Several routes to determining entropy generation in the early universe

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Abstract

We analyze how entropy could be generated via a semi classical argument as well as by multiple brane-anti brane combinations leading to an initial soliton-instanton formation. The supposition is that the two different types of methods give similar initial conditions for entropy and information/ computational bits of information in the initial universe. We close then with observations we think are pertinent to entropy increase . This is linkable to a table of computational bits as presented by Smoot in 2007.

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Introduction

We wish to present two alternative routes to generation of entropy. One is strictly a semi classical argument, linked to the exponential of early universe entropy being proportional to the spatial integral of energy minus an interaction between particles or clumps[1]. The other is a brane world summation of brane and anti brane components [2]which are in a very small space time geometry of no less than several orders of magnitude larger than Planck length, cubed. This second result is orthodox brane theory for the formation of an instanton, but it is remarkable that this same argument as given in the IUCAA meeting pre supposes the same increase in minimum geometrical length scale which the more semi classical argument gives.

We then will mention in passing how to form a five dimensional instanton in pre inflation space[3], and then the datum of a worm hole bridge between a prior to our present universe. This will be put in the context of an embedding of the instanton in pre inflation space, and its connections with the model given above of brane world instanton formation due to branes and anti branes. This among other thing has consequences to black hole physics and entropy via brane and anti branes[4] which we will mention as an area to be explored later.

The intersection of all these arguments does several things simultaneously. First, it establishes a minimum criteria for forming entropy. It secondly links entropy to instanton formation. We also mention in passing an argument linked to changes in energy density states, at the start of inflation with an increase as of entropy which would account for the dramatic growth of entropy after what we refer to as a causal discontinuity barrier breaking the instanton of energy density transmitted from a prior to a present universe.

The main point of the article which is presented toward the end is that there are bench marks as to information based complexity of cosmological evolution. This was presented by Smoot in the Challonge symposia of August 2007[5]. The main point in the end is that analyzing the particulars of cosmological models which purport to analyze initial phases of the big bang, and inputs of the big bang would be well advised as to try to answer or in part match the computational complexity bench marks Smoot is offering in the table reproduced at the end of this document.

Semi classical models of entropy generation

Kolb and Turner [6] have a temperature T related entropy density which can be treated as being written as :

$$s_{Density} = \frac{2\cdot\pi^2}{45}\cdot g_* \cdot T^3 \tag{1}$$

This pre supposes when we do it that we are able to state a total entropy as the entropy density times space time volume V_4

$$S_{total} \equiv s_{Density}\cdot V_4 \tag{2}$$

In this situation we are writing for initial conditions with a temperature $T \approx 10^{32} K$ for the initiation of quantum effects for quantum gravity as given by Weinberg (1972)[7].

$T \approx 10^{32} K \approx 1.3\times 10^{28} eV \sim 1.3\times 10^{19} GeV$. This gives us the option of comparing what we get in entropy with Seth Lloyds[8]

$$I = S_{total} / k_B \ln 2 = \left[\# operations\right]^{3/4} = \left[\rho\cdot c^5 \cdot t^4 / \hbar\right]^{3/4} \tag{3}$$

We will examine if or not the following is actually true in terms of time, i.e. can we write $I = (t/t_P)^2$? This is assuming that the density[9] $\rho \equiv T^{00} \sim \Lambda_{vacuum-energy}$ which is initially enormous, and which will be due in terms of a transfer of energy density from a prior universe to our present universe, which will be elaborated upon later in this document.

We can if we take the absolute value of Eqn. (3) and (2) above get for small volume values good estimates as to the relative volume of the phase space in early universe cosmology where Eqn. (2) and Eqn. (3) are congruent with each other. For our purposes, we will take time as greater than (or equal) to a Planck time interval, in line with the temperature dependence of entropy density mentioned in Eqn. (1) above.

We can compare this with Thanu Padamanadan's treatment of entropy[1] which is with regards to micro canonical ensemble as defined via

$$\begin{aligned} \exp(S_{total}) = g(E) = \frac{A}{N!} \cdot \int d^{3N} x \cdot \left[E - \frac{1}{2} \cdot \sum_{i \neq j} U(x_i, x_j) \right]^{\frac{3N}{2}} \\ \leq \frac{A}{N!} \cdot \int d^{3N} x \cdot [E]^{\frac{3N}{2}} \approx \left[\frac{A}{N!} \cdot \int d^{3N} x \right] \cdot \left[\frac{\Lambda_{Max} V_4}{8 \cdot \pi \cdot G} \right]^{\frac{3N}{2}} \end{aligned} \tag{4}$$

If $A \sim O(1)$, i.e. we re scale it as being of order unity, and $N \sim 10^{87}$ particles, and we re scale $\int d^{3N} x \sim V_4^N$ where we choose V_4 , and where we assume Eqn (2) and Eqn. (3) are equivalent and we assume that there is grounds for writing $\frac{\Lambda_{Max} V_4}{8 \cdot \pi \cdot G} \sim T^{00} V_4 \equiv \rho \cdot V_4 >> \frac{1}{2} \cdot \sum_{i \neq j} U(x_i, x_j)$, we can shed light on if or not it is still feasible to treat entropy, with $N \sim 10^{87}$ as a micro canonical ensemble phenomena, which we claim has implications for the formation of an instanton in early universe cosmology. Frankly we would want, in early universe cosmology that we have $\rho \cdot V_4 \neq \frac{1}{2} \cdot \sum_{i \neq j} U(x_i, x_j)$, but not by too much, so we can form an instanton.

Brane world picture of early universe entropy formation

This is adapted from a lecture given at the ICGC-07 conference by Samir Mathur[2]. The supposition is that branes and anti branes form the working component of an instanton. Which is part of what has been developed.

I.e. look at the case, first of massless radiation, and then we obtain for D space time dimensions, and E the general energy

$$S \sim E^{(D-1/D)} \tag{5}$$

This has

$$\frac{\Lambda_{Max} V_4}{8 \cdot \pi \cdot G} \sim T^{00} V_4 \equiv \rho \cdot V_4 = E_{total} \tag{6}$$

The question now becomes how do we go about defining what the necessary volume is re scaled via a quantum gravity changing of how to measure gravitational lengths which are for the threshold of quantum gravity . Traditionally the bench marking has been via the Planck length $l_P \sim 10^{-33} cm \xrightarrow[Quantum-Gravity-threshold]{} \tilde{\breve{N}}^{\alpha} \cdot l_P$. This re scaling of the minimum length needed for the importance of quantum gravity effects showing up in a grid of space time resolves, as information paradox of black hole physics. So far we have merely been working with a typical string gas model for entropy. Now, let us add in a supposition for $\breve{N}$ branes and anti branes to put in an instanton structure as to how we look at the entropy. Gilad Lifschytz [4] in 2004 codified thermalization equations of the black hole which was recovered from the model of branes and anti-branes, and in lieu of assuming an anti brane is merely in this situation the charge conjugate of say a Dp brane wrote an entropy along the lines of modifying Eqn. (5) above to read

$$S_{Total} \sim \breve{a} \cdot \left[\frac{E_{Total}}{2^n}\right]^{\lambda} \cdot \prod_{j=1}^{N} \left(\sqrt{M_{p\,j,0}} + \sqrt{M_{\bar{p}\,j,0}} \right) \tag{7}$$

This has when we do it E_{Total} as in Eqn. (5) above, and proportional to the cosmological vacuum energy parameter. Of course, in string theory, the energy is also defined via

$$E_{Total} = 4\lambda \cdot \sqrt{M_{p\,j,0} \cdot M_{\bar{p}\,j,0}} \tag{7a}$$

Furthermore, the values of $M_{p\,j,0}$, and $M_{\bar{p}\,j,0}$ refer to the mass of p branes and p anti branes,_as Gilad Lifschytz refers to it. This can be changed and rescaled to treating the mass and the energy of the brane contribution along the lines of Mathur's[2] CQG article where he has a string winding intepretation of energy along the lines of putting as much energy E into string windings as possible via, $[n_1 + \bar{n}_1]LT = [2n_1]LT = E/2$, where we are talking about n_1 wrappings of a string about a cycle of the torus , and $\bar{n}_1$ being 'wrappings the other way' , with the torus having a cycle of length L , which leads to an entropy defined in terms of an energy value of , if mass $m_i = T_P \prod L_j$ (with T_P being the tension of the i th brane, and L_j being spatial dimensions of a complex torus structure)

$$E_{Total} = 2\sum_i m_i n_i \tag{8}$$

This leads to entropy

$$S_{Total} = A \cdot \prod_i^N \sqrt{n_i} \tag{9}$$

Our claim is that this very specific value of entropy for Eqn. (9) above will in Planck interval of time at about the onset of inflation lead to

$$\left| \left[S_{Total} = A \cdot \prod_i^N \sqrt{n_i} \right] \Big/ k_B \ln 2 \right| \approx [\#operations]^{3/4} \approx 10^8 \tag{10}$$

Furthermore we also claim that the interaction of the branes and anti branes will form an instanton structure, which is implicit in the treatment outlined in Eqn. (7) , and that the numerical counting given in Eqn (9) merely reflects that branes and anti branes , even if charge conjugates of each other have the same 'wrapping number' n_i .

Specific numerical estimates for the minimum boundary of quantum gravity volume vs. classical gravity dominated effects

We begin with a temperature estimate of $T \approx 10^{45} K > T_{QG-Threshold} \sim 10^{32} K$. Then, Eqn. (4) above modified when we take the absolute value will lead to , if we look at when $N \approx 10^{86}$:

$$\left|S_{Total}\right| \sim \left|k_B \cdot \ln 2\right| \cdot [\#operations]^{3/4} \approx N\left|-\log\frac{N}{10} + \log V_4^3 + \log E^{3/2}\right| \sim 10^8 \qquad (11)$$

Leading to solving for E as follows $\Rightarrow \rho \cdot V_4 \neq \frac{1}{2} \cdot \sum_{i \neq j} U(x_i, x_j)$, and also that

$$V_4^3 \cdot E^{3/2} \sim 10^{85} \Rightarrow E \sim \frac{10^{57}}{V_4^2} \equiv \frac{\Lambda_{Max} \cdot V_4}{8 \cdot \pi \cdot G} \qquad (12)$$

We can and will reference what we can say about $\Lambda_{Max} \sim c_2 \cdot T^{\tilde{\beta}}$, as given by Park (2003), as a way to get an upper bound estimate upon V_4 for quantum gravity effects in inflation. We get an upper bond estimate of

$$V_4\big|_{Threshold-volume-for-quantum-effects} \sim 10^{-4} cm \qquad (13)$$

This is **way too large**, but it indicates that the interaction of material within the region of space being considered does not obey $\rho \cdot V_4 >> \frac{1}{2} \cdot \sum_{i \neq j} U(x_i, x_j)$. If this is what we have, we can then begin to look at if the instanton picture is true or not. We will first review what can be said about different variants of vacuum energy. I.e where the vacuum energy models of four and five dimensions could conceivably overlap. But to do this we will look at what these models are.

Comparing different models for how one can input thermal-radiation energy.

Begin first with looking at different value of the cosmological vacuum energy parameters, in four and five dimensions[9].

$$\left|\Lambda_{5-\dim}\right| \approx c_1 \cdot \left(1/T^{\alpha}\right) \qquad (14)$$

in contrast with the more traditional four dimensional version of the same, minus the minus sign of the brane world theory version The five dimensional version is actually connected with Brane theory, and higher dimensions, whereas the four dimensional is linked to more traditional De Sitter space time geometry, as given by Park[10] (2003)

$$\Lambda_{4-\dim} \approx c_2 \cdot T^{\beta} \tag{15}$$

This is such that If one looks at the range of allowed upper bounds of the cosmological constant, we have that the difference between what Barvinsky[11] (2006) recently predicted, and Park (2003)

$$\Lambda_{4-\dim} \propto c_2 \cdot T \xrightarrow[graviton-production]{} 360 \cdot m_P^2 << c_2 \cdot \left[T \approx 10^{32} K \right] \tag{16}$$

Needless to say, right after the gravitons are released one still is seeing a drop off of temperature contributions to the cosmological constant .Then we can write, for small time values $t \approx \delta^1 \cdot t_P$, $0 < \delta^1 \leq 1$ and for temperatures sharply lower than $T \approx 10^{12}\, Kelvin$, Beckwith[9] (2007), where for a positive integer n

$$\frac{\Lambda_{4-\dim}}{\left|\Lambda_{5-\dim}\right|} - 1 \approx \frac{1}{n} \tag{17}$$

If we have an order of magnitude equivalence between such representations, we can talk about a quantum regime of gravity which is consistent with regards to fluctuations in energy and also in the growth of entropy. We will use an order of magnitude estimate as to presenting what the vacuum energy should be in the neighborhood of Planck time in the advent of nucleation of a new universe

First principles argument as to large scale values of the absolute magnitude of the cosmological vacuum energy

Look at an argument provided by Thanu Padmanabhan[12], leading to the observed cosmological constant value suggested by Park. Assume that $l_P \sim 10^{-33} cm \xrightarrow[Quantum-Gravity-threshold]{} \tilde{N}^{\alpha} \cdot l_P$, but that when we make this substitution that $1 \leq \tilde{N}^{\alpha} \leq 10^2$ [13]

$$\begin{aligned} \rho_{VAC} &\sim \frac{\Lambda_{observed}}{8\pi G} \sim \sqrt{\rho_{UV} \cdot \rho_{IR}} \\ &\sim \sqrt{l_{Planck}^{-4} \cdot l_H^{-4}} \sim l_{Planck}^{-2} \cdot H_{observed}^{2} \end{aligned} \tag{18}$$

$$\Delta\rho \approx \text{ a dark energy density } \sim H_{observed}^{2} \big/ G \tag{19}$$

We can replace $\Lambda_{observed}, H^2_{observed}$ by $\Lambda_{initial}, H^2_{initial}$. In addition we may look at inputs from the initial value of the Hubble parameter to get the necessary e folding needed for inflation, according to

$$\begin{aligned} &E - foldings = H_{initial} \cdot \left(t_{End\,of\,\inf} - t_{beginning\quad of\,\inf} \right) \equiv N \geq 100 \\ &\Rightarrow H_{initial} \geq 10^{39} - 10^{43} \end{aligned} \tag{20}$$

Leading to

$$a(End - of - \inf)/a(Beginning - of - \inf) \equiv \exp(N) \tag{21}$$

If we set $\Lambda_{initial} \sim c_1 \cdot \left[T \sim 10^{32}\, Kelvin \right]$ implying a very large initial cosmological constant value, we get in line with what Park suggested for times much less than the Planck interval of time at the instant of nucleation of a vacuum state

$$\Lambda_{initial} \sim \left[10^{156} \right] \cdot 8\pi G \approx huge \quad number \tag{22}$$

.Question. Do we always have this value of Eqn. (22)? At the onset of Inflation? When we are not that far away from a volume of space characterized by l_P^3, or at most 100 or so times larger ? Contemporary big bang theories imply this. I.e. a very high level of thermal energy. We need to ask if this is something which could be transferred from a prior universe , i.e. could there be a pop up nucleation effect , i.e. emergent space time? Appendix 1 gives a way for this to occur. We will now examine a mechanism which would allow for this to happen. It involves transfer of energy from a prior to the present universe.

Worm hole transition from a prior to the present universe.

To model this, we use results from Crowell [14] (2005) on quantum fluctuations in space time which gives a model from a pseudo time component version of the Wheeler De Witt equation, with use of the Reinssner-Nordstrom metric to help us obtain a solution which passes through a thin shell separating two space times. The radius of the shell, $r_0(t)$ separating the two space times is of length l_P in approximate magnitude, leading to a domination of the time component for the Reissner – Nordstrom metric

$$dS^2 = -F(r) \cdot dt^2 + \frac{dr^2}{F(r)} + d\Omega^2 \tag{23}$$

This has:

$$F(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} \cdot r^2 \xrightarrow[T \to 10^{32}\, Kelvin \sim \infty]{} - \frac{\Lambda}{3} \cdot \left(r = l_P \right)^2 \tag{24}$$

This assume that the cosmological vacuum energy parameter has a temperature dependence as outlined by Park (2003) leading to if

$$\frac{\partial F}{\partial r} \sim -2 \cdot \frac{\Lambda}{3} \cdot \left(r \approx l_P\right) \equiv \eta(T) \cdot \left(r \approx l_P\right) \tag{25}$$

As a wave functional solution to a Wheeler De Witt equation bridging two space times. This solution bridging two space times is similar to that being made between these two space times with 'instantaneous' transfer of thermal heat ,as given by Crowell[14] (2005)

$$\Psi(T) \propto -A \cdot \left\{\eta^2 \cdot C_1\right\} + A \cdot \eta \cdot \omega^2 \cdot C_2 \tag{26}$$

This has $C_1 = C_1(\omega, t, r)$ as a pseudo cyclic and evolving function in terms of frequency, time, and spatial function, with the same thing describable about $C_2 = C_2(\omega, t, r)$ See **Appendix I** below as to what the entries in Eqn. (26) mean above.

Conclusion. Match up with Smoot's table

In a colloquium presentation done by Dr. Smoot in Paris [15] (2007); he alluded to the following information theory constructions which bear consideration as to how much is transferred between a prior to the present universe in terms of information 'bits'.

0) Physically observable bits of information possibly in present Universe - 10^{180}
1) Holographic principle allowed states in the evolution / development of the Universe - 10^{120}
2) Initially available states given to us to work with at the onset of the inflationary era- 10^{10}
3) Observable bits of information present due to quantum / statistical fluctuations -10^{8}

Our guess is as follows. That the thermal flux so implied by the existence of a worm hole accounts for perhaps 10^{10} bits of information. These could be transferred via a worm hole solution from a prior universe to our present , and that there could be , perhaps 10^{120} minus 10^{10} bytes of information temporarily suppressed during the initial bozonification phase of matter right at the onset of the big bang itself .

'Then after the degrees of freedom dramatically drops during the beginning of the descent of temperature from about $T \approx 10^{32}\, Kelvin$ to at least three orders of magnitude less, as we move out from an initial red shift

$$z \approx 10^{25}$$

to[16]

$$T \approx \sqrt{\varepsilon_V} \times 10^{28}\, Kelvin \sim T_{Hawkings} \cong \frac{\hbar \cdot H_{initial}}{2\pi \cdot k_B} \tag{75}$$

Whichever model we can come up with that does this is the one we need to follow, experimentally. And it gives us hope in confirming if or not we can eventually analyze the growth of structure in the initial phases of quantum nucleation of emergent space time[17] . We also need to consider the datum so referenced as to the irregularities as to the cooling down phase of inflation, as mentioned by Sakar,[18] which is below

"*Quasi-DeSitter spacetime during inflation has no "lumpiness" - it is necessarily very smooth. Nevertheless one can generate structure in the spectrum of quantum fluctuations originating from inflation by disturbing the slow-roll of the inflaton - in our model this happens because other fields to which the inflaton couples through gravity undergo symmetry breaking phase transitions as the universe cools during inflation*"

We intend to model this better, and to come up with experiments better than what Ruutu et al [17]. came up for early universe structure model paradigms.

Appendix I. Details as to forming Crowell's time dependent Wheeler De Witt equation, and its links to Worm holes

We will fill in the details inherent in Eqn. (26) above in the main text.. This will be to show some things about the worm hole we assert the instanton traverses en route to our present universe. Eqn (26) of the main text actually comes from the following version of the Wheeler De Witt equation with a pseudo time component added. From Crowell [14]

$$-\frac{1}{\eta r}\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{\eta r^2}\cdot\frac{\partial\Psi}{\partial r}+rR^{(3)}\Psi=\left(r\eta\phi-r\ddot{\phi}\right)\cdot\Psi \qquad (1)$$

This has when we do it $\phi \approx \cos(\omega\cdot t)$, and frequently $R^{(3)} \approx$ constant, so then we can consider

$$\phi \cong \int_0^\infty d\omega\left[a(\omega)\cdot e^{ik_\varpi x^\mu}-a^+(\omega)\cdot e^{-ik_\varpi x^\mu}\right] \qquad (2)$$

In order to do this, we can write out the following with regards to the solutions to Eqn (1) put up above.

$$\begin{aligned} C_1 &= \eta^2\cdot\left(4\cdot\sqrt{\pi}\cdot\frac{t}{2\omega^5}\cdot J_1(\omega\cdot r)+\frac{4}{\omega^5}\cdot\sin(\omega\cdot r)+(\omega\cdot r)\cdot\cos(\omega\cdot r)\right) \\ &+\frac{15}{\omega^5}\cos(\omega\cdot r)-\frac{6}{\omega^5}Si(\omega\cdot r) \end{aligned} \qquad (3)$$

And

$$C_2=\frac{3}{2\cdot\omega^4}\cdot\left(1-\cos(\omega\cdot r)\right)-4e^{-\omega\cdot r}+\frac{6}{\omega^4}\cdot Ci(\omega\cdot r) \qquad (4)$$

This is where $Si(\omega\cdot r)$ and $Ci(\omega\cdot r)$ refer to integrals of the form $\int_{-\infty}^{x}\frac{\sin(x')}{x'}dx'$ and $\int_{-\infty}^{x}\frac{\cos(x')}{x'}dx'$. It so happens that this is for forming the wave functional permitting an instanton forming , while we next should consider if or not the instanton so farmed is stable under evolution of space time leading up to inflation. We argue here that we are forming an instanton whose thermal energy is focused into a wave functional which is in the throat of the worm hole up to a thermal discontinuity barrier at the onset , and beginning of the inflationary era.

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