NTEP: Chapter 8. Nonlinear quantum electron equation

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The derivation of nonlinear quantum electron equation in the framework of nonlinear theory of elementary particles (NTEP) is presented. It can help to understand many aspects of the quantum description of elementary particles. In particular, it is shown that the fields self-action is "the mechanism", which introduces the mass into the quantum electron equation. This mechanism has a similarities with the Higgs mechanism of mass generation, however it is not needed a Higgs boson. The results of the experiments, which were set until now, to find the Higgs's boson, are negative. At the same time the NTEP has not difficulties, which will appear in Standard Model theory, if Higgs's boson is not discovered.

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1.0. Introduction. Unified nonlinear theories

Nonlinear theories of elementary particles originated in an attempt to unify the descriptions of motion with intrinsic characteristics of particles (first of all, of electron).

1.1. Fom classical to quantum nonlinear electron theory

In the electron theory before Mie's (Bialynicki-Birula, 1983), the electron was not considered to be a purely electromagnetic entity. Its description, for example, contained Poincare stresses and a mechanical mass. The first attempt to set up a theory, which can give a solution of this and other problems, was made by the German physicist Gustav Mie. Later, as an application of this theory, the well-known nonlinear Born-Infeld theory was developed (Born and Infeld, 1934), and encouraging numerical results were obtained. But the non-quantum nature was the basic defect of these theories.

G. Mie wanted the electromagnetic field only to be responsible for all properties of the electron. In particular, he wanted the electromagnetic current to be a consequence of electrodynamics postulates. In order to achieve this goal, Mie assumed that the four-vector potential must be directly included into the Lagrangian. Actually, this approach allows to achieve the generation of current. However, the potentials in Mie's theory acquired a physical meaning, and gauge invariance was lost. Other physicists found these properties unacceptable, and as a result of this, Mie's theory has been shelved for many decades.

In fact, gauge non-invariance and the non-quantized nature were serious defects of Mie's theory. However, in the next chapter, we will show that both deficiencies can be overcome on the basis of later results.

Another approach to the description of elementary particles - an theory of composite particles - has its origin in the neutrino theory of light of L. de Broglie. He assumed that the photon is a pair of "fusion" neutrinos (therefore, the theory is also known as the "theory of fusion"). The neutrino has an electric charge equal to zero, and spin equal to ½. Its resting mass was formerly considered to be zero. In the process of fusion, two neutrinos thus could form a neutral particle with zero mass and spin 1, as is the case with a photon. It was possible to obtain other particles with the spin a multiple of ½ through the fusion of several neutrinos.

By way of a solution to all these difficulties (Ivanenko, 1958), many authors have proposed that the nonlinear spinor equation be made the basis of the field theory. As indicated many times, Heisenberg and his associates were able to attain most noticeable successes in this direction (Coll. of articles, 1959; Heisenberg, 1957). First, the reciprocal transformation of the particles clearly indicates that they are excited states of some general substance. In accordance with the arguments of de Broglie, the simplest basic field, from which it is possible to construct all the others, should be a spinor field of Dirac particles with spin s = ½ . A clear example of the method of "joining" is the idea of construction of the neutrino theory of light by de Broglie (developed by Kronig, Jordan, A. A. Sokolov, and others).

If we generalize these ideas and adopt the point of view of a unified theory, then, obviously, its base should be some sort of nonlinear generalization of the Dirac equation. In fact, to yield excited states, the fundamental world spinor field should interact with something, but in the unified theory it can interact only with itself. Later D. Ivanenko (Ivanenko, 1938), established the form of all possible nonlinear generalizations of Dirac's equation, not including the derivatives, on which Heisenberg indeed leans in his papers. In his papers (see also the book (Heisenberg, 1966) Heisenberg expounded on the principal ideas and advances of unified nonlinear theory of matter. Let us summarize the principal achievements of this theory.

Taking into account the invariance under Pauli and Salam-Touschek transformations (from the neutrino theory), Heisenberg arrives at the Lagrangian

$$L_{NL} = \psi^+ \gamma_\nu \frac{\partial}{\partial \psi_\nu} \psi \pm \frac{l^2}{2} \sum_\mu \left(\psi^+ \gamma_\nu \gamma_5 \psi\right)^2 , \qquad (8.1.1)$$

from which he obtains the fundamental nonlinear spinor equation of matter

$$\gamma_\mu \partial_\mu \psi \pm \lambda \left(\psi^+ \gamma_5 \gamma_\nu \psi\right) \gamma_5 \gamma_\nu \psi = 0 , \qquad (8.1.2)$$

where l is the fundamental minimal length, $\lambda = hcl^2$ is the interaction constant, γ_5 and γ_ν are Dirac's matrices .

After establishing the fundamental nonlinear equation, it is necessary to consider the rules of quantization of the field. In this connection Heisenberg made a very bold and original step, by modifying the commutation rules through introducing a Dirac indefinite metric in Hilbert space so that these equations come into agreement with the new nonlinear equation.

Among the results of Heisenberg and his associates, let us recall their derivation of the fermion state with mass $k = 7.426/l$ (where $k = mc/h$), determined by the interaction constant (when the calculations are made with the new nonlinear term, the coefficient is 7.08 in the first approximation and 6.67 in the second approximation) $\lambda = hcl^2$ and several excited states with series of masses. It became also possible to obtain a value for the fine-structure constant in the form $\alpha = \frac{2\pi e^2}{hc} \cong \frac{1}{267}$.

Unfortunately, final mathematically solving of Heisenberg's equation proved to be a difficult problem. On the other hand, some ideas of Heisenberg were shown to be of special importance, and consequently deeply influenced the development of modern quantum field theory.

1.2. Unified quantum nonlinear Heisenberg's theory of matter and spontaneous symmetry breakdown

W. Heisenberg's goal was the description of all particles as bound states of a different number of some primary particles. In order to obtain all necessary particle spins, the primary particles must have spin ½. According to Heisenberg's supposition, the fundamental equation must have the highest possible symmetry. However the mass term in Dirac's equation disrupts the invariance of this equation in relation to a series of transformations (of transformation $\psi \to \gamma_5 \psi$, where γ_5 is the fifth matrix of Dirac; of scale transformation $x \to \theta x$, $\psi \to \theta^{-1/2}\psi$, where θ is a certain number, and others). W. Heisenberg considered that the mass of particles should appear in the theory automatically in the course of its decision. Therefore he proposed a nonlinear equation (8.1.2) without particle mass. Since the equation (8.1.2) has not a term with particle mass, it possess the highest possible symmetry. However it is very well known that the interactions of elementary particles are characterized by different symmetries (isotopic symmetry is lost upon transfer from the strong interaction to the electromagnetic, upon the subsequent transfer to the weak interaction the law of parity conservation ceases to work, etc). It is understandable that it is impossible to create a simple fundamental equation which will automatically have these different symmetries.

The theory of ferromagnetism, the author of which was Heisenberg, showed him a way to resolve this situation. It was the idea of spontaneous symmetry breaking (SSB): the fundamental equation can have a maximum symmetry, but other symmetries can be introduced by the spontaneous breaking of this symmetry.

One of the most important mechanisms of SSB within the framework of Heisenberg's program was proposed at the beginning of the 1960's by Nambu and Jona-Lazinio (Nambu and Jona-Lasinio, 1961a, 1961b). It was taken from the microscopic theory of superconductivity of Bardeen, Cooper and Shriffer (known as the BCSh mechanism).

Mathematically this was like the appearance of a new symmetry - so-called chiral symmetry, which is spontaneously broken. As a result of the breaking of chiral symmetry, in the model of Nambu and Jona-Lasinio mesons appeared, and fermions acquired significant mass.

Heisenberg’s equation (1.2) and the equation of superconductivity (nonrelativistic here):

$$\left[i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m} - \lambda\left(\psi^+ \hat{\gamma}\psi\right)\right]\psi = 0\,, \qquad (8.1.3)$$

have similarities. In Heisenberg's theory, in the case of attraction between primary particles, SSB also occurs as the result of formation of Cooper’s pairs of primary particles and their Bose condensation.

The generalization of the SSB model in the case of interaction of scalar and vector EM fields was examined by Higgs. In a statical limit, Higgs' model is completely analogous to the theory of Ginsburg-Landau’s superconductivity, being its relativistic generalization.

Thus, we come to the conclusion that in order to introduce the required symmetries and particle masses we must take the initial dynamic equations in a mass-free form and use the idea of spontaneous symmetry breakdown (SSB).

Early versions of a unified theory of weak and EM interactions were proposed by Weinberg and Salam. An essential element of this theory was the use of Higgs's model.

1.3. The SSB mechanism and mass generation

The possibility of calculation of the particle masses by means of the SSB is the characteristic property of SM. The mathematical description of this procedure is called Higgs's mechanism. This mechanism is repeatedly described in literature. Therefore, we will only consider the conclusions of the theory.

The Higgs field in SM has three important functions:

1) it breaks the gauge symmetries and gives masses to intermediate bosons (W and Z);

2) it breaks the chiral symmetry and gives masses to fermions;

3) it restores the unitarity of the theory.

The last role is very important: if Higgs's boson does not exist, the unitarity of theory in the general case will be broken. In this case it is necessary to exceed the limits of SM. According to present ideas this possibility gives: super-symmetry; the additional measurements of space-time; “great” unificaton of interactions; new internal particle structure of SM (technicolor, little Higgs, etc); superstring, membranes, and the like. But all these versions lie beyond the limitations of the experimental check.

In the Standard Model theory the Higgs's boson mass is not determined. Some estimations, which is based on experimental data, showed that the mass of Higgs's boson must lie approximately in the interval of 96-251 GeV. The results of the experiments, which were set until now for confirmation of Higgs's mechanism, are negative. With a 95% confidence level (ScienceDaily, 2009) the mass of the Higgs boson (within the framework of SM) must be in the limits: m(H) >114 GeV from straight searches on LEP II, and m(H) <160 GeV from the fit of precision measurements on LEP and Tevatron. Also the 1^{st} type of two-doublet Higgs model, in which the different bosons of Higgs are required, was not confirmed.

Other results show that the probability of the Higgs boson detection in a remained, comparatively small, region of energies from 114 to 160 GeV is limited. In connection with the difficulties, which will appear if Higgs's bosons is not discovered, an interest arises in other possible variations of the field theory, which can be accessible for experimental check.

Earlier (see (Kyriakos, 2010a)) we have shown that self-action of fields of a photon leads to occurrence of mass of a particle and transformation of a usual (massless) photon into an intermediate massive photon, which, due to spontaneous breakdown, can generate the massive spinor particles – electron and positron ((see (Kyriakos, 2010b))). This mechanism solves the problem of particle masses without the Higgs mechanism. Below we will examine the nonlinear theory of electron (positron) to show that in this case also the particles mass are generated by self-interaction of the particle fields.

2.0. Nonlinear electron equation of NTEP and its Lagrangian

"Is the quantum theory linear or is it a nonlinear theory?" - this question, set by W. Heisenberg in 1967 (Heisenberg, 1967), arose in connection with the fact that "practically every problem in theoretical physics is governed by nonlinear mathematical equations, except perhaps quantum theory, and even in quantum theory it is a rather controversial question whether it will finally be a linear or nonlinear theory". A number of works is devoted to the analysis of this contradiction (Parwani, 2005; Jordan, 2007; etc), but no final solution was found until now.

2.1. About specifics of NTEP as a nonlinear theory

NTEP discloses two types of nonlinearity. The first is related to the postulate of NTEP about the rotation transformation of a quantum of an EM wave. It is possible to consider the motion of rotation as a deviation from linearity, i.e. as a kind of nonlinearity. However, in this case, such nonlinear motions are of a specific type: they are created and described by harmonic functions and their superposition. This allows us to describe this type of nonlinearity by linear equations.

Actually, rotation, as a motion along the circle, can be represented by a sum of two linear, mutually perpendicular harmonic oscillations. The sum of a greater number of oscillations leads to curvilinear trajectories with a form known as Lissajous figures. Apparently, all these nonlinearities are conveniently and simply described by complex functions. It is possible to assume that Fourier theory reflects the possibility of a linear description of these nonlinear curves.

Since the Fourier transform is linear, this "rotation" or "harmonic curvilinearity" allows us to consider NTEP as a linear theory, i.e. a theory in which the principle of superposition is strictly fulfilled.

On the other hand, the rotation transformation of EM fields also gives us another type of nonlinearity. Here, we deal not only with rotation motions, but also with the fields which are "attached" to these motions in a strictly defined way. During the formation of EM particles (i.e. as the result of rotation of a quantum of an EM wave) the field configuration changes inside the particle's volume. In this case the self-interaction of particle fields appears, which is described by the nonlinear terms. Thus, strictly speaking, nonlinear field theory operates inside of a particle, and probably the principle of superposition is not valid in this case.

The simplest way to approach the nonlinear theory is the use of the electromagnetic representation of Dirac's lepton theory. Further we will derive the general type of the nonlinear equation of electron and construct its Lagrangian.

2.2. Self-action and the nonlinear equation of electron

The stability of a semi-photon (i.e. electron) is only possible because of the self-action of the semi-photon fields. This self-action forms the particle itself, and the particle's internal parameters must ensure this self-action. The basic parameters which determine the behaviour of a particle are the energy and momentum of the particle's fields. This shows how self-action can be introduced into the equation.

Since Dirac's equation does not have other parameters, the internal parameters of electron must be connected with the free term: $\hat{\beta} m_e c^2$. Linearizing the conservation law of energy-momentum $\varepsilon^2 - c^2 \vec{p}^2 - m_e c^2 = 0$ according to Dirac's method, namely $\varepsilon_{\pm} = \pm\sqrt{c^2 \vec{p}^2 + m_e^2 c^4} = \pm\left(c\hat{\vec{\alpha}}\vec{p} + \hat{\beta} m_e c^2\right)$, we obtain the linear equivalent of this relationship: the linear expression of the energy-momentum conservation law (in present case for the internal – *in* – field:

$$\hat{\beta}\ m_e c^2 = -\varepsilon_{in} - c\hat{\vec{\alpha}}\ \vec{p}_{in} = -e\varphi_{in} - e\hat{\vec{\alpha}}\ \vec{A}_{in}, \qquad \text{(A)}$$

(note that here $\varepsilon_{in} = e\varphi_{in}$ and $p_{in} = ec\vec{A}_{in}$ are not operators, but the energy and momentum of field; φ_{in} and $\vec{A}_{in}$ are the scalar and vector potentials correspondingly). Substituting (A) into Dirac's equation, we obtain the following equation:

$$\left[\hat{\alpha}_0\left(\hat{\varepsilon} - \varepsilon_{in}\right) + c\hat{\vec{\alpha}}\cdot\left(\hat{\vec{p}} - \vec{p}_{in}\right)\right]\psi = 0, \qquad (8.2.1)$$

Here, the inner energy ε_{in} and momentum p_{in} can be expressed using the inner energy density u and the inner momentum density $\vec{g}$ (or Poynting vector $\vec{S}$) of an EM wave:

$$\varepsilon_{in} = \frac{1}{8\pi}\iiint_{x,y,z}\left(\vec{E}^2 + \vec{H}^2\right)dxdydz = \int_0^{\tau} u\,d\tau, \qquad (8.2.2)$$

$$\vec{p}_{in} = \iiint_{x,y,z}\left[\vec{E}\times\vec{H}\right]dxdydz = \int_0^{\tau}\vec{g}\,d\tau = \frac{1}{c^2}\int_0^{\tau}\vec{S}\,d\tau, \qquad (8.2.3)$$

assuming that the upper limit of integration for the space is variable ($0 \le x, y, z < \infty$) or conditionally ($0 \le \tau < \infty$), where $d\tau = dxdydz$.

Taking into account the EM form of ψ- function (see (Kyriakos, 2010d)), we obtain the quantum forms of u and $\vec{S}$ as follows:

$$u = \frac{1}{8\pi}\left(\vec{E}^2 + \vec{H}^2\right) = \frac{1}{8\pi}\psi^{+}\hat{\alpha}_0\psi, \qquad (8.2.4)$$

$$\vec{S} = \frac{c}{4\pi}\left[\vec{E}\times\vec{H}\right] = c^2\vec{g} = -\frac{c}{8\pi}\ \psi^{+}\hat{\vec{\alpha}}\ \psi, \qquad (8.2.5)$$

Substituting expressions (8.2.2) and (8.2.3) into the electron equation (8.2.1), and taking into account (8.2.4) and (8.2.5), we will obtain the *nonlinear integro-differential equation in both electromagnetic and quantum forms.*

We assume that equation (8.2.1) *is the basic nonlinear equation of the electron, which describes both the electron's motion and structure.*

Actually, taking into account the relationship (A), the equation (8.2.1) is reduced to the usual Dirac's equation (8.2.1), which describes *motion* of an electron.

For the description of the electron field *structure* apparently it is necessary to solve the nonlinear equation. The difficulty of solving such equations is already noted by Heisenberg (Heisenberg, 1967). The solution is usually anticipated by the analysis of the properties of the equation symmetry and by the possibility of its conversion into the system of linear equations.

In order to study the properties of symmetry, let us find the approximate quantum form of the equation (8.2.1). Then the nonlinear equation of Heisenberg occurs unexpectedly, which properties of symmetry are well studied.

2.2.1. The derivation of the Heisenberg nonlinear equation as first approximation

Let us find the approximate quantum form of the equation (8.2.1).

Taking into account that the solution of Dirac's equation for a free electron is the plane wave

$$\psi = \psi_0 \exp[i(\omega t - ky)], \qquad (8.2.6)$$

we can approximately write (8.2.2) and (8.2.3) as follows:

$$\varepsilon_p = u\Delta\tau = \frac{\Delta\tau}{8\pi}\psi^+ \hat{\alpha}_0 \psi = \frac{\Delta\tau}{8\pi}\left(\vec{E}^2 + \vec{H}^2\right), \qquad (8.2.7)$$

$$\vec{p}_p = \vec{g}\ \Delta\tau = -\frac{\Delta\tau}{8\pi\ c}\ \psi^+ \hat{\vec{\alpha}}\ \psi = \frac{\Delta\tau}{4\pi c}\left[\vec{E}\times\vec{H}\right], \qquad (8.2.8)$$

where $\Delta\tau$ is the volume that contains the main part of the semi-photon's energy. If we assume that the fields of the particle apply to infinity, then apparently the cutting of integral will lead to the violation of the unitarity of theory. This must be taken into account in the use of this (approximate) equation for the description of particles.

Using (8.2.7) and (8.2.8) we can find the approximate form of the equation (8.2.1) as follows:

$$\frac{\partial\ \psi}{\partial\ t} - c\hat{\vec{\alpha}}\vec{\nabla}\psi + i\frac{\Delta\tau}{8\pi\ c}\left(\psi^+\hat{\alpha}_0\psi - \hat{\vec{\alpha}}\psi^+\hat{\vec{\alpha}}\psi\right)\psi = 0, \qquad (8.2.9)$$

If instead of using the α-set of Dirac's matrices we use the γ-set matrices, from the equation (8.2.9) we obtain the equation of Heisenberg in a form, which is known from the theory (Heisenberg, 1966; Paper translation collection, 1959):

$$\gamma_\mu \frac{\partial\psi}{\partial x_\mu} + \frac{1}{2}i\lambda\left[\gamma_\mu\psi\left(\overline{\psi}\gamma_\mu\psi\right) + \gamma_\mu\gamma_5\psi\left(\overline{\psi}\gamma_\mu\gamma_5\psi\right)\right] = 0, \qquad (8.2.10)$$

where in our case constant λ is $\lambda = \frac{\Delta\tau}{4\pi\ c}$.

The nonlinear equation (8.2.10) was postulated by Heisenberg. Unlike, the equation (8.2.9) was obtained in a logical and correct way, and the constant λ automatically appears in this equation as a self-action constant.

2.3. The Lagrangian of the nonlinear electron theory

The linear type Lagrangian is presented in quantum form in Dirac's electron theory as follows (Schiff, 1955):

$$L_D = \psi^+\left(\hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}} + \hat{\beta}m_e c^2\right)\psi, \qquad (8.2.11)$$

It is not difficult to find its electromagnetic form:

$$L_D = \frac{\partial\ u}{\partial\ t} + div\ \vec{S} - i\frac{\omega}{8\pi}\left(\vec{E}^2 - \vec{H}^2\right), \qquad (8.2.12)$$

(Note that in the case of a variation procedure we must distinguish the complex conjugate field vectors $\vec{E}^*, \vec{H}^*$ and $\vec{E}, \vec{H}$).

The Lagrangian of nonlinear theory can be obtained from the Lagrangian (8.2.11) using the same method that we used to find the nonlinear equation. Substituting relationship (A) into this equation, we obtain:

$$L_N = \psi^+\left(\hat{\varepsilon}\ - c\hat{\vec{\alpha}}\cdot\hat{\vec{p}}\right)\psi + \psi^+\left(\varepsilon_{in} - c\hat{\vec{\alpha}}\cdot\vec{p}_{in}\right)\psi, \qquad (8.2.13)$$

We will assume that (8.2.13) *represents the general form of the Lagrangian of nonlinear electron theory.*

2.4. The effective Lagrangian of the nonlinear electron theory

In order to understand the connection of this theory with the contemporary results, let us find electromagnetic and quantum approximations of Lagrangian (8.2.13), which corresponds to equation (8.2.9-8.2.10). Using (8.2.7) and (8.2.8), we can represent (8.2.11) in the following quantum form:

$$L_N = i\hbar\left[\frac{\partial}{\partial t}\left[\frac{1}{2}\left(\psi^+\psi\right)\right] - c\,div\left(\psi^+\hat{\vec{\alpha}}\psi\right)\right] + \frac{\Delta\tau}{8\pi}\left[\left(\psi^+\psi\right)^2 - \left(\psi^+\hat{\vec{\alpha}}\psi\right)^2\right], \qquad (8.2.14)$$

In order to obtain an EM form of (8.2.14), we initially substitute the normalized ψ-function using the expression $L'_N = \frac{1}{8\pi\ mc^2}L_N$. Then, using (8.2.4) and (8.2.5), we obtain the following electromagnetic approximation:

$$L'_N = i\frac{\hbar}{2m_e}\left(\frac{1}{c^2}\frac{\partial\ u}{\partial\ t} + div\ \vec{g}\right) + \frac{\Delta\tau}{m_e c^2}\left(u^2 - c^2\vec{g}^{\,2}\right), \qquad (8.2.15)$$

We can transform here the second term using the following known electrodynamics identity:

$$(8\pi)^2\left(U^2 - c^2\vec{g}^{\,2}\right) = \left(\vec{E}^2 + \vec{H}^2\right)^2 - 4\left(\vec{E}\times\vec{H}\right)^2 = \left(\vec{E}^2 - \vec{H}^2\right)^2 + 4\left(\vec{E}\cdot\vec{H}\right)^2, \quad (8.2.16)$$

Taking into account that $L_D = 0$, and using (8.2.12) and (8.2.16), we can represent (8.2.15) in the following form:

$$L'_N = \frac{1}{8\pi}\left(\vec{E}^2 - \vec{H}^2\right) + \frac{\Delta\tau}{(8\pi)^2 mc^2}\left[\left(\vec{E}^2 - \vec{H}^2\right)^2 + 4\ \left(\vec{E}\cdot\vec{H}\right)^2\right], \qquad (8.2.17)$$

As we can see, the approximation of the full Lagrangian of the nonlinear equation contains only invariants of Maxwell’s theory. It is similar to the known Lagrangian of photon-photon interaction (Akhiezer and Berestetskii, 1965), Born-Infeld Lagrangian ( ) and Gustav Mie Lagrangian ( ) also.

Now, let us analyze the quantum form of the Lagrangian density (8.2.17). The equation (8.2.14) can be written in the form:

$$L_Q = \psi^+\hat{\alpha}_\mu\partial_\mu\psi\ \ + \frac{\Delta\tau}{8\pi}\left[\left(\psi^+\hat{\alpha}_0\psi\right)^2 - \left(\psi^+\hat{\vec{\alpha}}\ \psi\right)^2\right], \qquad (8.2.18)$$

We can see that in quantum form, the electrodynamics correlation (8.2.16) takes the form of the known Fierz identity (Cheng and Li, 1984; 2000):

$$\left(\psi^+\hat{\alpha}_0\psi\right)^2 - \left(\psi^+\hat{\vec{\alpha}}\ \psi\right)^2 = \left(\psi^+\hat{\alpha}_4\psi\right)^2 + \left(\psi^+\hat{\alpha}_5\psi\right)^2, \qquad (8.2.19)$$

Using (8.2.19), we obtain from (8.2.18):

$$L_Q = \psi^+\hat{\alpha}_\mu\partial_\mu\psi\ \ + \frac{\Delta\tau}{8\pi}\left[\left(\psi^+\hat{\alpha}_4\psi\right)^2 - \left(\psi^+\hat{\alpha}_5\psi\right)^2\right], \qquad (8.2.20)$$

If instead of using the α-set of Dirac’s matrices we use the γ-set of matrices, *the Lagrangian* (8.2.20) *coincides with the Lagrangian of Nambu – Jona-Lazinio* (Nambu and Jona-Lazinio, 1961; 1961a).

The first presentation of the idea of this Lagrangian was made in (Nambu, 1960a, 1960b); the model system Nambu worked out with Jona-Lasinio (Nambu and Jona-Lasinio, 1961a, 1961b) is a concrete realization of the proposed SSB. It has the form similar to the Bardeen-Cooper-Schrieffer model

$$L = -\bar{\psi}\gamma^\mu\partial_\mu\psi + g\left[\left(\bar{\psi}\psi\right)^2 - \left(\bar{\psi}\gamma_5\psi\right)^2\right], \qquad (8.2.21)$$

which is invariant against the particle number and chiral transformations.

In the current Standard Model of particle physics, the Nambu–Jona-Lasinio (NJL) model may be regarded as an effective theory for the QCD with respect to generation of the so-called constituent masses. One is interested in the low energy degrees of freedom on a scale smaller than some cutoff $\Lambda \sim 1$ GeV. The short distance dynamics as well as the confinement may be treated as a perturbation. The problem has been extensively studied by many people.

Let us note some special features of the results, obtained in the NTEP in comparison with the results that were obtained in the contemporary theory.

Since the Lagrangian of Nambu – Jona-Lazinio is a Lagrangian of weak interaction of the type (V - A), the nonlinear theory – NTEP - covers not only electromagnetic, but also weak interactions. As we will show in future, this conclusion conforms to the fact that in the general case the Dirac equation describes massive neutrino with a conserved inner helicity.

The NTEP show that Lagrangian of Nambu – Jona-Lazinio is actually approximate. Therefore its use can cause different violations of the type of violations of unitarity. This is connected to the fact that the probability distribution density must behave under the Lorenz transformation as time component of the four-dimensional vector, whose divergence is equal to zero. But the Lagrangian of Nambu - Jona-Lazinio contains the strengths of electromagnetic field. As is known, from the strengths of electromagnetic field it is not possible to compose the bilinear combination, which forms the four-dimensional vector, whose divergence would be equal to zero.

However, this value can be constructed, relying on the integral values - energy and momentum of full Lagrangian (8.2.13), which compose a completely determined 4-vector. It is understandable that in order to avoid these difficulties there is no need to use some additional models; it is sufficient to use the precise Lagrangian (8.2.13).

As we noted, the Heisenberg equation has a high degree of symmetry because of the absence of mass, but a special mathematical mechanism SSB is required for the primary particles of equation (8.2.10) to become massive.

In our case the nonlinear integro-differential equation (8.2.1) does not contain mass, and “the mechanism”, through which the mass is introduced into the quantum field equations, is the relationship (A):

$$-\varepsilon_{in} - c\hat{\vec{\alpha}}\ \vec{p}_{in} = -e\varphi_{in} - e\hat{\vec{\alpha}}\ \vec{A}_{in} = \hat{\beta}\ m_e c^2\,, \qquad \text{(A')}.$$

This relationship, recorded here in the reverse order, clearly reflects the process of symmetry breaking, since we substitute the term of high degree of symmetry with a term of low degree of symmetry. Moreover, it is possible to show that the relationship (A’) reflects the result of the rotation transformation of the internal symmetry of particle, which is mathematically equivalent to the gauge transformation result (see (Kyriakos, 2010a)). The special feature of this mechanism is that it does not require the introduction of additional particles and at the same time it does not lead to the necessity to exceed the limits of SM.

Heisenberg poses a problem to obtain all the remaining particles in the form of bound states of a different number of primary particles on the basis of some primary spinor particles. If we consider the spinor particles as the primary building elements of matter, then (as we will show further) it is really possible, using spinor equations, to obtain the equations of all other particles.

3.0. Lagrangian of self-interaction of Dirac’s fermions in NTEP

The Lagrangian of fermions

$$L = \psi^{+}\left(\hat{\alpha}_o \hat{\varepsilon} \mp c\hat{\vec{\alpha}} \cdot \hat{\vec{p}} \pm \hat{\beta} mc^2\right)\psi \qquad (8.3.1)$$

can be represented as the sum:

$$L = L_0 + L'\,, \qquad (8.3.2)$$

where

$$L_0 = \psi^+ \left(\hat{\alpha}_o \hat{\varepsilon} \mp c \hat{\vec{\alpha}} \cdot \hat{\vec{p}} \right) \psi , \qquad (8.3.3)$$

is the Lagrangian of the mass-free particle, and

$$L' = \psi^+ \left(\pm \hat{\beta} mc^2 \right) \psi , \qquad (8.3.4)$$

corresponds to the Lagrangian of self-interaction, which generates the mass.

Let us note that because of the validity of Dirac's equation we have:

$$L = \psi^+ \left(\hat{\alpha}_o \hat{\varepsilon} \mp c \hat{\vec{\alpha}} \cdot \hat{\vec{p}} \pm \hat{\beta} mc^2 \right) \psi = 0$$

From this the relationship follows:

$$\psi^+ \left(\hat{\alpha}_o \hat{\varepsilon} \mp c \hat{\vec{\alpha}} \cdot \hat{\vec{p}} \right) \psi = \psi^+ \left(\pm \hat{\beta} mc^2 \right) \psi , \qquad (8.3.5)$$

which reflects the virial theorem. Let us analyze the physical meaning of the relationship (8.3.5).

Using the Lagrangian (8.3.1), we can obtain the Euler-Lagrange equation:

$$\begin{cases} rot \ \vec{E} + \frac{1}{c} \frac{\partial \ \vec{H}}{\partial \ t} = \vec{j}_\tau^{\,m} , \\ rot \ \vec{H} - \frac{1}{c} \frac{\partial \ \vec{E}}{\partial \ t} = \vec{j}_\tau^{\,e} , \end{cases} \qquad (8.3.6)$$

where

$$\vec{j}_\tau^{\,e} = i \frac{\omega}{4\pi} \vec{E} , \ \vec{j}_\tau^{\,m} = i \frac{\omega}{4\pi} \vec{H} , \qquad (8.3.7)$$

Using the EM form of the wave function ((see (Kyriakos, 2010c))), and taking into account equation (8.3.7), we obtain the following expression for the Lagrangian (8.3.4):

$$L' = i \frac{\omega}{8\pi} \left(\vec{E}^2 - \vec{H}^2 \right) = \frac{1}{2} \left(\vec{j}_\tau^{\,e} \vec{E} - \vec{j}_\tau^{\,m} \vec{H} \right), \qquad (8.3.8)$$

Thus, the Lagrangian of electron self-interaction corresponds to the interaction of a fermion's own current with a fermion's own fields.

4.0. On the mass of interacting particles

From the relationship (A') follows, that a particle's mass is equivalent to the energy-momentum of self-interaction of the particle. Basing on this conclusion, it is possible to analyze how the particle's mass changes during its interaction with other particles.

Let us examine Dirac's equation with external ("*ex*") field:

$$\left[\left(\hat{\alpha}_o \hat{\varepsilon} - c \hat{\vec{\alpha}} \cdot \hat{\vec{p}} \right) + \left(\hat{\alpha}_o \varepsilon_{ex} - c \hat{\vec{\alpha}} \cdot \vec{p}_{ex} \right) + \hat{\beta} \ mc^2 \right] \psi = 0 , \qquad (8.4.1)$$

The own mass of the electron corresponds to its inner field is:

$$\hat{\beta} \ m_e c^2 = \hat{\alpha}_o \varepsilon_{in} - c \hat{\vec{\alpha}} \cdot \vec{p}_{in} . \qquad (8.4.2)$$

In this case, we can rewrite (8.4.1) in the following form:

$$\left[\left(\hat{\alpha}_o \hat{\varepsilon} - c \hat{\vec{\alpha}} \cdot \hat{\vec{p}} \right) + \left(\hat{\alpha}_o \varepsilon_{ex} - c \hat{\vec{\alpha}} \cdot \vec{p}_{ex} \right) + \left(\hat{\alpha}_o \varepsilon_{in} - c \hat{\vec{\alpha}} \cdot \vec{p}_{in} \right) \right] \psi = 0 , \qquad (8.4.3)$$

Similarly to (8.4.2), we can also assert that a certain mass m_{ad} corresponds to the interaction of the external and internal fields:

$$\hat{\alpha}_o \varepsilon_{ex} - c \hat{\vec{\alpha}} \cdot \vec{p}_{ex} = \hat{\beta} \ m_{ad} c^2 , \qquad (8.4.4)$$

Using (8.4.4), we obtain:

$$\left[\left(\hat{\alpha}_o \hat{\varepsilon} - c \hat{\vec{\alpha}} \cdot \hat{\vec{p}} \right) + \hat{\beta} \ (m_e + m_{ad}) c^2 \right] \psi = 0 , \qquad (8.4.6)$$

or

$$\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\cdot\hat{\vec{p}}\right)\psi = -\hat{\beta}\ (m_e + m_{ad})c^2\psi\ . \qquad (8.4.7)$$

It follows from (8.4.6) that the mass m_{ad} is an addition to the electron's own mass m_e. Note, the value of m_{ad} must satisfy the specified resonance conditions relative to m_e.

It is also possible to represent the right side of equation (8.4.7) through the currents (8.3.15). Taking into account that $\omega = \frac{mc^2}{\hbar}$, we obtain:

$$mc^2\vec{E} = -i4\pi\hbar\vec{j}\ , \qquad (8.4.8')$$

or

$$mc^2\psi = -i4\pi\hbar\vec{j}\ , \qquad (8.4.8'')$$

Substituting (8.4.8) into (8.4.7), we find that:

$$\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\cdot\hat{\vec{p}}\right)\psi = -i\hat{\beta}\ 4\pi\hbar(j_e + j_{ad})\ , \qquad (8.4.9)$$

By comparing the above formulas we can make the following conclusions. Within the framework of NEPT we have:

1) the external field of Dirac's equation in quantum form can be considered to be an addition to an electron's own mass;

2) the external field of Dirac's equation in electromagnetic form can be considered to be an addition to an electron's own inner current, i.e., as a certain external current;

3) an external field can be considered to be an environment that has some polarization properties, and can be characterized by a variable electrical and magnetic permeability.

4) it follows from the above considerations that the interaction of an elementary particle with other particles can be considered to be an interaction of the elementary particle's charge and current with some electromagnetic medium.

The transition from Dirac's linear equation to the nonlinear equation of a semi-photon is accomplished through simple substitution: replacing the constant mass term by a functional that contains EM fields. These fields depend on three dimensional coordinates and time. This procedure corresponds to the transition from a point (linear) to a volume (nonlinear) representation of the electron theory.

It is interesting that this transition was already described within the framework of the old nonlinear electromagnetic theory. This was done by making it possible to solve certain nonlinear equations in both "point" and "volume" forms (see (Kyriakos, 2011)).

5.0 The interaction Hamiltonian of the electron nonlinear theory in EM form

In frameworks of NTEP the equations of interaction of the electron with other charged particles (or, in other words, the equations of the electron motion in the field of other particle) can be presented in form of the equations of the classical electrodynamics of medium:

$$\frac{1}{c}\frac{\partial\ \vec{E}}{\partial\ t} - rot\vec{H} = -\frac{4\pi}{c}\left(\vec{j}^{\,e} + \vec{j}^{\,e}_{ex}\right), \qquad (8.5.1)$$

$$\frac{1}{c}\frac{\partial\ \vec{H}}{\partial\ t} + rot\vec{E} = \frac{4\pi}{c}\left(\vec{j}^{\,m} + \vec{j}^{\,m}_{ex}\right), \qquad (8.5.2)$$

where $\vec{j}^{\,e}, \vec{j}^{\,m}$ are the electric and magnetic current densities of the particle, $\vec{j}^{\,e}_{ex}, \vec{j}^{\,m}_{ex}$ are the external current densities, which caused by the interaction of the given particle with other particles. In case if other particles (including also the virtual particles of the physical vacuum) form a medium, this equations can be presented as the electromagnetic theory of polarized medium (Jackson, 1999).

The Hamiltonian of Dirac's electron theory is following:

$$\hat{H} = c\hat{\vec{\alpha}} \cdot \hat{\vec{p}}\psi - \left[\hat{\beta}\, mc^2 + \left(\hat{\alpha}_o \varepsilon_{ex} - c\hat{\vec{\alpha}} \cdot \vec{p}_{ex}\right)\right]\psi\,, \tag{8.5.5}$$

Using (8.5.2) we can obtain the EM representation of (8.5.5), which we will conditionally write in the form:

$$\hat{H} = \pm rot\left(\vec{E}, \vec{H}\right) \mp \frac{4\pi}{c}\left(\vec{j}^{\,e,m} + \vec{j}_{ex}^{\,e,m}\right), \tag{8.5.6}$$

The expression (8.5.6) show that the connection of Hamiltonian with above currents (8.5.3) and (8.5.4) and correspondingly with the features of external medium ε_{ex} and μ_{ex} exists.

Conclusion

The negative result of the experiments for the confirmation of Higgs's mechanism, which set until now, implies that a different version of the generation of masses takes place in nature. The complete agreement of NTEP with SM and with the existing experimental results makes the NTEP version the basic candidate to the role of the theory, which is adequate to the reality.

Bibliography

Akhiezer, A.I. and Berestetskii,. W.B. (1965). *Quantum electrodynamics*. Moscow, Interscience publ., New York.

Bialynicki-Birula, I. (1983). Nonlinear Electrodynamics: Variations on a theme by Born and Infeld etc., World Scientific, Singapore.

Born, M. and Infeld, L. (1934b). Foundations of the new Field Theory, Proc. Roy. Soc. **A144**, 425 (1934).

Cheng T.-P. and Li, L.-F. (1984). *Gauge Theory of Elementary Particle Physics* Clarendon Press, Oxford.

Coll. of the articles (1959). *Nonlinear quantum field theory*.(Russian) Moscow, Foreign Literature Publishing House.

Heisenberg, W. (1966). *Introduction to the unified field theory of elementary particles*. London.

W. Heisenberg. (1967) Nonlinear Problems in Physics, *Physics Today* 20, 27 (1967)

Ivanenko, D. (1938). Phys. Zs. Sowjetunion, 13, 141 (1938)

Ivanenko, D. D. (1958). Planck celebrations in Berlin and Leipzig (Meetings and Conferences). Usp. Fiz. Nauk **66**, 523-542 (November, 1958)

Jackson, J.D. (1999). *Classical Electrodynamics*, 3rd ed., 808 pp. Wiley, New York.

Jordan, T. F. (2007). Why quantum dynamics is linear? http://arxiv.org/abs/quant-ph/0702171

Kirzhnits, D. A. (1978) Superconductivity and elementary particles. *UFN*, Tom 125, iss. 1, May.

Kyriakos, A.G. (2010a). Nonlinear Theory of Elementary Particles: 4. The Intermediate Bosons and Mass Generation Theory. http://vixra.org/abs/1009.0066

Kyriakos, A.G. (2010b). Nonlinear Theory of Elementary Particles: 5. The electron and positron equations (linear approach) http://vixra.org/abs/1011.0013

Kyriakos, A (2010c). Nonlinear Theory of Elementary Particles: 6.Electrodynamic sense of the quantum forms of Dirac electron theory. http://vixra.org/abs/1012.0016, Dec 2010

Kyriakos A.G. (2011) NTEP: Chapter 7. Classical nonlinear electron theories and their connection with QED http://vixra.org/abs/1102.0041

Nambu, Y. (1960a). Phys. Rev. Lett. **4**, 380.

Nambu, Y. (1960b). Proceedings of the Midwest Conference on Theoretical Physics (Purdue University, Lafayette, IN), p.1

Nambu Y. and Jona-Lasinio G., (1961a) *Phys. Rev*., **122**, No.1, 345-358.

Nambu Y. and Jona-Lasinio G., (1961b) *Phys. Rev*., **124**, 246.

Parwani, R.R. (2005). Why is Schrödinger's equation linear? *Braz. J*. Phys. vol.35 no.2b São Paulo June 2005.

Schiff L.T., (1955). *Quantum Mechanics*, 2nd ed., McGraw-Hill Book Co., Jnc, New York.

ScienceDaily (Mar. 22, 2009). New Experiments Constrain Higgs Mass http://www.sciencedaily.com/releases/2009/03/090313110741.htm

Volkov, M. K. and Radzhabov, A.E. (2006). Model Nambu - Jona-Lazinio and its development. *UFN*, volume 176, iss. 6, June 2006)