# Jiang and Wiles Who Has First Proved Fermat Last Theorem(1)

**Abstract**

**D.Zagier(1984) and K.Inkeri(1990) said[7] Jiang mathematics is true, but Jiang determinates the irrational numbers to be very difficult for prime exponent p>2.In 1991 Jiang studies the composite exponents n=15,21,33,…,3p and proves Fermat last theorem for prime exponent p>3[1].In 1986 Gerhard Frey places Fermat last theorem at elliptic curve ,now called a Frey curve.Andrew Wiles studies Frey curve.In 1994 Wiles proves Fermat last theorem[9,10].Conclusion:Jiang proof is direct and very simple,but Wiles proof is indirect and very complex. If China mathematicians and Academia Sinica had supported and recognized Jiang proof on Fermat last theorem,Wiles would not have proved Fermat last theorem,because in 1991 Jiang had proved Fermat last theorem[1].Wiles has received many prizes and awards, he should thank China mathematicians and Academia Sinica.To support and to publish Jiang Fermat last theorem paper is prohibited in Academia Sinica. Remark. Chun-Xuan Jiang,A general proof of Fermat last theorem(Chinese),Mimeograph papers,July 1978. In this paper using circulant matrix,circulant determinant and permutation group theory Jiang had proved Fermat last theorem for odd prime exponent.**

**1978 年 7 月 19 日下午在中科院数学所由王元组织蒋春暄费马大定理讨论会, (这次讨论会是国家科委主任方毅指示下进行的 ) 蒋春暄首先报告, 接着数学所发言, 陈绪明(现在加拿大) 发言: 你没理解蒋春暄讲话内容. 最后宣布散会.** 后来蒋春暄单位收到数学所来信, 领导对蒋春暄说, 内容大概如下: <你们单位好好教育蒋春暄, 为社会主义作些有益工作, 不要做些对社会主任无用的工作>。在这次讨论会上蒋春暄已经证明了费马大定理。如果数学所所长华罗庚对这件事关心, 组织有关专家邦助并发表. 费马大定理在上世纪七十年代就解决了。不会出现怀尔斯事件。蒋春暄最后证明费马大定理是在这次报告基础进一步完成的, 基本思路没有变化。这是一种证明费马大定理新的数学方法。华罗庚数学学派他们不相信中国人能证明费马大定理, 华罗庚对中国证明费马大定理人有句名言: 骑自行登月是不可能的。所以蒋春暄是做骑自行登月的事. 所以到今天, 中国不承认不支持, 连蒋春暄母校北京航空航天大学也不支持。2009 年蒋春暄因首先证明费马大定理获国际金奖, 中国不承认这个金奖, 蒋春暄证明费大定理得到部分人支持,没有人否定蒋春暄证明。一句话中国只承认怀尔斯证明费马大定理, 不承认中国蒋春暄证明费马大定理。2010 年 8 月出版王元主编<数学大辞典> **, 王元宣布费马大定理是由怀尔斯 1994 年解决的, 这件事总会解决, 利用网络来宣传这件数学大事, 可能要下代, 怀尔斯学派力量太强大,** 它是日本德国美国法国英国顶尖数学家成果, 最后由怀尔斯完成。蒋春暄单枪匹马斗不过他们, 但科学真理力量是巨大, 最后胜利一定是属于蒋春暄的。历史将会作出最后结论。蒋春暄证明费马大定理主要宣传他划时代 Automorphic function. **这和微分方程, 群论, 函数论, 代数,** 几何等学科都有联系, 三角函数

非常有用，它是三角函数推广。用它可解决自然界最复杂问题。这个问题研究几百年，最后由蒋春暄解决。蒋春暄用他发明新数学，这种新数学就包括费马大定理，不用任何数论知识，直接证明了费马大定理，这种证明一般数学家都能理解。说明这种数学非常有用。怀尔斯没有发明新数学，利用与费马大定理没有直接关系数学，硬把它和费马大定理联系在一起,间接证明费马大定理。

# Automorphic Functions And Fermat's Last Theorem(1)

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Abstract

In 1637 Fermat wrote: "*It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree: I have discovered a truly marvelous proof, which this margin is too small to contain.*"

This means: $x^n + y^n = z^n (n > 2)$ has no integer solutions, all different from 0(i.e., it has only the trivial solution, where one of the integers is equal to 0). It has been called Fermat's last theorem (FLT). It suffices to prove FLT for exponent 4. and every prime exponent $P$. Fermat proved FLT for exponent 4. Euler proved FLT for exponent 3.

In this paper using automorphic functions we prove FLT for exponents $3P$ and $P$, where $P$ is an odd prime. The proof of FLT must be direct. But indirect proof of FLT is disbelieving.

In 1974 Jiang found out Euler formula of the cyclotomic real numbers in the cyclotomic fields

$$\exp\left(\sum_{i=1}^{n-1} t_i J^i\right) = \sum_{i=1}^{n} S_i J^{i-1} \qquad (1)$$

where $J$ denotes a $n$th root of unity, $J^n = 1$, $n$ is an odd number, $t_i$ are the real numbers.

$S_i$ is called the automorphic functions(complex hyperbolic functions) of order $n$ with $n-1$ variables [1-7].

$$S_i = \frac{1}{n}\left[e^A + 2\sum_{j=1}^{\frac{n-1}{2}} (-1)^{(i-1)j} e^{B_j} \cos\left(\theta_j + (-1)^j \frac{(i-1)j\pi}{n}\right)\right] \qquad (2)$$

where $i$=1,2,…,$n$;

$$A=\sum_{\alpha-1}^{n-1} t_\alpha , \qquad B_j=\sum_{\alpha=1}^{n-1} t_\alpha (-1)^{\alpha j}\cos\frac{\alpha j\pi}{n},$$

$$\theta_j=(-1)^{j+1}\sum_{\alpha=1}^{n-1} t_\alpha (-1)^{\alpha j}\sin\frac{\alpha j\pi}{n}, \qquad A+2\sum_{j=1}^{\frac{n-1}{2}} B_j=0 \tag{3}$$

(2) may be written in the matrix form

$$\begin{bmatrix} S_1\\ S_2\\ S_3\\ \cdots\\ S_n\end{bmatrix}=\frac{1}{n}\begin{bmatrix} 1 & 1 & 0 & \cdots & 0\\ 1 & -\cos\frac{\pi}{n} & -\sin\frac{\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{2n}\\ 1 & \cos\frac{2\pi}{n} & \sin\frac{2\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{n}\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ 1 & \cos\frac{(n-1)\pi}{n} & \sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n}\end{bmatrix}\begin{bmatrix} e^A\\ 2e^{B_1}\cos\theta_1\\ 2e^{B_1}\sin\theta_1\\ \cdots\\ 2\exp B_{\frac{n-1}{2}}\sin\theta_{\frac{n-1}{2}}\end{bmatrix} \tag{4}$$

where $(n-1)/2$ is an even number.

From (4) we have its inverse transformation

$$\begin{bmatrix} e^A\\ e^{B_1}\cos\theta_1\\ e^{B_1}\sin\theta_1\\ \cdots\\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}})\end{bmatrix}=\begin{bmatrix} 1 & 1 & 1 & \cdots & 1\\ 1 & -\cos\frac{\pi}{n} & \cos\frac{2\pi}{n} & \cdots & \cos\frac{(n-1)\pi}{n}\\ 0 & -\sin\frac{\pi}{n} & \sin\frac{2\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{n}\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ 0 & -\sin\frac{(n-1)\pi}{2n} & -\sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n}\end{bmatrix}\begin{bmatrix} S_1\\ S_2\\ S_3\\ \cdots\\ S_n\end{bmatrix} \tag{5}$$

From (5) we have

$$e^A=\sum_{i=1}^{n} S_i , \qquad e^{B_j}\cos\theta_j=S_1+\sum_{i=1}^{n-1} S_{1+i}(-1)^{ij}\cos\frac{ij\pi}{n}$$

$$e^{B_j}\sin\theta_j=(-1)^{j+1}\sum_{i=1}^{n-1} S_{1+i}(-1)^{ij}\sin\frac{ij\pi}{n}, \tag{6}$$

In (3) and (6) $t_i$ and $S_i$ have the same formulas. (4) and (5) are the most critical formulas of proofs for FLT. Using (4) and (5) in 1991 Jiang invented that every factor of exponent $n$ has the Fermat equation and proved FLT [1-7] Substituting (4) into (5) we prove (5).

$$\begin{bmatrix} e^{A} \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix} = \frac{1}{n}\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & -\cos\frac{\pi}{n} & \cos\frac{2\pi}{n} & \cdots & \cos\frac{(n-1)\pi}{n} \\ 0 & -\sin\frac{\pi}{n} & \sin\frac{2\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & -\sin\frac{(n-1)\pi}{2n} & -\sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix} \times$$

$$\begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ 1 & -\cos\frac{\pi}{n} & -\sin\frac{\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{2n} \\ 1 & \cos\frac{2\pi}{n} & \sin\frac{2\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \cos\frac{(n-1)\pi}{n} & \sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix} \begin{bmatrix} e^{A} \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}$$

$$= \frac{1}{n}\begin{bmatrix} n & 0 & 0 & \cdots & 0 \\ 0 & \frac{n}{2} & 0 & \cdots & 0 \\ 0 & 0 & \frac{n}{2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \frac{n}{2} \end{bmatrix} \begin{bmatrix} e^{A} \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}$$

$$= \begin{bmatrix} e^{A} \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}, \qquad (7)$$

where $1+\sum_{j=1}^{n-1}(\cos\frac{j\pi}{n})^2 = \frac{n}{2}$, $\sum_{j=1}^{n-1}(\sin\frac{j\pi}{n})^2 = \frac{n}{2}$.

From (3) we have

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j) = 1. \qquad (8)$$

From (6) we have

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=\begin{vmatrix} S_1 & S_n & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_n & S_{n-1} & \cdots & S_1 \end{vmatrix}=\begin{vmatrix} S_1 & (S_1)_1 & \cdots & (S_1)_{n-1} \\ S_2 & (S_2)_1 & \cdots & (S_2)_{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ S_n & (S_n)_1 & \cdots & (S_n)_{n-1} \end{vmatrix}, \qquad (9)$$

where $(S_i)_j=\dfrac{\partial S_i}{\partial t_j}$ [7].

From (8) and (9) we have the circulant determinant

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=\begin{vmatrix} S_1 & S_n & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \vdots \\ S_n & S_{n-1} & \cdots & S_1 \end{vmatrix}=1 \qquad (10)$$

If $S_i\neq 0$, where $i=1,2,\cdots,n$, then (10) has infinitely many rational solutions.

Assume $S_1\neq 0$, $S_2\neq 0$, $S_i=0$ where $i=3,4,\cdots,n$. $S_i=0$ are $n-2$ indeterminate equations with $n-1$ variables. From (6) we have

$$e^A=S_1+S_2,\quad e^{2B_j}=S_1^2+S_2^2+2S_1S_2(-1)^j\cos\frac{j\pi}{n}. \qquad (11)$$

From (10) and (11) we have the Fermat equation

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=(S_1+S_2)\prod_{j=1}^{\frac{n-1}{2}}(S_1^2+S_2^2+2S_1S_2(-1)^j\cos\frac{j\pi}{n})=S_1^n+S_2^n=1 \qquad (12)$$

**Example**[1]. Let $n=15$. From (3) we have

$$A=(t_1+t_{14})+(t_2+t_{13})+(t_3+t_{12})+(t_4+t_{11})+(t_5+t_{10})+(t_6+t_9)+(t_7+t_8)$$

$$B_1=-(t_1+t_{14})\cos\frac{\pi}{15}+(t_2+t_{13})\cos\frac{2\pi}{15}-(t_3+t_{12})\cos\frac{3\pi}{15}+(t_4+t_{11})\cos\frac{4\pi}{15}$$
$$-(t_5+t_{10})\cos\frac{5\pi}{15}+(t_6+t_9)\cos\frac{6\pi}{15}-(t_7+t_8)\cos\frac{7\pi}{15},$$

$$B_2=(t_1+t_{14})\cos\frac{2\pi}{15}+(t_2+t_{13})\cos\frac{4\pi}{15}+(t_3+t_{12})\cos\frac{6\pi}{15}+(t_4+t_{11})\cos\frac{8\pi}{15}$$
$$+(t_5+t_{10})\cos\frac{10\pi}{15}+(t_6+t_9)\cos\frac{12\pi}{15}+(t_7+t_8)\cos\frac{14\pi}{15},$$

$$B_3=-(t_1+t_{14})\cos\frac{3\pi}{15}+(t_2+t_{13})\cos\frac{6\pi}{15}-(t_3+t_{12})\cos\frac{9\pi}{15}+(t_4+t_{11})\cos\frac{12\pi}{15}$$
$$-(t_5+t_{10})\cos\frac{15\pi}{15}+(t_6+t_9)\cos\frac{18\pi}{15}-(t_7+t_8)\cos\frac{21\pi}{15},$$

$$B_4=(t_1+t_{14})\cos\frac{4\pi}{15}+(t_2+t_{13})\cos\frac{8\pi}{15}+(t_3+t_{12})\cos\frac{12\pi}{15}+(t_4+t_{11})\cos\frac{16\pi}{15}$$
$$+(t_5+t_{10})\cos\frac{20\pi}{15}+(t_6+t_9)\cos\frac{24\pi}{15}+(t_7+t_8)\cos\frac{28\pi}{15},$$

$$B_5 = -(t_1+t_{14})\cos\frac{5\pi}{15} + (t_2+t_{13})\cos\frac{10\pi}{15} - (t_3+t_{12})\cos\frac{15\pi}{15} + (t_4+t_{11})\cos\frac{20\pi}{15}$$
$$-(t_5+t_{10})\cos\frac{25\pi}{15} + (t_6+t_9)\cos\frac{30\pi}{15} - (t_7+t_8)\cos\frac{35\pi}{15},$$
$$B_6 = (t_1+t_{14})\cos\frac{6\pi}{15} + (t_2+t_{13})\cos\frac{12\pi}{15} + (t_3+t_{12})\cos\frac{18\pi}{15} + (t_4+t_{11})\cos\frac{24\pi}{15}$$
$$+(t_5+t_{10})\cos\frac{30\pi}{15} + (t_6+t_9)\cos\frac{36\pi}{15} + (t_7+t_8)\cos\frac{42\pi}{15},$$
$$B_7 = -(t_1+t_{14})\cos\frac{7\pi}{15} + (t_2+t_{13})\cos\frac{14\pi}{15} - (t_3+t_{12})\cos\frac{21\pi}{15} + (t_4+t_{11})\cos\frac{28\pi}{15}$$
$$-(t_5+t_{10})\cos\frac{35\pi}{15} + (t_6+t_9)\cos\frac{42\pi}{15} - (t_7+t_8)\cos\frac{49\pi}{15},$$

$$A + 2\sum_{j=1}^{7} B_j = 0, \qquad A + 2B_3 + 2B_6 = 5(t_5+t_{10}). \tag{13}$$

Form (12) we have the Fermat equation

$$\exp(A + 2\sum_{j=1}^{7} B_j) = S_1^{15} + S_2^{15} = (S_1^5)^3 + (S_2^5)^3 = 1. \tag{14}$$

From (13) we have

$$\exp(A + 2B_3 + 2B_6) = [\exp(t_5+t_{10})]^5. \tag{15}$$

From (11) we have

$$\exp(A + 2B_3 + 2B_6) = S_1^5 + S_2^5. \tag{16}$$

From (15) and (16) we have the Fermat equation

$$\exp(A + 2B_3 + 2B_6) = S_1^5 + S_2^5 = [\exp(t_5+t_{10})]^5. \tag{17}$$

Euler proved that (14) has no rational solutions for exponent 3[8]. Therefore we prove that (17) has no rational solutions for exponent 5[1].

**Theorem 1.** [1-7]. Let $n = 3P$ ,where $P > 3$ is odd prime. From (12) we have the Fermat's equation

$$\exp(A + 2\sum_{j=1}^{3P-1} B_j) = S_1^{3P} + S_2^{3P} = (S_1^P)^3 + (S_2^P)^3 = 1. \tag{18}$$

From (3) we have

$$\exp(A + 2\sum_{j=1}^{\frac{P-1}{2}} B_{3j}) = [\exp(t_P + t_{2P})]^P. \tag{19}$$

From (11) we have

$$\exp(A + 2\sum_{j=1}^{\frac{P-1}{2}} B_{3j}) = S_1^P + S_2^P. \tag{20}$$

From (19) and (20) we have the Fermat equation

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}}B_{3j})=S_1^P+S_2^P=[\exp(t_P+t_{2P})]^P. \qquad (21)$$

Euler proved that (18) has no rational solutions for exponent 3[8]. Therefore we prove that (21) has no rational solutions for $P>3$ [1, 3-7].

**Theorem 2**. In 1847 Kummer write the Fermat's equation

$$x^P+y^P=z^P \qquad (22)$$

in the form

$$(x+y)(x+ry)(x+r^2y)\cdots(x+r^{P-1}y)=z^P \qquad (23)$$

where $P$ is odd prime, $r=\cos\frac{2\pi}{P}+i\sin\frac{2\pi}{P}$.

Kummer assume the divisor of each factor is a $P$ th power. Kummer proved FLT for prime exponent p<100 [8]..

We consider the Fermat's equation

$$x^{3P}+y^{3P}=z^{3P} \qquad (24)$$

we rewrite (24)

$$(x^P)^3+(y^P)^3=(z^P)^3 \qquad (25)$$

From (24) we have

$$(x^P+y^P)(x^P+ry^P)(x^P+r^2y^P)=z^{3P} \qquad (26)$$

where $r=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$

We assume the divisor of each factor is a $P$ th power.

Let $S_1=\frac{x}{z}$, $S_2=\frac{y}{z}$. From (20) and (26) we have the Fermat's equation

$$x^P+y^P=[z\times\exp(t_P+t_{2P})]^P \qquad (27)$$

Euler proved that (25) has no integer solutions for exponent 3[8]. Therefore we prove that (27) has no integer solutions for prime exponent $P$.

**Fermat Theorem**. It suffices to prove FLT for exponent 4. We rewrite (24)

$$(x^3)^P+(y^3)^P=(z^3)^P \qquad (28)$$

Euler proved that（25）has no integer solutions for exponent 3 [8]. Therefore we prove that (28) has no integer solutions for all prime exponent $P$ [1-7].

We consider Fermat equation

$$x^{4P}+y^{4P}=z^{4P} \qquad (29)$$

We rewrite (29)

$$(x^P)^4+((y^P)^4=(z^P)^4 \qquad (30)$$

$$(x^4)^P + (y^4)^P = (z^4)^P \qquad (31)$$

Fermat proved that (30) has no integer solutions for exponent 4 [8]. Therefore we prove that (31) has no integer solutions for all prime exponent $P$ [2,5,7].This is the proof that Fermat thought to have had.

**Remark**. It suffices to prove FLT for exponent 4. Let $n = 4P$, where $P$ is an odd prime. We have the Fermat's equation for exponent $4P$ and the Fermat's equation for exponent $P$ [2,5,7]. This is the proof that Fermat thought to have had. In complex hyperbolic functions let exponent $n$ be $n = \Pi P$, $n = 2\Pi P$ and $n = 4\Pi P$. Every factor of exponent $n$ has the Fermat's equation [1-7]. In complex trigonometric functions let exponent $n$ be $n = \Pi P$, $n = 2\Pi P$ and $n = 4\Pi P$. Every factor of exponent $n$ has Fermat's equation [1-7].Using modular elliptic curves Wiles and Taylor prove FLT[9,10].This is not the proof that Fermat thought to have had. The classical theory of automorphic functions, created by Klein and Poincare, was concerned with the study of analytic functions in the unit circle that are invariant under a discrete group of transformations. Automorphic functions are generalization of the trigonometric,hyperbolic,elliptic, and certain other functions of elementary analysis. The complex trigonometric functions and complex hyperbolic functions have a wide application in mathematics and physics.

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**References**

[1] Jiang, C-X, Fermat last theorem had been proved, Potential Science (in Chinese), 2.17-20 (1992), Preprints (in English) December (1991). http://www.wbabin.net/math/xuan47.pdf.

[2] Jiang, C-X, Fermat last theorem had been proved by Fermat more than 300 years ago, Potential Science (in Chinese), 6.18-20(1992).

[3] Jiang, C-X, On the factorization theorem of circulant determinant, Algebras, Groups and Geometries, 11. 371-377(1994), MR. 96a: 11023, http://www.wbabin.net/math/xuan45.pdf

[4] Jiang, C-X, Fermat last theorem was proved in 1991, Preprints (1993). In: Fundamental open problems in science at the end of the millennium, T.Gill, K. Liu and E. Trell (eds). Hadronic Press, 1999, P555-558. http://www.wbabin.net/math/xuan46.pdf.

[5] Jiang, C-X, On the Fermat-Santilli theorem, Algebras, Groups and Geometries, 15. 319-349(1998)

[6] Jiang, C-X, Complex hyperbolic functions and Fermat's last theorem, Hadronic Journal Supplement, 15. 341-348(2000).

[7] Jiang, C-X, Foundations of Santilli Isonumber Theory with applications to new cryptograms, Fermat's theorem and Goldbach's Conjecture. Inter, Acad. Press. 2002. MR2004c:11001, http://www.wbabin.net/math/xuan13.pdf. http://www.i-b-r.org/docs/jiang.pdf

[8] Ribenboim, P, Fermat last theorem for amateur, Springer-Verlag, (1999).

[9]Wiles,A,Modular elliptic curves and Fermat last theorem,Ann. of Math.,(2) 141(1995),443-551.

[10] Taylor,R. and Wiles,A., Ring-theoretic properties of certain Hecke algebras,Ann. of Math (2),141(1995),553-572.

# Solving Fermat: Andrew Wiles

**Andrew Wiles devoted much of his entire career to proving Fermat's Last Theorem, the world's most famous mathematical problem. In 1993, he made front-page headlines when he announced a proof of the problem, but this was not the end of the story; an error in his calculation jeopardized his life's work. Andrew Wiles spoke to NOVA and described how he came to terms with the mistake, and eventually went on to achieve his life's ambition.**

**NOVA:** Many great scientific discoveries are the result of obsession, but in your case that obsession has held you since you were a child.

**ANDREW WILES:** I grew up in Cambridge in England, and my love of mathematics dates from those early childhood days. I loved doing problems in school. I'd take them home and make up new ones of my own. But the best problem I ever found, I found in my local public library. I was just browsing through the section of math books and I found this one book, which was all about one particular problem—Fermat's Last Theorem. This problem had been unsolved by mathematicians for 300 years. It looked so simple, and yet all the great mathematicians in history couldn't solve it. Here was a problem, that I, a ten year old, could understand and I knew from that moment that I would never let it go. I had to solve it.

**NOVA:** Who was Fermat and what was his Last Theorem?

**AW:** Fermat was a 17th-century mathematician who wrote a note in the margin of his book stating a particular proposition and claiming to have proved it. His proposition was about an equation which is closely related to Pythagoras' equation. Pythagoras' equation gives you:

$$x^2 + y^2 = z^2$$

You can ask, what are the whole number solutions to this equation, and you can see that

$$3^2 + 4^2 = 5^2$$

and

$$5^2 + 12^2 = 13^2$$

And if you go on looking then you find more and more such solutions. Fermat then considered the cubed version of this equation:

$$x^3 + y^3 = z^3$$

He raised the question: can you find solutions to the cubed equation? He claimed that there were none. In fact, he claimed that for the general family of equations:

$$x^n + y^n = z^n \text{ where n is bigger than 2}$$

it is impossible to find a solution. That's Fermat's Last Theorem.

**NOVA:** So Fermat said because he could not find any solutions to this equation, then there were no solutions?

**AW:** He did more than that. Just because we can't find a solution it doesn't mean that there isn't one. Mathematicians aren't satisfied because they know there are no solutions up to four million or four billion, they really want to know that there are no solutions up to infinity. And to do that we need a proof. Fermat said he had a proof. Unfortunately, all he ever wrote down was: "I have a truly marvelous demonstration of this proposition which this margin is too narrow to contain."

**NOVA:** What do you mean by a proof?

**AW:** In a mathematical proof you have a line of reasoning consisting of many, many steps, that are almost self-evident. If the proof we write down is really rigorous, then nobody can ever prove it wrong. There are proofs that date back to the Greeks that are still valid today.

**NOVA:** So the challenge was to rediscover Fermat's proof of the Last Theorem. Why did it become so famous?

**AW:** Well, some mathematics problems look simple, and you try them for a year or so, and then you try them for a hundred years, and it turns out that they're extremely hard to solve. There's no reason why these problems shouldn't be easy, and yet they turn out to be extremely intricate. The Last Theorem is the most beautiful example of this.

**NOVA:** But finding a proof has no applications in the real world; it is a purely abstract question. So why have people put so much effort into finding a proof?

**AW:** Pure mathematicians just love to try unsolved problems—they love a challenge. And as time passed and no proof was found, it became a real challenge. I've read letters in the early 19th century which said that it was an embarrassment to mathematics that the Last Theorem had not been solved. And of course, it's very special because Fermat said that he had a proof.

**NOVA:** How did you begin looking for the proof?

**AW:** In my early teens I tried to tackle the problem as I thought Fermat might have tried it. I reckoned that he wouldn't have known much more math than I knew as a teenager. Then when I reached college, I realized that many people had thought about the problem during the 18th and 19th centuries and so I studied those methods. But I still wasn't getting anywhere. Then when I became a researcher, I decided that I should put the problem aside. It's not that I forgot about it—it was always there—but I realized that the only techniques we had to tackle it had been around for 130 years. It didn't seem that these techniques were really getting to the root of the problem. The problem with working on Fermat was that you could spend years getting nowhere. It's fine to work on any problem, so long as it generates interesting mathematics along the way—even if you don't solve it at the end of the day. The definition of a good mathematical problem is the mathematics it generates rather than the problem itself.

**NOVA:** It seems that the Last Theorem was considered impossible, and that mathematicians could not risk wasting getting nowhere. But then in 1986 everything changed. A breakthrough by Ken Ribet at the University of California at Berkeley linked Fermat's Last Theorem to another unsolved problem, the Taniyama-Shimura conjecture. Can you remember how you reacted to this news?

**AW:** It was one evening at the end of the summer of 1986 when I was sipping iced tea at the house of a friend. Casually in the middle of a conversation this friend told me that Ken Ribet had proved a link between Taniyama-Shimura and Fermat's Last Theorem. I was electrified. I knew that moment that the course of my life was changing because this meant that to prove Fermat's Last Theorem all I had to do was to prove the Taniyama-Shimura conjecture. It meant that my childhood dream was now a respectable thing to work on. I just knew that I could never let that go.

**NOVA:** So, because Taniyama-Shimura was a modern problem, this meant that working on it, and by implication trying to prove Fermat's Last Theorem, was respectable.

**AW:** Yes. Nobody had any idea how to approach Taniyama-Shimura but at least it was mainstream mathematics. I could try and prove results, which, even if they didn't get the whole thing, would be worthwhile mathematics. So the romance of Fermat, which had held me all my life, was now combined with a problem that was professionally acceptable.

**NOVA:** At this point you decided to work in complete isolation. You told nobody that you were embarking on a proof of Fermat's Last Theorem. Why was that?

**AW:** I realized that anything to do with Fermat's Last Theorem generates too much interest. You can't really focus yourself for years unless you have undivided concentration, which too many spectators would have destroyed.

**NOVA:** But presumably you told your wife what you were doing?

**AW:** My wife's only known me while I've been working on Fermat. I told her on our honeymoon, just a few days after we got married. My wife had heard of Fermat's Last Theorem, but at that time she had no idea of the romantic significance it had for mathematicians, that it had been such a thorn in our flesh for so many years.

**NOVA:** On a day-to-day basis, how did you go about constructing your proof?

**AW:** I used to come up to my study, and start trying to find patterns. I tried doing calculations which explain some little piece of mathematics. I tried to fit it in with some previous broad conceptual understanding of some part of mathematics that would clarify the particular problem I was thinking about. Sometimes that would involve going and looking it up in a book to see how it's done there. Sometimes it was a question of modifying things a bit, doing a little extra calculation. And sometimes I realized that nothing that had ever been done before was any use at all. Then I just had to find something completely new; it's a mystery where that comes from. I carried this problem around in my head basically the whole time. I would wake up with it first thing in the morning, I would be thinking about it all day, and I would be thinking about it when I went to sleep. Without distraction, I would have the same thing going round and round in my mind. The only way I could relax was when I was with my children. Young children simply aren't interested in Fermat. They just want to hear a story and they're not going to let you do anything else.

**NOVA:** Usually people work in groups and use each other for support. What did you do when you hit a brick wall?

**AW:** When I got stuck and I didn't know what to do next, I would go out for a walk. I'd often walk down by the lake. Walking has a very good effect in that you're in this state of relaxation, but at the same time you're allowing the sub-conscious to work on you. And often if you have one particular thing buzzing in your mind then you don't need anything to write with or any desk. I'd always have a pencil and paper ready and, if I really had an idea, I'd sit down at a bench and I'd start scribbling away.

**NOVA:** So for seven years you're pursuing this proof. Presumably there are periods of self-doubt mixed with the periods of success.

**AW:** Perhaps I can best describe my experience of doing mathematics in terms of a journey through a dark unexplored mansion. You enter the first room of the mansion and it's completely dark. You stumble around bumping into the furniture, but gradually you learn where each piece of furniture is. Finally, after six months or so, you find the light switch, you turn it on, and suddenly it's all illuminated. You can see exactly where you were. Then you move into the next room and spend another six months in the dark. So each of these breakthroughs, while sometimes they're momentary, sometimes over a period of a day or two, they are the culmination of—and couldn't exist without—the many months of stumbling around in the dark that proceed them.

**NOVA:** And during those seven years, you could never be sure of achieving a complete proof.

**AW:** I really believed that I was on the right track, but that did not mean that I would necessarily reach my goal. It could be that the methods needed to take the next step may simply be beyond present day mathematics. Perhaps the methods I needed to complete the proof would not be invented for a hundred years. So even if I was on the right track, I could be living in the wrong century.

**NOVA:** Then eventually in 1993, you made the crucial breakthrough.

**AW:** Yes, it was one morning in late May. My wife, Nada, was out with the children and I was sitting at my desk thinking about the last stage of the proof. I was casually looking at a research paper and there was one sentence that just caught my attention. It mentioned a 19th-century construction, and I suddenly realized that I should be able to use that to complete the proof. I went on into the afternoon and I forgot to go down for lunch, and by about three or four o'clock, I was really convinced that this would solve the last remaining problem. It got to about tea time and I went downstairs and Nada was very surprised that I'd arrived so late. Then I told her I'd solved Fermat's Last Theorem.

**NOVA:** *The New York Times* exclaimed "At Last Shout of 'Eureka!' in Age-Old Math Mystery," but unknown to them, and to you, there was an error in your proof. What was the error?

**AW:** It was an error in a crucial part of the argument, but it was something so subtle that I'd missed it completely until that point. The error is so abstract that it can't really be described in simple terms. Even explaining it to a mathematician would require the mathematician to spend two or three months studying that part of the manuscript in great detail.

**NOVA:** Eventually, after a year of work, and after inviting the Cambridge mathematician Richard Taylor to work with you on the error, you managed to repair the proof. The question that everybody asks is this; is your proof the same as Fermat's?

**AW:** There's no chance of that. Fermat couldn't possibly have had this proof. It's 150 pages long. It's a 20th-century proof. It couldn't have been done in the 19th century, let alone the 17th century. The techniques used in this proof just weren't around in Fermat's time.

**NOVA:** So Fermat's original proof is still out there somewhere.

**AW:** I don't believe Fermat had a proof. I think he fooled himself into thinking he had a proof. But what has made this problem special for amateurs is that there's a tiny possibility that there does exist an elegant 17th-century proof.

**NOVA:** So some mathematicians might continue to look for the original proof. What will you do next?

**AW:** There's no problem that will mean the same to me. Fermat was my childhood passion. There's nothing to replace it. I'll try other problems. I'm sure that some of them will be very hard and I'll have a sense of achievement again, but nothing will mean the same to me. There's no other problem in mathematics that could hold me the way that this one did. There is a sense of melancholy. We've lost something that's been with us for so long, and something that drew a lot of us into mathematics. But perhaps that's always the way with math problems, and we just have to find new ones to capture our attention. People have told me I've taken away their problem—can't I give them something else? I feel some sense of responsibility. I hope that seeing the excitement of solving this problem will make young mathematicians realize that there are lots and lots of other problems in mathematics which are going to be just as challenging in the future.

**NOVA:** What is the main challenge now?

**AW:** The greatest problem for mathematicians now is probably the Riemann Hypothesis. But it's not a problem that can be simply stated.

**NOVA:** And is there any one particular thought that remains with you now that Fermat's Last Theorem has been laid to rest?

**AW:** Certainly one thing that I've learned is that it is important to pick a problem based on how much you care about it. However impenetrable it seems, if you don't try it, then you can never do it. Always try the problem that matters most to you. I had this rare privilege of being able to pursue in my adult life, what had been my childhood dream. I know it's a rare privilege, but if one can really tackle something in adult life that means that much to you, then it's more rewarding than anything I can imagine.

**NOVA:** And now that journey is over, there must be a certain sadness?

**AW:** There is a certain sense of sadness, but at the same time there is this tremendous sense of achievement. There's also a sense of freedom. I was so obsessed by this problem that I was thinking about it all the time—when I woke up in the morning, when I went to sleep at night—and that went on for eight years. That's a long time to think about one thing. That particular odyssey is now over. My mind is now at rest.