THREE FUNDAMENTAL MASSES DERIVED BY DIMENSIONAL ANALYSIS

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Abstract

Three new mass dimension quantities have been derived by dimensional analysis, in addition to the famous Planck mass $m_P \sim 10^{-8}$ *kg*. These masses have been derived by means of fundamental constants – the speed of light (c), the gravitational constant (G), the Plank constant ($\hbar$) and the Hubble constant (H). The enormous mass $m_1 \sim 10^{53}$ *kg* practically coincides with the Hoyle-Carvalho formula for the mass of the observable universe. The extremely small mass $m_2 \sim 10^{-33}$ *eV* has been identified with the minimum quantum of energy, which seems close to the graviton mass. It is noteworthy that the Planck mass appears geometric mean of the masses m_1 and m_2. The mass $m_3 \sim 10^7$ *GeV* could not be unambiguously identified at present time. Besides, the order of magnitude of the total density of the universe has been estimated by this approach.

Key words: Planck mass, dimensional analysis, mass of the universe, minimum quantum of energy

1. Introduction

The Planck mass $m_P \sim \sqrt{\frac{\hbar c}{G}}$ has been introduced from Planck (1906) by means of three fundamental constants – the speed of light in vacuum (c), the gravitational constant (G) and the reduced Plank constant ($\hbar$). Since the constants c, G and $\hbar$ represent three very basic aspects of the universe (i.e. the relativistic, gravitational and quantum phenomena), the Plank mass appears to a certain degree a unification of these phenomena. The Plank mass have many important aspects in modern physics. One of them is that the energy equivalent of Planck mass $E_P = m_P c^2 \sim \sqrt{\frac{\hbar c^5}{G}} \sim 10^{19}$ *GeV* appears unification energy of the fundamental interactions (Georgi et al., 1974). Also, the Planck mass can be approximately derived by setting it as a mass, whose Compton wavelength and Schwartzchild radius are equal (Bergmann, 1992).

The Plank mass formula has been derived by dimensional analysis using fundamental constants c, G and $\hbar$. The dimensional analysis is a conceptual tool often applied in physics to understand physical situations involving certain physical quantities (Bridgman, 1922; Kurth, 1972; Bhaskar and Nigam, 1990; Petty, 2001). It is routinely used to check the plausibility of the derived equations and computations. When it is known, the certain quantity with which other determinative quantities would be connected, but the form of this connection is unknown, a dimensional equation is composed for its finding. In the left side of the equation, the unit of this quantity q_0 with its dimensional exponent has been placed. In the right side of the equation, the product of units of the determinative quantities q_i rise to the unknown exponents n_i has been placed $[q_0] \sim \prod_{i=1}^{n} [q_i]^{n_i}$, where n is positive integer and the exponents n_i are rational numbers. Most often, the dimensional analysis is applied in the mechanics and other fields of the modern physics, where there are many problems with a few determinative quantities. Many interesting and important problems related to the fundamental constants have been considered from (Levy-Leblond, 1977; Duff, 2002; Duff et al., 2002; Barrow, 2002; Fritzsch, 2009).

The discovery of the linear relationship between recessional velocity of distant galaxies, and distance $v = Hr$ from Hubble (1929) introduces new fundamental constant in physics and cosmology – the famous Hubble constant (H). Even seven years before, Friedman (1922) derived his equations from the Einstein (1916) field equations, showing that the universe might expand at a rate calculable by the equations. Hubble constant determines the age of the universe H^{-1}, the Hubble distance cH^{-1}, the critical density of the universe $\rho_c = \frac{3H^2}{8\pi G}$ (Peebles, 1971), and other large-scale properties of the universe.

Because of the importance of the Hubble constant, in the present paper we include *H* in the dimensional analysis together with *c*, *G* and $\hbar$ aiming to find the new mass dimension quantities $m_i \sim \prod_{j=1}^{3} q_j^{n_j}$, where every triad q_1, q_2, q_3 consists of three constants *c*, *G*, $\hbar$ and *H*. Thus, the Hubble constant will represent the cosmological phenomena in new derived fundamental masses. According to the recent cosmology, the Hubble 'constant' slowly decreases with the age of the universe, but there are indications that other constants, especially gravitational and fine structure constants also vary with comparable rate during the expansion (Dirac, 1937; Wu and Wang, 1986; Webb et al., 2001). That is why, the Hubble constant could deserve being treated on an equal level with the other three constants used by Planck.

2. Three fundamental masses derived by dimensional analysis

Below, we obtain a mass dimension quantity m_1 constructed from the fundamental constants – the speed of light (*c*), the gravitational constant (*G*) and the Hubble constant (*H*) using dimensional analysis. A quantity m_1 having mass dimension could be constructed by means of the fundamental constants *c*, *G* and *H*:

$$m_1 = kc^{n_1} G^{n_2} H^{n_3} \quad (1),$$

where n_1, n_2 and n_3 are unknown exponents to be determined by matching the dimensions of both sides of the equation, and *k* is dimensionless parameter of an order of magnitude of a unit.

As a result we find the system of linear equations:

$$\begin{aligned} & n_1 + 3n_2 = 0 \\ & -n_1 - 2n_2 - n_3 = 0 \\ & -n_2 = 1 \end{aligned} \quad (2)$$

The unique solution of the system is $n_1 = 3, n_2 = -1, n_3 = -1$. Replacing obtained values of the exponents in equation (1) we find formula (3) for the mass m_1:

$$m_1 \sim \frac{c^3}{GH} \quad (3)$$

First of all, the formula (3) has been derived by dimensional analysis from Valev (2009). This formula practically coincides with the Hoyle formula for the mass of the observable universe $M = \frac{c^3}{2GH}$ (Kragh, 1999) and perfectly coincides with Carvalho (1995) formula for the mass of the observable universe, obtained by totally different approach.

Evidently, the Hoyle formula coincides with the mass of the Hubble sphere M_H, i.e. mass of the sphere having radius equal to the Hubble distance cH^{-1} and density equal to the total density of the universe $\overline{\rho} \approx \rho_c$:

$$M_H = \frac{4}{3}\pi \frac{c^3}{H^3} \frac{3H^2}{8\pi G} = \frac{c^3}{2GH} \quad (4)$$

The recent experimental values of *c*, *G* and *H* are used from Mohr and Taylor (1999): $c = 299\,792\,458$ $m\,s^{-1}$, $G = 6.673 \times 10^{-11}$ $m^3\,kg^{-1}\,s^{-2}$ and $H \approx 70$ $km\,s^{-1}\,Mps^{-1}$ from Mould et al., (2000). Replacing this values in (3) we obtain $m_1 \sim 1.76 \times 10^{53}$ *kg*. Therefore, the enormous mass m_1 would be identified with the mass of the observable universe.

Analogously, by means of the fundamental constants *c*, $\hbar$ and *H*, a quantity m_2 having dimension of a mass could be constructed:

$$m_2 = kc^{n_1}\hbar^{n_2}H^{n_3} \tag{5}$$

We determine the exponents $n_1 = -2, n_2 = 1, n_3 = 1$ by the dimensional analysis again. Replacing the obtained values of the exponents in equation (5) we find formula (6) for the mass m_2:

$$m_2 \sim \frac{\hbar H}{c^2} \tag{6}$$

Replacing the recent values of the constants *c*, $\hbar$ and *H* in (6) we obtain $m_2 \sim 2.70\times10^{-69}$ *kg* $=1.52\times10^{-33}$ *eV*. This exceptionally small mass coincides with the minimal measurable gravitational self energy of a particle (Sivaram, 1982) which is accepted as minimum quantum of energy $\hbar H \sim 10^{-33}$ eV from Alfonso-Faus (2012, 2013). This quantity takes substantial place in the estimations of total information and entropy of the universe (Gkigkitzis et al., 2013; Haranas and Gkigkitzis, 2013). Thus, the mass m_2 seems close to the graviton mass obtained by different methods (Woodward et al., 1975; Gershtein et al., 1998; Valev, 2005; Alves et al. 2009). The mass m_2 is in several orders of magnitude smaller than the upper limit of the graviton mass, obtained by astrophysical constraints from (Goldhaber and Nietto, 1974). From equation (6) we find that the reduced Compton wavelength $\lambda\!\!\!^{-}{}_2$ of this mass is equal to the Hubble distance cH^{-1}:

$$\lambda\!\!\!^{-}{}_2 = \frac{\hbar}{m_2 c} = cH^{-1} \sim 1.3\times10^{26}\, m \tag{7}$$

From formulae (3) and (6) we find an interesting relation (8):

$$\sqrt{m_1 m_2} = \sqrt{\frac{c^3}{GH}\frac{\hbar H}{c^2}} = \sqrt{\frac{\hbar c}{G}} \equiv m_P = 2.17\times10^{-8}\, kg \tag{8}$$

Therefore, the Planck mass appears geometric mean of the Hubble mass and the mass of the observable universe. As the physical quantity mass is among the most important properties of the matter, the formula (8) hints at a deep relation of the micro particles and the entire universe.

Besides, the ratios (9) take place:

$$\frac{m_1}{m_P} = \frac{m_P}{m_2} = \frac{cH^{-1}}{r_P} = \frac{H^{-1}}{t_P} = \sqrt{\frac{c^5}{G\hbar H^2}} \sim 8\times10^{60} \qquad (9),$$

where $r_P = \sqrt{\frac{G\hbar}{c^3}}$ is the Planck length, t_P is the Plank time, cH^{-1} is the Hubble distance, and H^{-1} is the Hubble time.

The third quantity m_3, having mass dimension could be constructed by means of the fundamental constants *G*, $\hbar$ and *H*:

$$m_3 = kG^{n_1}\hbar^{n_2}H^{n_3} \tag{10}$$

We determine the exponents $n_1 = -\frac{2}{5}, n_2 = \frac{3}{5}, n_3 = \frac{1}{5}$ by dimensional analysis again. Replacing the obtained values of the exponents in equation (10) we find formula (11) for the mass m_3 :

$$m_3 \sim \sqrt[5]{\frac{H\hbar^3}{G^2}} \tag{11}$$

Replacing the recent values of the constants *G*, $\hbar$ and *H*, the mass m_3 takes value m_3 ~ 1.43×10^{-20} *kg* ≈ 8.0×10^{6} *GeV*. This mass is a dozen of orders of magnitude lighter than the Planck mass and several orders of magnitude heavier than the heaviest known particles like the top quark $m_t \approx 174.3$ *GeV* (Mangano and Trippe, 2000). On the other hand, the energy m_3c^2 ~ 8×10^{6} GeV appears medial for the important GUT scale E_{GUT} ~ 10^{16} GeV and electroweak scale E_{EW} ~ 10^{2} GeV. Therefore, the mass/energy m_3 could not be unambiguously identified at the present time, and it could be considered as heuristic prediction of the suggested approach.

Below, we demonstrate the heuristic power of the suggested approach approximately estimating the total density of the universe by dimensional analysis. Actually, a quantity ρ having dimension of density could be constructed by means of the fundamental constants *c*, *G* and *H*:

$$\rho = kc^{n_1}G^{n_2}H^{n_3} \tag{12),}$$

where *k* is a dimensionless parameter of the order of magnitude of unit.

By the dimensional analysis, we have found the exponents $n_1 = 0, n_2 = -1, n_3 = 2$. Therefore:

$$\rho \sim \frac{H^2}{G} \approx 7.93\times10^{-26}\ kg\ m^{-3} \tag{13}$$

The recent Cosmic Microwave Background (*CMB*) observations show that the total density of the universe $\overline{\rho}$ is (Balbi et al., 2000 ; de Bernardis et al., 2000 ; Spergel et al., 2003):

$$\overline{\rho} = \Omega\rho_c \approx \rho_c = \frac{3H^2}{8\pi G} \sim 10^{-26}\ kg\ m^{-3} \tag{14}$$

Evidently, the density ρ derived by means of the fundamental constants *c*, *G* and *H* coincides with formula (14) for the total density of the universe with an accuracy of a dimensionless parameter of an order of magnitude of a unit. Besides, the formula (13) could be derived by means of other triad of fundamental constants, namely *G*, $\hbar$ and *H*.

3. Conclusions

Three new mass dimension quantities m_i have been derived by dimensional analysis, in addition to the Planck mass $m_P \sim \sqrt{\frac{\hbar c}{G}}$ ~ 2.17×10^{-8} *kg*. Four fundamental constants – the speed of light in vacuum (*c*), the gravitational constant (*G*), the reduced Plank constant ($\hbar$) and the Hubble constant (*H*) have been involved in the dimensional analysis. The first derived mass dimension quantity $m_1 \sim \frac{c^3}{GH}$ ~ 10^{53} *kg* practically coincides with the Hoyle-Carvalho formula for the mass of the universe obtained by totally different approach. The exceptionally small mass dimension quantity $m_2 \sim \frac{\hbar H}{c^2}$ ~ 10^{-33} *eV* has been identified with the minimum quantum of energy, which seems close to the graviton mass. It is amazing that the Planck mass appears geometric mean of the masses m_1 and m_2, i.e. $m_P = \sqrt{m_1 m_2}$. The third derived mass $m_3 \sim \sqrt[5]{\frac{H\hbar^3}{G^2}} \sim 10^7$ *GeV* could not be identified unambiguously at present time. The identification of the two derived masses reinforces the trust in the suggested approach.

According to the Big Bang cosmology, the Hubble constant decreases with the age of the universe. Therefore, the mass of the universe $m_1 \sim \frac{c^3}{GH}$ increases, whereas the Hubble mass $m_2 \sim \frac{\hbar H}{c^2}$ and mass $m_3 \sim \sqrt[5]{\frac{H\hbar^3}{G^2}}$ decrease with time. Nevertheless, the Planck mass remains a geometric mean of the Hubble mass and mass of the observable universe.

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