Generalized Fermat's Last Theorem $R^n = y_1^3 + y_2^3$

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Abstract

In this paper we prove $R^2 = y_1^3 + y_2^3$ has infinitely many nonzero integer solutions. We prove $R^n = y_1^3 + y_2^3 (n > 2)$ has no nonzero integer solutions.

We define the supercomplex number [1,2,3]

$$W = x_1 + x_2 J + x_3 J^2 \qquad (1)$$

where J denotes a $3-th$ root of unity, $J^3 = 1$,

Then from (1)

$$W^n = (x_1 + x_2 J + x_3 J^2)^n = y_1 + y_2 J + y_3 J^2 \qquad (2)$$

Then from (2) we have the modulus of supercomplex number

$$R^n = |x_i|^n = |y_i| \qquad (3)$$

where

$$R^n = x_1^3 + x_2^3 + x_3^3 - 3x_1 x_2 x_3, \qquad (4)$$

$$|y_i| = y_1^3 + y_2^3 + y_3^3 - 3y_1 y_2 y_3, \qquad (5)$$

We prove that (3) has infinitely many nonzero integer solutions.
We define the stable group [1,4]

$$G = \{g_2, g_3\} \qquad (6)$$

where

$$g_2 = \begin{pmatrix} 1\,2\,3 \\ 1\,2\,3 \end{pmatrix}, g_3 = \begin{pmatrix} 1\,2\,3 \\ 1\,2\,3 \end{pmatrix} \text{g.}$$

Theorem 1. Suppose $n = 2$ and $y_3 = 0$. Then from (3) and (5)

$$R^2 = y_1^3 + y_2^3 \qquad (7)$$

when $n = 2$ from (2)

$$y_1 = x_1^2 + 2x_2 x_3, \quad y_2 = x_3^2 + 2x_1 x_2, \quad y_3 = x_2^2 + 2x_1 x_3 \qquad (8)$$

which are the homogeneous and irreducible polynomials.

$$g_3: \quad x_1 \to x_1, \quad x_2 \to x_3, \quad x_3 \to x_2$$
$$g_3: \quad y_1 \to y_1, \quad y_2 \to y_3, \quad y_3 \to y_2 \qquad (9)$$

$$g_3 y_2 = y_3 = 0 \qquad (10)$$

If $y_3 = 0$ has nonzero integer solutions, then $y_2 = 0$ also has nonzero integer solutions, and vice versa.

Put $x_1 = P^2$, $x_2 = 2P, x_3 = -2, y_3 = 0$, where P is an odd number.

From (7) and (9)

$$(g_3 R)^2 = (g_3 y_1)^3 + (g_3 y_2)^3, \qquad (11)$$

$$R^2 = y_1^3 + y_3^3 \qquad (12)$$

Put $x_1 = P^2$, $x_2 = -2, x_3 = 2P, y_2 = 0$, where P is an .odd number.

Suppose $y_1 = 0$ and $n = 2$. From (3) and (5)

$$R^2 = y_2^3 + y_3^3 \qquad (13)$$

Put $x_1 = 2P$, $x_2 = -2, x_3 = P^2, y_1 = 0$, where P is an odd number. (7), (11) and (12) are the same equation. We prove that every

$$y_1 = 0, \quad y_2 = 0, \quad y_3 = 0 \qquad (14)$$

has infinitely many nonzero integer solutions.

Hence (7), (12) and (13) have infinitely many nonzero integer solutions.

Theorem 2. Suppose $n = 3$ and $y_3 = 0$. Then from (3) and (5)

$$R_1^3 = y_1^3 + y_2^3 \qquad (15)$$

when $n = 3$ from (2)

$$y_1 = x_1^3 + x_2^3 + x_3^3 - 3x_1x_2x_3, \ \ y_2 = 3(x_1x_3^2 + x_2x_1^2 + x_3x_2^2), \ \ y_3 = 3(x_1x_2^2 + x_2x_3^2 + x_3x_1^2), \qquad (16)$$

which are the homogeneous and irreducible polynomials.

From (6)

$$g_3: \quad x_1 \to x_1, \quad x_2 \to x_3, \quad x_3 \to x_2$$
$$g_3: \quad y_1 \to y_1, \quad y_2 \to y_3, \quad y_3 \to y_2 \qquad (17)$$

$$g_3 y_2 = y_3 = 0 \qquad (18)$$

If $y_3 = 0$ has no nonzero integer solutions then $y_2 = 0$ has no nonzero integer solutions, and vice versa [1,5]

Euler prove that (15) has no nonzero integer solutions. Hence y_2 and $y_3 = 0$ have no nonzero integer solutions.

From (15) and (17) we have

$$(g_3 R)^2 = (g_3 y_1)^3 + (g_3 y_2)^3, \tag{19}$$

$$R^3 = y_1^3 + y_3^3 \tag{20}$$

From (18) $y_2 = 0$ has no nonzero integer solutions, Hence (20) has no nonzero integer solutions, Euler prove that (20) has no nonzero integer solutions, hence y_3 and $y_2 = 0$ have no nonzero integer solutions.

Suppose $n = 3$ and $y_1 = 0$ from (3) and (5)

$$R^3 = y_2^3 + y_3^3 \tag{21}$$

Euler prove (21) has no nonzero integer solutions, hence $y_1 = 0$ also has no nonzero integer solutions.

We prove that every

$$y_1 = 0, \quad y_2 = 0, \quad y_3 = 0 \tag{22}$$

has no nonzero integer solutions. Hence we prove that (15), (20) and (21) are the same equation and have no nonzero integer solutions.

Theorem 3. when $n > 3$, y_1, y_2 and y_3 are homogenous and irreducible polynomials.

Suppose $y_3 = 0$. From (3) and (5)

$$R_1^n = y_1^3 + y_2^3 \tag{23}$$

From (18) $y_3 = 0$ has no nonzero integer solutions. Hence (23) has no nonzero integer solution.

From (17) and (23) we have

$$(g_3 R)^n = (g_3 y_1)^3 + (g_3 y_2)^3, \tag{24}$$

$$R^n = y_1^3 + y_3^3 \tag{25}$$

From (18) $y_2 = 0$ has no nonzero integer solutions, Hence (25) has no nonzero integer solutions.

Suppose $n > 3$ and $y_1 = 0$. From (3) and (5)

$$R_1^n = y_2^3 + y_3^3 \tag{26}$$

We prove that every

$$y_1 = 0, \quad y_2 = 0 \text{ and } y_3 = 0 \qquad (27)$$

has no nonzero integer solutions.

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