THE PARALLEL POSTULATE **IS DEPENDED** ON THE OTHER AXIOMS .

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This article was sent to some specialists in Euclidean Geometry for criticism . The geometrical solution of this problem is based on the four Postulates for Constructions in Euclid geometry .

EUCLID ELEMENTS FOR A PROOF OF THE PARALLEL POSTULATE (AXIOM)

The first Definitions (D) of Terms in Geometry :

1 . D1. A point is that which has no part . (Position)
2 . D2. A line is a breathless length . (For straight line , The Whole is equal to the Parts)
3 . D3. The extremities of lines are points . (Equation) .
4 . D4. A straight line lies equally with respect to the points on itself . (Identity)
5 . D A midpoint C divides a Segment AB (of a straight line) in two . CA = CB
Any point C divides all straight lines through this in two .
6 . D A straight line AB divides all Planes through this in two .
7 . D A Plane ABC divides all Spaces through this in two .

Common Notions .

CN1. Things which equal the same thing also equal one another .
CN2. If equals are added to equals , then the wholes are equal .
CN3. If equals are subtracted from equals , then the remainders are equal .
CN4. Things which coincide with one another equal one another .
CN5. The whole is greater than the part .

The five Postulates (P) for construction are :

1 . P1. To draw a straight line from any point A to any other B .
2 . P2. To produce a finite straight line AB continuously in a straight line.
3 . P3. To describe a circle with any centre and distance . *P_1 , P_2 are Unique* .
4 . P4. That , all Right angles are equal to each other .
5 . P5. That , if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, if produced indefinitely , meet on that side on which are the angles less than the two right angles , **or** (for three points on a Plane)
5a. The same is Player`s Postulate which states that , From any point M , not on a straight line AB , only one line MM´ can be drawn Parallel to AB .

Since a straight line passes through two points only and because point M is the third then The Parallel Postulate it is valid on a Plane (three points only) .

AB is a straight line through points A , B . **M** is a point not on AB since MA + MB > AB .

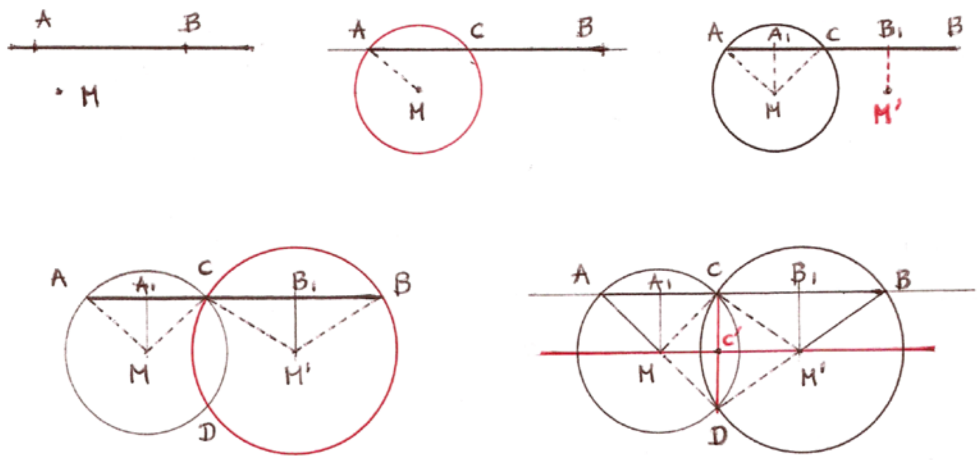
To prove that , one and only one line MM ′ can be drawn Parallel to AB .

A	B	A	B
▫	▫	—▫————————▫—	
M ▫		M ▫ — — —	M′

To prove the above Axiom is necessary to show :

a. The parallel to AB is the locus of all points at a constant distance **h** from the line AB , *and for point M is* MA_1 ,

b. The locus of all these points is a straight line .



METHOD

STEP 1.
Draw the circle (M , MA) be joined meeting line AB in C .

Since MA = MC , point M is on mid-perpendicular of AC . Let A_1 be the midpoint of AC, (*it is* $A_1A+A_1C = AC$ *because* A_1 *is on the straight line AC . Triangles* MAA_1 , MCA_1 *are equal because the three sides are equal , therefore angle* $< MA_1A = MA_1C$ (CN1) *and since the sum of the two angles* $< MA_1A + MA_1C = 180^{\circ}$ (CN2 , 6D) *then angle* $< MA_1A = MA_1C = 90^{\circ}$.(P4) *so* , MA_1 is the minimum fixed distance **h** of point M to AC.

STEP 2.
Let B_1 be the midpoint of CB ,(*it is* $B_1C+B_1B = CB$ *because* B_1 *is on the straight line CB*) and draw $B_1M' = h$ equal to A_1M on the mid-perpendicular from point B_1 to CB .

Draw the circle (M′, M′B = M′C) intersecting the circle (M , MA = MC) at point D.(P3)

Since M′C = M′B , point M′ lies on mid-perpendicular of CB. (CN1)
Since M′C = M′D , point M′ lies on mid-perpendicular of CD. (CN1)
Since MC = MD , point M lies on mid-perpendicular of CD. (CN1)

Because points M and M′ lie on the same mid-perpendicular (This mid-perpendicular is drawn from point C′ to CD and it is the midpoint of CD), and because only one line MM′ passes through points M ,M ′ then **line MM′ coincides with this mid-perpendicular** (CN4)

STEP 3 .
Draw the perpendicular of CD at point C′. (P3, P1)

a Because $MA_1 \perp AC$ and also $MC' \perp CD$ then angle < $A_1MC' = A_1CC'$. (*CN 2 ,CN3 ,E.I.15*)
Because $M'B_1 \perp CB$ and also $M'C' \perp CD$ then angle < $B_1M'C' = B_1CC'$. (*CN2 , CN3, E.I.15*)

b **The sum of angles $A_1CC' + B_1CC' = 180^\circ = A_1MC' + B_1M'C'$. (** 6.D **) , and since Point C′ lies on straight line MM′ , therefore the sum of angles in shape $A_1B_1M'M$ are** < $MA_1B_1 + A_1B_1M' +$ **[** $B_1M'M + M'MA_1$ **] = $90^\circ + 90^\circ + 180^\circ = 360^\circ$ (** *CN2* **) ,** ***i.e. The sum of angles in a Quadrilateral is 360° and in Rectangle all equal to 90° . (m)***

c The right-angled triangles MA_1B_1 , $M'B_1A_1$ are equal because $A_1M = B_1M'$ and A_1B_1 common , ***therefore side $A_1M' = B_1M$*** (CN1). Triangles A_1MM',$B_1M'M$ are equal because have the three sides equal each other , ***therefore angle < $A_1MM' = B_1M'M$*** , and since their sum is 180° as before (6D) , ***so angle < $A_1MM' = B_1M'M = 90^\circ$*** (CN2).

d Since angle < $A_1MM' = A_1CC'$ and also angle < $B_1M'M = B_1CC'$ (P4),***therefore quadrilaterals $A_1CC'M$, $B_1CC'M'$, $A_1B_1M'M$ are Rectangles*** **(**CN3**) .** From the above three rectangles and because all points (M , M′ and C′) equidistant from AB , this means that C′C is also the minimum equal distance of point C′ to line AB **or , h = $MA_1 = M'B_1 = CD / 2 = C'C$** **(**CN1**)** **Namely , line MM′ is perpendicular to segment CD at point C′ and this line coincides with the mid-perpendicular of CD at this point C′ and points M , M′ ,C′ are on line MM′. Point C′ equidistant , h , from line AB , as it is for points M , M′, so the locus of the three points is the straight line MM′ , so the two demands are satisfied ,**
($h = C'C = MA_1 = M'B_1$ and also $C'C \perp AB$, $MA_1 \perp AB$, $M'B_1 \perp AB$) . (*ο.ε.δ.*)

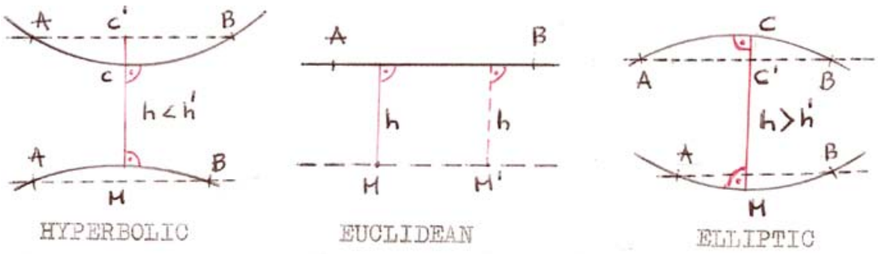
e The right-angle triangles A_1CM ,MCC' are equal because side $MA_1 = C'C$ and MC common so angle <$A_1CM = C'MC$, and ***The Sum of angles $C'MC + MCB_1 = A_1CM + MCB_1 = 180^\circ$***

All above is a Proof of the Parallel postulate due to the fact that the parallel postulate is dependent of the other four axioms (now is proved as a theorem from the other four) .Since AB is common to ∞ Planes and only one Plane is passing through point M (Plane ABM from the three points A , B , M , then the Parallel Postulate is valid for all Spaces which have this common Plane , as Spherical, n-dimensional Spaces geometry .It was proved that it is a necessary logical consequence of the others axioms, agree also with the Properties of physical objects ,d + 0 = d ,d * 0 = 0, now is possible to decide through mathematical reasoning ,that the geometry of the physical universe is Euclidean .Since the essential difference between Euclidean geometry and the two non-Euclidean geometries *Spherical and hyperbolic geometry*, is the nature of parallel line ,the parallel postulate so ,

The consistent System of the < non-Euclidean geometry > have to decide the direction of the existing mathematical logic .

The above consistency proof is applicable to any line Segment AB on line AB , (segment AB is the first dimentional unit , as AB = 0 → ∞) , and M any Point not on line AB , (MA + MB > AB on three points only which consist the Plane) , and answers to the cry about the < crisis in the foundations of Euclid geometry > .

TYPES OF GEOMETRY



1 Any single point A constitutes a Unit without any Position (non-dimentional= Empty Space) Simultaneously zero , finite and infinite . The unit meter of Point is equal to 0 .

2 Any single point B , not coinciding with A , constitutes another one Unit which has also dimension zero . Only one straight line (i.e. the Whole is equal to the Parts) passes through points A and B ,which consists another un-dimensional Unit ,since is consisted of infinite points with dimension zero . A line Segment AB between points A and B (either points A and B are near zero or are extended to the infinite) , **consists the first Unit with one dimensional** , ***the length AB*** , beginning from Unit A and a regression ending in Unit B .
$AB = 0 \rightarrow \infty$, is the one-dimensional Space . The unit meter of AB is $m = 2. (AB/2) = AB$

3 Adding a third point C , outside the straight line AB , then is constituted a new Unit (the Plane) without position, since is consisted of infinite points , without any position. Shape ABC enclosed between parts AB , AC , BC is of two dimensional, the enclosed area ABC. Following harmony of unit meter AB=AC=BC , then Area $ABC = 0 \rightarrow \infty$, is the two-dimensional Space with unit meter equal to $m = 2.(\pi.AB/\sqrt{2})^2 = \pi .AB^2$, i.e. one square equal to the area of the unit circle .

4 Four points A,B,C,D (. . . .) not coinciding , consist a new Unit (the Space or Space Layer) without position also , which is extended between the four Planes and all included , forming Volume ABCD . Following the same harmony of the first Unit , shape ABCD is the Regular Tetrahedron with volume $ABCD = 0 \rightarrow \infty$, and it is the three-dimensional Space .
The dimension of Volume is 4 - 1 = 3 . The unit measure of volume is the side X of cube X^3 twice the volume of another random cube of side a=AB such as $X^3 = 2.a^3$ and $X = \sqrt[3]{2}.a$ Geometry measures Volumes with side X related to the problem of doubling of the cube .

In case that point D is on a lower Space Layer , then all Properties of Space , or Space Layer are transferred to the corresponding Unit , i.e. to Plane or to the Straight line or to the Point . This *Concentrated(Compact) Logic of geometry* [CLG] exists for all Space – Layers and is very useful in many geometrical and physical problems. (exists , Quality = Quantity , since all the new Units are produced from the same , *the first one* , dimensional Unit AB) .

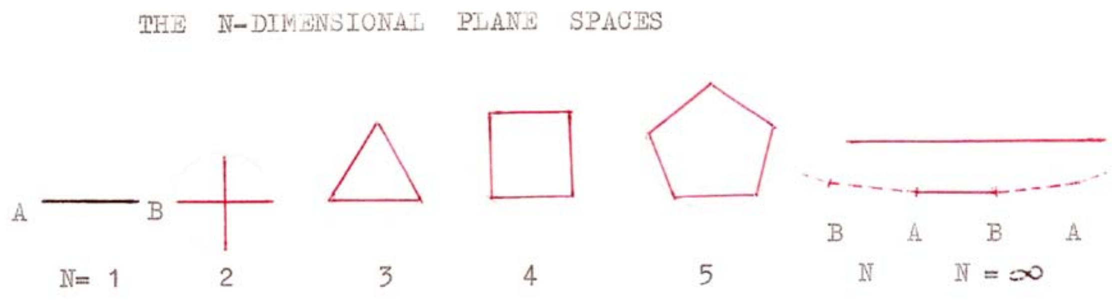
5 N points represent the N-1 dimentional Space or the N-1 Space Layer ,DL, and has analogous properties and measures .Following the same harmony for unit AB , ($AB = 0 \rightarrow \infty$) then shape ABC..M (i.e. the ∞ spaces AB = 1, 2,..nth) is the Regular Solid in Sphere $ABC..M = 0 \rightarrow \infty$. This N Space Layer is limiting to ∞ as $N \rightarrow \infty$.
Proceeding inversely with roots of any unit $AB = 0 \rightarrow \infty$ (i.e. the Sub-Spaces are the roots of AB , $\sqrt[2]{AB}$, $\sqrt[3]{AB}$,.. $\sqrt[n]{AB}$ then it is $\sqrt[n]{AB} = 1$ as $n \rightarrow \infty$) , and since all roots of unit AB are the vertices of the Regular Solids in Spheres then this n Space Layer is limiting to 0 as $n \rightarrow \infty$
The dimensionality of the physical universe is unbounded (∞) but simultaneously equal to (0) as the two types of Spaces and Sub-Spaces show .

Because the unit-meters of the N-1 dimensional Space Layers coincide with the vertices of the nth-roots of the first dimensional unit segment AB as $AB = \infty \rightarrow 0$, (the vertices of the n-sided Regular Solids) , therefore the two Spaces are coinciding (the Space Layers and the Sub-Space Layers are in superposition) .

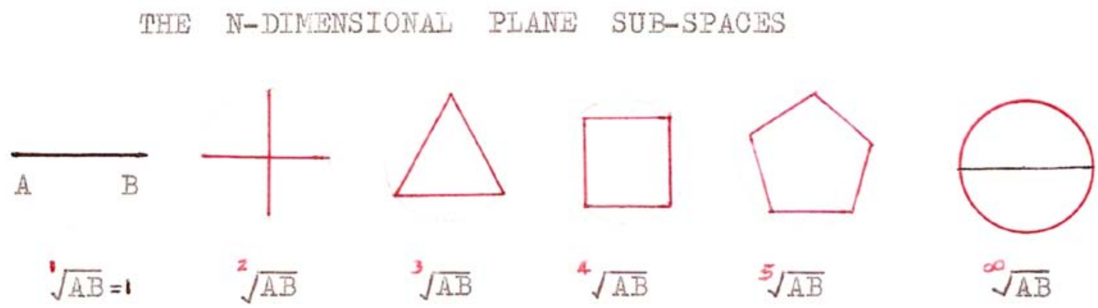
That is to say , ***Any point on the Nth Space or Space-Layer*** , *of any unit* $AB = 0 \rightarrow \infty$, ***jointly exists*** *partly or whole,* ***with all Subspaces of higher than N Spaces*** , $N =(N+1) -1=(N+2) -2=(N+N)-N ..= (N+\infty) - \infty$, *where* $(N+1) ,\ldots(N+\infty)$ *are the higher than N Spaces* , ***and with all Spaces of lower than N Subspaces*** , $N = (N-1) +1 = (N-2)+2 = (N-N)+N = (N -\infty) +\infty$, *where* $(N-1) , (N-2), (N-N) , (N-\infty)$ *are the lower than N Spaces* .

The boundaries of N points , *corresponding to the Space , have their unit meter of the Space and is* ***a Tensor of N dimension*** *(i.e. the unit meters of the N roots of unity AB) ,* ***simultaneously*** *,because belonging to the Sub-Space of the Unit Segments > N ,* ***have also the unit meter of all spaces*** *.*

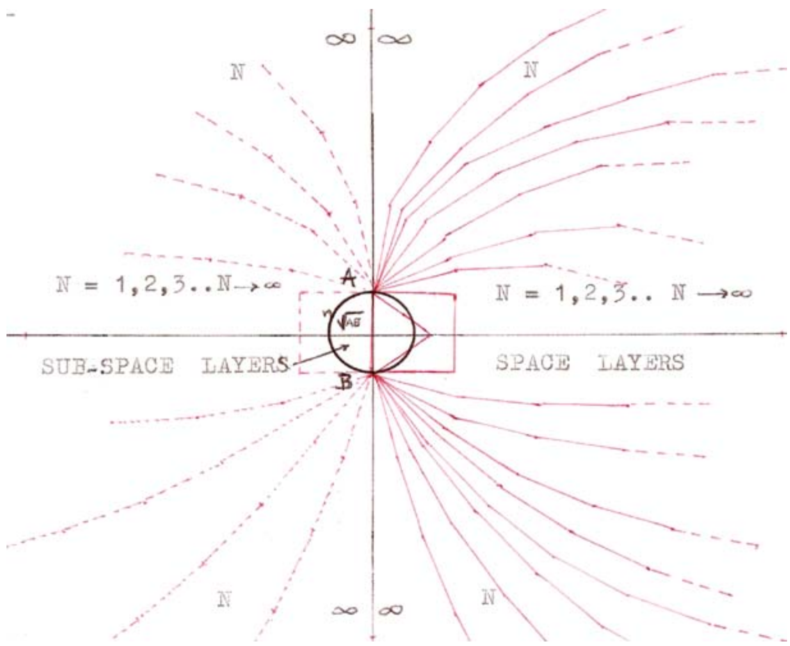
1. The Space Layers (Regular Solids) with sides equal to AB = 0 → ∞
   The Increasing Plane Spaces with the same Unit .



2 . The Sub-Space Layers (Regular Solids on AB) as Roots of AB = 0 → ∞
   The Decreasing Plane Spaces with the same Unit .



3. The superposition of Plane Space Layers and Sub-Space Layers :
   The simultaneously co-existence of Spaces and Sub-Spaces of any Unit AB = 0 → ∞ , i.e. Euclidean , Elliptic , Spherical , Parabolic , Hyperbolic , Geodesics , metric and non-metric geometries . The Interconnection of Homogeneous and Heterogeneous Spaces , and Subspaces of the Universe .



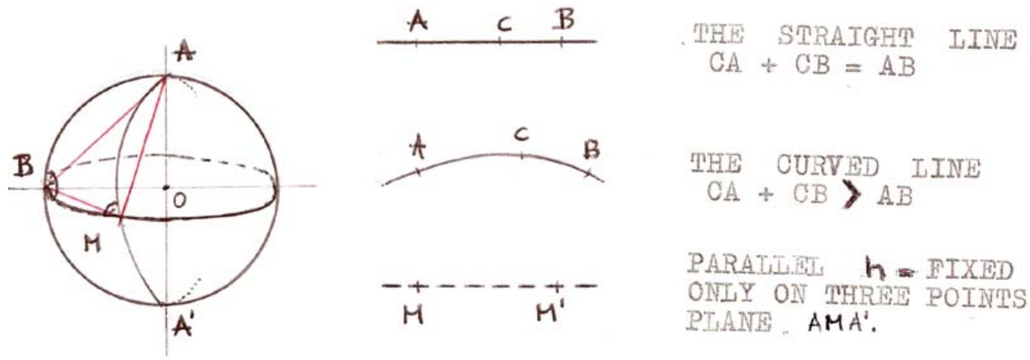
6 A linear shape is the shape of N points on a Plane bounded with straight lines .
A circle is the shape on a Plane with all points equally distance from a fix point O .
A curved line is the shape on a Plane with points not equally distance from a fix point O .
Curved shapes are those on a Plane bounded with curved lines .

Rotating the above axial-centrifugally (machine AB $\perp$ AC) is obtained Flat Space , Conics , Sphere , Curved Space , multi Curvature Spaces , Curved Hyperspace etc .
The fact that curvature changes from point to point , is not a property of one Space only but that of the common area of more than two Spaces , namely the result of the Position of Points.
Euclidean manifold (Point , sectors , lines , Planes , all Spaces etc) and the one dimentional Unit AB is the same thing (according to Euclid ἑν το πάν) .

Since Riemannian metric and curvature is on the great circles of a Sphere which consist a Plane, say AMA΄ , where the Parallel Postulate is consistent with three points only , therefore the great circles are not lines (this is because it is MA + MA΄ > AA΄) and the curvature of Space is that of the circle in this Plane , i.e. that of the circle (O, OA) .
Because Parallel Axiom is for three points only , which consist a Plane , then the curvature of < empty space > is 0 , (*has not intrinsic curvature*) .

The physical laws are correlated with the geometry of Spaces and can be seen , using CLG , in Plane Space as it is shown in figures 1 , 2 , 3 .
A marvelous Presentation of the method can be seen ***on Dr Geo-Machine Macro-constructions.***

Perhaps , Inertia is the Property of a certain Space Layer , and the Interaction of Spaces at the Commons or those have been called *Concentrated Logic* , creates the motion .



The parallel axiom (the postulate) on any Segment AB is experimentally verifiable, and in this way it is Dependent of the other Axioms and is logically consistent , and since this is true then must be accepted , The Parallel Postulate is a Parallel Axiom ,
So , all Nature (the Universe) is working according to the Principles (the patterns) , the Properties and the dialectic logic of Euclidean geometry.

Hyperbolic and Projective geometry transfers the Parallel Axiom to problem of a point M and a Plane AB-C instead of problem of three points only , which such it is.

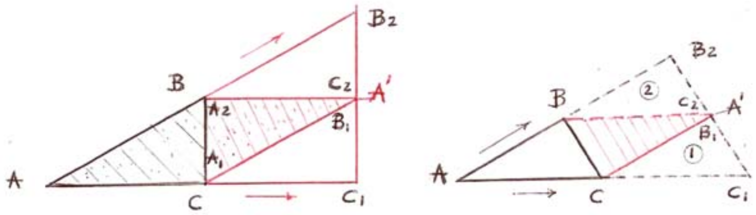
Vast (the empty space) is simultaneously ∞ and 0 for every unit AB , as this is for numbers.
Uniformity (Homogenous) of Empty Space creates , all the one dimensional units , the Laws of conservation for Total Impulse , moment of Inertia in Mechanics , independently of the Position of Space and regardless the state of motion of other sources. (*Isotropic Spaces*)
Uniformity (Homogenous) of Empty Time creates , the Laws of conservation of the Total Energy regardless of the state of motion (*Time is not existing here)*, since Timing is always the same and Time Intervals are not existing .

In Special Relativity events from the origin are determined by a velocity and a given unit of time , and the position of an observer is related with that velocity after the temporal unit.
Since all Spaces and Substances co-exist , then Past , Present and Future simultaneously exist on different Space Layer . Odd and Even Spaces have common Properties , so Gravity belonging to different Layer as that of particles , is not valid in atom Layer .
Spaces may be simultaneously Flat or Curved or multi-Curved , according to the Concentrated , (Compact) Logic of the Space , so the changing curvature from point to point is possible .

RATIONAL FIGURED NUMBERS OR FIGURES . 20 / 3 / 2010

This document is related to the definition of " Heron " that gnomon is as that which , when added to anything , a number or figure , makes the whole similar to that to which it is added . In general the successive gnomonic numbers for any polygonal number , say , of n sides have n-2 for their common difference . The odd numbers successively added were called gnomons . See Archimedes (Heiberg 1881, page 142 ,ε΄.) The Euclidean dialectic logic of an axiom is that which is true in itself. τών δέ άνίσων , όμοίων δέ τά κέντρα τών βαρέων όμοίως έσσείται κείμενα . όμοίως δέ λέγομες σαμεία κείσθαι ποτί τά όμοία σχήματα . άφ΄ ών επί τάς ίσας γωνίας άγόμεναι εύθείαι ποιέοντι γωνίας ίσας ποτί τάς όμολόγους πλευράς . (Επιπέδων ίσορροπιών ή κέντρα βαρών επιπέδων α΄).

This logic exists in nature and is reflected to our minds as dialectic logic of mind . Shortly for ancient Greeks was , (μηδέν εν τη νοήσει ειμή πρότερον εν τοί αισθήσοι) .Since the first dimentional Unit is Segment AB , it is obvious that all Rational Segments are multiples of AB *potentially the first polygonal number of any form*, and the first is 2AB = AB +AB , which shows that multiplication and Summation is the same action with the same common base , the Segment AB .



To Prove :

The triangle with sides AC_1 ,AB_2 ,C_1B_2 twice the length of initial segments AC ,AB ,CB preserves the same angles < A = BAC , B = ABC , C = ACB of the triangle .

Proof :

a. Remove triangle ABC on line AC such that point A coincides with point C (A_1) . Triangles CB_1C_1 , ABC are equal , so CA΄ = AB , C_1A΄ = CB

b. Remove triangle ABC on line AB such that point A coincides with point B (A_2) . Triangles BB_2C_2 , ABC are equal , so BC_2 = AC , B_2C_2 = BC

c. The two circles (C ,CB_1 = AB) and (B ,BC_2 = AC) determine by their intersection point A΄, *so* ***triangles CBA΄, CBA are equal*** *, and also equal* ***to the triangles*** ***CC_1B_1 , BB_2C_2*** *, and this proposition states that sides* *CB_1 = CA΄ , BC_2 = BA΄ .* Point A΄ must simultaneously lie on circles ***(C_1 , C_1B_1) , (B_2 , B_2C_2)*** , **which is not possible unless point A΄ coincides with points B_1 and C_2 .**

d. This logic exists in Mechanics as follows :
The linear motion of a Figure or a Solid is equivalent to the linear motion of the gravity centre because all points of them are linearly displaced , so

1^{st}	Removal	----	$BB_1 = AC$, $CB_1 = AB$, $BC = BC$
2^{nd}	Removal	----	$CC_2 = AB$, $BC_2 = AC$, $BC = BC$
$1^{st} + 2^{nd}$	Removal	----	$CB_1 = AB$, $BC_2 = AC$, $BC = BC$

Since all degrees of freedom of the System should not be satisfied therefore points B_1 , C_2 , A' coincide .

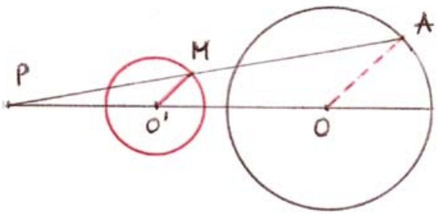
e. Since circles (C_1 ,$C_1B_1 = C_1A' = CB$) , (B_2 , $B_2C_2 = B_2A' = CB$) pass through one point A', then $C_1A'B_2$ is a straight line , *this because $C_1A' + A'B_2 = C_1B_2$* , and A' is the midpoint of segment B_2C_1 .

f. By reasoning similar to what has just been given , it follows that the area of a triangle with sides twice the initials , is four times the area of the triangle .

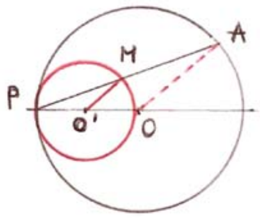
g. Since the sum of angles < $C_1A'C + CA'B + BA'B_2 = 180^\circ$ (6D) and equal to the sum of angles CBA + CAB + ACB ***then the Sum of angles of any triangle ABC is 180°*** , which is not depended on the Parallel Theorem or else-where .

Verification :
Let be the sides $a=5$, $b=4$, $c=3$ of a given triangle and from the known formulas of area $S = (a + b + c) / 2 = 6$, Area $= \sqrt{6.1.2.3} = 6$
For $a=10$, $b=8$, $c=6$ then $S = 24/2 = 12$ and Area $= \sqrt{12.2.4.6} = 24 = 4 \times 6$ (four times)

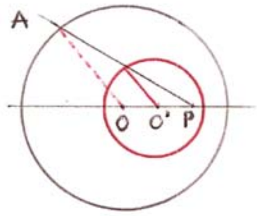
A GIVEN POINT **P** , AND ANY CIRCLE **(O , OA)** .



Point P outside the circle.



Point P on circle.



Point P in circle.

To Prove :

The locus of midpoints M of segments PA , is a circle with center O' at the middle of PO and radius $O'M = OA / 2$ *where* ,

P is any point on a Plane
A is any point on circle (O , OA)
M is mid point of segment PA ,

Proof :

Let O' and M be the midpoints of PO , PA . According to the previous given for Gnomon , the sides of triangle POA are twice the size of $PO'M$, or $PO = 2.PO'$ and $PA = 2.PM$ therefore as before , $OA = 2.O'M$, **or $O'M = OA/2$** . Assuming M found , and
Since O' is a fixed point , and $O'M$ is constant , then (O' , $O'M = OA/2$) is a circle .

For point P on the circle :

The locus of the midpoint M of chord PA is the circle (O',$O'M = PO / 2$) and it follows that triangles OMP , OMA are equal which means that angle < $OMP = OMA = 90^\circ$, i.e. the right angle < $PMO = 90^\circ$ and exists on diameter PO (on arc PO) , and since the sum of the other two angles < MPO + MOP exist on the same arc PO = PM + MO , it follows that the sum of angles in a rectangle triangle is $90 + 90 = 180^\circ$

THE TWO ANGLES PROBLEM
ANY TWO ANGLES α =AOB , β = A′O′B′ WITH PERPENDICULAR SIDES ARE EQUAL

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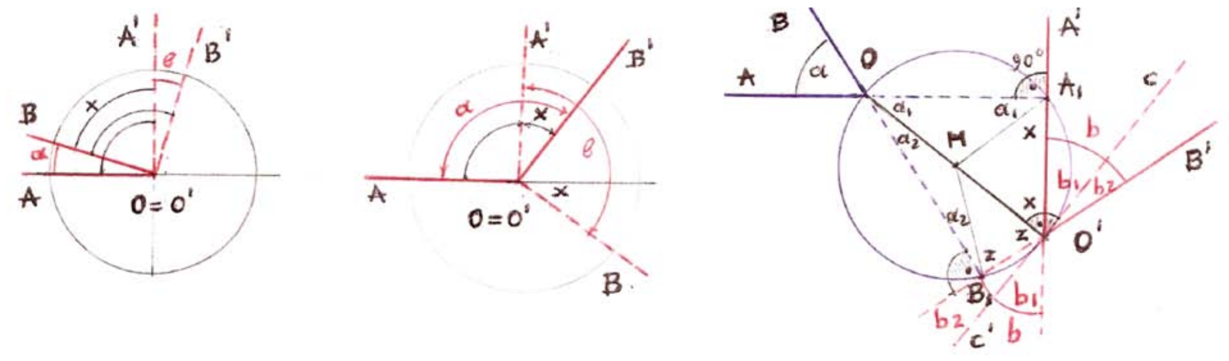


O = O ′
angle $\alpha \leq 90^{\circ}$
Rotation of $\alpha=\beta$

O = O ′
angle $\alpha > 90^{\circ}$
Rotating of $\alpha=\beta$

O # O′
any angle
Displacing of $\alpha = \beta$

Let AOB = a be any given angle and angle A′O′B′ = b such that AO ⊥ O′A′ , OB ⊥ O′B′ .
To proof that angle b is equal to a .

Proof :

CENTRE O′ = O , $\alpha \leq 90^{\circ}$
Angle < AOA′ = 90° = AOB + BOA′ = α + x (1)
Angle < BOB ′ = 90° = BOA′ + A′OB′ = x + β (2) , subtracting (1) , (2) → **angle $\beta = \alpha$**

CENTRE O′ = O , 90° < α < 180°
Angle < AOA′ = 90° = AOB′ - B′OA′ = α - x (1)
Angle < BOB ′ = 90° = A′OB - A′OB′ = β - x (2) , subtracting (1) , (2) → **angle $\beta = \alpha$**

CENTRE O′ # O .
Draw circle (M , MO = MO′) with OO′ as diameter intersecting OA ,O′B′ produced to points A_1,B_1.
Since the only perpendicular from point O to O′A′ and from point O′ to OB is on circle (M , MO) then , points A_1 , B_1 are on the circle and angles O′A_1O , O′B_1O are equal to 90° .

The vertically opposite angles a = $a_1 + a_2$, b = $b_1 + b_2$ where O′C ⊥ OO′ .

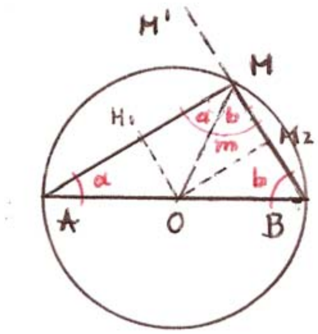
Since MO = MA_1 then angle < MOA_1 = MA_1O = a_1 .
Since MA_1 = MO ′ then angle < MA_1O′ = MO′A_1 = x
Since MO′ = MB_1 then angle < MO′B_1 = MB_1O′ = z

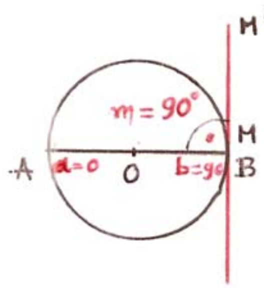
Angle MO′C = 90° = x + b_1 = z + b_2 .
Angle O′A_1O = 90° = x + a_1 = x + b_1 → $a_1 = b_1$
Angle O′B_1O = 90° = z + a_2 = z + b_2 . → $a_2 = b_2$

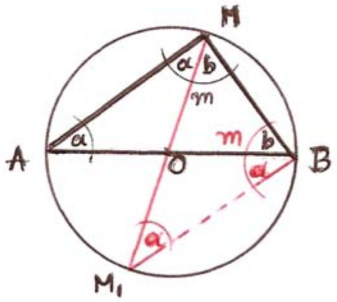
By summation $a_1 + a_2 = b_1 + b_2$ **or** **b = a** (ο.ε.δ)

i.e. any two angles a , b having their sides perpendicular among them are equal .
From upper proof is easy to derive the Parallel axiom , and more easy from the Sum of angles on a right-angled triangle .

POINT M ON A CIRCLE OF DIAMETER AB = 0 → ∞







AB = Diameter   M → B , AB ⊥ BM′   Δ [AMB = MBM1]

Let M be any point on circle (O ,OM = OA = OB) , M_1,M_2 the middle points of MA , MB and in second figure MM′ ⊥ BA at point B (angle AMM′ = 90 °) . In third figure MM_1 is diameter.

To Show :

1 . Angle < AMB = MAB + MBA = a + b = m
2 . Triangles MBM_1 , MBA are always equal and angle < MBM_1 = AMB = 90 °
3 . The Sum of angles on triangle MAB are < AMB + MAB + MBA = 180° .

Proof :

1 . Since OA = OM and M_1A = M_1M and OM_1 common , then triangles OM_1A , OM_1M are equal and angle < OAM = OMA = BAM = a → (a)

Since OM = OB and M_2B = M_2M and OM_2 common , then triangles OM_2B , OM_2M are equal and angle < OBM = OMB = ABM = b → (b)

By summation (a) , (b) **BAM + ABM** = (OMA + OMB) = **AMB = a + b = m** ..(c)

i.e. When a Point M lies on the circle of diameter AB , then the sum of the two angles at points A , B is constantly equal to the other angle at M . Concentrated logic of geometry exists at point B , because as on segment AB of a straight line AB , which is the one dimensional Space , *springs the law of Equality , the equation AB = OA + OB* ***i .e. The whole is equal to the parts*** *, so the same is valid for angles of all points on the circumference of the circle (O,OM) ,* [as Plane ABM and all angles there exist in the two dimensional Space] *, and it is* ***m = a + b*** .

In figure (2) , when point M approaches to B , the Side BM′ of angle < ABM tends to the perpendicular on BA and ***when point M coincides with point B , then angle < ABM = 90° and < OAM = BAM = 0 , therefore angle < AMB = 90°*** and equation (c) becomes :

BAM + ABM = AMB → 0 + 90° = AMB → **AMB = 90°** , (**i.e. AM ⊥ BM**) and the sum of angles is (BAM + ABM) + AMB = 90° + 90 ° = 180° , **or BAM + ABM + AMB = 180°**

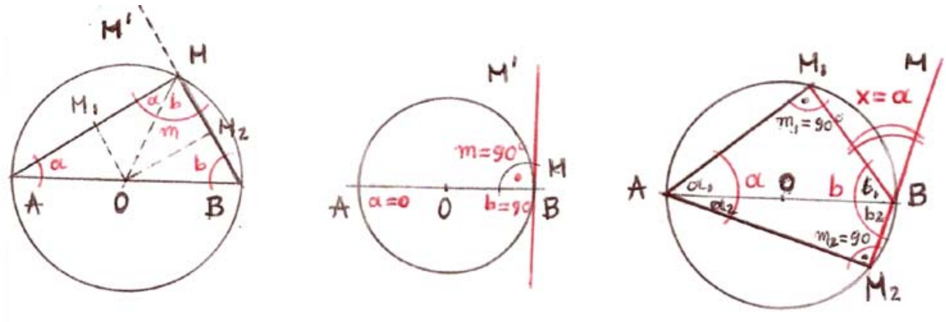
2 . Triangles MBA , MBM_1 are equal because they have diameter MM_1 = AB , MB common and angle < OBM = OMB = b (*from isosceles triangle OMB*) .
Since Triangles MBA , MBM_1 are equal therefore angle < MM_1B = MAB = a ,
and from isosceles triangle OM_1B , angle < OBM_1 = OM_1B = a
The angle at point B is equal to MBM_1 = MBA + ABM_1 = b + a = m = AMB
Rotating diameter MM_1 through centre O so that points M , M_1 coincides with B , A then angle < MBM_1 = MBA + ABM_1 = BBA + ABA = 90° + 0 = 90° and equal to AMB

i.e. The required convention for angle MBM_1 = AMB = m = a + b = 90 °. (o.ε.δ)

3 . Since the Sum of angles a + b = 90▫ , and also m = 90 ▫ then a + b + m = 90+90 = 180 ▫.
It is needed to show that angle **m** is always constant and equal to 90▫ for all points on the circle.
Sine angle at point B is always equal to $MBM_1 = MBO + OBM_1 = b + a = m = AMB$, by Rotating triangle MBM_1 so that points M ,B coincide then $MBM_1 = BBA + ABA = 90+0 = m$

Since angle < AMB = a + b = m and is always equal to angle < MBM_1 , *of the rotating unaltered triangle MBM_1 , and since at point B angle < MBM_1 of the rotating triangle MBM_1 is 90 ▫ ,* then is always valid , angle < AMB = MBM_1 = 90 ▫ (ο.ε.δ) , 22 / 4 /2010 .

ANY TWO ANGLES HAVING THEIR SIDES PERPENDICULAR AMONG THEM ARE EQUAL OR SUPLEMENTARY



AB = Diameter M → B , AB ⊥ BM′ $AM_1 \perp BM_1$, $AM_2 \perp BM_2 = AM_2 \perp BM$

Let angle < M_1AM_2 = a and angle < M_1BM_2 = b , which have side $AM_1 \perp BM_1$ and side $AM_2 \perp BM_2 \perp BM$

To show :
1 . Angle < M_1AM_2 = M_1BM = a
2 . Angle < M_1AM_2 + M_1BM_2 = a + b = 180▫ .
3 . The Sum of angles in Quadrilateral AM_1BM_2 is 360▫ .
4 . The Sum of angles in Any triangle AM_1M_2 is 180▫ .
Proof :

1 . In figure 3 , since $AM_1 \perp BM_1$ and $AM_2 \perp BM_2$ or the same $AM_2 \perp BM$, then according to prior proof , AB is the diameter of the circle passing through points M_1 , M_2 , and exists $a_1 + b_1 = m_1 = 90$▫ , $a_2 + b_2 = m_2 = 90$▫ and by summation $(a_1 + b_1) + (a_2 + b_2) = 180$▫ or $(a_1 + a_2) + (b_1 + b_2) = a + b = 180$▫ , and since also x + b = 180▫ therefore **angle < x = a**
2 .Since the Sum of angles M_1BM_2 + M_1BM = 180▫ then a + b = 180 ▫
3 .The sum of angles in quadrilateral AM_1BM_2 is a + b + 90 +90 = 180 + 180 = 360 ▫
4 .Since any diameter AB in Quadrilateral divides this in two triangles , it is very easy to show that diamesus M_1M_2 form triangles AM_1M_2 , BM_1M_2 equal to 180▫ each .
so ,
1 Any angle between the diameter AB of a circle is right angle (90▫) .
2 Two angles with vertices the points A , B of a diameter AB , have perpendicular sides and are equal or supplementary .
3 Equal angles exist on equal arcs , and central angles are twice the inscribed angles.
4 The Sum of angles of any triangle is equal to two right angles .

i.e two Opposite angles having their sides perpendicular between them , are Equal or Supplementary between them . This property has been used in proofs of Parallel Postulate

Conclusions , and how Useful is this invention , is left to the reader . marcos

The essential difference between Euclidean and non-Euclidean geometries is the nature of parallel lines .
1 .
Euclid’s fifth postulate , the parallel postulate , states that , within a two-dimensional plane ABM for a given line AB and a point M , which is not on AB , ***there is exactly one line through M*** that does not intersect AB
In Euclid geometry , in case of two straight lines that are both perpendicular to a third line , the lines remain at a constant distance from each other and are known as parallels .

Now is proved that , a point M on the Nth Space , of any first dimensional Unit AB = 0 → ∞ , jointly exists , with all Sub-Spaces of higher than N Spaces , and with all Spaces of lower than N Subspaces . This is the Structure of Euclidean geometry.
As in fundamental theorem of Algebra Equations of Nth degree can be reduced to all N-a or N +a degree , by using the roots of the equations , in the same way Multi –Spaces are formed on AB .
Nano-scale-Spaces , unorganig and organig , Cosmic-scale-Spaces are now unified in our world scale.
Euclidean Empty Space is Homogenously Continues, but all first dimensional Unit- Spaces Heterogenous and this because all Spaces constitute another Units (*the Nth Space Tensor is the boundaries of N points* ***).***
All above referred and many others , are springing from the first acceptance for point , and the approaching of Points . By multiplication is created another one very important logical notion for the laws concerning Continues or Not Continues Transformations in Space and in Time for Mechanics , Physics Chemistry and motions generally . From this logic Yields that a limited and not an unlimited Universe can Spring anywhere .Since Non-existence is found everywhere then Existence is found and is Done everywhere .
If Universe follows Euclidean geometry , then this is not expanded indefinitely at escape velocity, but is moving in Changeable Spaces with all types of motions , < a twin symmetrically axial – centrifugal rotation > into a Steady Space (System AB ⊥ AB = 0 → AB → ∞) , with all types of curvatures . (***It is a Moving and Changeable Universe into a Steady Formation***) .

2 .
Hyperbolic geometry, by contrast ,states that there are ***infinitely many lines through M*** , not intersecting AB

In Hyperbolic geometry , the two lines “ curve away ” from each other , increasing in distance as one moves further from the points of intersection with the common perpendicular , which have been called ultra-parallels.
The simplest model for Hyperbolic geometry is the pseudo-sphere of Beltrami-Klein , which is a portion of the appropriate curvature of Hyperbolic Space , and the Klein model , by contrast , calls a segment as line and the disk as Plane . ???
Mobius strip and Klein bottle (complete one-sided objects of three and four dimensions) transfers the parallel Postulate to a problem of one point M and a Plane , because all curves and other curve lines are not lines (For any point on a straight line exists < the whole is equal to the parts which is an equality > and not the inequality which has to do with Plane , *three points only and anywhere* .

If Universe follows Hyperbolic geometry then this is expanded indefinitely , which contradicts to the homogenous and isotropic Empty Spaces and also to the laws of conservation of Energy .

3 .
Elliptic geometry , by contrast , states that , ***all lines through point M*** , intersect AB .

In Elliptic geometry the two lines “ curve toward ” each other and eventually intersect .
The simplest model for Elliptic geometry is a sphere , where lines are “ great circles ” ???
For any great circle (which is not a straight line ???) and a point M which is not on the circle all circles (not lines ???) through point M will intersect the circle .

If Universe follows Elliptic geometry then this is expanded to a halt and then this will stark to shrink possibly not to explode as is said , but to change the axial-centrifugal motion to the initial Rectilinear .

Spherical Geometry , changes a line to a " great circle " , that is , a circle of maximum radius , saying that the sum of the angles of a triangle is no longer 180▫ .
Remarks :
1 . A line **is not** a great circle , so anything is built on this logic is false .
2 . The fact that the sum of angles on any triangle is 180▫ ***, is springing for the first time*** , in this article (Rational Figures) , proof g. page 8 , and after 1 / 5 / 2010 → page 11 - 12 .

Hyperbolic Geometry , is replacing the Parallel Postulate , accepting that , for any line AB and a point M not on AB , there are at least two (minimum) distinct lines through M , which do not intersect AB . (Hyperbolic geometry , Wikipedia , the free encyclopedia) .
Remarks :
1 . This admission of two or more than two parallel lines , instead of one of Euclid's , does not proofs the truth of the admission . The same to Euclid's .
2 . ***The proposed Method in this article*** , based on the prior four axioms only , ***proofs*** , (not using any admission) that ,
Through point M on any Plane ABM (three points only) , passes only one line of which all points equidistant from AB , as point M , i.e. ***the right is to Euclid Geometry*** .

Question : Which axiom is not satisfied by Hyperbolic or other geometry ? 15 / 4 / 2010

It has been proved that quadrilateral MA_1CC' is Rectangle (d, p.3) and from equality of triangles MA_1C , $MC'C$ then angle < $C'MC = MCA_1$.
Since the sum of angles < $MCA_1 + MCB = 180$▫ , also , ***the sum of angles < C´MC + MCB = 180*** ▫ ***which answers to Postulate*** **P5** , as this has been set (e , p.3) .

Hyperbolic geometry, *Lobachevsky, non-Euclidean geometry, Wikipedia the free encyclopedia,* states that there are TWO or more lines parallel to a given line AB through a given point M not on AB .
If this is true , *for second angle C_2MC* , exists also the sum of angles < $C_2MC + MCB = 180$▫ , which is Identity ($C_2MC = C'MC$) , i.e. all (*the called parallels*) lines coincide with the only one parallel line MM´ , and so again the right is to Euclid geometry .

Definitions , Axioms or Postulates create a geometry, but in order this geometry to be right must follow the logic of Nature , which is the meter of all logics , and has been found to be the first dimensional Unit AB = 0 → ∞ (figure 3, page 5) i.e. the reflected Model of the Universe .

Lobachevsky's and Rieman's Postulate may seem to be good attempts to prove Euclid's Fifth Postulate by contradiction , and recently by " *compromising the opposites* " in the Smarandache geometries . *Non of them contradicts any of the other Postulates of what actually are or mean* .

From any point M on a straight line AB , ***springs the logic of the equation*** (the whole AB is equal to the parts MA , MB) , which is rightly followed (*intrinsically*) ***in Euclidean geometry only*** , in contradiction to the others which are based on a confused and muddled false notion (the great circle or segment is line , disk as planes and others) , ***so all non-Euclidean geometries contradict to the second (D2) definition and to the first (P 1) Euclidean Postulate .***

Question : Is it possible to show that the sum of angles on any triangle is 180▫ ? 1 / 5 / 2010
Yes by Using Euclidean Spaces and Subspaces .
Since the two dimensional Spaces exists on Space and Subspace then this problem of angles must be on the boundaries of the two Spaces i.e. on the circumference of the circle and on any tangent of the circle and also to that point where Concentrated Logic of geometry exists for all units , as straight lines etc . It was proofed at first that , the triangle with hypotenuse the diameter of the circle is a right angled triangle and then the triangle of the Plane of the three vertices and that of the closed area of the circle (the Subspace) , measured on the circumference is 180 ▫ .

Some Answers to dear professors ,

Question 1 : Which axiom is not satisfied by Hyperbolic or other geometry ? 15 / 4 / 2010

It has been proved that quadrilateral MA_1CC' is Rectangle (d, p.3) and from equality of triangles MA_1C , $MC'C$ then angle < $C'MC = MCA_1$.
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If this is true , *for second angle C_2MC* , exists also the sum of angles < $C_2MC + MCB = 180$▫ , which is Identity ($C_2MC = C'MC$) , i.e. all (*the called parallels*) lines coincide with the only one parallel line MM´ , and so again the right is to Euclid geometry .

Definitions , Axioms or Postulates create a geometry, but in order this geometry to be right must follow the logic of Nature , which is the meter of all logics , and has been found to be the first dimensional Unit AB = 0 → ∞ (figure 3, page 5) i.e. the reflected Model of the Universe .

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Question 2 : Why Hyperbolic geometry satisfies the same 4 axioms as Euclidean geometry , and the error in my Euclidean derivation of the 5^{th} axiom . 24 / 4 / 2010
An analytical trial is done to answer this question .

Postulate 1 : states that ,
" Let it have been postulated to draw a straight-line from any point to any point " .

As this can be done by placing the Ruler on any point A to any point B , then this is not in doubt by any geometry . The world " line" in Euclid geometry is straight line (the whole is equal to the parts) and axioms require that line to be as this is (*Black color is Black and White color is White*)
In ancient Greek < Ευθεία γραμμή έστιν , ήτις εξ΄ ίσου τοίς εαυτής σημείοις κείται . >

Postulate 2 : states that ,
" And to produce a finite straight-line "

Marking points A , B which are a line segment AB , and by using a Ruler then can produce AB in both sides continuously , not in doubt by any geometry .

Postulate 3 : states that ,
" And to draw a circle with any centre and radius "

Placing the sting of a Compass at any point A (centre) and the edge of pencil at position B and (as in definition 15 for the circle) Radiating all equal straight lines AB , is then obtained the figure of the circle (the circumference and the inside) , not in doubt by any geometry .

Postulate 4 : states that ,
" And all right angles are equal to one another "

In definition D8 is referred as *Plane Angle* , to be the inclination of two lines in a plane meeting one another , and are not laid down straight-on with respect to one another *, i.e. the angle at one part of a straight line .*

In definition D9 is stated " And when the lines containing the angle are straight then the angle is called *rectilinear* " and this because straight lines divide the plane , and as plane by definition is 360° then the angles on a straight line are equal to 180°
In definition D10 is stated that a perpendicular straight line stood upon (another) straight-line makes adjacent angles (which are) equal to one another , each of the equal angles *is a right-angle* and this because as the two adjacent angles are equal and since their sum is 180° , then the two right-angles are 90° each and since this happen to any two perpendicular straight-lines , then *all right angles* are equal to one another , not in doubt by any geometry .

Postulate 5 :

This postulate is referred to the Sum of the two internal angles on the same side ***of a straight- line falling across two*** (other) ***straight lines*** , being produced to infinity , and be equal to 180° .

Because this postulate , *beside all attempts to prove it* , was standing for centuries , mathematicians created new geometries *to step aside* this obstruction .

In my *proposed article* the followings have been geometrically proved :

1 . From any point M to any line AB (the three points consist a Plane) is constructed by using the prior four Postulates , a system of three rectangles MA_1CC' , $C'CB_1M'$, MA_1B_1M' .

2 . The Sum of angles $C'MC$ and MCB is $< C'MC + MCB = 180^\circ$, which satisfies postulate P5 of Euclid geometry , and as this is now proved , *then it is an axiom* .

3 ***. The extended Structure of Euclidean-geometry to all Spaces (Spaces and Sub-spaces) resettles truth to this geometry , and by the proposed solution which is applicable to any point M , not on line AB , answers to the temporary settled age-old question for this problem .***

4 . Mathematical interpretation and all relative Philosophical reflections based on the non-Euclid geometry theories , must properly revised and resettled in the truth one .

5. ……

marcos

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EUCLID'S ELEMENTS IN GREEK
The great text of J . L .Heiberg (1883-1886) and the English translation by Richard Fizpatrik .
ELEMENTS BOOK 1 .

THE PARALLEL POSTULATE IS DEPENDED ON THE OTHER AXIOMS by
Markos Georgallides

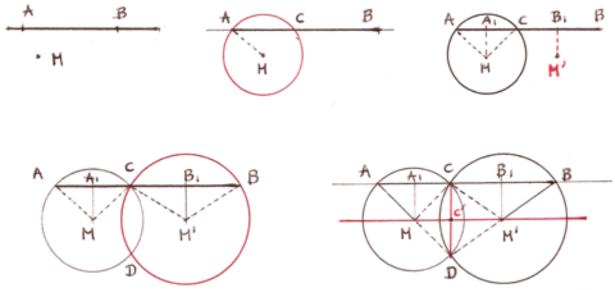
GREEK MATHEMATICS by, Sir Thomas L .Heath - Dover Publications,
Inc ,New York.63-3571.

THE SUCCESSION OF PROOFS FOR THE PARALLEL POSTULATE

AB ***is a straight line*** through points A , B and **M** is a point not on AB .

A ▫ B ▫   A ▫———— B ▫

M ▫   M ▫—— —— M′



METHOD

1 . Draw the circle (M , MA) be joined meeting line AB in C and let A_1 , B_1 be the midpoint of CA , CB .

2 . On mid-perpendicular B_1M ′ find point M′ such that M′B_1 = MA_1 and draw the circle (M′, M′B = M′C) intersecting the circle (M , MA = MC) at point D .

3 . Draw mid-perpendicular of CD at point C′ .

4 . **To show** that line MM′ is a straight line passing through point C ′ and it is such that **MA_1 = M′B_1 = C′C = h** , i.e. a constant distance **h** from line AB or , also **The Sum of angles C′MC + MCB_1 = A_1CM + MCB_1 = 180°**

Proofed Succession :

1 . The mid-perpendicular of CD passes through points M , M ′ . (*page 3* , *step 2*)

2 . Angle < A_1MC′ = A_1 MM′ = A_1CC′ , Angle < B_1M′C′ = B_1M′M = B_1CC′
<A_1MC′ =A_1CC′ because their sides are perpendicular among them i.e. MA_1⊥CA ,MC′⊥CC′.

2a. In case < A_1MM′ +A_1CC′=180° and B_1M′M +B_1CC′ = 180° then < A_1MM′ = 180°- A_1CC′, B_1M′M = 180° - B_1CC′ , and by summation < A_1MM′ + B_1M′M = 360° - A_1CC′- B_1CC′ or **Sum of angles <A_1MM′ + B_1M′M** = 360 - (A_1CC′ + B_1CC′) = 360-180° = **180°** ,(2, *page13*)

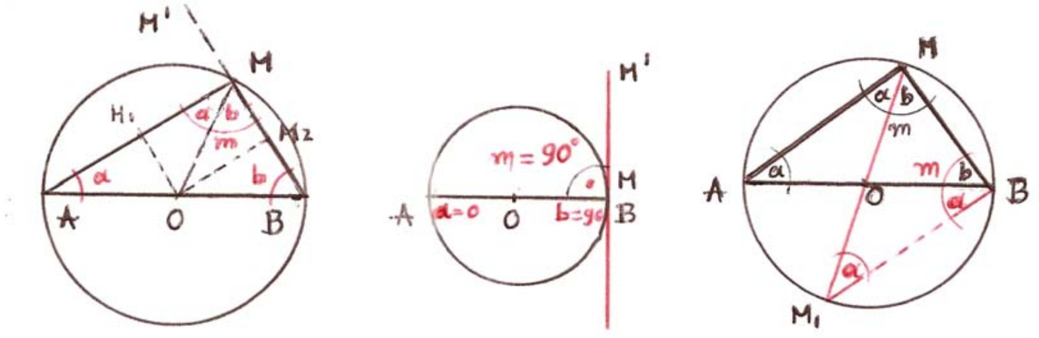
3 . The sum of angles A_1MM′ + B_1M′M = 180° because the equal sum of angles A_1CC′ + B_1CC′ = 180° , so the sum of angles in quadrilateral MA_1B_1 M′ is equal to 360° .

4 . The right-angled triangles MA_1B_1 , M′B_1A_1 are equal , so diagonal MB_1 = M′A_1 and since triangles A_1MM′ , B_1M′M are equal , then angle A_1MM′ = B_1M′M and since their sum is 180° , therefore angle < **A_1MM′ = MM′B_1 = M′B_1A_1 = B_1A_1M = 90°**

5 . Since angle A_1CC′ =B_1CC′ = 90° , then quadrilaterals A_1CC′M , B_1CC′M′ are rectangles and for the three rectangles MA_1CC′ , CB_1M′C′, MA_1B_1M′ exists **MA_1 = M′B_1 = C′C**

6 . The right-angled triangles MCA_1 , MCC ′ are equal , so angle < A_1CM = C′MC and since the sum of angles < A_1CM + MCB_1 = 180° then also C′MC + MCB_1 = 180°
Which is the second to show , as this problem has been set at first by Euclid .

POINT M ON A CIRCLE OF DIAMETER AB = 0 → ∞



AB = Diameter   M → B , AB ⊥ BM′   Δ [AMB = MBM1]

Let M be any point on circle (O , OM = OA = OB) , M_1 , M_2 the middle points of MA ,MB and in second figure MM′ ⊥ BA at point B , (angle AMM′ = 90°) .

To show that the Sum of angles < AMB , MAB , MBA of any triangle ABM is 180° .
Proof :

1 . Since OA = OM and $M_1A = M_1M$ and OM_1 common , then triangles OM_1A , OM_1M are equal and angle < OAM = OMA = BAM = (a)

Since OM = OB and $M_2B = M_2M$ and OM_2 common , then triangles OM_2B , OM_2M are equal and angle < OBM = OMB = ABM = (b)

By summation (a) and (b) → **BAM + ABM** = (OMA + OMB) = **AMB** .... (c)

i.e. When a Point M lies on the circle of diameter AB , then the sum of the two angles at points A , B is constantly equal to the other angle at M . Concentrated logic of geometry exists at point B , because as on segment AB of a straight line AB , which is the one dimensional Space , *springs the law of Equality , the equation AB = OA + OB* ***i .e. The whole is equal to the parts*** *, so the same is valid for angles of all points on the circumference of the circle (O,OM) ,* [as Plane ABM and all angles there exist in the two dimensional Space] *, and it is* ***m = a + b*** .

2 . In figure (2) , when point M approaches to B , the Side BM′ of angle < ABM tends to the perpendicular on BA and ***when point M coincides with point B , then angles < ABM = 90° < OAM = BAM = 0 , AMB = 90°*** and equation (c) becomes :

BAM + ABM = AMB → 0 + 90° = AMB → **AMB = 90°** (d) , (**i.e. AM ⊥ BM**)
and the sum of angles is (BAM + ABM) + AMB = 90° + 90 ° = 180° ,
or BAM + ABM + AMB = 180° (**o.ε.δ**)

2a. To show , the Sum of angles a + b = constant = 90° = m . (**figure 3)**
M is any point on the circle and MM_1 is the diameter . Triangles MBA ,MBM_1 are equal and by rotating diameter MM_1 through centre O , the triangles remain equal .

Proof :

a. Triangles MBA , MBM_1 are equal because they have MM_1 = AB , MB common and angle < OBM = OMB = b (*from isosceles triangle OMB*) so MA = BM_1 .

b. Since Triangles MBA , MBM_1 are equal therefore angle < MM_1B = MAB = a , and from isosceles triangle OM_1B , angle < ABM_1 = OBM_1 = OM_1B = a

c. The angle at point B is always equal to MBM_1 = MBO + OBM_1 = b + a = m = AMB
Rotating triangle MBM_1 so that points M ,B coincide then MBM_1 = ABB +ABA = 90+0 = m

Since angle AMB = a + b = m and is equal to angle <MBM$_1$, *of the rotating unaltered triangle MBM$_1$ and* which at point B has angle m = 90 ▫ , then is valid angle <AMB = MBM$_1$ = 90 ▫ i.e. the required convention for angle AMB = m = a + b = 90 ▫. (ο.ε.δ) , 22 / 4 /2010 .

2b. When point M moves on the circle , Euclidean logic is as follows .
Accepting angle ABM′ = b at point B , automatically point M is on the straight line BM′ and the equation at point B is for (a = 0 , b = 90▫ , m= 90▫) → 0 + 90▫ = m and also equal to , 0 + b – b + 90▫ = m or the same → **b + (90▫ - b) = m** …… (**B**)
In order that point M be on the circle of diameter AB , is necessary → **m = b + a** ...(**M**) where **a** is an angle such that straight line AM (the direction AM) cuts BM′ , and is b + (90▫ - b) = m = b + a or → 90▫ - b = a and → **a + b = 90▫ = constant ,**
i.e. the demand that the two angles a , b satisfy equation (M) is that their sum must be constant and equal to 90 ▫ . (ο.ε.δ)

3 . In figure 3 ,*according to prior proof*, triangles MBA,MBM$_1$ are equal .Triangles AM$_1$B,AMB are equal because AB is common , MA = BM$_1$ and angle < MAB = ABM$_1$, so AM$_1$ = MB .
Triangles ABM$_1$, ABM are equal because AB is common MB = AM$_1$ and AM = BM$_1$ therefore angle < BAM$_1$ = ABM = b and so , angle MAM$_1$ = a + b = MBM$_1$
Since angle AMB = AM$_1$B = 90▫ then AM ⊥ BM and AM$_1$ ⊥ BM$_1$.
Triangles OAM$_1$, OBM are equal because side OA = OB , OM = OM$_1$ and angle < MOB = AOM$_1$, therefore segment M$_1$A = MB .
Rotating diameter MM$_1$ through O to a new position Mx , M$_1$x any new segment MxB = M$_1$xA the angle < MxBM$_1$x = MxBA + ABM$_1$x and segment BMx = AM$_1$x .
Simultaneously rotating triangle MxBM$_1$x through B such that BMx ⊥ AB then angle < MxBM$_1$x = BBA + ABA = 90▫ + 0 = 90▫ ,
i.e. in any position Mx of point M angle < AMxB = MxBM$_1$x = 90▫

i.e two Equal or Supplementary between them Opposite angles have their sides perpendicular between them . (*the opposite to that in page 11*)

Followings the proofs , Any angle between the diameter of a circle is right angle (90 ▫) , central angles are twice the inscribed angles , angles in the same segments are equal to one another and then applying this logic on the circumscribed circle of any triangle ABM , then is proofed that the Sum of angles of any triangle is equal to two right angles or < BAM + ABM + AMB = 180▫

Conclusions , and how Useful is this invention , is left to the reader . marcos

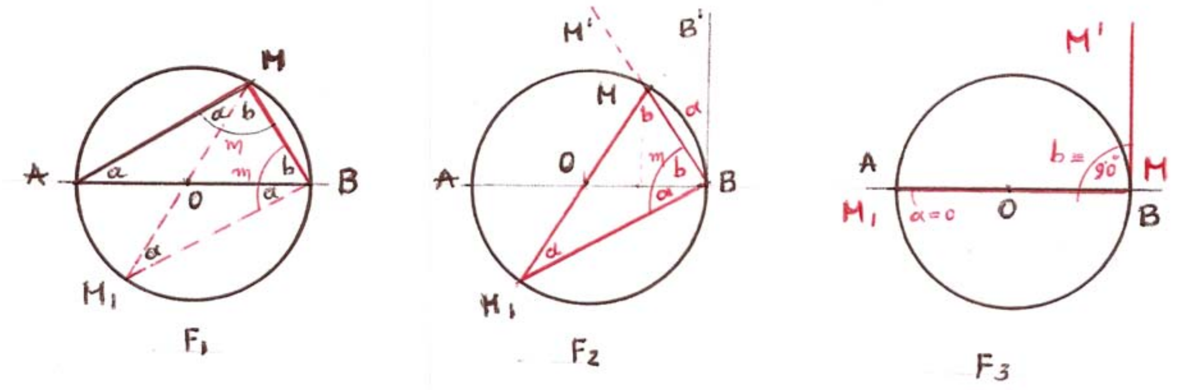
Question 3 **:** Is any error in the argument that angles with Perpendicular sides are equal ? 17/5/10

In Proofed Succession (a) page 3 , is referred that two angles with perpendicular sides are equal (or supplementary) . To avoid any pretext , a clear proof is given to this presupposition in (1) page 11 showing also that ,

1 Any angle between the diameter AB of a circle is right angle (90▫) .
2 Any two angles with vertices the points A , B of a diameter AB , have perpendicular sides and are also equal or supplementary .
3 Equal angles exist on equal arcs , and central angles are twice the inscribed angles.
4 The Sum of angles of any triangle is equal to two right angles .
so ,

There is not any error in argument of proofs .
The 5th Postulate is Depended on (derived from) the prior four axioms .

POINT M ON A CIRCLE OF DIAMETER AB = 0 → ∞



To show that angle < AMB = m = 90°
BB′ ⊥ BA (angle ABB ′ = 90°) , MM " ⊥ AB

F1 : It has been proved (page17) that triangles AMB,MBM_1 are equal and angle <AMB = MBM_1= m for all positions of M on the circle.
Since triangles OMB ,OAM_1 are equal then chord BM = AM_1 and **arc BM = AM_1 .**

F2 : The rotation of diameter MM_1 through centre O is equivalent to the new position Mx of point M and simultaneously is the rotation of angle < M′MxBM_1 = M′BM_1 through point B , and this because arc BM = AM_1 , BMx = AM_{1x} , i.e. when point M moves with BMM′ to a new position Mx on the circle , diameter MOM_1 = MxOM_{1x} is rotated through O , the points M,M_1 are sliding on sides BMM′, BM_1 because point M_1 to the new position M_{1x} is such that AM_{1x} = BMx and **angle < M′BM_1 is then rotated through B .** (*analytically below*)

F3 : When diameter MM_1 is rotated through O , point M lies on arc MB = AM_1 and angle < M′BM_1 is not altered (*this again because MB = AM_1*) and **when point M is at B , point M_1 is at point A ,** because again arc BM = AM_1 = 0 , and angle a = BM_1M = 0 ,

angle < M_1BM = M_1BM' = ABM′ = 90° = m = a + b

Conclusion 1 .
Since angle < AMB is always equal to MBM_1 = M′BM_1 and angle M′BM_1 = 90° therefore **angle < AMB = a + b = m = 90°**

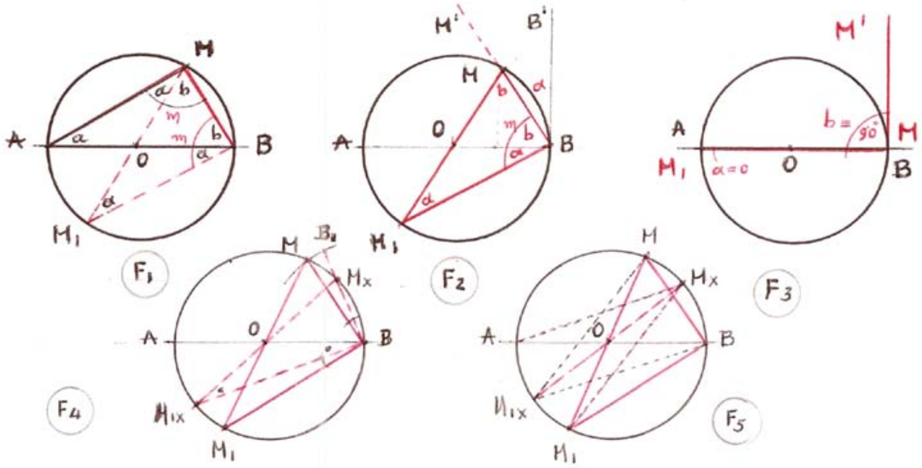
Conclusion 2 .
Since angle < ABB′ = 90° = ABM + MBB′ = b + a , therefore angle MBB′ = a , i.e. the two angles < BAM , MBB′ which have **AM ⊥ BM and AB ⊥ BB′ are equal between them .**

Conclusion 3 .
Any angle < MBB′ on chord BM and tangent BB′ of the circle (O , OA = OB) , where is (BB′ ⊥ BA) **, is equal to the inscribed one** , on chord BM .

Conclusion 4 .
Drawing the perpendicular MM " on AB , then angle BMM " = MAB = MBB ′ , *because they have their sides perpendicular between them* , i.e since the two **lines BB′ , MM " are parallel and are cut by the transversal MB** then the **alternate interior angles MBB′ , BMM " are equal** .

Conclusion 5 .
In Mechanics , the motion of point M is equivalent to , a curved one on the circle , two Rotations through points O , B , and one rectilinear in the orthogonal system M′BM_1 = MBM_1 .

Why angle MBM_1 is unaltered when plane MBM_1 is rotated through B to new position M_xBM_{1x} ??



Let Plane (MBM_1) , (F4) be rotated through B , to a new position B_1BM_{1x} such that :

1. Line $BM \rightarrow BB_1$ intersects circle (O,OB) at point M_x and the circle ($B,BM = BB_1$) , at point B_1.
2. Line $BM_1 \rightarrow BM_{1x}$ extended intersects circle (O,OB) at the new point M_{1x} .
3. Angle $< M_1BM_{1x} = MBB_1 = MBM_x$, is angle of rotation .

Since angle $< M_1BM_{1x} = MBM_x$, therefore angle $< M_1BM$ is unaltered by rotation ***i.e.***
Angle < $M_1BM = M_{1x}BM_x$ and diameter MM_1 is sliding uniformly on their sides .

Data + Remarks .

1. Diameter MM_1 is sliding in angle M_1BM which means that points M_1 , M lie on the circle (O,OB) and on lines BM_1 , BM respectively , and also sliding to the other sides BM_{1x} , BM_x of the equal angle $< M_{1x}BM_x$. Any line segment $M_{1x}M_w = MM_1$ is also diameter of the circle.
2. Only point M_x is simultaneously on circle (O,OB) and on line BB_1 .
3. The circle with point M_{1x} as centre and radius $M_{1x}M_w = MM_1$ intersect circle (O , OB) at only one unique point M_w .
4. Since angle $< M_{1x}BB_1 = M_1BM$ and since segment $M_{1x}M_w = MM_1$ then chord $M_{1x}M_w$ must be also on sides of angles $M_{1x}BB_1$, M_1BM , i.e. Point M_w must be on line BB_1
5. Ascertain 2 and 4 contradict because this property belongs to point M_x , unless this unique point M_w coincides with M_x **and chord M_xM_{1x} is diameter of circle (O,OB) .**

Point M_x is simultaneously on circle (O , OB) , on angle $< M_{1x}BB_1 = M_{1x}BM_x$ and is sliding on line BB_1 . We know also that ***the unique point M_w*** has the same properties as point M_x , i.e. point M_w must be also on circle (O , OB) and on line BB_1 , and the diameter $M_{1x}M_w$ is sliding also on sides of the equal angles $M_{1x}BB_1$, $M_{1x}BM_x$, M_1BM .

Since point M_w is always a unique point on circle (O,OB) and also sliding on sides of angle $M_1BM = M_{1x}BM_x$ and since point M_x is common to circle (O,OB) and to line $BB_1 = BM_x$, therefore , points M_w , M_x coincide and chord M_xM_{1x} is diameter on the circle (O,OB) i.e. The Rotation of diameter MM_1 through O , to a new position M_xM_{1x} , is equivalent to the Rotation of Plane (MBM_1) through B and exists angle $< MBM_1 = M_xBM_{1x}$, so

angle $< MBM_1 = M_xBM_{1x} = AMB = 90^\circ = m = a + b$ ……… ο.ε.δ

Since angle $< MBM_x = M_1BM_{1x}$ is the angle of rotation , and since also arc $MM_x = M_1M_{1x}$ (this because triangles OMM_x , OM_1M_{1x} are equal) then :

Equal inscribed angles exist on equal arcs .