Sifting the Doppler Effect

Roald C. Maximo

rc-maximo@uol.com.br

Jan. 2011

Abstract

What many people do not realize is that in the root of all motion detection by means of sound or electromagnetic waves, be it by interferometry or direct wave length measurements, there is, invariably, the all important Doppler effect. Experimenters who conducted experiments in interferometry, which involve phase comparisons, resorted frequently to a naive analogy with boats in river flows to substantiate their calculations.That analogy does not take into account that the source and observer are moving together in the same direction. And that makes all the difference! Apparently, the prime importance of the Doppler effect has been consistently neglected. Thinking in terms of Doppler makes things easier!. So, let's, firstly, try to bring it back to its rightful stand.

1) The general Doppler equations are

$$F = F_0 \cdot \frac{\vec{c} + \vec{v_r}}{\vec{c} + \vec{v_s}} \quad [1] \qquad \text{or} \qquad \lambda = \lambda_0 \cdot \frac{\vec{c} + \vec{v_s}}{\vec{c} + \vec{v_r}} \quad [2]$$

as related respectively to frequency and wavelength where:

c = light velocity vector
v_s = light source speed vector
v_r = receiver or observer speed vector

Compounding the vector additions for [2] above we get

$$\lambda = \lambda_0 \cdot \sqrt{\frac{c^2 + v_s^2 - 2 \cdot c \cdot v_s \cdot \cos(\theta_s)}{c^2 + v_r^2 + 2 \cdot c \cdot v_r \cdot \cos(\theta_r)}} \quad [3]$$

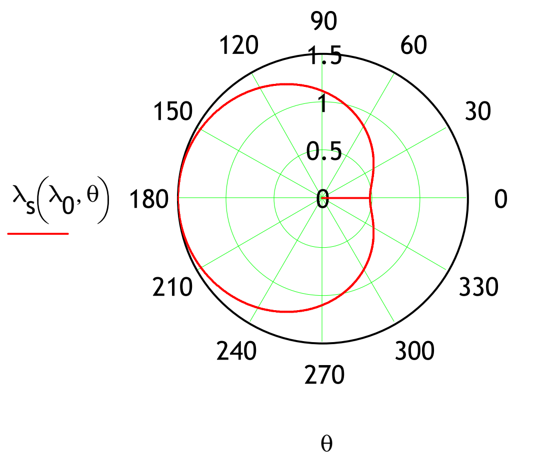
<== source
<== receiver

Equations [1], [2] and [3] are telling us that, in laboratory experiments, the source and observer are integral parts of an ensemble and must be handled as such when performing calculations. Neglecting this fact has frequently conducted to false interpretations and poor judgements of some otherwise trivial physics phenomena.
Prior to elaborating on the main reason of this paper I want to make a brief tour through some aspects seldom, if ever, divulged about the general Doppler phenomenon. Though the forward wave length compression and backward wavelength dilation of a moving light or sound source have been exhaustively demonstrated, Eq.[3] shows that there are some subtle aspects that arise in whether the source is moving and the observer is static or whether the observer is moving and the source is static or both are moving and in which relative direction. Let's first have a look at source and observer independently

$$\lambda_s(\lambda_0,\theta) = \lambda_0 \cdot \frac{\sqrt{c^2 + v_s^2 - 2 \cdot c \cdot v_s \cdot \cos(\theta)}}{c} \qquad [4]$$

The graph below is a plot of Eq.(4) and shows the wave length as a function of direction of an isotropic monochromatic E/M radiator moving at half the speed of light.
λ_0 taken as 1 m

Graph 1 (source)



Some sample wave lengths:

$\lambda_s(1 \cdot m, 0 \cdot deg) = 0.5\,m$

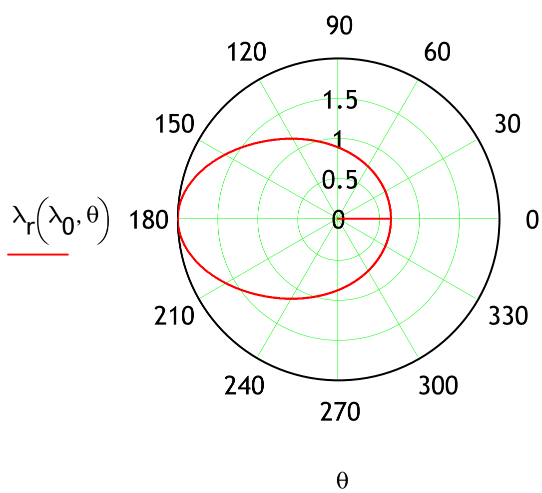
$\lambda_s(1 \cdot m, 180 \cdot deg) = 1.5\,m$

$\lambda_s(1 \cdot m, 90 \cdot deg) = 1.118\,m$

$$\lambda_r(\lambda_0,\theta) = \lambda_0 \cdot \frac{c}{\sqrt{c^2 + v_r^2 + 2 \cdot c \cdot v_r \cdot \cos(\theta)}} \qquad [5]$$

This graph shows a plot of Eq. (5) which represents the perceived wavelength for an observer moving at half the speed of light at an angle θ through a plane wave front of wavelength $\lambda_0 = 1$m
Note the marked difference between both graphs. There is a distinct asymmetry between the two situations.

Graph 2 (receiver)



Some sample wave lengths:

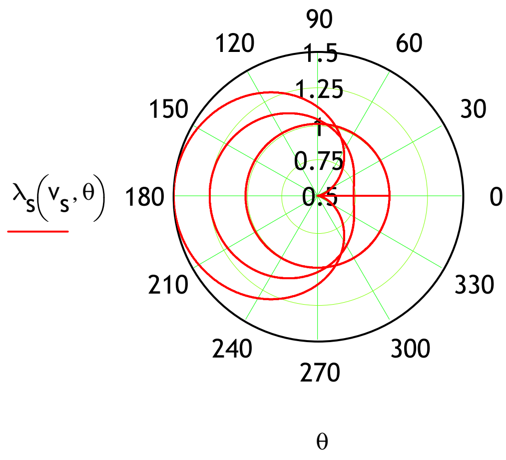
$\lambda_r(1 \cdot m, 0 \cdot deg) = 0.667\,m$

$\lambda_r(1 \cdot m, 180 \cdot deg) = 2\,m$

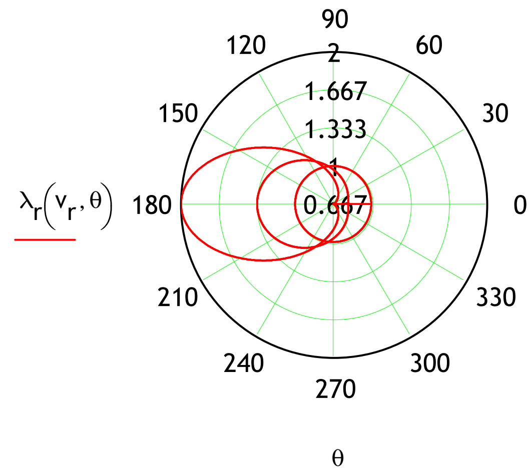
$\lambda_r(1 \cdot m, 90 \cdot deg) = 0.894\,m$

The graphs below depict the situations above for a source and a receiver traveling at speeds **0**, **C/4** and **C/2** respectively

Graph 3 (source)



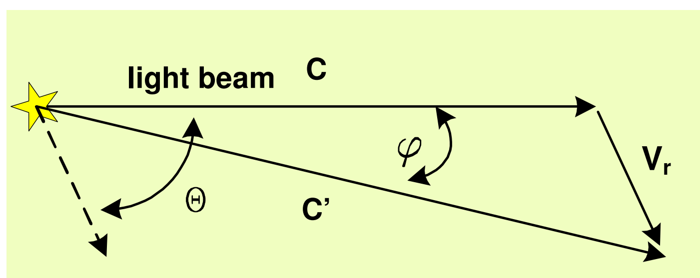
Graph 4 (receiver)



2) The case for light aberration

In the case of light aberration, the source does not belong to the observer's system of reference and light becomes a detached plane wave front freely propagating with velocity **c** though a universal frame of reference (ether?).

$$c' = \vec{c} + \vec{v} = \sqrt{c^2 + v_r^2 + 2 \cdot c \cdot v_r \cdot \cos(\theta_r)} \qquad \textbf{[6]}$$



$$\varphi = \operatorname{asin} \cdot \left(\frac{v \cdot \sin(\theta)}{\vec{c} + \vec{v_r}} \right) \qquad \textbf{[7]}$$

$\theta_r = 90 \cdot \text{deg}$ <== For the simplest case of light rays arriving from a perpendicular direction we get the trivial solution:

The Earth orbital speed is $v_r = 2.978 \cdot 10^4 \cdot \frac{m}{s}$

$$\varphi = \operatorname{asin}\left(\frac{v_r \cdot \sin(\theta_r)}{\sqrt{c^2 + v_r^2 + 2 \cdot c \cdot v_r \cdot \cos(\theta_r)}} \right) \qquad \textbf{[8]} \qquad \varphi = 20.489394 \cdot \text{arcsec}$$

A series expansion of [8] yields

$$\varphi = \sin(\theta_r)\cdot\frac{v_r}{c} - \sin(\theta_r)\cdot\frac{v_r^3}{2\cdot c^3} + \sin(\theta_r)^3\cdot\frac{v_r^3}{6\cdot c^3} \;..\; 0$$

3) The detection of absolute movement **"Thinking in terms of Doppler makes things clearer"**

Any attempt to detect absolute motion must involve electro magnetic waves and some form of phase comparison. And phase is directly linked to the Doppler effect. An electromagnetic wave of wave length λ_0 traveling a distance L from source to detector at speed c will have phase:

$$\psi_0 = \frac{2\cdot\pi\cdot L}{\lambda_0} \qquad \textbf{[9]}$$

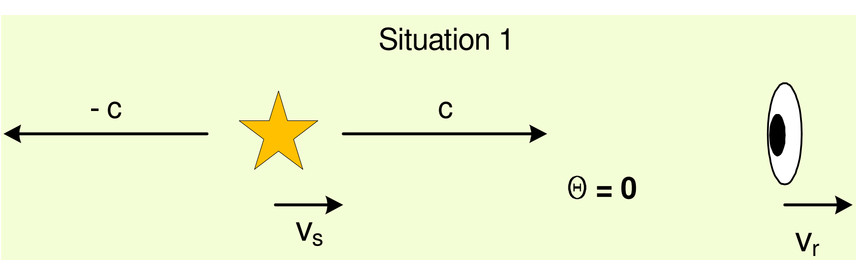
where L is the spacing between source and detector

$$\psi = \frac{2\cdot\pi\cdot L}{\lambda} \qquad \frac{\psi_0}{\psi} = \frac{\lambda}{\lambda_0} \qquad \psi = \psi_0\cdot\frac{\lambda_0}{\lambda} \qquad \textbf{[10]}$$

When source and observer are integral parts of an ensemble, ex. in a laboratory experiment, both speeds v_s and v_r and both angles θ_s and θ_r are always equal. Substituting Eq. [3] for λ in Eq. [10]and making $v_s = v_r = v$ and $\theta_s = \theta_r = \theta$

$$\psi = \psi_0\cdot\frac{1}{\sqrt{\dfrac{c^2 + v^2 - 2\cdot c\cdot v\cdot\cos(\theta)}{c^2 + v^2 + 2\cdot c\cdot v\cdot\cos(\theta)}}} \qquad \textbf{[11]}$$

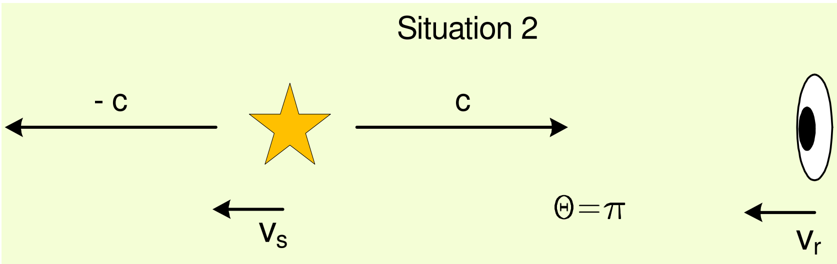
There are four important situations to be considered here, where θ is the angle the light makes in relation to the measuring ensemble direction of movement.
In situation 1, the observer is leading and the source is trailing.



For $\theta = 0$, Eq.[11] has solution:

$$\psi_f = \psi_0\cdot\frac{c+v}{c-v} \qquad \theta = 0 \qquad \textbf{[12]}$$

In situation 2, the source is leading and the observer is trailing.



For $\theta = \pi$, Eq.[11] has solution:

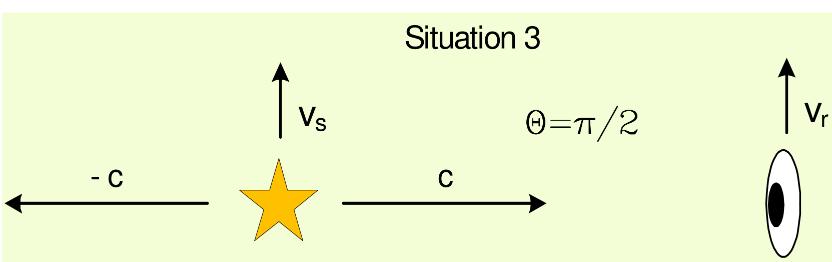
$$\psi_b = \psi_0 \cdot \frac{c - v}{c + v} \qquad \theta = \pi \qquad \textbf{[13]}$$

The well known Sagnac effect takes advantage of the phase difference between two opposing positions corresponding to situations 1 and 2 simultaneously. A simple Sagnac device is made in the form of two semi-circular paths with origin in the light source and ending in a phase detector. It makes for a very sensitive first order turn detector now widely used in navigation. It is obvious that a linear configuration would give the same results were in not for the difficulty of measuring the phase difference between two points at a linear distance 2 * L or even L.

$$\psi_0 \cdot \frac{c + v}{c - v} - \psi_0 \cdot \frac{c - v}{c + v}$$

expands to $4 \cdot \psi_0 \cdot \frac{v}{c} + 4 \cdot \psi_0 \cdot \frac{v^3}{c^3} .. 0$ $\qquad \psi = \frac{4 \cdot \pi \cdot L}{\lambda_0} \cdot \frac{v}{c}$ **[14]** Which is the Sagnac equation in the linear case

In situation 3, source and observer move side by side at 90 degrees to the light.

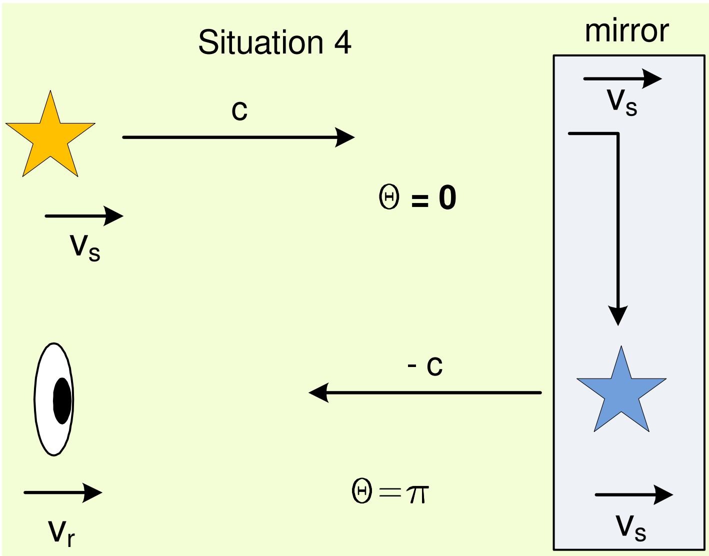


For $\theta = \pi/2$, Eq.[11] has solution:

$$\psi_{\perp} = \psi_0$$

no speed related phase shift!

In situation 4, source and observer move together and occupy the same point. Light travels forward and is reflected back by a mirror. Phase is compared between the outgoing and returning light rays. And there is a catch! The emitted ray follows Eq.[12] of situation 1 and the reflected wave follows Eq.[13] of situation 2 but the value of ψ to be inserted in Eq.[13] must be the one, already phase shifted and calculated by [12], arriving at the mirror. And there is no speed related phase shift.This has been consistently neglected.



For a reflected wave the solution is:

$$\psi_{\supset} = \psi_0 \cdot \frac{c+v}{c-v} \cdot \frac{c-v}{c+v} = \psi_0 \qquad \textbf{[15]}$$

no speed related phase shift!

Erroneous M/M:

All attempts to determine the absolute velocity of the Earth in space by means of M/M type interferometers did not take into account this detail and based the calculations on something akin to the sum of Eqs. [12] [13] which, definitely, isn't the case. Surely it would yield a second order result similar to that expected by M/M but the experiment's layout does not conduct to such a result.

$$\psi = \psi_0 \cdot \frac{c+v}{c-v} + \psi_0 \cdot \frac{c-v}{c+v} \quad \text{expands to} \quad \psi = 2 \cdot \psi_0 + \frac{4 \cdot v^2 \cdot \psi_0}{c^2} + \frac{4 \cdot v^4 \cdot \psi_0}{c^4} \,..\, 0 \qquad \textbf{[16]}$$

That error, easily made, leads to incorrect expectations. Michelson and Morley used the data below in their experiment and their calculations, on the new perspective, reduces to:

$\lambda_0 = 5 \cdot 10^{-7} \cdot m$ $\quad$ $L = 10 \cdot m$ $\quad$ $v = 3 \cdot 10^4 \cdot \frac{m}{s}$ $\quad$ $\psi_0 = \frac{2 \cdot \pi \cdot L}{\lambda_0}$

$$\psi_{MM} = \frac{2 \cdot \pi \cdot L}{\lambda_0} \cdot \frac{v^2}{c^2} \qquad \textbf{[17]}$$

$\frac{\psi_{MM}}{2 \cdot \pi} = 0.2$ $\quad$ <= expected M/M fringe shift

$\frac{4 \cdot \psi_0}{2 \cdot \pi} \cdot \frac{v^2}{c^2} = 0.801$ $\quad$ <= still flawed expected M/M fringe shift if calculated on a Doppler basis

Imagine how nice it would be if it were possible to measure speed by means of the reflected wave from a mirror physically coupled to the transmitter! An ultrasound emitter with a wavelength of 0.1 mm and a reflector at 1 m distance would give us a nice 0.5 fringe shift for a speed of only 6 km/h!

As seen, there is no need for a fudge factor, such as Lorentz's length contraction, to explain the M/M null result!