[Pdf] Submitted on 25, Feb, 2011.

The partition function Z ***and Lagrangian multiplier*** $1/(q\Phi)$ ***of a particle-charged system***

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Comments: 7 pages.

abstract: In a system consisting of a large number of charged particles, the potential energy of the in-system particles satisfy probability distribution. Starting from such consideration, we defined the partition function Z and get the particles in-system distribution function N_i, and obtained the Lagrangian multiplier $1/(q\Phi)$ by using the Lagrange's multiplication method.

Category: Classical Physics.

The partition function Z and Lagrangian multiplier $1/(q\Phi)$ of a particle-charged system

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Abstract

In a system consisting of a large number of charged particles, the potential energy of the in-system particles satisfy probability distribution. Starting from such consideration, we defined the partition function Z and get the particles in-system distribution function N_i, and obtained the Lagrangian multiplier $1/(q\Phi)$ by using the Lagrange's multiplication method.

Key word: Partition function, Lagrangian multiplier.

PACS: 05.20.-y

1 Introduction

In a system, the charged particle acquires potential energy $q\Phi$ by coupling with its neighbors. Such potential energy may satisfies the probability distri-

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bution function W. By using the Lagrange's multiplication method and the Stirling formula, we defined the system's partition function Z. The potential function Φ exists in the system with charged particles, based on which the charged particles acquire average potential $\langle q\Phi \rangle$. Hence we get the system's Lagrangian multiplier $1/(q\Phi)$, from which we derived the partition function Z and distribution function N_i. This made it possible for us to further establish the probabilistic statistical distribution of the charged particles in a system which features concurrent existence of temperature T and potential Φ

2 The Lagrangian Multiplier of Charged Particles

When ith charge particle coupling with a system, exist potential function [1] $\varphi_i = \int_{a_i}^{b_i} \mathbb{E}_i \cdot\ dk$. The particle carrying charge q would have energy as shown

$$\epsilon_i = q\varphi_i. \tag{1}$$

This charge particle with momentum p_i, and would have energy [2]

$$\epsilon_i = p_i c. \tag{2}$$

For the same particle, its energy may be indicated either by (1) and (2), and this is because energy of the same particle is observed from different point of view, and hence we have

$$p_i c = q\varphi_i. \tag{3}$$

Under the assumption that the system has N_i particles numbers which respectively carries energy $p_i c$, and that the system has a total number of N particles and energy $\mathcal{E}$, then we have

$$\mathcal{E} = \sum_i N_i p_i c, \quad N = \sum_i N_i. \tag{4}$$

We now subdivide the system's phase space Ω into ν cells, each of which has ith cell and unit volume ω_i has N_i particles. Make g_i the probability, then N particles would have probability W[3]

$$W = \Pi_i \frac{N!}{N_i!} g_i^{N_i}. \tag{5}$$

Applying the Stirling's asymptotic formula [4] $\ln N! \approx (N + 1/2)\ln N - N + 1/2 \ln(2\pi)$, when $N_i \gg 1$. By (5) we have

$$\delta \ln W = \sum_i (\ln g_i - \ln N_i)\delta N_i = 0. \tag{6}$$

By calculating the variation of (4) and Lagrangian multipliers [5], α and γ, from (4) and (6), become

$$\sum_i (\ln(g_i/N_i) - \alpha p_i c + \gamma)\delta N_i = 0, \tag{7}$$

$$N_i = g_i \exp(-\alpha p_i c)\exp(\gamma), \tag{8}$$

from (4) hence, we have

$$N = \exp(\gamma) \sum_i g_i \exp(-\alpha p_i c). \tag{9}$$

from the (9), let we define the partition function Z [6, 7], we have

$$Z = \sum_i g_i \exp(-\alpha p_i c), \tag{10}$$

from (10), formula (9) become $\exp(\gamma) = N/Z$, we make (8) into

$$N_i = (g_i/Z)\; N \exp(-\alpha p_i c), \tag{11}$$

let $A = g_i/Z$, formula (11) become

$$N_i\ \mathrm{d}p_i = NA \exp(-\alpha p_i c)\ dp_i, \tag{12}$$

we had known

$$NA \exp(-\alpha p_i c)\ dp_i = d(N p_i A \exp(-\alpha p_i c)) + N\alpha p_i c A \exp(-\alpha p_i c)\ dp_i, \tag{13}$$

from (12) and above formula (13) the $A\exp(-\alpha p_i c)$ is the probability function, and number of particles in the system would be

$$N = \int_0^\infty N_i\ dp_i = N\alpha \int_0^\infty pcA \exp(-\alpha pc)\ dp, \tag{14}$$

hence, the pc average value

$$\langle pc \rangle = \int_0^\infty pcA \exp(-\alpha pc)\, dp, \tag{15}$$

where above (14), we have

$$N = N\alpha \langle pc \rangle. \tag{16}$$

Owing to that the system has electric charge $Q = Nq$ and energy $\mathcal{E} = Q\Phi = Nq\Phi$, the average energy of individual particles

$$\langle\epsilon\rangle = \mathcal{E}/N = q\Phi, \tag{17}$$

for same particle, formula (2), (15) and (17) have average energy $\langle\epsilon\rangle = \langle pc\rangle$, would be

$$\langle\epsilon\rangle = \langle pc\rangle = q\Phi. \tag{18}$$

Therefore formula (16) become

$$\alpha = \frac{1}{q\Phi}. \tag{19}$$

By now, (10) and (11) may be rewritten as

$$Z = \sum_i g_i \exp(-\frac{p_i c}{q\Phi}), \tag{20}$$

$$N_i = \frac{g_i}{Z} N \exp(-\frac{p_i c}{q\Phi}). \tag{21}$$

This is the charged particle's Lagrangian multiplier $\alpha = 1/(q\Phi)$ in the statistical sense, which will play a vitally important role in the forthcoming discussion on the particle's energy transmission and its probability distribution. There are large numbers of charged particle when potential function Φ exist in a system, the probability distribution exist in potential energy of these particle. In a system of charged particle we obtain distribution function N_i and the partition function Z.

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