A simple presentation of derivation of harmonic oscillator and a different derivation of the Pythagorean theorem

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Abstract. Sergio Rojas wrote an article about derivation of classical harmonic oscillator. This derivation is more clear for students. Some sentences are added here which still more visualize his derivation. A derivation of the Pythagorean theorem from kinetic energy law is added. Such derivations are a way, how to improve visualization of fundamental theories of physics, and to visualize their derivations and problems.

1. Visualization of harmonic motion

Sergio Rojas writes in his article [1], how to more clearly introduce the harmonic oscillator in middle school. The principle is that the part d^2x/dt^2 can be rewritten as:

$$d^2x/dt^2 = vdv/dx\,, \tag{1}$$

where v is velocity, t is time and x is distance in x-axis. For further simplification of his derivation it is possible to add some things. Namely, Rojas' derivation is made, in essence, by conservation of energy, where elastic potential energy is changing to kinetic energy and oppositely. Not only that this is easier for students, whose do not understand mathematics perfectly, physical (mathematical) background can also be easier to visualize. It is not only enough that mathematical derivation can be done, but also that it can be visualized.

Let us simplify that the spring lies horizontally. So we get the expression where the total energy W_{total} is constant:

$$mv^2/2 + kx^2/2 = W_{\text{total}} = \text{const}\,, \tag{2}$$

with m the mass and k the spring constant. Or differently written

$$W_{\text{kinetic}} + W_{\text{potential}} = W_{\text{total}} = \text{const}\,. \tag{3}$$

Equation (2) can be written also as

$$v^2 + (2\pi x/t_0)^2 = 2W_{\text{total}}/m\,, \tag{4}$$

where t_0 equals to time of one oscillation. It can be also written as

$$[vt_0/(2\pi)]^2 + x^2 = t_0^2 W_{\text{total}}/(2\pi^2 m)\,. \tag{5}$$

It can also be easily shown, why this gives only a harmonic motion, so that it is not necessary to go through a long derivation, made by Rojas. Let us draw a graph, where elongation of spring is on y-axis and velocity of spring-end v is on x-axis. In general we obtain an elliptic curve. But when we correctly calibrate x and y with m and k, we obtain a circular curve.‡ Its radius means square root of the total energy. If we suppose that both axes are equivalent, we obtain that this is a steady circulation. Namely point at angle 90 degrees is no more privileged than an angle of 13 degrees.

So the above consideration quickly visualizes a rather long derivation. Such square-parts describe also many other types of energy, for instance, magnetic and electric fields at electromagnetic radiation. So the above visualization is more general.

2. The Pythagorean theorem

In principle, formula (2) is a form of the Pythagorean theorem. Thus, we can see that quadratic parts give us the Pythagorean theorem in partly abstract space. But

‡ Equations (4) and (5) can also help at visualization of calibration.

something is possible to see also in our real space, namely how energy creates the Pythagorean theorem. Let us imagine a train which is moving in x direction with velocity v_x. In the train is one trolley which is moving perpendicularly to the direction of the train, therefore it is moving in the y direction with the velocity v_y according to the observer in the train. Before someone pushed this trolley, it had kinetic energy mv_x^2. After its push he gave it additional energy mv_y^2. So a rest observer see velocity of this trolley v as

$$mv^2 = mv_x^2 + mv_y^2 \,. \tag{6}$$

We see, that this is calculated with the assumption that the direction of the additional moving is orthogonal. We did not assumed the Pythagorean theorem, but the above equation gives it.

It is possible to oppose that such a result cannot be obtained at enough large velocities, where special relativity should be respected. Indeed, summation of two orthogonal velocities gives:

$$v^2 = v_x^2 + v_y^2(1 - v_x^2/c^2) \,, \tag{7}$$

where c is the speed of light. Different formula (7) as (6) is a consequence of smaller speed of time in a very fast train. If this is respected, the energy law still ever gives the Pythagorean theorem. At my interpretation of special relativity theory [2] time in the moving train has the same speed as our time, only masses in the train are increased. But because of its larger masses, it seems to us that time run is slower for him.

The Pythagorean theorem is no more valid in curved space and this is only a generalization of special relativity. In truth, space time does not exist by itself or without any cause. It is created by elementary particles and by interactions and relations among them [2, 4]. So, I suppose that the Pythagorean law is only a consequence of this. But, I admit, I am not sure. There are many proofs of the Pythagorean theorem [3]. No one includes kinetic energy. If we look at the simplest examples of the Pythagorean theorem, it is a consequence of the fact that area of a rectangle equals to the product of two its orthogonal sides. This is true only in non-curved space. If true space is no more fundamental than curved space, then the form of kinetic energy is a cause for the Pythagorean theorem. But, is it really?

3. Quantum mechanics

Some of the fundamental principles of the quantum mechanics are the Heisenberg principle of uncertainty, superposition of pure states, entanglement, wave functions and importance of information. As above, we can ask ourselves, what is the most fundamental one.

The formula for the harmonic oscillator in the lowest energy state can help us. Its wave function has the shape of the Gaussian curve. Its wave function in momentum representation has also shape of the Gaussian curve. This is consequence of the fact that the Fourier transformation of of the Gaussian curve is also the Gaussian curve. This is the fact, where the Heisenberg uncertainty principle stands. Therefore wave functions are consequences of the uncertainty principle. Here we should ask ourselves, why the uncertainty principle is important. According to Brukner-Zeilinger interpretation the information is the basic principle of quantum mechanics. Therefore the Heisenberg uncertainty principle is more fundamental than wave function.

So this topic with harmonic oscillator is important, although the derivation of the classical and quantum harmonic oscillator are not the same. (But formula (1) can be used to visualize also much other formulae.) Of course, Brukner also found some of information and wave function connection [5]. He did not succeed to describe principle of uncertainty, but some path should exist. Harmonic curves exist also in quantum mechanics. Time oscillation is typical for wave functions. It is almost never mentioned what is additional parameter, which explains oscillation with circulation, but Brukner shows this [5]. So when we do not find information about spin, this information is oriented in different direction as we measure spin.

4. Conclusion

So, the article fulfill the Rojas' article. Besides, it find a connection with Pythagoras theorem, it searches importance of harmonic oscillator in quantum mechanics and indicates use of Rojas' derivation for quantum harmonic oscillator.

It is a problem today, that a promising theory of quantum gravity is not yet obtained. One of problems is also that the pre-theories of quantum gravity are not enough clearly presented. So a number of peoples, which know them is small. If this number was larger, the possibility to find a correct theory of quantum gravity would be larger. One example of a better visualization of one of pre-theories, special relativity theory, is also [2]. It seems that all formulae of fundamental theories can be better visualized.

References

[1] Rojas S 2010 A non-standard approach to introduce simple harmonic motion *preprint* ed-ph/1011.0687
[2] Kokosar J 2010 Reasons for Relativistic Mass and Its Influence on Duff's Claims that Dimensionful Quantities Are Physically Nonexistent *preprint* http://vixra.org/abs/1012.0006
[3] http://en.wikipedia.org/wiki/Pythagorean_theorem
[4] Markopoulou F 2008 Space does not exist, so time can *preprint* http://www.fqxi.org/community/forum/topic/376
[5] Brukner Č and Zeilinger A 2003 Information and Fundamental Elements of the Structure of Quantum Theory *Time, Quantum, Information* ed L Castell and O Ischebeck (Berlin, Springer) p 323 (*preprint* quant-ph/0212084)