Deciphering and Fathoming Negative Probabilities in Quantum Mechanics?

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Abstract. As currently understood since its discovery, the bare Klein-Gordon theory consists of negative quantum probabilities which are considered to be physically meaningless if not outright obsolete. Despite this annoying setback, these negative probabilities are what led the great Paul Dirac in 1928 to the esoteric discovery of the Dirac equation. The Dirac equation led to one of the greatest advances in our understanding of the physical *World*. In this reading, we ask the seemingly senseless question, "Do negative probabilities exist in quantum mechanics?". In an effort to answer this question, we arrive at the conclusion that depending on the choice one makes of the quantum probability current, one will obtain negative probabilities. We thus propose a new quantum probability current of the Klein-Gordon theory. This quantum probability current leads directly to positive definite quantum probabilities. Because these negative probabilities are in the bare Klein-Gordon theory, intrinsically a result of negative energies, the fact that we-here arrive at a theory with positive probabilities, it means that negative energy particles are not to be considered problematic as is the case in the bare Klein-Gordon theory. From an abstract-objective stand-point; in comparison with positive energy particles, the corollary is that negative energy particles should have equal chances to exist. As to why these negative energy particles do not exist, this is redolent to asking why is it that Dirac's antimatter does not exist in equal proportions with matter. This problem of why negative energy particles not exist in equal proportions with positive energy particles is a problem that needs to be solved by a future theory.

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1. Introduction

If one accepts the bare Klein-Gordon theory as it is currently understood since its discovery in 1927 by Oskar Klein (1894-1977, *of* Sweden) and Walter Gordon (1893-1938, *of* Germany), then, there is no doubt that they will accept without fail that negative quantum mechanical probabilities do exist in the bare Klein-Gordon theory. Solemnly, by a combination of a deep and rare curiosity, fortune, and serendipity, than by natural design, the existence of these negative probabilities in the Klein-Gordon theory is what led the eminent British physicist Paul Maurice Adrian Dirac (1902-1984) to his landmark discovery of the Dirac equation (Dirac 1928*a*,*b*). Needless to say but perhaps as a way of expressing our deepest admiration of this great achievement, the Dirac equation ranks amongst the greatest and most noble equations of physics. Eighty three years on since its discovery (*i.e.*, 1928-2011), the Dirac equation is an equation whose wealth of knowledge cannot be said to have been completely deciphered but is in the process thereof.

Like other deep-thinking physicists of his time, right from *the-word-go*, Dirac objected to the negative probabilities implied by the Klein-Gordon theory. This led him to silently embark on a noble journey whose final destination was to successfully solve this persistent and nagging problem of negative probability. Dirac had hoped that by eliminating the negative probabilities, he would concurrently eliminate the negative energies, alas, that did not happen, only the negative probabilities is what vanished. On completion of his seemingly divine journey, he arrived at his esoteric equation, which is thought to describe only the electron.

Why do we say Dirac silently embarked on his quest for the Dirac equation? Well the answer is that for example, during a break at the 1927 Solvay conference attended by the great Danish physicist Neils Henrik David Bohr (1885-1962) and Dirac amongst others; Dirac was asked by Neils Bohr what he was working on, to which he replied: *"I'm trying to take the square root of something ..."* meaning the square root of the Klein-Gordon equation – this is strange because it meant taking the square root of an operator, this is unheard of. Later, Dirac recalled that he continued on by saying he was trying to find a relativistic quantum theory of the electron, to which Bohr commented, *"But Klein has already solved that problem."* Dirac then tried to explain he was not satisfied with the (Klein-Gordon) solution because it involved a 2nd order equation in the time and space derivatives. Dirac was simple not open; he was a man of notoriously very few words, he meant every word he said; he was economic with his words.

Further, Dirac was a man of great mathematical subtleness; it is this quality which led him to the Dirac equation. He believed that one must follow the mathematics to where it would lead and in so doing he unlocked a great wealth of ideas such as magnetic monopoles, the variation of the gravitational constant amongst others. In 1942; Dirac, like before; he delved once again into the unchartered waters of negative probability when he wrote a paper entitled: *"The Physical Interpretation of Quantum Mechanics"* where he introduced the concept of negative probabilities (Dirac 1942). Introduction these, he said:

> *"Negative energies and probabilities should not be considered as [mere] nonsense. They are well-defined concepts mathematically, like a negative of money."*

Fifty five years later after Dirac's musings, *i.e.* in 1987 toward the end of his life, another great mind, the flamboyant and charismatic American physicist, Richard Feynman (1924-1987), took the idea further when he argued that, no one objects to using negative numbers in calculations, although "minus three apples" is not a valid concept in real life [so, it should be reasonable to consider negative probabilities too]. Further into the shores of the unknown, he argued not only how negative probabilities could possibly be useful in probability calculations, but as well how probabilities above unity may be useful.

The ideas of Dirac and Feynman on negative probability have not gained much support. To me, negative probabilities, even if they may be well defined mathematical concepts as Dirac believes, they are to me physically meaningless and obsolete; they signify something sinister about the theory in question. I like to view these ideas of negative probability as nothing more than highlighting and dramatising the desperation by physicists to make sense of nonsense all in an effort to find a natural explanation of nonsense. I think nonsense is nonsense and should be left that way; one should simple let the sleeping dogs lay.

The root of negative probabilities is the Klein-Gordon theory. If it could be shown that the Klein-Gordon theory is devoid of these, it would render Dirac and Feynman's effort worthless. This would mean the chapter of negative probabilities is closed altogether. The endeavour of this reading is to point out that the Klein-Gordon theory is devoid of these negative probabilities.

If only physicists had extended the British-German physicist Max Born (1882-1970)'s idea that the magnitude of the wavefunction gives the probability; that is, extend this idea so that it applies to all quantum mechanical wavefunctions, then, we would never have landed on these rough, bizarre and uncertain shores of negative probabilities. As will be argued, what physicists have done is to carry over the probability current density found in the Schrödinger theory directly into the Klein-Gordon theory, in which process the probability of the Klein-Gordon theory is constrained in a manner that allows for negative probabilities. Our suggestion, if correct as we would like to believe, is that instead of carrying over the probability current density found in the Schrödinger theory into the Klein-Gordon, we need to do things the other way round, that is, we have to carry over the probability of the Schrödinger theory into the Klein-Gordon theory. Simple, we must generalize Born's idea, that is:

> *"Born's idea that the wavefunction represents the probability amplitude and its magnitude represents the probability; this idea must be generalized so that it is applicable to any general wavefunction that purports to describe material particles."*

In this way, we constrain the resultant probability current density of the Klein-Gordon theory and not the probability. In the end, we obtain a Klein-Gordon theory that is devoid of negative probabilities; this off cause leads us to an objective *World* since all the probabilities are positive. Notice that Schrödinger's wavefunction together with Dirac's wavefunction all conform to Born's idea but the Klein-Gordon wavefunction does not. This is where we believe the problem in the negative probabilities lies.

Now, to windup this section, we shall give the synopsis of this reading; it is as follows. In the next section, we present the Schrödinger quantum mechanical probability theory as it is understood in the present day. In section (3), we also present the Klein-Gordon probability theory as it is understood in the present day. In section (4), we go onto the main theme of the present reading, where we demonstrate that if one makes an appropriate choice of the Klein Gordon probability current, one obtains a Klein-Gordon theory that is free from negative probabilities. While sections (2) and (3) may seem trivial to the quantum mechanically erudite reader, it is worthwhile that we mention that we have taken the decision to go through these sections (*i.e.* 2 *&* 3) for nothing other than instructive purposes. The well versed reader will obviously have to skip these and go straight to section (4). In section (5), we give the overall discussion and conclusions drawn thereof (if any).

2. Schrödinger Theory

While in search of the Schrödinger equation, Schrödinger first arrived at the Klein-Gordon equation but discarded it because it did not give the correct predictions for the hydrogen atom. This great Austrian physicist, Erwin Rudolf Josef Alexander Schrödinger (1887-1961), was largely motivated to successfully search for the Schrödinger equation after a thoughtful remark by the eminent Professor, Peter Joseph William Debye (1884-1966, of Austria), at the end of a lecture that he delivered on *de* Broglie's waves at the University *of* Vienna where he [Schrödinger] was working.

Professor Debye who was the head of the physics research group, on hearing of the *de* Broglie waves, he asked Schrödinger to explain these to the rest of the research group. So Schrödinger weighed up to the task. At the end of the lecture, Professor Debye remarked that it seemed childish to talk of waves without a corresponding wave equation for these waves. This proved to be Schrödinger's great moment of inspiration that would immortalize his name in the annals of human history.

In his 1924 doctoral thesis, which was nearly turned down [thanks to the prominent French physicist Paul Langevin (1872-1946)'s wisdom and Einstein's influence and stature], Louis *de* Broglie only proposed that there is a duality between waves and matter; he gave a formula for the matter waves which stated that the wavelength of material particles is inversely proportional to the momentum of the matter particle in question. However, in his proposal, he did not propose the corresponding wave equation for these matter waves. Logic dictates that every wave must be described by a corresponding wave equation. The *deep-and-agile* Schrödinger saw immediately the depth of Professor Debye's question and it is said he went into "hiding" for about six months in search of the Schrödinger equation which he successfully found at the end of his esoteric sojourn which was not without tribulation and trials. The equation he found is:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = -i\hbar\frac{\partial\Psi}{\partial t}, \qquad (1)$$

where the symbols have their usual meaning. In presenting his equation in 1926, Schrödinger interpreted the magnitude of the wavefunction as giving the density of the electronic charge of the atom. Born (1927) gave a radically different interpretation, where this quantity [magnitude of the wavefunction] is assumed to represent the probability that the atom is in a given state. In this way, Born ushered physics into the depth and realm of probability calculus, and to this day, physicists do not agree on how to interpret this wavefunction but the general consensus is that, it is a probability function.

2.1. Schrödinger Probability Current Density

For instructive purposes, we present here the usual way in which one arrives at the expression of the Schrödinger probability current density. To do this, we have to take the Schrödinger equation, divide it throughout by $-i\hbar$ and then multiply the resultant by the complex conjugate of the wavefunction, that is:

$$\Psi^{*}\frac{\partial \Psi}{\partial t}=-\frac{i\hbar}{2m}\left(\Psi^{*}\nabla^{2}\Psi-\frac{2im}{\hbar^{2}}V\Psi^{*}\Psi\right). \quad (2)$$

Further, taking the complex conjugate of this same equation and then multiplying it by the wavefunction, one arrives at:

$$\Psi\frac{\partial \Psi^{*}}{\partial t}=\frac{i\hbar}{2m}\left(\Psi\nabla^{2}\Psi^{*}+\frac{2im}{\hbar^{2}}V\Psi^{*}\Psi\right) \quad (3)$$

and now adding these two equations, one obtains:

$$\Psi^{*}\frac{\partial \Psi}{\partial t}+\Psi\frac{\partial \Psi^{*}}{\partial t}=-\frac{i\hbar}{2m}\left(\Psi^{*}\nabla^{2}\Psi+\Psi\nabla^{2}\Psi^{*}\right)=\nabla\cdot\left[-\frac{i\hbar}{2m}\left(\Psi^{*}\nabla\Psi-\Psi\nabla\Psi^{*}\right)\right]=\nabla\cdot\mathbf{J}_{\rho}^{S} \quad (4)$$

where $\mathbf{J}_{\rho}^{S}$ is the Schrödinger probability current. The left hand side of this equation gives the rate of change of the probability, *i.e.*:

$$\Psi^{*}\frac{\partial \Psi}{\partial t}+\Psi\frac{\partial \Psi^{*}}{\partial t}=\frac{\partial \rho}{\partial t} \quad (5)$$

where $\rho = \Psi^{*}\Psi$ is the probability function. Combining these results, one is lead to the continuity equation:

$$\frac{\partial \rho}{\partial t}+\vec{\nabla}\cdot\mathbf{J}_{\rho}^{S}=0. \quad (6)$$

This is not the only continuity equation that can be written out of the Schrödinger equation. In the next section, we shall write another continuity equation out of the Schrödinger equation not in terms of the probability but in terms of the probability amplitude.

2.2. Schrödinger Probability Amplitude Current

To write down the Schrödinger equation in terms of the probability amplitude, we know that, for any general smooth and differentiable time varying function (or field) $\Psi = \Psi(\mathbf{r},t)$, the following holds:

$$\Psi \equiv \frac{1}{3}\vec{\nabla}\cdot\left(\int \Psi d\mathbf{r}\right)=\frac{1}{3}\vec{\nabla}\cdot\left(\int \Psi \frac{d\mathbf{r}}{dt}dt\right)=\frac{1}{3}\vec{\nabla}\cdot\left(\int \Psi\mathbf{v}dt\right)=\frac{1}{3}\vec{\nabla}\cdot\left(\int \Psi dt\right)\mathbf{v}, \tag{7}$$

where $\boldsymbol{v}$ is the velocity of the field (function) and $d\mathbf{r}=dx\mathbf{i}+dy\mathbf{j}+dz\mathbf{k}$ is line element of the position vector, where the **i**, **j**, **k**'s are the usual orthogonal unit vectors along the *x,y* and *z-axis* respectively. Using this, one can recast after a few basic algebraic operations, the Schrödinger equation into the continuity equation:

$$\frac{\partial\Psi}{\partial t}+\vec{\nabla}\cdot\mathbf{J}_{\Psi}^{KG}=0. \tag{8}$$

where:

$$\mathbf{J}_{\Psi}^{KG}=\frac{i\hbar}{2m}\vec{\nabla}\Psi-\frac{i\mathbf{v}}{3\hbar}\int V\Psi dt. \tag{9}$$

What this means is that there is a corresponding conserved probability amplitude current for the probability current. Though this is a very trivial result, if it is correct (as we believe) and acceptable, it will be a new result.

3.0. Klein-Gordon Quantum Mechanical Probability

For a particle of rest mass m_0 and wavefunction Ψ, the Klein-Gordon equation describing this particle is given by:

$$\frac{\partial^2\Psi}{\partial t^2}-\nabla^2\Psi=\left(\frac{m_0c^2}{\hbar}\right)^2\Psi. \tag{10}$$

This equation is named after the physicists Oskar Klein and Walter Gordon, who in 1927 proposed that it describes relativistic electrons. The Klein–Gordon equation was first considered as a quantum mechanical wave equation by Schrödinger in his search for an equation describing *de* Broglie waves. In his final presentation in January 1926 where he proposed the Schrödinger equation, Schrödinger discarded this equation because it did not give the correct predictions for the hydrogen atom.

Now, we would like to develop for the Klein-Gordon equation the usual expressions for probability and probability current similar to the Schrödinger case. This is a task that is considered a bit tricky as compared to the Schrödinger case because the Klein-Golden is second differential equation. If we take the probability $\rho = \Psi^*\Psi$ and then differentiate it with respect to time, what we get is:

$$\frac{\partial\rho}{\partial t}=\Psi^*\frac{\partial\Psi}{\partial t}+\Psi\frac{\partial\Psi^*}{\partial t}, \tag{11}$$

and if we are to follow the Schrödinger prescription, we should be able to arrive at a continuity equation containing the probability and the corresponding probability current by substituting the time derivatives of the wavefunction. Now, most textbooks will tell you that one is not able to proceed to find the continuity equation for the above equation from the Schrödinger prescription simple because the Klein-Gordon equation does not have a first-order derivative that would enable a straight substitution. So what is typically done is to work backwards, that is start from the known Schrödinger probability current density $\mathbf{J}_{\rho}^{S}$ and proceed from there to see if one can find a corresponding probability density. As we all know, one does arrive at a continuity equation, namely:

$$\frac{\partial \rho_{KG}}{\partial t} + \vec{\nabla} \cdot \mathbf{J}_{\rho}^{S} = \boldsymbol{0}, \qquad (\boldsymbol{12})$$

where the Klein-Gordon probability is given by:

$$\rho_{KG} = \frac{i\hbar}{\boldsymbol{2} m_{\boldsymbol{0}} c^{\boldsymbol{2}}} \left(\Psi^{*} \frac{\partial \Psi}{\partial t} - \Psi \frac{\partial \Psi^{*}}{\partial t} \right). \qquad (\boldsymbol{13})$$

There is no need for us to go through the full derivation of the Klein-Gordon probability continuity equation as this can readily be found in most textbooks of quantum mechanics. Further, there is no need to demonstrate that this probability leads to negative probabilities for particles of negative energy as this is well anchored in most quantum mechanics textbooks. What we shall do is to point out that there is a *loophole* in this derivation and this *loophole* is deeply embedded in the fact that:

"The Klein-Gordon equation is not a first order differential equation which would allow for a smooth and straight forward substitution of the time derivative of the magnitude of the Klein-Gordon wave function into equation (11) directly from the Klein-Gordon equation, so as to derive the probability continuity equation; because of this, one has to seek other alternative means."

Why not force the Klein-Gordon equation to produce a probability current under these conditions? The legitimate rules of mathematics allow for this, why not go for it?! This is our *borne-of-contention.*

It is perhaps important that we mention here that in 1934, the Austrians, Wolfgang Pauli (1904-1982) and Victor Frederick Weisskopf (1908-2002) discovered what is hailed as a suitable interpretation of the Klein-Gordon equation within the scope of quantum field theory. Treating it [Klein-Gordon equation] like a field equation analogous to Maxwell's equations for an electromagnetic field, they quantized it, so that ψ became an operator (Pauli & Weisskopf 1934). This made the Klein-Gordon more acceptable.

As will be seen latter in section (4), if the main reason for adopting the Klein-Gordon probability function that leads to negative probabilities is that it emerges from the continuity equation constructed out of the second order differential Klein-Gordon equation;

then, this way of arriving at the probability function can be challenged as there is another way to arrive at a continuity equation from the Klein-Gordon equation, this equation involves the magnitude of the Klein-Gordon wavefunction. *After all*, no one has made a direct measurement to test the correctness or lack thereof the Klein-Gordon probability and the Klein-Gordon probability current, it is just but a working interpretation.

3.1. Klein-Gordon Probability Amplitude Current

Just as we have done in the Schrödinger case, we will write down the continuity equation of the Klein-Gordon equation which involves the probability amplitude and not the probability. By integrating it with respect to time throughout, we recast the Klein-Gordon equation in to the form:

$$\frac{\partial \Psi}{\partial t} - \vec{\nabla} \cdot \left[\vec{\nabla}\left(\int \Psi dt\right)\right] = \left(\frac{m_0 c^2}{\hbar}\right)^2 \int \Psi dt, \tag{14}$$

and once in this form, we construct a continuity equation. It is not difficult to deduce that this equation can be written as:

$$\frac{\partial \Psi}{\partial t} = \vec{\nabla} \cdot \left[\vec{\nabla}\left(\int \Psi dt\right) + \left(\frac{m_0 c^2}{\hbar}\right)^2 \left(\iint \Psi dt dt\right)\mathbf{v}\right]. \tag{15}$$

Thus setting:

$$\mathbf{J}_{\Psi}^{KG} = \vec{\nabla}\left(\int \Psi dt\right) + \left(\frac{m_0 c^2}{\hbar}\right)^2 \left(\iint \Psi dt dt\right)\mathbf{v}. \tag{16}$$

It is easy to see that:

$$\frac{\partial \Psi}{\partial t} + \vec{\nabla} \cdot \mathbf{J}_{\Psi}^{KG} = 0. \tag{17}$$

This is our desired equation. What this means is that the probability amplitude has a corresponding current. In the language of Einstein's Special Theory *of* Relativity, it means we can talk of a four probability amplitude comprising the probability amplitude and the probability amplitude current.

4. New Klein-Gordon Quantum Mechanical Probability

Now we come to the main theme of this reading. First things first, we need to state one thing which is *clear to all*; which is that, the Klein-Gordon equation is not cast in stone, the meaning of which is that it can be written in different but mathematically equivalent forms provided one applies permissible and legitimate mathematical operations to it. We want to have this equation written with the time derivate to first order. This can be achieved by integrating this equation throughout with respect to time as has been done in equation (14). For convenience, we shall rewrite equation (14) here, that is:

$$\frac{\partial \Psi}{\partial t} = \nabla^2 \left(\int \Psi dt \right) + \left(\frac{m_0 c^2}{\hbar} \right)^2 \int \Psi dt. \qquad (18)$$

Now, multiplying this equation throughout by the complex conjugate of the wavefunction, that is:

$$\Psi^* \frac{\partial \Psi}{\partial t} = \Psi^* \int \nabla^2 \Psi dt + \left(\frac{m_0 c^2}{\hbar} \right)^2 \Psi^* \int \Psi dt. \qquad (19)$$

Further, taking the complex conjugate of this same equation and then multiplying it by the wavefunction, one arrives at:

$$\Psi \frac{\partial \Psi^*}{\partial t} = \Psi \int \nabla^2 \Psi^* dt + \left(\frac{m_0 c^2}{\hbar} \right)^2 \Psi \int \Psi^* dt, \qquad (20)$$

and now adding these two equations, one obtains:

$$\Psi^* \frac{\partial \Psi}{\partial t} + \Psi \frac{\partial \Psi^*}{\partial t} = \left(\Psi^* \int \nabla^2 \Psi + \Psi \int \nabla^2 \Psi^* \right) dt + \left(\frac{m_0 c^2}{\hbar} \right)^2 \left(\Psi^* \int \Psi + \Psi \int \Psi^* \right) dt. \qquad (21)$$

The left hand side is obviously equal to the time derivative of the probably density function $\rho = \psi^* \psi$ *i.e.* $d\rho/dt = d(\psi^* \psi)/dt$. To simplify the right hand side, we have to make use of the identity in equation (7); doing so, we will have:

$$\left(\Psi^* \int \nabla^2 \Psi + \Psi \int \nabla^2 \Psi^* \right) dt + \left(\frac{m_0 c^2}{\hbar} \right)^2 \left(\Psi^* \int \Psi + \Psi \int \Psi^* \right) dt \equiv$$

$$\left[\int \left\{ \left(\Psi^* \int \nabla^2 \Psi + \Psi \int \nabla^2 \Psi^* \right) dt \right\} dt + \left(\frac{m_0 c^2}{\hbar} \right)^2 \int \left\{ \left(\Psi^* \int \Psi + \Psi \int \Psi^* \right) dt \right\} dt \right] \mathbf{v} =$$

$$\rho_Q \mathbf{v} = \mathbf{J}_\rho^{KG}, \qquad (22)$$

where $\boldsymbol{v}$ is the velocity of the probability wave packet and:

$$\rho_Q = \int \left\{ \left(\Psi^* \int \nabla^2 \Psi + \Psi \int \nabla^2 \Psi^* \right) dt \right\} dt + \left(\frac{m_0 c^2}{\hbar} \right)^2 \int \left\{ \left(\Psi^* \int \Psi + \Psi \int \Psi^* \right) dt \right\} dt, \qquad (23)$$

is what we shall call the probability charge density and $\mathbf{J}_{\boldsymbol{\rho}}^{KG}$ is the new Klein-Gordon probability current density which leads us to positive definite probabilities. All our efforts lead us to recast the Klein-Gordon equation into the continuity equation:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \mathbf{J}_{\rho}^{KG} = \boldsymbol{0}. \qquad (\boldsymbol{24})$$

In this manner, just as in the Schrödinger case, the magnitude of the wavefunction gives the positive definite probability. We feel and believe this approach is the correct approach to understanding the Klein-Gordon equation. It contains no negative probabilities but real and objective probabilities just as in the Schrödinger case. It is our opinion that it is much easier to try and understand $\mathbf{J}_{\boldsymbol{\rho}}^{KG}$ as the new Klein-Gordon probability current density, than to try and justify negative probabilities as Dirac, Feynman and many others have done (with great pain).

5. Discussion and Conclusions

We are of the view that the reader will concur with us that -- from the rather trivial presentation made herein; the existence of negative quantum mechanical probabilities depends on the choice of the probability current that one has made. In moving from Schrödinger's theory (which officially was first to be discovered) to the Klein-Gordon theory, it is the probability current that is held sacrosanct, the meaning or suggestion of which is that it must be the important quantity, otherwise there would no reason to preserve it. We have suggested otherwise, that it is the probability function in the Schrödinger theory that must be held sacrosanct when we move over to the Klein-Gordon theory. This way of looking at the Klein-Gordon theory solves the negative probabilities faced by the Klein-Gordon theory; because of this, we believe this reading is a significant contribution in physics insofar as deciphering and fathoming the meaning of Klein-Gordon's negative probabilities.

It is important to note that no single experiment has been performed to date to directly measure the probability and the probability current. For example, the wavefunction of the hydrogen atom as deduced from the Schrödinger equation is known. No one has measured directly that the electron in the hydrogen atom is found at the position that it is expected with the predicted frequency. The probability interpretation is an interpretation that appears to work, especially when dealing with ensembles. What this means is that the currently accepted Klein-Gordon probability can be revised as we have done. If what is required of this probability is that it satisfies the continuity equation, then we have shown that there exists such a positive definite probability satisfying the continuity equation.

If our suggestion is correct and acceptable (as we believe it to be), then, one is lead to wonder what trajectory physics might have taken if what we have just presented were known to Dirac and his contemporaries. This is so especially given that Dirac was largely motivated by the desire to get reed of the negative probabilities that appear in the Klein-Gordon theory. There is nothing exotic or new about the ideas that we have presented, it is just a different way of looking at things. The only thing that appears to the make this of importance is that it allows us to settle the nagging problem of negative probabilities.

Clearly, if the present presentation was available and acceptable to Dirac before the advent of his equation; then, if he [Dirac] was to discover the Dirac equation as he did, he would have arrived at it from a different point of departure altogether; I wonder what his motivation would have been. Trying to fathom what his point of departure would have been, leads me into the oblivious -- redolent to chasing after the rainbow. Perhaps, out of the desiderata of mathematical curiosity, beauty and elegance, he could have simple sought for an equation linear in both the time and space derivatives.

On the other hand, since negative probabilities are intrinsically tied to negative energy-mass particles, the non-existence of negative probabilities would mean that negative energy-mass particles have no problem in principle. This invariably means that negative energy-mass particles must be considered without any prejudice whatsoever as they have equal legitimacy to exist. Our only worry would be what these negative energy-mass particles are; are they Dirac's antimatter, or Dirac's sea of invisible energy-mass particles? Current thinking is that negative energy-mass particles are antiparticles. These antiparticles have positive energy and the reason for this is that they are thought to be negative energy-mass particles moving back in time, in which case they would appear to have positive energy-mass. The perfect symmetry of Dirac's theory allows a negative energy-mass particle that is moving forward in time to look identical to a positive energy-mass particle moving back in time.

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