Jiang and Wiles Proofs on Fermat Last Theorem(4)

Abstract

D.Zagier(1984) and K.Inkeri(1990) said[7] Jiang mathematics is true, but Jiang determinates the irrational numbers to be very difficult for prime exponent p>2.In 1991 Jiang studies the composite exponents n=15,21,33,…,3p and proves Fermat last theorem for prime exponent p>3[1].In 1986 Gerhard Frey places Fermat last theorem at elliptic curve that is Frey curve.Andrew Wiles studies Frey curve.In 1994 Wiles proves Fermat last theorem[9,10].Conclusion:Jiang proof is direct and very simple,but Wiles proof is indirect and very complex. If China mathematicians and Academia Sinica had supported and recognized Jiang proof on Fermat last theorem,Wiles would not have proved Fermat last theorem,because in 1991 Jiang had proved Fermat last theorem[1].Wiles has received many prizes and awards, he should thank China mathematicians and Academia Sinica.To support and to publish Jiang Fermat last theorem paper is prohibited in Academia Sinica.

Remark. Chun-Xuan Jiang,A general proof of Fermat last theorem(Chinese),Mimeograph papers,July 1978. In this paper using circulant matrix,circulant determinant and permutation group theory Jiang had proved Fermat last theorem for odd prime exponent.

1978 年 7 丹 19 日下午在中科院数学所由王元组织蒋春暄费马大定理讨论会, (这次讨论会是国家科委主任方毅指示下进行的) 蒋春暄首先报告，接着数学所发言，陈绪明(现在加拿大)发言：你没理解蒋春暄讲话内容. 最后宣布散会. 后来蒋春暄单位收到数学所来信, 领导对蒋春暄说, 内容大概如下: <你们单位好好教育蒋春暄, 为社会主义作些有益工作, 不要做些对社会主任无用的工作>。在这次讨论会上蒋春暄已经证明了费马大定理。如果数学所所长华罗庚对这件事关心, 组织有关专家邦助并发表. 费马大定理在上世纪七十年代就解决了。不会出现怀尔斯事件。蒋春暄最后证明费马大定理是在这次报告基础进一步完成的, 基本思路没有变化。这是一种证明费马大定理新的数学方法。华罗庚数学学派他们不相信中国人能证明费马大定理, 华罗庚对中国证明费马大定理人有句名言: 骑自行登月是不可能的。所以蒋春暄是做骑自行登月的事. 所以到今天, 中国不承认不支持, 连蒋春暄母校北航也不支持。2009 年蒋春暄因首先证明费马大定理获国际金奖, 中国不承认这个金奖, 蒋春暄证明费大定理得到部分人支持,没有人否定蒋春暄证明。一句话中国只承认怀尔斯证明费马大定理, 不承认中国蒋春暄证明费马大定理。2010 年 8 月出版王元主编<数学大辞典>，**王元宣布费马大定理是由怀尔斯 1994 年解决的，这件事总会解决，利用网络来宣传这件数学大事，可能要下代，怀尔斯学派力量太强大，**它是日本德国美国法国英国顶尖数学家成果, 最后由怀尔斯完成。蒋春暄单枪匹马斗不过他们, 但科学真理力量是巨大, 最后胜利一定是属于蒋春暄的。历史将会作出最后结论。蒋春暄证明费马大定理主要宣传他划时代 Automorphic function. **这和微分方程，群论，函数论，代数，**几何等学科都有联系, 三角函数非常有用,

它是三角函数推广。**用它可解决自然最复杂问题。这个问题研究几百年，最后由蒋春暄解决。**

Automorphic Functions And Fermat's Last Theorem(4)

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Absract

1637 Fermat wrote: "*It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree: I have discovered a truly marvelous proof, which this margin is too small to contain.*"

This means: $x^n + y^n = z^n (n > 2)$ has no integer solutions, all different from 0(i.e., it has only the trivial solution, where one of the integers is equal to 0). It has been called Fermat's last theorem (FLT). It suffices to prove FLT for exponent 4. and every prime exponent P. Fermat proved FLT for exponent 4. Euler proved FLT for exponent 3.

In this paper using automorphic functions we prove FLT for exponents $3P$ and P, where P is an odd prime. We find the Fermat proof. The proof of FLT must be direct. But indirect proof of FLT is disbelieving..

In 1974 Jiang found out Euler formula of the cyclotomic real numbers in the cyclotomic fields

$$\exp\left(\sum_{i=1}^{n-1} t_i J^i\right)=\sum_{i=1}^{n} S_i J^{i-1} \qquad (1)$$

where J denotes a n th root of negative unity, $J^n=-1$, n is an odd number, t_i are the real numbers.

S_i is called the automorphic functions(complex trigonometric functions) of order n with $n-1$ variables [1-7].

$$S_i=\frac{(-1)^{i-1}}{n}[e^A+2\sum_{j=1}^{\frac{n-1}{2}}(-1)^{(i-1)j}e^{B_j}\cos(\theta_j+(-1)^j\frac{(i-1)j\pi}{n})] \qquad (2)$$

where i=1,2,3,…,n;

$$A=\sum_{\alpha=1}^{n-1} t_\alpha(-1)^\alpha, \qquad B_j=\sum_{\alpha=1}^{n-1} t_\alpha(-1)^{(j-1)\alpha}\cos\frac{\alpha j\pi}{n}, \qquad (3)$$

$$\theta_j=(-1)^{j+1}\sum_{\alpha=1}^{n-1} t_\alpha(-1)^{(j-1)\alpha}\sin\frac{\alpha j\pi}{n}, \qquad A+2\sum_{j=1}^{\frac{n-1}{2}} B_j=0$$

(2) may be written in the matrix form

$$\begin{bmatrix}S_1\\S_2\\S_3\\\cdots\\S_n\end{bmatrix}=\frac{1}{n}\begin{bmatrix}1 & 1 & 0 & \cdots & 0\\ -1 & \cos\frac{\pi}{n} & \sin\frac{\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{2n}\\ 1 & \cos\frac{2\pi}{n} & \sin\frac{2\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{n}\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ 1 & \cos\frac{(n-1)\pi}{n} & \sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n}\end{bmatrix}\begin{bmatrix}e^A\\2e^{B_1}\cos\theta_1\\2e^{B_1}\sin\theta_1\\\cdots\\2\exp B_{\frac{n-1}{2}}\sin\theta_{\frac{n-1}{2}}\end{bmatrix} \qquad (4)$$

where $(n-1)/2$ is an even number.

From (4) we have its inverse transformation

$$\begin{bmatrix}e^A\\e^{B_1}\cos\theta_1\\e^{B_1}\sin\theta_1\\\cdots\\\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}})\end{bmatrix}=\begin{bmatrix}1 & -1 & 1 & \cdots & 1\\ 1 & \cos\frac{\pi}{n} & \cos\frac{2\pi}{n} & \cdots & \cos\frac{(n-1)\pi}{n}\\ 0 & \sin\frac{\pi}{n} & \sin\frac{2\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{n}\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ 0 & \sin\frac{(n-1)\pi}{2n} & -\sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n}\end{bmatrix}\begin{bmatrix}S_1\\S_2\\S_3\\\cdots\\S_n\end{bmatrix} \qquad (5)$$

From (5) we have

$$e^{A}=\sum_{i=1}^{n}S_i(-1)^{i+1},\qquad e^{B_j}\cos\theta_j=S_1+\sum_{i=1}^{n-1}S_{1+i}(-1)^{(j-1)i}\cos\frac{ij\pi}{n}$$

$$e^{B_j}\sin\theta_j=(-1)^{j+1}\sum_{i=1}^{n-1}S_{1+i}(-1)^{(j-1)i}\sin\frac{ij\pi}{n},\qquad (6)$$

In (3) and (6) t_i and S_i have the same formulas. (4) and (5) are the most critical formulas of proofs for FLT. Using (4) and (5) in 1991 Jiang invented that every factor of exponent n has the Fermat equation and proved FLT [1-7] .Substituting (4) into (5) we prove (5).

$$\begin{bmatrix} e^{A} \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}=\frac{1}{n}\begin{bmatrix} 1 & -1 & 1 & \cdots & 1 \\ 1 & \cos\frac{\pi}{n} & \cos\frac{2\pi}{n} & \cdots & \cos\frac{(n-1)\pi}{n} \\ 0 & \sin\frac{\pi}{n} & \sin\frac{2\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \sin\frac{(n-1)\pi}{2n} & -\sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix}\times$$

$$\begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ -1 & \cos\frac{\pi}{n} & \sin\frac{\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{2n} \\ 1 & \cos\frac{2\pi}{n} & \sin\frac{2\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \cos\frac{(n-1)\pi}{n} & \sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix}\begin{bmatrix} e^{A} \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}$$

$$=\frac{1}{n}\begin{bmatrix} n & 0 & 0 & \cdots & 0 \\ 0 & \frac{n}{2} & 0 & \cdots & 0 \\ 0 & 0 & \frac{n}{2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \frac{n}{2} \end{bmatrix}\begin{bmatrix} e^{A} \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}$$

$$= \begin{bmatrix} e^{A} \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}, \qquad (7)$$

where $1+\sum_{j=1}^{n-1}(\cos\frac{j\pi}{n})^2=\frac{n}{2}$, $\sum_{j=1}^{n-1}(\sin\frac{j\pi}{n})^2=\frac{n}{2}$.

From (3) we have

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=1. \qquad (8)$$

From (6) we have

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=\begin{vmatrix} S_1 & -S_n & \cdots & -S_2 \\ S_2 & S_1 & \cdots & -S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_n & S_{n-1} & \cdots & S_1 \end{vmatrix}=\begin{vmatrix} S_1 & (S_1)_1 & \cdots & (S_1)_{n-1} \\ S_2 & (S_2)_1 & \cdots & (S_2)_{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ S_n & (S_n)_1 & \cdots & (S_n)_{n-1} \end{vmatrix}, \qquad (9)$$

where $(S_i)_j=\frac{\partial S_i}{\partial t_j}$ [7].

From (8) and (9) we have the circulant determinant

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=\begin{vmatrix} S_1 & -S_n & \cdots & -S_2 \\ S_2 & S_1 & \cdots & -S_3 \\ \cdots & \cdots & \cdots & \vdots \\ S_n & S_{n-1} & \cdots & S_1 \end{vmatrix}=1 \qquad (10)$$

If $S_i\neq 0$, where $i=1,2,\cdots,n$, then (10) has infinitely many rational solutions.

Assume $S_1\neq 0$, $S_2\neq 0$, $S_i=0$ where $i=3,4,\cdots,n.\ S_i=0$ are $n-2$ indeterminate equations with $n-1$ variables. From (6) we have

$$e^{A}=S_1-S_2,\ e^{2B_j}=S_1^2+S_2^2+2S_1S_2(-1)^{j-1}\cos\frac{j\pi}{n}. \qquad (11)$$

From (3) and (11) we have the Fermat equation

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=(S_1-S_2)\prod_{j=1}^{\frac{n-1}{2}}(S_1^2+S_2^2+2S_1S_2(-1)^{j-1}\cos\frac{j\pi}{n})=S_1^n-S_2^n=1 \qquad (12)$$

Example[1]. Let $n=15$. From (3) we have

$$A=-(t_1-t_{14})+(t_2-t_{13})-(t_3-t_{12})+(t_4-t_{11})-(t_5-t_{10})+(t_6-t_9)-(t_7-t_8)$$

$$B_1=(t_1-t_{14})\cos\frac{\pi}{15}+(t_2-t_{13})\cos\frac{2\pi}{15}+(t_3-t_{12})\cos\frac{3\pi}{15}+(t_4-t_{11})\cos\frac{4\pi}{15}$$
$$+(t_5-t_{10})\cos\frac{5\pi}{15}+(t_6-t_9)\cos\frac{6\pi}{15}+(t_7-t_8)\cos\frac{7\pi}{15},$$
$$B_2=-(t_1-t_{14})\cos\frac{2\pi}{15}+(t_2-t_{13})\cos\frac{4\pi}{15}-(t_3-t_{12})\cos\frac{6\pi}{15}+(t_4-t_{11})\cos\frac{8\pi}{15}$$
$$-(t_5-t_{10})\cos\frac{10\pi}{15}+(t_6-t_9)\cos\frac{12\pi}{15}-(t_7-t_8)\cos\frac{14\pi}{15},$$
$$B_3=(t_1-t_{14})\cos\frac{3\pi}{15}+(t_2-t_{13})\cos\frac{6\pi}{15}+(t_3-t_{12})\cos\frac{9\pi}{15}+(t_4-t_{11})\cos\frac{12\pi}{15}$$
$$+(t_5-t_{10})\cos\frac{15\pi}{15}+(t_6-t_9)\cos\frac{18\pi}{15}+(t_7-t_8)\cos\frac{21\pi}{15},$$
$$B_4=-(t_1-t_{14})\cos\frac{4\pi}{15}+(t_2-t_{13})\cos\frac{8\pi}{15}-(t_3-t_{12})\cos\frac{12\pi}{15}+(t_4-t_{11})\cos\frac{16\pi}{15}$$
$$-(t_5-t_{10})\cos\frac{20\pi}{15}+(t_6-t_9)\cos\frac{24\pi}{15}-(t_7-t_8)\cos\frac{28\pi}{15},$$
$$B_5=(t_1-t_{14})\cos\frac{5\pi}{15}+(t_2-t_{13})\cos\frac{10\pi}{15}+(t_3-t_{12})\cos\frac{15\pi}{15}+(t_4-t_{11})\cos\frac{20\pi}{15}$$
$$(t_5-t_{10})\cos\frac{25\pi}{15}+(t_6-t_9)\cos\frac{30\pi}{15}+(t_7-t_8)\cos\frac{35\pi}{15},$$
$$B_6=-(t_1-t_{14})\cos\frac{6\pi}{15}+(t_2-t_{13})\cos\frac{12\pi}{15}-(t_3-t_{12})\cos\frac{18\pi}{15}+(t_4-t_{11})\cos\frac{24\pi}{15}$$
$$-(t_5-t_{10})\cos\frac{30\pi}{15}+(t_6-t_9)\cos\frac{36\pi}{15}-(t_7-t_8)\cos\frac{42\pi}{15},$$
$$B_7=(t_1-t_{14})\cos\frac{7\pi}{15}+(t_2-t_{13})\cos\frac{14\pi}{15}+(t_3-t_{12})\cos\frac{21\pi}{15}+(t_4-t_{11})\cos\frac{28\pi}{15}$$
$$+(t_5-t_{10})\cos\frac{35\pi}{15}+(t_6-t_9)\cos\frac{42\pi}{15}+(t_7-t_8)\cos\frac{49\pi}{15},$$

$$A+2\sum_{j=1}^{7}B_j=0,\qquad A+2B_3+2B_6=5(-t_5+t_{10}).\tag{13}$$

Form (12) we have the Fermat equation

$$\exp(A+2\sum_{j=1}^{7}B_j)=S_1^{15}-S_2^{15}=(S_1^5)^3-(S_2^5)^3=1.\tag{14}$$

From (13) we have

$$\exp(A+2B_3+2B_6)=[\exp(-t_5+t_{10})]^5.\tag{15}$$

From (11) we have

$$\exp(A+2B_3+2B_6)=S_1^5-S_2^5.\tag{16}$$

From (15) and (16) we have the Fermat equation

$$\exp(A+2B_3+2B_6)=S_1^5-S_2^5=[\exp(-t_5+t_{10})]^5.\tag{17}$$

Euler proved that (14) has no rational solutions for exponent 3[8]. Therefore we prove that (17) has no rational solutions for exponent 5[1].

Theorem 1. Let $n=3P$,where $P>3$ is odd prime. From (12) we have the Fermat's equation

$$\exp(A+2\sum_{j=1}^{3P-1}B_j)=S_1^{3P}-S_2^{3P}=(S_1^P)^3-(S_2^P)^3=1. \tag{18}$$

From (3) we have

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}}B_{3j})=[\exp(-t_P+t_{2P})]^P. \tag{19}$$

From (11) we have

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}}B_{3j})=S_1^P-S_2^P. \tag{20}$$

From (19) and (20) we have the Fermat equation

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}}B_{3j})=S_1^P-S_2^P=[\exp(-t_P+t_{2P})]^P. \tag{21}$$

Euler proved that (18) has no rational solutions for exponent 3[8]. Therefore we prove that (21) has no rational solutions for $P>3$ [1, 3-7].

Theorem 2. We consider the Fermat's equation

$$x^{3P}-y^{3P}=z^{3P} \tag{22}$$

we rewrite (22)

$$(x^P)^3-(y^P)^3=(z^P)^3 \tag{23}$$

From (24) we have

$$(x^P-y^P)(x^{2P}+x^Py^P+y^{2P})=z^{3P} \tag{24}$$

Let $S_1=\frac{x}{z}$, $S_2=\frac{y}{z}$. From (20) and (24) we have the Fermat's equation

$$(x^{2P}+x^Py^P+y^{2P}=z^{2P}[\exp(t_P-t_{2P})]^P \tag{25}$$

$$x^P-y^P=[z\times\exp(-t_P+t_{2P})]^P \tag{26}$$

Euler proved that (23) has no integer solutions for exponent 3[8]. Therefore we prove that (26) has no integer solutions for prime exponent P.

Fermat Theorem. It suffices to prove FLT for exponent 4. We rewrite (22)

$$(x^3)^P-(y^3)^P=(z^3)^P \tag{27}$$

Euler proved that(23)has no integer solutions for exponent 3 [8]. Therefore we prove that (27) has no integer solutions for all prime exponent P [1-7].

We consider Fermat equation

$$x^{4P}-y^{4P}=z^{4P} \tag{28}$$

We rewrite (28)

$$(x^P)^4 - ((y^P)^4 = (z^P)^4 \qquad (29)$$

$$(x^4)^P - (y^4)^P = (z^4)^P \qquad (30)$$

Fermat proved that (29) has no integer solutions for exponent 4 [8]. Therefore we prove that (30) has no integer solutions for all prime exponent P [2,5,7].This is the proof that Fermat thought to have had.

Remark. It suffices to prove FLT for exponent 4. Let $n = 4P$, where P is an odd prime. We have the Fermat's equation for exponent $4P$ and the Fermat's equation for exponent P [2,5,7]. This is the proof that Fermat thought to have had. In complex hyperbolic functions let exponent n be $n = \Pi P$, $n = 2\Pi P$ and $n = 4\Pi P$. Every factor of exponent n has the Fermat's equation [1-7]. In complex trigonometric functions let exponent n be $n = \Pi P$, $n = 2\Pi P$ and $n = 4\Pi P$. Every factor of exponent n has Fermat's equation [1-7]. Using modular elliptic Curves Wiles and Taylor prove FLT[9,10]. This is not the proof that Fermat thought to have had. The classical theory of automorphic functions, created by Klein and Poincare, was concerned with the study of analytic functions in the unit circle that are invariant under a discrete group of transformation. Automorphic functions are the generalization of trigonometric, hyperbolic elliptic and certain other functions of elementary analysis. The complex trigonometric functions and complex hyperbolic functions have a wide application in mathematics and physics.

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Wiles' proof of Fermat's Last Theorem

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Sir Andrew John Wiles

Wiles' proof of Fermat's Last Theorem is a proof of the modularity theorem for semistable elliptic curves, which, together with Ribet's theorem, provides a proof for Fermat's Last Theorem. Wiles first announced his proof in June 1993 in a version that was soon recognized as having a serious gap. The widely accepted version of the proof was released by Andrew Wiles in September 1994, and published in 1995. The proof uses many techniques from algebraic geometry and number theory, and has many ramifications in these branches of mathematics. It also uses standard constructions of modern algebraic geometry, such as the category of schemes and Iwasawa theory, and other 20th century techniques not available to Fermat.

The proof itself is over 100 pages long and consumed seven years of Wiles' research time. Among other honors for his accomplishment, he was knighted.

Contents

[hide]

[edit] Progress of the previous decades

Fermat's Last Theorem states that no nontrivial integer solutions exist for the equation

$$a^n + b^n = c^n$$

if n is an integer greater than two.

In the 1950s and 1960s a connection between elliptic curves and modular forms was conjectured by the Japanese mathematician Goro Shimura based on some ideas that Yutaka Taniyama posed. In the West it became well known through a 1967 paper by André Weil. With Weil giving conceptual evidence for it, it is sometimes called the Shimura-Taniyama-Weil conjecture. It states that every rational elliptic curve is modular.

On a separate branch of development, in the late 1960s, when Yves Hellegouarch came up with the idea of associating solutions (a, b, c) of Fermat's equation with a completely different mathematical object: an elliptic curve.[1] The curve consists of all points in the plane whose coordinates (x, y) satisfy the relation.

$$y^2 = x(x - a^n)(x + b^n)$$

Such an elliptic curve would enjoy very special properties, which are due to the appearance of high powers of integers in its equation and the fact that $a^n + b^n = c^n$ is a nth power as well.

In 1982–1985, Gerhard Frey called attention to the unusual properties of the same curve as Hellegouarch, now called a Frey curve. This provided a bridge between Fermat and Taniyama by showing that a counterexample to Fermat's Last Theorem would create such a curve that would not be modular. Again, the conjecture says that each elliptic curve with rational coefficients can be constructed in an entirely different way, not by giving its equation but by using modular functions to parametrize coordinates x and y of the points on it. Thus, according to the conjecture, any elliptic curve over $\mathbb{Q}$ would have to be a modular elliptic curve, yet if a solution to Fermat's equation with non-zero a, b, c and p greater than 2 existed, the corresponding curve would not be modular, resulting

in a contradiction. The link between Fermat's Last Theorem and the Taniyama–Shimura conjecture is a little subtle: in order to derive the former from the latter, one needs to know a small amount more, or as mathematicians would have it, "an epsilon more".

In 1985, Jean-Pierre Serre proposed that a Frey curve could not be modular and provided a partial proof of this. This showed that a proof of the semistable case of the Taniyama-Shimura conjecture would imply Fermat's Last Theorem. Serre did not provide a complete proof and what was missing became known as the epsilon conjecture or ε-conjecture. Serre's main interest was in an even more ambitious conjecture, Serre's conjecture on modular Galois representations, which would imply the Taniyama–Shimura conjecture. Although in the preceding twenty or thirty years a lot of evidence had been accumulated to form conjectures about elliptic curves, the main reason to believe that these various conjectures were true lay not in the numerical confirmations, but in a remarkably coherent and attractive mathematical picture that they presented. Moreover, it could have happened that one or more of these conjectures were actually false.

In the summer of 1986, Ken Ribet succeeded in proving the epsilon conjecture. (His article was published in 1990.) He demonstrated that, just as Frey had anticipated, a special case of the Taniyama–Shimura conjecture (still unproven at the time), together with the now proven epsilon conjecture, implies Fermat's Last Theorem. Thus, if the Taniyama–Shimura conjecture holds for a class of elliptic curves called semistable elliptic curves, then Fermat's Last Theorem would be true.

[edit] General approach of proof

Given an elliptic curve E over the field Q of rational numbers $E(\bar{Q})$, for every prime power l^n, there exists a homomorphism from the absolute Galois group

$$\mathrm{Gal}(\bar{Q}/Q)$$

to

$$GL_2(Z / l^n Z),$$

the group of invertible 2 by 2 matrices whose entries are integers $(\mathrm{mod}\ l^n)$. This is because $E(\bar{Q})$, the points of E over $\bar{Q}$, form an abelian group, on which $\mathrm{Gal}(\bar{Q}/Q)$acts; the subgroup of elements x such that $l^n x = 0$ is just $(Z/l^n Z)^2$, and an automorphism of this group is a matrix of the type described.

Less obvious is that given a modular form of a certain special type, a Hecke eigenform with eigenvalues in Q, one also gets a homomorphism from the absolute Galois group

$$\mathrm{Gal}(\bar{Q}/Q) \to GL_2(Z/l^n Z).$$

This goes back to Eichler and Shimura. The idea is that the Galois group acts first on the modular curve on which the modular form is defined, thence on the Jacobian variety of the curve, and finally on the points of l^n power order on that Jacobian. The resulting representation is not usually 2-dimensional, but the Hecke operators cut out a 2-dimensional piece. It is easy to demonstrate that these representations come from some elliptic curve but the converse is the difficult part to prove.

Instead of trying to go directly from the elliptic curve to the modular form, one can first pass to the $(\mathrm{mod}\ l^n)$ representation for some l and n, and from that to the modular form. In the case l=3 and n=1, results of the Langlands-Tunnell theorem show that the (mod 3) representation of any elliptic curve over Q comes from a modular form. The basic strategy is to use induction on n to show that this is true for l=3 and any n, that ultimately there is a single modular form that works for all n. To do this, one uses a counting argument, comparing the number of ways in which one can lift a $(\mathrm{mod}\ l^n)$ Galois representation to $(\mathrm{mod}\ l^{n+1})$ and the number of ways in which one can lift a $(\mathrm{mod}\ l^n)$ modular form. An essential point is to impose a sufficient set of conditions on the Galois representation; otherwise, there will be too many lifts and most will not be modular. These conditions should be satisfied for the representations coming from modular forms and those coming from elliptic curves. If the original (mod 3) representation has an image which is too small, one runs into trouble with the lifting argument, and in this case, there is a final trick, which has since taken on a life of its own with the subsequent work on the Serre Modularity Conjecture. The idea involves the interplay between the (mod

3) and (mod 5) representations. See Chapter 5 of the Wiles paper for this 3/5 switch.

[edit] Wiles' proof

Shortly after learning of the proof of the epsilon conjecture, it was clear that a proof that all rational semistable elliptic curves are modular would also constitute a proof of Fermat's Last Theorem. Wiles decided to conduct his research exclusively towards finding a proof for the Taniyama-Shimura conjecture. Many mathematicians thought the Taniyama-Shimura conjecture was inaccessible to proof because the modular forms and elliptic curves seem to be unrelated.

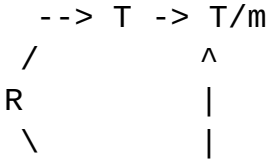
Wiles opted to attempt to "count" and match elliptic curves to counted modular forms. He found that this direct approach was not working, so he transformed the problem by instead matching the Galois representations of the elliptic curves to modular forms. Wiles denotes this matching (or mapping) that, more specifically, is a ring homomorphism:

$$R_n \to T_n.$$

R is a deformation ring and T is a Hecke ring.

Wiles had the insight that in many cases this ring homomorphism could be a ring isomorphism. (Conjecture 2.16 in Chapter 2, § 3) Wiles had the insight that the map between R and T is an isomorphism if and only if two abelian groups occurring in the theory are finite and have the same cardinality. This is sometimes referred to as the "numerical criterion". Given this result, one can see that Fermat's Last Theorem is reduced to a statement saying that two groups have the same order. Much of the text of the proof leads into topics and theorems related to ring theory and commutation theory. The Goal is to verify that the map $R \to T$ is an isomorphism and ultimately that $R=T$. This is the long and difficult step. In treating deformations, Wiles defines four cases, with the flat deformation case requiring more effort to prove and is treated in a separate article in the same volume entitled "Ring-theoretic properties of certain Hecke algebra".

Gerd Faltings, in his bulletin, on p. 745. gives this commutative diagram:



--> Z3 -> F3

or ultimately that $R = T$, indicating a complete intersection. Since Wiles cannot show that $R=T$ directly, he does so through $Z3$, $F3$ and T/m via lifts.

In order to perform this matching, Wiles had to create a class number formula (CNF). He first attempted to use horizontal Iwasawa theory but that part of his work had an unresolved issue such that he could not create a CNF. At the end of the summer of 1991, he learned about a paper by Matthias Flach, using ideas of Victor Kolyvagin to create a CNF, and so Wiles set his Iwasawa work aside. Wiles extended Flach's work in order to create a CNF. By the spring of 1993, his work covered all but a few families of elliptic curves. In early 1993, Wiles reviewed his argument beforehand with a Princeton colleague, Nick Katz. His proof involved the Kolyvagin-Flach method,[2] which he adopted after the Iwasawa method failed.[3] In May 1993 while reading a paper by Mazur, Wiles had the insight that the 3/5 switch would resolve the final issues and would then cover all elliptic curves (again, see Chapter 5 of the paper for this 3/5 switch). Over the course of three lectures delivered at Isaac Newton Institute for Mathematical Sciences on June 21, 22, and 23 of 1993, Wiles announced his proof of the Taniyama–Shimura conjecture, and hence of Fermat's Last Theorem. There was a relatively large amount of press coverage afterwards.[4]

After announcing his results, Katz was a referee on his manuscript and he asked Wiles a series of questions that led Wiles to recognize that the proof contained a gap. There was an error in a critical portion of the proof which gave a bound for the order of a particular group: the Euler system used to extend Flach's method was incomplete. Wiles and his former student Richard Taylor spent almost a year resolving it.[5][6] Wiles indicates that on the morning of September 19, 1994 he realized that the specific reason why the Flach approach would not work directly suggested a new approach with the Iwasawa theory which resolved all of the previous issues with the latter and resulted in a CNF that was valid for all of the required cases. On 6 October Wiles sent the new proof to three colleagues including Faltings. The new proof was published and, despite its size, widely accepted as likely correct in its major components.[7][8]

In his 1995 108 page article, Wiles divides the subject matter up into the following chapters (preceded here by page numbers):

475 3. Some results on subgroups of $GL_2(k)$

Chapter 2

479 1. The Gorenstein property

489 2. Congruences between Hecke rings

503 3. The main conjectures

517 Chapter 3 : Estimates for the Selmer group

Chapter 4

525 1. The ordinary CM case

533 2. Calculation of η

541 Chapter 5 : Application to elliptic curves

545 Appendix: Gorenstein rings and local complete intersections

Gerd Faltings provided some simplifications to the 1995 proof, primarily in switch from geometric constructions to rather simpler algebraic ones.[9][10] The book of the Cornell conference also contained simplifications to the original proof.[11]

[edit] Culmination of the work of many

Because Wiles had incorporated the work of so many other specialists, it had been suggested in 1994 that only a small number of people were capable of fully understanding at that time all the details of what Wiles has done.[12] The number is likely much larger now with the 10-day conference and book organized by Cornell et al.,[11] which has done much to make the full range of required topics accessible to graduate students in number theory. The paper provides a long Bibliography and Wiles mentions the names of many mathematicians in the text. The list of some of the many other mathematicians whose work the proof incorporates includes Felix Klein, Robert Fricke, Adolf Hurwitz, Erich Hecke, Barry Mazur, Dirichlet, Richard Dedekind, Robert Langlands, Jerrold B. Tunnell, Jun-Ichi Igusa, Martin Eichler, André Bloch, Tosio Kato, Ernst S. Selmer, John Tate, P. Georges Poitou, Henri Carayol, Emil Artin, Jean-Marc Fontaine, Karl Rubin, Pierre Deligne, Vladimir Drinfel'd and Haruzo Hida and to those mathematicians who have searched (or continue to search) for a more elementary proof.

[edit] Aftermath

In 1998, the full modularity theorem was proven by Christophe Breuil, Brian Conrad, Fred Diamond, and Richard Taylor using many of the methods that Andrew Wiles used in his 1995 published papers.

A computer science challenge given in 2005 is "Formalize and verify by computer a proof of Fermat's Last Theorem, as proved by A. Wiles in 1995."[13]

[edit] Reading and notation guide

The Wiles paper is over 100 pages long and often uses the peculiar symbols and notations of group theory, algebraic geometry, commutative algebra, and Galois theory.

One might want to first read the 1993 email of Ken Ribet,[14][15] Hesselink's quick review of top-level issues gives just the elementary algebra and avoids abstract algebra.[16], or Daney's web page which provides a set of his own notes and lists the current books available on the subject. Weston attempts to provide a handy map of some of the relationships between the subjects.[17] F. Q. Gouvêa provides an award-winning review of some of the required topics.[18][19][20][21] Faltings' 5-page technical bulletin on the matter is a quick and technical review of the proof for the non-specialist. For those in search of a commercially available book to guide them, he recommended that those familiar with abstract algebra read Hellegouarch, then read the Cornell book,[11] which is claimed to be accessible to "a graduate student in number theory". Note that not even the Cornell book can cover the entirety of the Wiles proof.[4]

The work of almost every mathematician who helped to lay the groundwork for Wiles did so in specialized ways, often creating new specialized concepts and yet more new jargon. In the equations, subscripts and superscripts are used extensively because of the numbers of concepts that Wiles is sometimes dealing with in an equation.

- See the glossaries listed in Lists of mathematics topics#Pure mathematics, such as Glossary of arithmetic and Diophantine geometry . Daney provides a proof-specific glossary.
- See Table of mathematical symbols and Table of logic symbols
- For the deformation theory, Wiles defines restrictions (or cases) on the deformations as Selmer (sel), ordinary(ord), strcit(str) or flat(fl) and he uses the abbreviations list here. He usually uses these as a subscript but he occasionally uses them as a superscript. There is also a fifth case: the implied "unrestricted" case but note that the superscript "unr" is not an abbreviation for unrestricted.
- Q^{unr} is the unramified extension of Q. A related but more specialized topic used is crystalline cohomology. See also Galois cohomology.
- Some relevant named concepts: Hasse-Weil zeta function, Mordell–Weil theorem, Deligne-Serre theorem

- Grab bag of jargon mentioned in paper: cover and lift, finite field, isomorphism, surjective function, decomposition group, j-invariant of elliptical curves, Abelian group, Grossencharacter, L-function, abelian variety, Jacobian[*disambiguation needed*], Néron model, Gorenstein ring, Torsion subgroup (including torsion points on elliptic curves here [22] and here [23]), Congruence subgroup, eigenform, Character (mathematics), Irreducibility (mathematics), Image (mathematics), dihedral, Conductor, Lattice (group), Cyclotomic field, Cyclotomic character, Splitting of prime ideals in Galois extensions (and decomposition group and inertia group), Quotient space, Quotient group

[edit] Notes

1. ^ Hellegouarch, Yves (2001). *Invitation to the Mathematics of Fermat-Wiles*. Academic Press. ISBN 978-0123392510.
2. ^ Singh, Simon. *Fermat's Last Theorem*, 2002, p. 259.
3. ^ Singh, Simon. *Fermat's Last Theorem*, 2002, p. 260.
4. ^ *a b* AMS book review Modular forms and Fermat's Last Theorem by Cornell et. al., 1999
5. ^ A Year Later, Snag Persists In Math Proof 1994-06-28
6. ^ June 26-July 2; A Year Later Fermat's Puzzle Is Still Not Quite Q.E.D. 1994-07-03
7. ^ NOVA Video, The Proof October 28, 1997, See also Solving Fermat: Andrew Wiles
8. ^ The Proof of Fermat's Last Theorem Charles Daney, 1996
9. ^ Fermat's Last Theorem at MacTutor
10. ^ Fermat's Last Theorem 1996
11. ^ *a b c* G. Cornell, J. H. Silverman and G. Stevens, *Modular forms and Fermat's Last Theorem*, ISBN 0-387-94609-8
12. ^ History of Fermat's Last Theorem Andrew Granville, Jun 24, 1993
13. ^ Computer verification of Wiles' proof of Fermat's Last Theorem
14. ^ FAQ: Wiles attack June 1993
15. ^ Fermat's Last Theorem a Theorem at last August 1993
16. ^ How does Wiles prove Fermat's Last Theorem? by Wim H. Hesselink
17. ^ Research Summary Topics
18. ^ A Marvelous Proof Fernando Gouvêa, The American Mathematical Monthly, vol. 101, 1994, pp. 203-222
19. ^ The Mathematical Association of America's Lester R. Ford Award
20. ^ Year of Award: 1995
21. ^ MAA Writing Awards, 1995
22. ^ http://mat.uab.es/~xarles/elliptic.html
23. ^ http://planetmath.org/encyclopedia/ArithmeticOfEllipticCurves.html

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ISSN 0003486X. OCLC 37032255. http://math.stanford.edu/~lekheng/flt/wiles.pdf. See also this smaller and searchable PDF text version. The smaller PDF gets the volume number correct: it is 141, not 142.

[edit] External links

- Weisstein, Eric W., "Fermat's Last Theorem" from MathWorld.
- "The Proof". http://www.pbs.org/wgbh/nova/proof/. The title of one edition of the PBS television series NOVA, discusses Andrew Wiles' effort to prove Fermat's Last Theorem broadcast on BBC Horizon and UTV/Documentary series as Fermat's Last Theorem at Google Video (Adobe Flash video) 1996
- Wiles, Ribet, Shimura-Taniyama-Weil and Fermat's Last Theorem
- Are mathematicians finally satisfied with Andrew Wiles' proof of Fermat's Last Theorem? Why has this theorem been so difficult to prove?, *Scientific American*, October 21, 1999

Retrieved from "http://en.wikipedia.org/wiki/Wiles'_proof_of_Fermat's_Last_Theorem"
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