[**Pdf**] submitted on 19, Mar, 2011.

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Comments: 7 pages.

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Category: Classical Physics.

Probability distribution function of the particle number in a system with concurrent existence of temperature T and potential Φ

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Abstract

In a system, coupling between the large number of charged particles will induce potential Φ. When temperature T and potential Φ concurrently exist in the system, the particle potential energy and kinetic energy would satisfy the probabilistic statistical distribution. Based on such consideration, we established the quantum statistical distribution for the particle. When temperature $T \to 0$, and the potential is extremely low, all the particles in the system would approach the ground-state-level distribution.

Key words: probability distribution.

PACS: 05.30.-d

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1 introduction

Starting from the existing quantum statistics method, the author of this paper found that when *ith* particle simultaneously carry thermodynamical kinetic energy $p_i^2/2m$ and coupling-induced potential energy $p_i c = q\varphi_i$, the energy of these particles would satisfy the quantum statistical probability W. When temperature T and potential Φ concurrently exist in the system, and by using the Lagrange's method of multiplication, we get the particle numbers statistical distribution function N_i, respectively in spin condition s=0, s=1/2 and s=1. When potential $\Phi = 0$, the particle distribution would show general statistical distribution jointly defined by Boltzmann, Bose and Fermi. When the system is exposed to extremely low temperature $T \to 0$ and the potential $\Phi \to 0$, all the particles in the system would approach the same statistical distribution state, namely, the ground-state-level distribution.

2 The Distribution Function of Particles number in Temperature T and Potential Φ

In a system *ith* charged particle couples with others of large amount particles, the particle would produce potential φ_i [1], the particle carries energy $p_i c = q\varphi_i$ [2] and this particle would have other kinetic energy $p_i^2/(2m)$ in temperature T. Owing to that the system has a total number of N particles that carry the afore-mentioned energy, when particle number N_i of energy, we get

$$\mathcal{E}_1 = \Sigma_i N_i p_i^2/(2m), \quad \mathcal{E}_2 = \Sigma_i N_i p_i c, \quad N = \Sigma_i N_i. \tag{1}$$

If the N_i particles distribute cell G_i in the phase space, the first particle would have G_i distribution patterns, the second particle would have $G_i - a$ distribution patterns and so forth, the final term $[G_i-(N_i-1)a]$, a is constant. Therefore, the N_i particles would have the following distribution probability in G_i. We have probability W [3, 4] ,

$$\begin{aligned} W &= \Pi_i[G_i(G_i-a)...(G_i-(N_i-1)a)]/N_i! \\ &= \Pi_i[G_i(G_i-a)...(G_i-(N_i-1)a)](G_i-N_ia)!/[N_i!(G_i-N_ia)!] \\ &= \Pi_i G_i!/[N_i!(G_i-N_ia)!], \end{aligned} \tag{2}$$

by Stirling's approximate formula [5] $\ln N! \approx (N+1/2)\ln N - N + 1/2\ln(2\pi)$, when $N \gg 1$, so as to calculate.

By multiplying (1) and (2) by the Lagrangian multipliers [6] α, β and γ. Behind make their variation, we have

$$\Sigma_i[\ln(G_i/N_i - a) - \beta p_i^2/(2m) - \alpha p_i c + \gamma]\delta N_i = 0, \tag{3}$$

set $\beta = 1/(kT)$, and $\alpha = 1/(q\Phi)$ [2], then (3) becomes

$$N_i = G_i/[\exp(\frac{p_i^2}{2mkT} + \frac{p_i c}{q\Phi} - \gamma) + a]. \tag{4}$$

So as make $a = 1, -1$, we have

$$N_i = G_i/[\exp(\frac{p_i^2}{2mkT} + \frac{p_i c}{q\Phi} - \gamma) \pm 1], \tag{5}$$

in a system there are concurrent temperature T and potential Φ, formula (5) are Bose and Fermi statistical distributions functions.

When make $a = 0$ we have

$$N_i = G_i / \exp(\frac{p_i^2}{2mkT} + \frac{p_i c}{q\Phi} - \gamma), \tag{6}$$

formula (6) is Boltzmann statistical distribution function, in a system there are temperature and potential.

When temperature T and potential Φ simultaneously exist in the system, the charged particle numbers would satisfy the Bose, Fermi or Boltzmann distribution function as expressed in (5) or (6). If the system is at high temperature T and low density of particles number, at $p_i^2/(2mkT) \gg p_i c/(q\Phi)$, the (5) two kinds of statistical distributions would approach (6) the Boltzmann statistics. When the particles would be at the temperature T and have the potential Φ, we only use (5) to study the particles' distribution. Now let us to set the phase space $\Omega_i = 4\pi v_i p_i^2$, and let the volume of individual phase cells be h^3, in which h is the Planck constant, then the number of phase cells $G_i = 4\pi v_i p_i^2/h^3$ [3]. For (5), Bose and Fermi distributions become

$$N_i \; dp_i = \frac{4\pi v_i p_i^2}{h^3} / [\exp(\frac{p_i^2}{2mkT} + \frac{p_i c}{q\Phi} - \gamma) \pm 1] dp_i, \tag{7}$$

the (7) we have particle numbers

$$N = \int_0^\infty N_i dp = \int_0^\infty \frac{4\pi v p^2}{h^3} \frac{1}{\exp(\frac{p^2}{2mkT} + \frac{p\,c}{q\Phi} - \gamma) \pm 1} dp. \tag{8}$$

From (6), Bltzmann distribution function become

$$N_i dp_i = \frac{4\pi v p^2}{h^3} \frac{1}{\exp(\frac{p^2}{2mkT} + \frac{p\,c}{q\Phi} - \gamma)} dp_i, \tag{9}$$

for (9) have particle numbers

$$N=\int_0^\infty N_i dp=\int_0^\infty \frac{4\pi v p^2}{h^3}\frac{1}{\exp(\dfrac{p^2}{2mkT}+\dfrac{p\,c}{q\Phi}-\gamma)}\,dp. \tag{10}$$

When the system's temperature $T\rightarrow 0$, these particles disordered thermal motions would also approach zero, and the system's potential $\Phi\rightarrow 0$, these particles produce ordered motion, namely, the particles would stay in the ground state. The particles at spin 1 would locate in the identical phase cell, and such particles would show the following particle number:

$$N=\frac{1}{\exp\,(\dfrac{p^2}{2mkT}+\dfrac{p\,c}{q\Phi})-1}+\int_0^\infty \frac{4\pi v p^2}{h^3}\frac{1}{\exp(\dfrac{p^2}{2mkT}+\dfrac{p\,c}{q\Phi}-\gamma)-1}dp. \tag{11}$$

In the potential Φ the particles produce ordered motion. For concurrent existence of temperature T and potential Φ, and the particles at spin 1/2, hence we have the following particle number:

$$N=\int_0^\infty \frac{4\pi v p^2}{h^3}\frac{1}{\exp(\dfrac{p^2}{2mkT}+\dfrac{p\,c}{q\Phi}-\gamma)+1}dp. \tag{12}$$

It is not difficult to see that in a system made up by charged particles, in the potential Φ there is the probability distribution of the particles ordered motion. However, if the system's particles do not carry any charges or the particles at $\Phi=0$ would show a distribution pattern that normally exists only when temperature T exists, there is they are general Boltzmann, Bose and Fermi probability distribution functions [7], these particles only there are kinetic energy and particle numbers are

$$gv/h^3 \int_0^\infty d^3p(p^2/2m)/[\exp(\beta p^2/2m - \gamma) + a] \quad (a = 0, -1, 1). \tag{13}$$

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