On the Resolution *of* the Azimuthally Symmetric Theory *of* Gravitation's λ-Parameters

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Abstract

In Newtonian gravitational physics, as currently understood, the spin of a gravitating body has no effect on the nature of the gravitational field emergent from this gravitating body. This position has been questioned by the Azimuthally Symmetric Theory of Gravitation (ASTG-model; in Nyambuya 2010). From the ASTG-model – which is a theory resulting from the consideration of the azimuthally symmetric solutions of the well known and well accepted Poisson-Laplace equation for gravitation, it has been argued that it is possible to explain the unexpected perihelion shift of Solar planetary orbits. However, as it stands in the present, the ASTG-model suffers from the apparent diabolic defect that there are unknown parameters (λ's) in the theory that up to now have not been able to be adequately deduced from theory. If this defect is not taken care of, it would consume the theory altogether, bringing it to a complete standstill, to nothing but an obsolete theory. Effort in resolving this defect has been made in the genesis reading of the theory *i.e.* in Nyambuya (2010*a*). This initial effort in trying to resolve this problem is not complete. In this short reading, we present what we believe is a significant improvement to the resolution of this problem. If this effort proves itself correct, then the ASTG-model is set on a sure pedal to make predictions without having to relay on observations to deduce these unknown parameters. Other than resolving the λ-parameter problem, this reading is designed to serve as an exposition of the ASTG-model as it currently stands.

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1 Introduction

It is bona-fide scientific consensus that Einstein's General Theory *of* Relativity (GTR), completed and published in 1915, is today regarded by the majority of researchers as the unquestionable doctrine and standard paradigm of gravitation. Further, it is a widely held view that Einstein's theory has so far passed all the "crucial" experimental tests to which it has been subjected to. Strangly, this vigorously defended, dominant and prevalent view is held in full view of the fact that the gravitational waves predicted by it [*i.e.* the GTR] at its inspection, have up to now, not been positively and directly detected (see *e.g.* Will 2006). "Massive" financial investment has gone into the search for gravitational waves and much against the desideratum, this investment has so far not brought any significant results. Furthermore, despite the impressive array of experimental success, there is an uncomfortable side of gravitation that most of the unquestioning GTR-advocates seem to pay little attention to. There exists puzzling gravitational anomalies in the Solar system such as the secular increase in the Earth-Sun (Krasinsky & Brumberg 2004, Standish 2005), the Earth-Moon distance (Williams & Boggs 2009, Williams & Dickey 2003, Williams et al. 2001), the Pioneer Anomaly (Anderson et al. 1998, Turyshev & Toth 2010) and the Earth Flyby Anomalies (see *e.g.* Anderson et al. 2008, and references therein).

Much to the surprise of the assuming researcher, these anomalies have forcefully surfaced, and it is prudent to say that they seem to be beyond the abilities of Einstein's GTR. That is, in its bare and unmodified form, Einsteins GTR strongly appears unable to offer an explanation to these ponderous anomalous observations. From these vexing observations, it would appear that the time for amendments or replacement of Einsteins GTR is something whose time has perhaps arrived or is imminent. The just stated opinion, is nothing but our limited and narrow view born out of the pine and need for the advancement of our knowledge horizons on the frontiers of gravitational physics.

Far from the shores and provinces of mundane astronomy, we learn that when the pre-eminent British physicist, Paul A. M. Dirac (1902 – 1984), formulated his theory (Dirac 1928*a*,8), a theory which predicted the existence of antimatter, and, a theory which he called the theory of the Electron; its immediate success and crowning achievement was its natural explanation of the existence of quantum spin and as-well (and more so), the gyromagnetic ratio of the Electron g_e. Despite its great success, the (then and only available) theory of Schrödinger could not explain the existence of spin and the gyromagnetic ratio of the Electron. Dirac's then new theory did explain these problems but new ones arose.

Classical theory coupled with Schrödinger's theory would only give the value $g_e = 1$, and experiment would give a value that is about twice this. Dirac's theory readily explained the experimental value of the Electron gyromagnetic ratio of $g_e = 2$. However, while Dirac's theory gave a value of exactly $g_e = 2$, the exact experimental value found is $g_e = 2.0011596521811 \pm 0.00000000000075$ [see any good book on Quantum Electrodynamics (QED)]. What this meant is that Dirac's theory had an anomalous gyromagnetic ratio of, $g_e - 2 = 0.0011596521811 \pm 0.00000000000075$, which it could not account for. This paltry sum is very easy to dismiss with the simple remark that it is marginally insignificant and perhaps an experimental error.

Interestingly, the agile and formidable quantum theorist of the time was not at all happy with this seemingly paltry anomalous gyromagnetic ratio of the Electron; they wanted it explained to "the dot" and to the "bone-marrow". The ingenious effort to explain this to "the dot" and to the "bone-marrow" led to great advances in quantum physics *i.e.*, this lead to the historic construction of QED, which is widely held and admired by the esoteric as the most accurate theory ever invented by the human-mind.

Perhaps, like the great quantum theorists that pioneered QED, the GTR theorists must take a leaf and follow in the footsteps of these great pioneers and demand that the GTR must explain all gravitational phenomenon to "the dot" and to the "bone-marrow" so that any deviation and or anomaly must be taken very seriously, much on the same if not better level than that which led to the ingenious formulation of QED and the great advances thereof.

This invariably means that all anomalous Solar astrometries must be taken very seriously as "embarrassing" holes in the beautiful fabric of the GTR. Actually, to be frank and factual, the experimental success of the GTR is in the regime of weak gravitational fields (see *e.g.* Will 2006). This is the same regime in which anomalous Solar astrometries are observed. That said; in search of furthering its universal applicability by expanding on the regime on which the GTR has been shown to be valid, the current quest is to test it in the regime of strong gravitation (see *e.g.* Will 2006). In the strong gravity regime, interesting to note is that, every test of the GTR on this level is either a potentially deadly test that would bring its "authority" to question or further cement it, or perhaps, it is a possible probe for new physics. This is because the predictions of the GTR are fixed. Other than the controversial and delicate cosmological constant, the theory contains little room for adjustable constants, so not much can be changed.

Further, if the GTR is the correct description of *Nature*, it is expected to give excellent results in the strong gravity regime because this is the regime which it is designed to deal with. If the GTR passes the future tests in the strong gravity regime, then, despite the holes due to the anomalous Solar astrometries, it would be very difficult to think of an alternative to it, therefore, it (possibly) would have to be patched so that it successfully stands in the face of the seemingly inexplicable new data on the anomalous Solar astrometries.

Ironically, while we may seek to test the GTR in this regime of strong gravitation, from the foregoing, it is not entirely correct to say it has successfully passed the weak gravity regime unless off cause the origins of the measured anomalous Solar astrometries (Pioneer and Flyby anomalies; the secular increase of the Earth-Sun and Earth-Moon distance) are not gravitational in nature. It appears unlikely that these anomalous Solar astrometries may in the end turnout not to be gravitational in nature.

Furthermore, we ask; is there a way to explain these anomalous Solar astrometries? Though more work is under-way, we have hinted that the Azimuthally Symmetric Theory of Gravitation (hereafter ASTG-model) is in-principle capable of explaining the Flyby anomalies (Nyambuya 2010*c*, hereafter Paper IV) and the secular increase of the Earth-Sun and Earth-Moon distance (Nyambuya 2010*a*, hereafter Paper I). The ASTG-model is nothing more than the azimuthally symmetric Poisson-Laplace theory applied to gravitation. Given that the GTR is designed such that in the spherically symmetric case of the weak field approximation, the theory must reduce to the well known spherically symmetric Poisson-Laplace theory, it would appear logical to assume that the ASTG-model is a subset of the GTR. This gravitational Law of Einstein (with the latter modification of the cosmological constant $\Lambda g_{\mu\nu}$ term, where Λ is the controversial cosmological constant and $g_{\mu\nu}$ is the metric tensor of spacetime) is:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu} + \Lambda g_{\mu\nu}, \tag{1}$$

where $\kappa = 8\pi G/c^4$ is Einstein's constant of gravitation, $G = 6.667 \times 10^{-11}\,\mathrm{kg^{-1}m^3s^{-2}}$ is Newton's universal constant of gravitation, $c = 2.99792458 \times 10^8\,\mathrm{ms^{-1}}$ is the speed of light in vacuum, $T_{\mu\nu}$ is the stress and energy tensor, $R_{\mu\nu}$ is the Ricci tensor, R the Ricci scalar. As afore-stated, Einstein was able to show that in the spherically symmetric case of weak gravitational fields, his equation does reduce to the wave equation:

$$\boldsymbol{\nabla}^2 g_{\mu\nu}(r) - \frac{1}{c^2}\frac{\partial g_{\mu\nu}(r)}{\partial t^2} = \kappa T_{\mu\nu}, \tag{2}$$

(see *e.g.* Kenyon 1990, or any good book on the GTR). It is from this equation (2) that gravitational waves where predicted; they [gravitational waves] are expected to propagate in empty space where $T_{\mu\nu} = 0$. When Einstein discovered his equation, the constant was unknown. In order to deduce this constant, he demanded of his equation that it must reduce to Newtonian gravitation, the invariable meaning of which is that to first order approximation, this theory must agree with the Newtonian phenomenology. For this to be so, it meant that the above equation must reduce to Poisson's equation. Inspection of its 10 unique components, meant the 00-component which is associated with mass-energy density distribution, is what must reduce to the Poisson equation.

Taking a spherically symmetric metric, as deduced from the Schwarzchild metric (and latter justified by gravitational red-shift experiments), it is found that: $g_{00}(r) = 1 + 2\Phi(r)/c^2$; Einstein inserted this into the above equation and made his posteriori justified simplifying assumptions that led him to: $\boldsymbol{\nabla}^2\Phi(r) - \ddot{\Phi}/c^2 = \kappa\rho(r)c^4/2$. He [Einstein] assumed that the time variation of φ was in the Newtonian limit (which is the regime of low energy and spacetime curvature), so small that one would neglect the term $\ddot{\Phi}$ entirely. This meant that the resultant equation or emergent theory from his reduction thesis is given by: $\boldsymbol{\nabla}^2\Phi(r) = \kappa\rho(r)c^4/2$. By comparing this with the spherically symmetric Poisson-Laplace equation:

$$\boldsymbol{\nabla}^2\Phi(r) = 4\pi G\rho(r), \tag{3}$$

he [Einstein] was able to deduce that $\kappa = 8\pi G/c^4$. In (3), $\rho(r)$ is the density of matter encased in sphere of radius r and the operator $\boldsymbol{\nabla}^2$ written for a spherical coordinate system [see figure (1) for the coordinate set-up] is given by:

$$\boldsymbol{\nabla}^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin\theta}\frac{\partial^2}{\partial\varphi^2}, \tag{4}$$

where the symbols have their usual meanings. From the above presentation, it would appear as though the ASTG-model is nothing but a subset of Einstein's GTR in the weak field limit.We are going to argue in §(2) that this is not true, the ASTG-model is a fully-fledged theory whose results are different from Einstein GTR for a rotating gravitating mass.

2 Kerr Metric

Like the Schwarzchild metric, the Kerr metric is an exact vacuum solution of Einstein's field equation (1), it is a generalization of the Schwarzschild metric in the case of a rotating massive gravitating body. It was discovered in 1963 by the New Zealander Roy Kerr. This metric describes the geometry of spacetime around a rotating

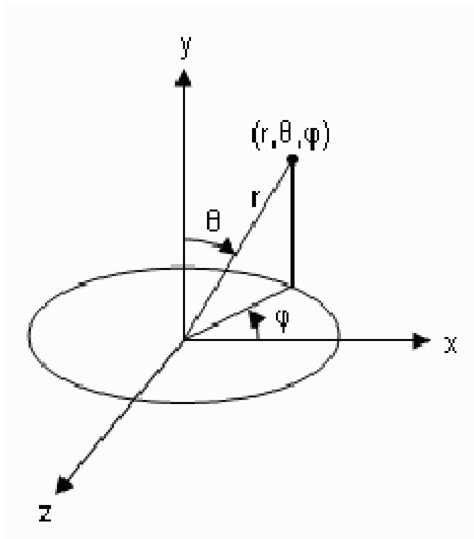

Figure (1): *This figure shows a generic spherical coordinate system, with the radial coordinate denoted by r, the zenith (the angle from the North Pole; the colatitude) denoted by θ, and the azimuth (the angle in the equatorial plane; the longitude) by φ.*

massive body. It is often used to describe a rotating black hole. Such a black hole has a different surface to that of the Schwarzchild black hole because there exists two closed paraboloid surfaces where the metric exhibits the singular condition; the size and shape of these surfaces depends on the black hole's mass and angular momentum. This same metric can also be used to study slow spinning massive bodies such as the Sun and the stars.

In this section, we are going to use the Kerr metric to study slow spinning massive bodies, the aim of which is to show that this metric does not give the same results as that obtained from the ASTG-model were one is able to infer the existence of outflows as a gravitational phenomenon that is inherently and intrinsically tied to the spin of the star. If the just said is true, then the ASTG-model can not be a subset of the GTR, but a fully fledged theory in its own right.

To demonstrate our assertion that the Kerr metric and the ASTG-model give different results for a spinning gravitating star, we have to go to the Kerr metric itself. We direct the reader unfamiliar with the Kerr metric to any good book on the GTR were a treatment of the Kerr metric is conducted. What is important for our purposes in this metric is the 00-component of the Kerr metric. This 00-component is given by:

$$g_{00} = 1 - \frac{2G\mathcal{M}_{\rm star}}{rc^2}\left(1 + \frac{\omega_{\rm star}^2 \mathcal{R}_{\rm star}^4 \cos^2\theta}{rc^2}\right)^{-1}, \qquad (5)$$

where $\mathcal{M}_{\rm star}$ is the mass of the spinning gravitating star, $\mathcal{R}_{\rm star}$ is its radius and $\omega_{\rm star}$ is its spin angular frequency.

For our purposes, it suffices to take on the 00-component of the Kerr metric and the reason for this is that we shall assume the weak field case were the gravitating bodies are not nearing their Schwarzchild limit, and, at the same time, these objects are not fast spinning objects nearing their critical spin state were the equatorial centrifugal forces are much stronger than the gravitational force so that the star beings to be torn apart by these centrifugal forces. In this case, the 00-component of the Kerr metric is a good approximation of the gravitational potential around a spinning gravitating massive body. Clearly and obviously, from this metric 00-component, the azimuthally symmetric gravitational potential is given by:

$$\Phi(r,\theta) = -\frac{2G\mathcal{M}_{\rm star}}{rc^2}\left(1+\frac{\omega^2\mathcal{R}_{\rm star}^4\cos^2\theta}{rc^2}\right)^{-1}, \tag{6}$$

Now, using the fact that the radial component of the gravitational force is given by $F_r(r,\theta) = -d\Phi(r,\theta)/dr$, it follows that:

$$F_r(r,\theta) = -\frac{G\mathcal{M}_{\rm star}}{r^2}\left(1+\frac{\omega^2\mathcal{R}_{\rm star}^4\cos^2\theta}{rc^2}\right)^{-1}, \tag{7}$$

and now, as in the case of the ASTG-model, we shall ask; when is $F_r(r,\theta) > 0$? that is, "*where in the gravitational field of the spinning gravitating mass, is the radial component of the gravitational force repulsive?*". The answer is simple; this will occur when the term in the square bracket in equation (7) is negative, and this is when:

$$r < \left(\frac{\omega\mathcal{R}_{\rm star}^2}{c}\right)\cos\theta = \alpha\mathcal{R}_{\rm star}\cos\theta \qquad \text{where} \quad \alpha = \frac{\omega\mathcal{R}_{\rm star}^2}{c}. \tag{8}$$

Because αc is a measure of the speed of the particles on the surface of the rotating star $\alpha < 1$ and, because of this, what the above invariably means is that the repulsive gravitational field can never protrude outside of the surface of the spinning star as is the case in the ASTG-model. Obviously, this means that for a rotating star in general relativity, repulsive polar gravitational fields are not predicted as is the case in the ASTG-model. Therefore, it follows that the ASTG-model and the GTR do give different results for spinning gravitating masses, hence the ASTG-model can not be viewed as a subset of the GTR. For the interested reader that may have the time to go through the reading (Nyambuya 2010*d*, hereafter Paper (V)), they will realise that in this reading [Paper (V)], gravity is cast not as a metric theory as is the case in Einstein's GTR, but as a scaler theory *i.e.*:

$$\boldsymbol{\nabla}^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 4\pi G\rho, \tag{9}$$

where Φ is the scaler gravitational potential. The metric of spacetime ($g_{\mu\nu}$) in Paper (V) describes not the gravitational field, but the electromagnetic, the weak and the strong nuclear forces. In the light of the ideas presented in Paper (V), the ASTG-model can be viewed not as a subset of the GTR but as a fully-fledged theory because the ASTG-model will emerge from (9) as the static solution for the azimuthally symmetric case. Whether or not the reader does or does not concur with the ideas presented in Paper (V), this does not affect the ASTG-model because this model can be viewed as a pure Poisson-Laplace theory. We should make this clear to the reader, that the ASTG-model is being developed from the vantage point of the ideas presented in Paper (V).

3 ASTG-model in a Nutshell

In the present section, we give an overview of the ASTG-model. We shall start from Newtons Law of universal gravitation, which can be written in a more general and condensed form as Poisson's Law *i.e.*, $\nabla^2\Phi = 4\pi G\rho$. For a spherically symmetric setting where $\Phi = \Phi(r)$, the solution to Poisson's equation outside the vacuum space (where $\rho = 0$) of a central gravitating body of mass $\mathcal{M}$ is given by the traditional inverse distance Newtonian gravitational potential which is given by:

$$\Phi(r) = -\frac{G\mathcal{M}_{\rm star}}{r}, \qquad (10)$$

where r is the radial distance from the center of the gravitating body. The Poisson equation for the case ($\rho = 0$) is known as the Laplace equation. The Poisson equation is an extension of the Laplace equation. Because of this, we have resorted to calling the Poisson equation as the Poisson-Laplace equation. In the case where there is material surrounding this central mass, we must take this into account by considering all the mass encased in the sphere of radius r *i.e.*, $\mathcal{M} = \mathcal{M}(r)$. In more general terms:

$$\mathcal{M}(r) = \int_0^r \int_0^{2\pi} \int_0^{2\pi} r^2 \rho(r,\theta,\varphi) d\theta d\varphi dr. \qquad (11)$$

To take the circumstellar mass into account, we must, in (10), make the replacement: $\mathcal{M}_{\rm star} \longrightarrow \mathcal{M}(r)$. As already argued in Paper I, if the gravitating body in question is spinning, we ought to consider an Azimuthally Symmetric Gravitational Field (ASGF). For this ASGF, we need to take into account the distribution of matter for the non-empty space scenario. We did solve (3) in Nyambuya (2010*b*) [hereafter Paper (III)] the azimuthally symmetric setting for both cases of empty and non-empty space. We shall present these in the subsequent subsections.

3.1 Empty Space Solutions

The Poisson-Laplace equation for empty space has been "solved" for a spinning gravitating system and the solution to it is:

$$\Phi(r,\theta;\lambda) = -c^2 \sum_{\ell=0}^{\infty} \lambda_\ell \left(\frac{G\mathcal{M}_{star}}{rc^2}\right)^{\ell+1} P_\ell(\cos\theta), \qquad (12)$$

where $\lambda_\ell : \ell = \{0, 1, 2, ..., \infty\}$, is an infinite set of dimensionless parameters with $\lambda_0 = 1$ and the rest of the parameters (*i.e.* for which $\ell > 1$) generally take values different from unity. Notice that unlike in Papers (I) and (II), we have not written the gravitational potential as $\Phi(r,\theta)$ but as $\Phi(r,\theta;\lambda)$ – why? This is to express the fact that the λ's are not mere constants but significant and important parameters of the theory: *i.e.* $\lambda = \lambda(\mathcal{R}, \omega)$. In the normal Poisson-Laplace theory the λ's are pure constants, they are un-changing. In the ASTG-model, the λ's are not constants but dynamic variables dependent on the angular frequency and the radius of the rotating gravitating body.

This is a significant difference which sets apart the ASTG-model from pure Poisson-Laplace theory.

Also, this fact the λ's are in the ASTG-model dynamic variables dependent on the angular frequency and the radius of the rotating gravitating body, set the ASTG-model apart from any other known theory of gravitation. At the inspection of the ASTG-model, we were prudent so as to distance ourself from the thinking that the ASTG-model is a new theory of gravitation. With the new light gained that the λ's are significant and very important dynamic variables, this view has changed: the ASTG-model is indeed a new theory of gravitation and must be viewed as such.

3.2 Non-Empty Space Solutions

We have argued in Paper (III) that the Poisson-Laplace equation for non-empty space for a spinning gravitating system is:

$$\Phi(r,\theta;\lambda) = -c^2 \sum_{\ell=0}^{\infty} \lambda_\ell \left(\frac{G\mathcal{M}(r)}{rc^2}\right)^{\ell+1} P_\ell(\cos\theta). \tag{13}$$

To arrive at the non-empty space solution, one simply has to replace $\mathcal{M}(r)$ in (12) with $\mathcal{M}(r)$. It is a straight forward replacement but comes along with the constraint that the mass distribution must be dependent on the gravitational potential by the relation:

$$\rho(r,\theta;\lambda) = -\frac{1}{4\pi G}\left(\frac{2}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\right)\frac{\partial\Phi(r,\theta;\lambda)}{\partial r}. \tag{14}$$

From the above, clearly; it is implied that the spin of the gravitating body affects the distribution of matter around the nascent star. In a nutshell, we have presented the ASTG-model for the empty and non-empty space cases. In the subsequent section, we shall present the main task of this reading, namely, the revision of the λ-parameters. This revision we hope places the ASTG-model on a sure pedestal to become a fully fledged theory capable of making independent and testable predictions. It is important to mention that in non-empty space case, there are two scenarios to consider and these are the case where the spinning star co-rotates with the core in which it is embedded and the second is when the core and the star have different rotational angular frequencies.

4 Proposal for the λ-Parameters

In this section, first, we improve on the Solar value $\lambda_1^{\odot}$ calculated in Paper (I) and; thereafter, we proceed to make a revision of this parameter to give what we feel is a more satisfying proposal of this parameter in general. That is, not only for the Solar value but any general spinning gravitating body.

4.1 Improvement on the Solar Value $\lambda_1^{\odot}$

In Papers (I), (II) and (III), it has already been over emphasised that one of the draw backs of the ASTG-model is that it is heavily dependent on observations for the values of λ have to be determined from observations.Without knowledge of the λ's, one is unable to produce the hard numbers required to make any numerical quantifications. Clearly, a theory incapable of making any numerical quantifications is, useless; to be more polite, it is of no use since true science is about measurements and not ideas that are beyond the realm of measurement. In an effort to solve this problem, in Paper (I), it has been proposed that:

$$\lambda_\ell = \left(\frac{(-1)^{\ell+1}}{(\ell^\ell)!(\ell^\ell)}\right)\lambda_1, \tag{15}$$

and using this proposal, it was found that for the Sun $\lambda_1^\odot = 24.00 \pm 7.00$. Our first task on the λ-parameters is to improve on this value. This value was determined by means of a not so robust mathematical calculation in Paper (I). It is possible to use graphical means. Using a graph is more powerful and gives one a good feeling especially if one were to obtain a well behaved graph indicating a clear correlation. To begin on this task, we direct the reader to equation (47) in Paper (I), that is:

$$P_p = A_p\lambda_1^\odot + B_p\lambda_2^\odot, \tag{16}$$

where the symbols are defined there-in Paper (I). Now, from (15), it follows that $\lambda_1^\odot = \lambda_2^\odot/96$ and substituting this into the above, one is led to:

$$P_p = (A_p - B_p/96)\lambda_1^\odot, \tag{17}$$

Setting $X_p = (A_p - B_p/96)$, implies $P_p = \lambda_1^\odot X_p$ and since P_p and X_p are known and $\lambda_1^\odot$ is unknown, a plot of P_p vs X_p should produce a straight line whose slope is $\lambda_1^\odot$. The values A_p, B_p, P_p and X_p are tabled in Table (I) and the corresponding graph is plotted in figure (2). From the graph, we get:

$$\lambda_1^\odot = 21.00 \pm 4.00. \tag{18}$$

Obviously, by inspection of the graph in figure (2), it is clear that this value $\lambda_1^\odot = 21.00 \pm 4.00$ is more accurate than our earlier value $\lambda_1^\odot = 24.00 \pm 7.00$, hence we now adopt the former *i.e.* $\lambda_1^\odot = 21.00 \pm 4.00$.

That we are able to obtain a very good linear graph as shown in figure (2), this points to the fact that our choice of the λ's is a good one. More than just pointing to a good choice, it points to the ASTG-model containing in it, a grain of truth. This exercise, to demonstrate that our choice of the λ-parameter, is a good one, is one of the main aims of the present reading. We believe that demonstrating this fact using a well behaved graph as that in figure (2) gives impetus and credence to our choice of the λ-parameters and more so, to the ASTG-model.

4.2 Improvement on the Choice of λ_1

Contrary to conventional wisdom, in Paper (III), it has been argued that the ASTG-model is able to explain outflows as a gravitational phenomenon. Pertaining to their association with star formation activity, it is believed that molecular outflows are a necessary part of the star formation process because their existence may explain the apparent angular momentum imbalance. It is well known that the amount of

Table I: *Column (1) gives the planet while columns (3 to 5) give values of A_p, B_p, P_p and X_p for the corresponding planets respectively. The values of A and B are adapted from table (I) in Paper (I).*

Planet	A	B	P	X
Mercury	3.50×10^{0}	1.72×10^{2}	43.1000 ± 0.5000	1.71×10^{0}
Venus	5.19×10^{-1}	2.88×10^{1}	8.0000 ± 5.0000	4.89×10^{-1}
Earth	1.57×10^{-1}	3.80×10^{-1}	5.0000 ± 1.0000	1.53×10^{-1}
Mars	7.02×10^{-2}	2.43×10^{-2}	1.3624 ± 0.0005	7.00×10^{-2}
Jupiter	3.02×10^{-3}	1.00×10^{-5}	0.0700 ± 0.0040	3.32×10^{-3}
Saturn	7.59×10^{-4}	1.72×10^{-7}	0.0140 ± 0.0020	7.93×10^{-4}

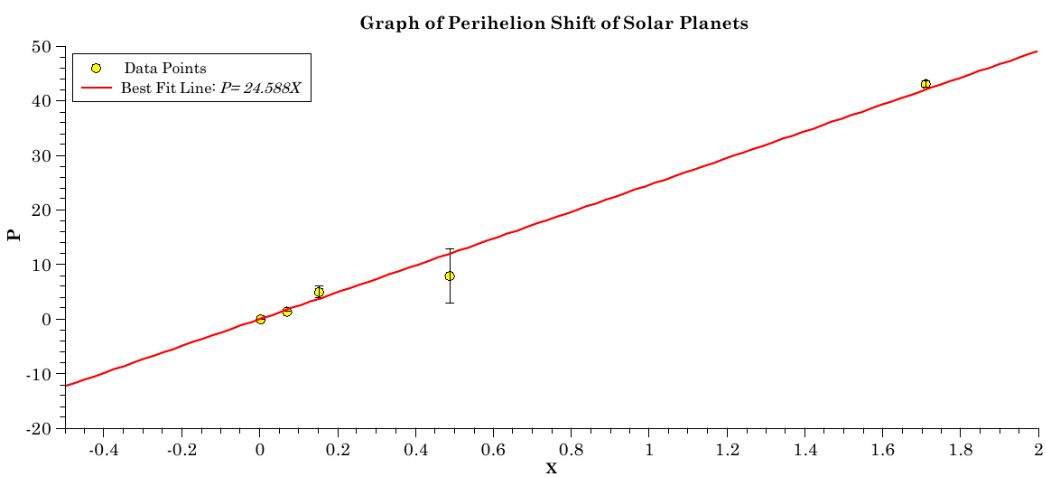

Figure (2): *A plot of the perihelion shift data points of Solar planetary orbits. This graph is (in our opinion and view) the most convincing piece of evidence that the choice of the λ's in Nyambuya* (2010a), *may very well be a correct.*

initial angular momentum in a typical star-forming cloud core is several orders of magnitude too large to account for the observed angular momentum found in formed or forming stars (see *e.g.* Larson 2003*a*,3). The sacrosanct Law of Conservation of angular momentum informs us that this angular momentum can not just disappear into the oblivion of interstellar spacetime. So, the question is where does this angular momentum go to? It is here that outflows are thought to come to the rescue as they can act as a possible agent that carries away the excess angular momentum.

Additionally and perhaps more importantly, this angular momentum, if it where to remain as part of the nascent star, it would, *via* the strong centrifugal forces[1], tear the star apart. This however does not explain, why they [outflows] exist and how they come to exist but simple posits them as a vehicle needed to explain the mystery of "The Missing Angular Momentum Problem" in star forming systems and the existence of stars in their intact and compact form as firery balls of gas. At the equator of the spinning star, once the gravitational acceleration $g_{\text{star}} = -G\mathcal{M}/\mathcal{R}^2$ star, becomes less than centrifugal acceleration (*i.e.* $g_{\text{star}} = a_c$). This condition will commence once the period of the spin angular momentum $\mathcal{P}$, of the star:

$$\mathcal{P} = \frac{3}{2}\left(\frac{\mathcal{R}^3_{\text{star}}}{G\mathcal{M}_{\text{star}}}\right)^{1/2} = \left(\frac{3\pi}{G\rho_{\text{star}}}\right)^{1/2} = \mathcal{P}_{\text{on}} \tag{19}$$

where ρ_{star} is the density of the spinning star. In Paper (III), we drew from the tacit thesis that *"... outflows possibly save the star from the detrimental centrifugal forces ..."*, the suggestion that $\lambda_1 = \zeta(a_c/g_{\text{star}})^{\zeta_0}$ where ζ and ζ_0 where assumed to be pure constants. In this way, we moved a step closer to resolving the λ-parameter problem.

Now, for a co-rotating star embedded in a core of radius $\mathcal{R}_{\text{core}}$, mass $\mathcal{M}_{\text{core}}$ which a density profile $\rho(r) \propto r^{-\alpha_\rho}$ where $\alpha_\rho : (0 \leq \alpha_\rho \leq 3)$ is the density index, outflows will occur in the region described by the cone $\theta : [125.3 < \theta < 54.7]$ *&* $[234.7 < \theta < 305.3]$ and this cone has a cap described by:

$$r < \left[\lambda_1\left(\frac{2G\mathcal{M}_{\text{core}}}{c^2\mathcal{R}_{\text{core}}}\right)\mathcal{R}_{\text{core}}\right]|\cos\theta|^{\frac{1}{2-\alpha}\rho} = \lambda_1\mathcal{R}^s_{\text{core}}\,|\cos\theta|^{\frac{1}{2-\alpha}\rho} \tag{20}$$

where $\mathcal{R}^s_{\text{core}}$ is the Schwarzchild radius of the core. For a star in empty space, the region of repulsive gravitational field (for a star whose the Schwarzchild radius is $\mathcal{R}^s_{\text{star}}$) is given described by the cone $\theta : [125.3 < \theta < 54.7]$ *&* $[234.7 < \theta < 305.3]$ and this cone has a cap described by:

$$r < \lambda_1\mathcal{R}^s_{\text{star}}\cos\theta, \tag{21}$$

If outflows are there to save the nascent star from the wrath and ruthlessness of the centrifugal forces, we argued in Paper (III) that, it is logical to imagine that at the moment the centrifugal forces are about to rip the star apart, outflows will switch-on, thus shedding off this excess spin angular momentum. The centrifugal forces have their maximum toll on the equatorial surface of the star hence if the centrifugal forces are to rip the nascent star apart, this would start at the equator of the nascent star. The centrifugal force on the surface of the star acting on a particle of mass

[1]The centrifugal acceleration is given by: $a_c = \omega^2\mathcal{R}$ where $\omega = 2\pi/\mathcal{P}$ is the spin angular frequency and $\mathcal{P}$ is the period of the spin.

m is $F_c = m\omega_{\text{star}}^2 \mathcal{R}_{\text{star}} = ma_c$ and the gravitational force on the same particle is $F_g = G\mathcal{M}m/\mathcal{R}_{\text{star}}^2 = mg_{\text{star}}$. Now lets define the quotient $\mathcal{Q} = F_c/F_g = a_c/g_{\text{star}}$. If the particle where to stay put on the surface of the star, then we will have $F_c - F_g < 0) \Longrightarrow \mathcal{Q} < 1$; and if the particle where to fly off the surface, we will have $F_c - F_g > 0$) $\mathcal{Q} > 1$. The critical condition before the star begins to be torn apart is $F_c - F_g = 0 \Longrightarrow \mathcal{Q} = 1$. All the above can be summarized as:

$$\mathcal{Q}_{\text{outf}} := \begin{cases} < 1 & \Longrightarrow \quad \text{No Outflow Activity} \\ = 1 & \Longrightarrow \quad \text{Critical Condition} \\ > 1 & \Longrightarrow \quad \text{Outflow Activity} \end{cases} \tag{22}$$

We have dumbed the parameter $\mathcal{Q}_{\text{outf}}$, the Outflow Control Quotient (OCQ). The OCQ determines the necessary conditions for outflows to switch on. The OCQ can be expressed in terms of the specific spin angular momentum $\mathcal{S}_{\text{star}} = \omega_{\text{star}}\mathcal{R}_{\text{star}}^2$ and the term $\mathcal{S}_{\text{star}}^* = (G\mathcal{M}_{\text{star}}\mathcal{R}_{\text{star}})^{\frac{1}{2}}$, that is:

$$\mathcal{Q}_{\text{outf}} = \frac{\mathcal{S}_{\text{star}}}{\mathcal{S}_{\text{star}}^*}. \tag{23}$$

The term $\mathcal{S}_{\text{star}}^*$ is actually the critical spin angular momentum, that is, the spin angular momentum which is the star or gravitating object where to attain, it would switch on the polar repulsive gravitational field.

Now, let λ_1^{on} be the λ_1-value when the polar repulsive gravitational field switches on. Since $\mathcal{Q}_{\text{outf}} = 1$ at the time when the polar repulsive gravitational field switches on, it follows that $\lambda_1^{\text{on}} = \zeta$. Now, considering the non-empty space case, *i.e.* (21), if polar repulsive gravitational field should switch-on when $\mathcal{Q}_{\text{outf}} = 1$, then, at that moment of switching-on, we must have $\lambda_1^{\text{on}}\mathcal{R}_{\text{star}}^s = \zeta\mathcal{R}_{\text{star}}^s = \mathcal{R}_{\text{star}}^s$. From this, it follows that:

$$\zeta = \frac{\mathcal{R}_{\text{star}}}{\mathcal{R}_{\text{star}}^s}. \tag{24}$$

We see that in this case, ζ is not a constant as initially supposed in Paper (III). If ζ where a constant, it is not difficult for one to deduce that the polar repulsive gravitational field would not switch on in immediately when $\mathcal{Q}_{\text{outf}} = 1$. This means that the star will first have to be torn apart until the radius of that star reaches the appropriate size for outflows to switch on. However, if we demand that once $\mathcal{Q}_{\text{outf}} = 1$, the polar repulsive gravitational field should switch-on in immediately, then we have no choice but to accept that $\mathcal{Q}_{\text{outf}}$ is given as in (23). The new proposal (23) is a very important revision of the ASTG-model.

If we accept the new proposal (23), then, there remains one unknown in the parameter λ_1 and this unknown is ζ_0. From all the data that we have, it follows that:

$$\lambda_1^{\text{on}} = \left(\frac{\mathcal{P}_{\text{star}}^2}{4\pi\mathcal{R}_{\text{star}}^3/G\mathcal{M}_{\text{star}}}\right)^{\zeta_0}\left(\frac{\mathcal{R}_{\text{star}}}{2G\mathcal{M}_{\text{star}}/c^2}\right) = \left(\frac{\mathcal{P}_{\text{star}}}{\mathcal{P}_{\text{on}}}\right)^{2\zeta_0}\frac{\mathcal{R}_{\text{star}}}{\mathcal{R}_{\text{star}}^s}, \tag{25}$$

Now, using the values for the Sun $\mathcal{M}_\odot$ and $\mathcal{R}_\odot$, one finds that $\zeta_0 = 0.92$. It does not look nor feel natural to have such a un-natural index. In close proximity to this un-natural index, the most natural would be $\zeta_0 = 1$. If we are going to take $\zeta_0 = 1$, then, we are going to be forced to add a factor 12/5 to λ_1 in (24) so that the resultant expression gives the correct Solar value for λ_1, that is:

$$\lambda_1 = \zeta \left(\frac{\mathcal{S}_{\text{star}}}{\mathcal{S}^*_{\text{star}}} \right)^{2\zeta_0}, \tag{26}$$

In the foreseeable event that one is un-happy with the approach leading to (25), they can stick to (24) and use the value $\zeta_0 = 0.92$. Whatever choice one make *i.e.* (24) with $\zeta_0 = 0.92$ or (25), one thing is clear, the ASTG-model is set on a sure pedestal to make independent testable predictions. In the end, the correct expression for λ_1 will be decided by observations. For now until then, we propose the adoption of:

$$\lambda_1 = \left(\frac{\mathcal{S}}{\mathcal{S}_*} \right)^{\frac{1}{5}} \frac{\mathcal{R}_{\text{star}}}{\mathcal{R}_s} = \left(\frac{\mathcal{R}_{\text{star}}}{\mathcal{R}_s} \right) \mathcal{Q}^{\frac{1}{5}}_{\text{outf}}. \tag{27}$$

Before we close, perhaps we should take this time to answer some criticism that has and can be levelled against the ASTG-model. For example, Dr. A. J. Walsh's has expressed deep-seated worries about the ASTG-model (Private communication). He believes or holds that *"to posit that the gravitational field can have in its potential a non-radial term is an extraordinary claim"*. He goes on to say *"extraordinary claims require extraordinary proof"*. As argued herein, the contrary appears to be the case. Newton's theory is a result of considering the spherically symmetric case of the Poisson-Laplace equation. This yields an inverse square law that Sir Isaac Newton discovered some 345 years ago in the summer[2] of 1666 AD. The Poisson-Laplace equation can be solved exactly for three symmetries, *i.e.* (1) the spherically symmetric case: $\Phi = \varphi(r)$, (2) the azimuthally symmetric case: $\Phi = \varphi(r, \varphi)$, and (3) the polar symmetric case: $\Phi = \varphi(r, \varphi, \phi)$. Given that since the time Sir Isaac Newton enunciated his universal Law of Gravitation, gravity has (in most cases) been modelled as an inverse square law, it is natural and very much understandable for one to hold that "to posit that the gravitational field can have in its potential a non-radial term is an extraordinary claim". But we must take into consideration that our belief – no matter how strong; can not and can never dictate to *Nature* what is possible or impossible.

Further criticism; Dr. P. D. Smith of the University of South Africa in the Department of Mathematics (Private communication) also holds that the ASTG-model can not be, simply because this would have ground breaking implications – true that; it would carry along with it ground breaking implications – actually, the ASTG-model carries along with it ground breaking implications some which have already been demonstrated in Nyambuya (2010*b*) where a gravitational connection as been made to molecular outflows. Furthermore, Dr. Smith believes because Newton's Law of Gravitation is what has been used to send satellites to distant planets and that its has been tested over the centuries, it can not be found critically and so much wanting – true that. But we have the puzzling and unexpected flyby anomalies discovered from space missions; we have the mysterious Pioneer anomaly; we have the vexing secular increase in the Sun-Earth distance. All these but point to the very fact that

[2]As legend has it, while he (Sir Isaac Newton) was sitting under an apple tree he was struck by a falling apple. This event caused him to go into a deep mediatory-state (while in contemplative mood) where upon he pondered and excogitated deeply and from this meditation and excogitatory-state, he discovered the inverse square law.

something subtle is not right with Sir Isaac Newton's model (and as-well Einstein's GTR). It (Newton's theory) must – at the very least; be further replaced (and also Einstein's GTR must be replaced if it successfully fails to face the new data). It must be said that all these seemingly inexplicable and ponderous observations; the ASTG-model appears to have some seemingly plausible explanations. Work on this is on-going and will be made publicly available soon.

5 Discussion and Conclusion

We have given an exposition of the ASTG-model. This exposition is meant to give the reader a current wholesome picture of the model. Second, we have revised the Solar value $\lambda_1^{\odot}$, from 24.00 ± 7.00 to 21.00 ± 4.00; this is a 10% improvement in the error margin. The revised value is much more accurate as it has been obtained from a superior method compared to that used to determine the old value. The graphical method used to determine the new value gave (in our view) a well behaved linear graph which strongly suggest a correlation, the invariable implication of which is that, the ASTG-model strongly appears to contains in it, an element of truth; otherwise the question naturally arises, how does it come about that such a reasonable linear fit arises?

Other than revising the Solar value $\lambda_1^{\odot}$, we have revised the general expression for the λ_1-parameter. Previously, we proposed that λ_1 be equal to $\zeta \mathcal{Q}^{\zeta_0}$ (see Paper III) where was assumed to be a fixed constant, but now we have as a variable; to be specific $\zeta = \mathcal{R}_{\rm star}/\mathcal{R}_s$. This revised expression of the λ_1-parameter is well designed such that when the centrifugal forces begin their toll at the equator of the spinning gravitating body, the polar repulsive gravitational force switches on, in which process it sheds off the excess spin angular momentum. In this previous case where ζ was fixed, the polar repulsive gravitational force would not switch on immediately when the star begins to be torn apart at the equator. In this case, the polar repulsive gravitational force would switch on when $\mathcal{R}_{\rm star} = \zeta \mathcal{R}_s$, that is, the radius of the star would have to be re-adjust and comply with this relation before the polar repulsive gravitational field is switched on.

Surely, from the standpoint that the repulsive polar gravitational field solemnly exists so as to shed off the excess spin angular momentum, it would not sound correct that the star switches on the repulsive polar gravitational field much latter when the centrifugal forces have already started their crusade to rip apart the star. We believe the improvement made in the present version, does, logically sound good and reasonable. On the deeper side of things, what this all means is that, the long held supposition that (see *e.g.* Larson 2003*b*):

> "*Outflows possibly exist as a vehicle or a means by which accreting stars shed-off excess spin angular momentum so as to save themselves from the detriment, wrath and ruthlessness of the centrifugal forces*[3]";

may very well be correct. To verify this, we will need to study outflows under the

[3]This is not a direct quotation from R. B. Larson's paper but in essence, the idea expressed in Larson's paper is pretty much the same as in the present statement.

proposed ASTG-model of outflows and see if we get reasonable agreement with observations.

Given that the spherically symmetric Poisson-Laplace equation emerges out of Einstein's GTR in the weak field approximation may lead one to think that the ASTG-model is a subset of Einstein's GTR. We have argued that this is not the case. We have argued that the ASGT-model must be viewed a fully fledge theory on its own right. Yes, in the spherically symmetric case, it shares a common ground with Einstein's GTR but this common ground diverges when we consider the azimuthally symmetric case. We have argued that Einstein's GTR give different result to those predicted by the ASTG-model. This is interesting in that this can be used as testing grounds of Einstein's GTR and the ASTG-model. For the interested reader, we have pointed out that our approach to the Poisson-Laplace theory is guided by the Unified Field Theory proposed in Paper (V). Importantly, the ASTG-model as it stands does not depend on the correctness of the theory proposed in Paper (V). In this theory, gravitation is no longer a metric theory but a scaler theory and the electromagnetic, weak and the strong nuclear force emerge in the metric description of spacetime.

In-closing, allow us to say that, we believe that the ASTG-model is now set on a sure and interesting pedestal to make testable predictions without the need to resort to observations in-order to deduce the (dynamic) λ-parameters. The chronic and diabolic problem of the λ-parameters appears to have been resolved (at least for now). We are now going to focus our remaining energy and thrust on finding grounds to test this theory further and as-well compare and contrast the theory with Einstein's GTR.

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