On the Measurement, Statistics and Uncertainty

İbrahim Mutlay*

Hayzen Engineering Co., Ankara, Turkey

June 10, 2009

Abstract

It is intended here to propose descriptive explanations for the basic statistical concepts. Although most of them are highly familiar to us, their conventional descriptions have vague sides. Especially it was focused on the absolute probabilistic uncertainty which was characterized by momentum of the measurement device and the system which was measured.

Keywords: Measurement problem, statistical methods, uncertainty

*Corresponding author. Tel: +90 312 394 55 12; fax: +90 312 394 55 14 E-mail address: ibrahimmutlay@gmail.com

Statistics, in principle, can be assumed as origin of all the natural sciences. Measurement and evaluation of a physical phenomenon which is theorized by statistics, constitutes the basic architect of the science. Besides, usage of statistical tools in diverse fields of science is a usual approach. We frequently use the concepts such as probability, uncertainty etc. in the attempts of understanding the real world using statistics of measurements. However, description of the highly ordinary concepts even probability may be topic of serious controversies especially in quantum mechanics. Most of the problems in the foundations of quantum mechanics are associated with lack of understanding the fundamental tools of the statistics. It is clear that finding explanations to the basic theme of the statistics would be useful in the interpretation of quantum mechanics.

Measurement is a process quantifying the physical reality and generating data. Basis of our knowledge comprises this data and its structured forms. Statistics is the science of data corresponding to value of observables generated by the measurements and its evaluation. Statistical methodology consists of collection of data from the experimentation (measurement), processing them via some mathematical tools and converting them to a set of apprehensible knowledge. Thus, let $\mathbb{M}$ be a space of all the possible outcome of a phenomenon which is understood by experimentation and let $\mathbb{C}$ be a set of apprehensible information deduced from $\mathbb{M}$. By the definition, statistics is a map $f : \mathbb{M} \mapsto \mathbb{C} \quad \forall \mathbb{M}, \mathbb{C} \in \mathbb{R}$. This map consists of the models which are elements of the science. We derive these models from statistics and use for the predictions. Accuracy of the results of these predictions and models is determined by the deviations. Deviation is ambiguous departures from expected value of a model. It is common in physics and statistics to express the models as

$$\psi(q_i, t) = \mu(q_i, t) + \sigma(q_i, t) \quad \forall \psi, \mu, \sigma \in \mathbb{R} \tag{1}$$

Let $\psi(q_i, t)$ for some spatial coordinates q_i and time t be the observable inferred from an experimentation; $\mu(q_i, t)$, average of the data or predictable value and $\sigma(q_i, t)$ is the deviation term. In classical methodology, $\mu(q_i, t)$ is the average of elements of the set of $\mathbb{M}$ and it is predictable. In other words, it can be predicted using the fundamental laws of physics for the limit value of the measurements. For that reason, averaging whole data especially for the huge set of data will provide the same result with the physical predictions. However it is assumed that there may be some deviations from unknown sources characterized by $\sigma(q_i, t)$. Mathematical behavior of $\sigma(q_i, t)$ obeys the statistical distribution laws but is predictable to some extent, for example, $\sum \sigma = 0$ can be supposed for the most of the systems. In another view, statistics is the science used to investigate the blackbox phenomenons and to construct models for them. We call a phenomenon as blackbox if we do not know physical nature of the phenomenon. In order to modeling the blackbox we observe the inputs and corresponding outputs of the blackbox using a measurement technique. Obtained data set is employed to build a model of the blackbox phenomenon. Model of the blackbox can be used for the predictions of future responses for some other inputs.

Above descriptions can be found in ordinary textbooks in fact and their mathematical background has been well understood. Neverthless it is unfortunate to say that physical meaning for the statistical concepts is quite confusing. Hence it is so important to find the consistent interpretations for them. To that goal, we proposed here more descriptive scheme for the observables of real world:

$$\psi(q_i,t) = \mu(q_i,t) + \eta(q_i,t) + \Lambda \quad \forall\eta, \Lambda \in \mathbb{R} \tag{2}$$

All the human perception is based on $\psi(q_i,t)$ and it is originated from physical phenomenon. We humans use $\psi(q_i,t)$ to sense Universe and its happenings. $\psi(q_i,t)$ consists of three different part; analytical value $\mu(q_i,t)$, deviation term $\eta(q_i,t)$ and uncertainty action Λ. $\mu(q_i,t)$ is the subject of physics and completely predictable by using the rules of natural laws. However level of scientific consciousness may not be enough for the estimation of $\psi(q_i,t)$. In other words, some "hidden variables" causing the deviations from $\psi(q_i,t)$ may exist in the phenomenon. If that deviation can be predicted via empirical ways using statistics or from first principles using statistical physics (not by the deterministic physical techniques such as classical mechanics) it corresponds to the deviation term $\eta(q_i, t)$. In fact, only reason to the noisy nature of $\eta(q_i, t)$ is due to the insufficient scientific consciousness and for a given phenomenon deviation term will diminish in future with increasing our scientific abilities. However, there will be a deviation term always for any physical phenomenon for anytime. Let $\mathbb{K}$ is knowledge or scientific consciousness and science which is a irreversibly progressing process be a dynamical map s. In addition, let available information as the input of the science be $\mathbb{I}$. It will be $s : \mathbb{I} \mapsto \mathbb{K}$ and for any scientific procedure inequality of $\mathbb{K} \geqslant s\mathbb{I}$ is realized. Consequently, it can be said that scientific consciousness of humankind will never decrease. Additionally, in future $\mathbb{K}$ will be available information for new scientific procedures and statements in the set of s will not lower. This means that deviation term will exist forever. Λ is an absolute uncertainty term and differs considerably from $\eta(q_i, t)$. Λ which is completely unpredictable arises from the coupling between physical behavior of the measurement device and the blackbox. In order to understand the Λ, we require some deeper insight to the measurement process. In physical world, any measurement can be carried out only using physical interactions. In a daily example, namely, in vision process, this interaction is occured between photon and matter in the way of scattering. Value of Λ is related with the level of the coupling between the measurement and the blackbox. The coupling can be quantified by the momentum of the measurement device and the blackbox. If $p_b \gg p_{md}$, then Λ term can be assumed negligible. Where p_b and p_{md} are momentum of the blackbox and the measurement device respectively. In other cases, Λ term is significant and there is an absolute uncertainty in the measurement. Such an uncertainty is independent from (q_i, t) coordinates and is free from mathematical structures such as distribution laws. We can not acquire a predictable pattern for Λ using any scientific approach. For a given system of the measurement device and the blackbox, Λ terms can be eliminated only via $p_b/p_{md} \to \infty$.

We apply the scheme reported here on some familiar examples. Tossing coins up and dice throwing are classical case studies for statistics and they are widely examined in textbooks for the elucidation of probability. In these classical examinations result of the tossing process is hypothesized as unpredictable. However, studies showed that tossing dice is not a random process [1]. Similarly, for the tossing up example, we can make $\eta(q_i, t)$ almost negligible and estimate the result of the process exactly using the initial conditions such as three dimensional launching, hydrodynamical effects and also environmental interactions such as perturbative and dissipative forces from the hydrodynamical vicinity and solid surfaces [1] etc. However we can not toss up deterministically in general since the human abilities are not enough for the control of initial and environmental conditions for such an aim [1]. Λ term would be meaningless for these examples because of that momentum of the photons scattered from the coin is exteremely small relative to that of the coin. Another useful example will be Heisenberg's Uncertainty Principle. According to the Uncertainty Principle, if uncertainty in the position and momentum is Δq and Δp respectively, then it will be $\Delta q \Delta p \geq \hbar/2$. In quantum mechanics, we generally measure position and momentum of the particles using some other (similar) particles. This is the reason why uncertainty is unavoidable in quantum mechanics.

It was discussed on the fundamentals of statistics, measurement and uncertainty. Statistics was defined as the tool providing the models for natural sciences. Accuracy of these models was determined by deviation and uncertainty terms. Deviations were assumed completely deterministic while there is always an absolute uncertainty in the models. Uncertainty was inherently probabilistic and it could not predicted due to its nature. Such an unpredictable uncertainty came from the physical coupling of the momentum of the measurement device and measured body. In order to eliminate the uncertainty, it was proposed to lower the momentum of the measurement device.

References

[1] J. Nagler, P. Richter, Phys. Rev. E 78 (2008) 036207.