ converts into a generalized classical state $\sigma = \sigma(\{x\}_N; \{p\}_N; t)$.

Further approximations are needed to arrive to a relativistic field theory. The first is the assumption that all the dynamical properties of interest can be obtained from the one-particle classical states $\sigma(x, p; t)$, tracing out all the other $6(N-1)$ dynamical variables. Authors as J. JACKSON have correctly noted that classical electrodynamics already fails for finding the equation of motion of two fully interacting charges.²¹ TRUMP & SCHIEVE emphasized that «*relativistic field theory is essentially a one-body theory*».¹⁶ Thanks to the new canonical theory, we can understand now why: *the derivation of relativistic field theory invokes the one-particle limit*, when tracing out two-body, three-body and higher order j -body corrections to the system's properties.

Subsequently, we invoke a pure-state approximation to these one-particle classical states. Simplifying the expressions and computing the classical traces, we will finish with a classical Liouvillian equation of motion with interactions given by improved nonlinear gravitational potentials $h_{\mu\nu}(x; t)$.

From the one-particle space $(x; t)$ —which, recall, is only an approximation to the more general N -particle space— we can derive the special relativistic spacetime (\check{x}, \check{t}) by applying a change of parametrization from fundamental time to dimensional time

$$\check{t} = \check{t}(t) .$$

Expert readers will notice that this canonical parametrization resembles one of the equations of motion in the classical SHP theory^{16–18} However, the above canonical parametrization eliminate the dynamical redundancies associated to the «*off-shell*» constraint used in the SHP theory.

The improved nonlinear gravitational field potentials on spacetime $h_{\mu\nu}(\check{x}, \check{t})$ can be cast into the usual nonlinear field theory of gravity (FTG) form¹⁹ $\check{h}_{\mu\nu}(\check{x}, \check{t})$ plus certain correction terms that eliminate the well-known deficiencies of the field theory: self-action divergences, violations of causality and inertia, and others

$$h_{\mu\nu}(\check{x}, \check{t}) = \check{h}_{\mu\nu}(\check{x}, \check{t}) + \text{corrections} .$$

Once FTG is at hand,¹⁹ we can obtain general relativity introducing additional geometric approximations —somehow as geometrical optics is an approximation to physical optics—. First, we must approximate the source of the gravitational field by the energy-momentum-stress tensor for matter alone $\check{T}^{\mu\nu} \rightarrow \check{T}^{\mu\nu}_{(\text{matter})}$ —as a consequence of this «*dematerialization*» of the gravitational field, no reliable positive energy-momentum-stress tensor for gravitation will be found within general relativity; this defect of general relativity is well-known, now we can understand the physical origin of such defect—. Next, we introduce an effective metric $\check{g}_{\mu\nu}(\check{x}, \check{t})$ as the sum of the flat spacetime metric $\check{\eta}_{\mu\nu}(\check{x}, \check{t})$ and of the gravitational field potentials $\check{h}_{\mu\nu}(\check{x}, \check{t})$ and rewrite the expressions; this introduces no additional approximation, but a change in the looking of some expressions only.

In a third step, we 'renormalize' the spacetime to curved coordinates defined by

$$\check{g}_{\mu\nu} d\check{x}^\mu d\check{x}^\nu \equiv \check{\eta}_{\mu\nu} d\check{x}^\mu d\check{x}^\nu$$

and, finally, approximate the effective metric $\check{g}_{\mu\nu}(\check{x}, \check{t})$ in the FTG expressions by the curved spacetime metric $\tilde{g}_{\mu\nu}(\check{x}, \check{t})$ —the difference being roughly of the order of the gravitational field potential—. ¹⁹ At this stage of the derivation, we finally recover the usual general relativistic expressions; for instance, we obtain the HILBERT & EINSTEIN *metric* equations

$$\tilde{\mathfrak{R}}_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{\mathfrak{R}} = -\frac{8\pi G}{c^4} \check{T}_{\mu\nu}^{(\text{matter})}$$

with all their known geometrical deficiencies —apart from the cited above for the lack of gravitational energy-momentum-stress tensor, we must add spacetime singularities, the problem of the systems of reference, violation of the usual conservation laws, and the impossibility to obtain a consistent quantization of such [geo]metric theory— cured now, because *the derivation allows us to consider general relativity only within its empirical range of validity*.

- 2 Applying a sequence of approximations very similar to those described in the previous note for the derivation of FTG—except that now we do not take the classical limit— we can derive an improved quantum field theory from canonical science. Consider as example quantum electrodynamics. Also in this case, the improved quantum field potentials $\hat{A}^\mu(\check{x},\check{t})$ can be cast into the usual field-theoretic form $\hat{\check{A}}^\mu(\check{x},\check{t})$ plus correction terms that eliminate the known deficiencies of quantum field theory associated to self-action divergences and others. There exist, however, additional approximations and steps that are specific of the derivation of a quantum field theory.

One of them is the decomposition of the velocity operators into DIRAC components $c\alpha$ plus «Zitterbewegung» corrections. This is a kind of light-cone 'renormalization' of spacetime coordinates and originates the characteristic *dummy spacetime* (\check{x},\check{t}) of relativistic quantum field theory. Authors as BACRY emphasize that the 'xs' and 'ts' in quantum fields «are just dummy variables without physical meaning». ²⁰ Several textbooks correctly notice that there exists no position operator in quantum field theory. ⁵ We can understand now, thanks to the new canonical theory, which is the physical origin of this deficiency of quantum field theory: *the derivation of quantum field theory relies on disregarding the «Zitterbewegung» corrections to one-particle currents, which means that the Hermitian character of the position operator \hat{x} is lost and, as a consequence, position is downgraded to the status of unobservable parameter \check{x} .* A loss of physical meaning can be also shown for the concept of time \check{t} used in quantum field theory, although the demonstration follows another way because there is not operator «time» involved here.

A needed step in the derivation of quantum field theory is the final change from the quantum pure state description for independent particles, but now in *dummy spacetime*, to a second-quantized formalism for field operators $\hat{\psi}(\check{x},\check{t})$. This involves no additional approximation.