On the Ramifications of the Schwarzschild Space-Time Metric

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In a previous paper I derived the general solution for the simple point-mass in a true Schwarzschild space. I extend that solution to the point-charge, the rotating point-mass, and the rotating point-charge, culminating in a single expression for the general solution for the point-mass in all its configurations when $\Lambda = 0$. The general exact solution is proved regular everywhere except at the arbitrary location of the source of the gravitational field. In no case does the black hole manifest. The conventional solutions giving rise to various black holes are shown to be inconsistent with General Relativity.

1 Introduction

In a previous paper [1] I showed that the general solution of the vacuum field for the simple point-mass is regular everywhere except at the arbitrary location of the source of the field, $r = r_0$, $r_0 \in (\Re - \Re^-)$, where there is a quasiregular singularity. I extend herein the general solution to the rotating and charged configurations of the point-mass and show that they too are regular everywhere except at $r = r_0$, obviating the formation of the Reissner-Nordstrom, Kerr, and Kerr-Newman black holes. Consequently, there is no basis in General Relativity for the black hole.

The sought for complete solution for the point-mass must reduce to the general solution for the simple point-mass in a natural way, give rise to an infinite sequence of particular solutions in each particular configuration, and contain a scalar invariant which embodies all the factors that contribute to the effective gravitational mass of the field's source for the respective configurations.

2 The vacuum field of the point-charge

The general metric, in polar coordinates, for the vacuum field is, in relativistic units,

$$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)(d\theta^2 + \sin^2\theta d\varphi^2)\,, \quad (1)$$

where analytic $A, B, C > 0$. The general solution to (1) for the simple point-mass is,

$$ds^2 = \left[\frac{(\sqrt{C_n}-\alpha)}{\sqrt{C_n}}\right]dt^2 - \left[\frac{\sqrt{C_n}}{(\sqrt{C_n}-\alpha)}\right]\frac{{C_n'}^2}{4C_n}dr^2 - C_n(d\theta^2 + \sin^2\theta d\varphi^2)\,, \quad (2)$$

$$C_n(r) = \left[\left(r - r_0\right)^n + \alpha^n\right]^{\frac{2}{n}}\,,\ \alpha = 2m\,,\ \ r_0 \in (\Re - \Re^-)\,,$$

$$n \in \Re^+\,,\quad r_0 < r < \infty\,,$$

where $C_n(r)$ satisfies the Metric conditions of Abrams (MCA) [2]* for the simple point-mass,

1. $C_n'(r) > 0$, $r > r_0$;
2. $\lim\limits_{r\to\infty} \dfrac{C_n(r)}{\left(r - r_0\right)^2} = 1$;
3. $C_n(r_0) = \alpha^2$.

The Reissner-Nordstrom [3] solution is,

$$ds^2 = \left(1 - \frac{\alpha}{r} + \frac{q^2}{r^2}\right)dt^2 - \left(1 - \frac{\alpha}{r} + \frac{q^2}{r^2}\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,, \quad (3)$$

which is conventionally taken to be valid for all $\frac{q^2}{m^2}$. It is also alleged that (3) can be extended down to $r = 0$, giving rise to the so-called Reissner-Nordstrom black hole. These conventional allegations are demonstrably false.

The conventional analysis simply looks at (3) and makes two mathematically invalid assumptions, viz.,

1. *The parameter r is a radius of some kind in the gravitational field*;
2. *r down to $r = 0$ is valid*.

The nature and range of the r-parameter must be established by mathematical rigour, *not* by mere assumption.

Transform (1) by the substitution

$$r^* = \sqrt{C(r)}. \quad (4)$$

*Abrams' equation (A.1) should read:

$$-8\pi T_1^1 = \frac{-1}{C} + \frac{C'^2}{4BC^2} + \frac{A'C'}{2ABC} = 0\,,$$

and his equation (A.6),

$$\frac{2C''}{C'} - [\ln(ABC)]' = 0\,.$$

The errors are apparently escapees from the proof reading.

Equation (4) carries (1) into

$$ds^2 = A^*(r^*)dt^2 - B^*(r^*)dr^{*2} - r^{*2}(d\theta^2 + \sin^2\theta d\varphi^2)\,. \quad (5)$$

Using (5) to determine the Maxwell stress-energy tensor, and substituting the latter into the Einstein-Maxwell field equations in the usual way, yields,

$$\begin{aligned} ds^2 &= \left(1 - \frac{\alpha}{r^*} + \frac{q^2}{r^{*2}}\right)dt^2 - \\ &- \left(1 - \frac{\alpha}{r^*} + \frac{q^2}{r^{*2}}\right)^{-1} dr^{*2} - r^{*2}(d\theta^2 + \sin^2\theta d\varphi^2)\,. \end{aligned} \quad (6)$$

Substituting (4) into (6),

$$\begin{aligned} ds^2 &= \left(1 - \frac{\alpha}{\sqrt{C}} + \frac{q^2}{C}\right)dt^2 - \left(1 - \frac{\alpha}{\sqrt{C}} + \frac{q^2}{C}\right)^{-1} \times \\ &\times \frac{C'^2}{4C}dr^2 - C(d\theta^2 + \sin^2\theta d\varphi^2)\,. \end{aligned} \quad (7)$$

The proper radius R_p on (1) is,

$$R_p(r) = \int \sqrt{B(r)}\,dr\,. \quad (8)$$

The parameter r therefore does not lie in the spacetime M_q of the point-charge.

Taking $B(r)$ from (7) into (8) gives the proper distance in M_q,

$$\begin{aligned} R_p(r) &= \int \left(1 - \frac{\alpha}{\sqrt{C(r)}} + \frac{q^2}{C(r)}\right)^{-\frac{1}{2}} \frac{C'(r)}{2\sqrt{C(r)}}dr = \\ &= \sqrt{C(r) - \alpha\sqrt{C(r)} + q^2} + \\ &+ m\ln\left|\frac{\sqrt{C(r)} - m + \sqrt{C(r) - \alpha\sqrt{C(r)} + q^2}}{K}\right|, \end{aligned} \quad (9)$$

$$K = \text{const.}$$

The valid relationship between r and $R_p(r)$ is,

$$\text{as } r \to r_0\,, \quad R_p(r) \to 0\,,$$

so by (9),

$$r \to r_0 \;\Rightarrow \sqrt{C(r_0)} = m \pm \sqrt{m^2 - q^2}\,,$$

$$K = \pm\sqrt{m^2 - q^2}\,.$$

When $q = 0$, (9) must reduce to the Droste/Weyl [4, 5] solution, so it requires,

$$\sqrt{C(r_0)} = m + \sqrt{m^2 - q^2}\,. \quad (10)$$

Then by (9),

$$K = \sqrt{m^2 - q^2}\,,\; q^2 < m^2\,. \quad (11)$$

Clearly, r_0 is the lower bound on r.

Putting (11) into (9) gives,

$$\begin{aligned} R_p(r) &= \sqrt{C(r) - \alpha\sqrt{C(r)} + q^2} + \\ &+ m\ln\left|\frac{\sqrt{C(r)} - m + \sqrt{C(r) - \alpha\sqrt{C(r)} + q^2}}{\sqrt{m^2 - q^2}}\right|. \end{aligned} \quad (12)$$

Equation (7) is therefore singular *only* when $r = r_0$ in which case $g_{00} = 0$. Hence, the condition $r \to r_0 \Rightarrow R_p \to 0$ is equivalent to $r = r_0 \Rightarrow g_{00} = 0$.

If $C' = 0$ the structure of (7) is destroyed, since $g_{11} = 0\ \forall\ r > r_0 \Rightarrow B(r) = 0\ \forall\ r > r_0$ in violation of (1). Therefore $C'(r) \neq 0$ for $r > r_0$.

For (7) to be asymptotically flat,

$$r \to \infty \Rightarrow \frac{C(r)}{(r - r_0)^2} \to 1\,. \quad (13)$$

Therefore,

$$\lim_{r\to\infty} \frac{C(r)}{(r - r_0)^2} = 1\,. \quad (14)$$

Since $C(r)$ behaves like $(r - r_0)^2$, must make (7) singular only at $r = r_0$, and $C'(r) \neq 0$ for $r > r_0$, $C(r)$ is strictly monotonically increasing, therefore, $C'(r) > 0$ for $r > r_0$. Thus, to satisfy (1) and (7), $C(r)$ must satisfy,

1. $C'(r) > 0$, $r > r_0$;
2. $\lim_{r\to\infty} \frac{C(r)}{(r - r_0)^2} = 1$;
3. $\sqrt{C(r_0)} = \beta = m + \sqrt{m^2 - q^2}$, $q^2 < m^2$.

I call the foregoing the Metric Conditions of Abrams (MCA) for the point-charge. Abrams [6] obtained them by a different method — using (1) and the field equations directly.

In the absence of charge (7) must reduce to the general Schwarzschild solution for the simple point-mass (2). The only functions that satisfy this requirement and MCA are,

$$C_n(r) = \left[(r - r_0)^n + \beta^n\right]^{\frac{2}{n}}\,,$$

$$\beta = m + \sqrt{m^2 - q^2}\,, \quad q^2 < m^2\,,$$

$$n \in \Re^+\,, \quad r_0 \in (\Re - \Re^-)\,,$$

where n and r_0 are arbitrary. Therefore, the general solution for the point-charge is,

$$\begin{aligned} ds^2 &= \left(1 - \frac{\alpha}{\sqrt{C}} + \frac{q^2}{C}\right)dt^2 - \left(1 - \frac{\alpha}{\sqrt{C}} + \frac{q^2}{C}\right)^{-1} \times \\ &\times \frac{C'^2}{4C}dr^2 - C(d\theta^2 + \sin^2\theta d\varphi^2)\,, \end{aligned} \quad (15)$$

$$C_n(r) = \left[\left(r - r_0\right)^n + \beta^n\right]^{\frac{2}{n}},$$

$$\beta = m + \sqrt{m^2 - q^2}, \quad q^2 < m^2,$$

$$n \in \Re^+, \quad r_0 \in (\Re - \Re^-),$$

$$r_0 < r < \infty.$$

When $n = 1$ and $r_0 = 0$, Abrams' [6] solution for the point-charge results.

Equation (15) is regular $\forall\ r > r_0$. There is no event horizon and therefore no Reissner-Nordstrom black hole. Furthermore, the Graves-Brill black hole and the Carter black hole are also invalid.

By (15) the correct rendering of (3) is,

$$ds^2 = \left(1 - \frac{\alpha}{r} + \frac{q^2}{r^2}\right) dt^2 - \left(1 - \frac{\alpha}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{16}$$

$$q^2 < m^2, \quad m + \sqrt{m^2 - q^2} < r < \infty,$$

so Nordstrom's assumption that $\sqrt{C(0)} = 0$ is invalid.

The scalar curvature $f = R_{ijkm}R^{ijkm}$ for (1) with charge included is,

$$f = \frac{8\left[6\left(m\sqrt{C} - q^2\right)^2 + q^4\right]}{C^4}.$$

Using (15) the curvature is,

$$f = \frac{8\left[6\left(m\left[\left(r - r_0\right)^n + \beta^n\right]^{\frac{1}{n}} - q^2\right)^2 + q^4\right]}{\left[\left(r - r_0\right)^n + \beta^n\right]^{\frac{8}{n}}}.$$

The curvature is always finite, even at r_0. No curvature singularity can arise in the gravitational field of the point-charge. Furthermore,

$$f(r_0) = \frac{8\left[6\left(m\beta - q^2\right)^2 + q^4\right]}{\beta^8},$$

where $\beta = m + \sqrt{m^2 - q^2}$. Thus, $f(r_0)$ is a scalar invariant for the point-charge. When $q = 0$, $f(r_0) = \frac{12}{\alpha^4}$, which is the scalar curvature invariant for the simple point-mass.

From (15) the circumference χ of a great circle is given by,

$$\chi = 2\pi\sqrt{C(r)}.$$

The proper radius is given by (12). Then the ratio $\frac{\chi}{R_p} > 2\pi$ for finite r and,

$$\lim_{r\to\infty} \frac{\chi}{R_p} = 2\pi,$$

$$\lim_{r\to r_0} \frac{\chi}{R_p} \to \infty,$$

which shows that $R_p(r_0)$ is a quasiregular singularity and cannot be extended.

Consider the Lagrangian,

$$L = \frac{1}{2}\left[\left(1 - \frac{\alpha}{\sqrt{C_n}} + \frac{q^2}{C_n}\right)\left(\frac{dt}{d\tau}\right)^2\right] - \frac{1}{2}\left[\left(1 - \frac{\alpha}{\sqrt{C_n}} + \frac{q^2}{C_n}\right)^{-1}\left(\frac{d\sqrt{C_n}}{d\tau}\right)^2\right] - \frac{1}{2}\left[C_n\left(\left(\frac{d\theta}{d\tau}\right)^2 + \sin^2\theta\left(\frac{d\varphi}{d\tau}\right)^2\right)\right]. \tag{17}$$

Restricting motion to the equatorial plane without loss of generality, the Euler-Lagrange equations from (17) are,

$$\left(1 - \frac{\alpha}{\sqrt{C_n}} + \frac{q^2}{C_n}\right)\frac{d^2\sqrt{C_n}}{d\tau^2} + \left(\frac{\alpha}{2C_n} - \frac{q^2}{C_n^{\frac{3}{2}}}\right)\left(\frac{dt}{d\tau}\right)^2 - \left(\frac{\alpha}{2C_n} - \frac{q^2}{C_n^{\frac{3}{2}}}\right)\left(1 - \frac{\alpha}{\sqrt{C_n}} + \frac{q^2}{C_n}\right)^{-2}\left(\frac{d\sqrt{C_n}}{d\tau}\right)^2 - \sqrt{C_n}\left(\frac{d\varphi}{d\tau}\right)^2 = 0, \tag{18}$$

$$\left(1 - \frac{\alpha}{\sqrt{C_n}} + \frac{q^2}{C_n}\right)\frac{dt}{d\tau} = k = \text{const}, \tag{19}$$

$$C_n\frac{d\varphi}{d\tau} = h = \text{const}. \tag{20}$$

Also, $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ becomes,

$$\left(1 - \frac{\alpha}{\sqrt{C_n}} + \frac{q^2}{C_n}\right)\left(\frac{dt}{d\tau}\right)^2 - \left(1 - \frac{\alpha}{\sqrt{C_n}} + \frac{q^2}{C_n}\right)^{-1}\left(\frac{d\sqrt{C_n}}{d\tau}\right)^2 - C_n\left(\frac{d\varphi}{d\tau}\right)^2 = 1. \tag{21}$$

It follows from these equations that the angular velocity ω of a test particle is,

$$\omega^2 = \left(\frac{\alpha}{2C_n^{\frac{3}{2}}} - \frac{q^2}{C_n^2}\right) = \left[\frac{\alpha}{2\left[\left(r - r_0\right)^n + \beta^n\right]^{\frac{3}{n}}} - \frac{q^2}{\left[\left(r - r_0\right)^n + \beta^n\right]^{\frac{4}{n}}}\right]. \tag{22}$$

Then,

$$\lim_{r\to r_0} \omega = \sqrt{\frac{\alpha}{2\beta^3} - \frac{q^2}{\beta^4}}, \tag{23}$$

where $\beta = m + \sqrt{m^2 - q^2}$, $q^2 < m^2$.

Equation (22) is Kepler's 3rd Law for the point-charge. It obtains the finite limit given in (23), which is a scalar invariant for the point-charge. When $q = 0$, equations (22) and (23) reduce to those for the simple point-mass,

$$\omega = \sqrt{\frac{\alpha}{2C_n^{\frac{3}{2}}}}\,,$$

$$\lim_{r \to r_0} \omega = \frac{1}{\alpha\sqrt{2}}\,.$$

In the case of a photon in circular orbit about the point-charge, (21) yields,

$$\omega^2 = \frac{1}{C_n}\left(1 - \frac{\alpha}{\sqrt{C_n}} + \frac{q^2}{C_n}\right), \tag{24}$$

and (18) yields,

$$\omega^2 = \frac{1}{\sqrt{C_n}}\left(\frac{\alpha}{2C_n} - \frac{q^2}{C_n^{\frac{3}{2}}}\right). \tag{25}$$

Equating the two, denoting the stable photon radial coordinate by r_{ph}, and solving for the curvature radius $\sqrt{C_{ph}} = \sqrt{C_n(r_{(ph)})}$, gives (since when $q = 0$, $\sqrt{C_{ph}} \neq 0$),

$$\sqrt{C_{ph}} = \sqrt{C_n(r_{(ph)})} = \frac{3\alpha + \sqrt{9\alpha^2 - 32q^2}}{4}\,, \tag{26}$$

which is a scalar invariant. In terms of coordinate radii,

$$r_{ph} = \left[\frac{\left(3\alpha + \sqrt{9\alpha^2 - 32q^2}\right)^n}{4^n} - \beta^n\right]^{\frac{1}{n}} + r_0\,, \tag{27}$$

which depends upon the values of n and r_0.

When $q = 0$ equations (26) and (27) reduce to the corresponding equations for the simple point-mass,

$$\sqrt{C_n(r_{ph})} = \frac{3\alpha}{2}, \tag{28}$$

$$r_{ph} = \left[\left(\frac{3\alpha}{2}\right)^n - \alpha^n\right]^{\frac{1}{n}} + r_0\,. \tag{29}$$

The proper radius associated with (28) and (29) is,

$$R_{p(ph)} = \frac{\alpha\sqrt{3}}{2} + \alpha \ln\left(\frac{1 + \sqrt{3}}{\sqrt{2}}\right), \tag{30}$$

which is a scalar invariant for the simple point-mass. Putting (26) into (12) gives the invariant proper radius for a stable photon orbit about the point-charge.

3 The vacuum field of the rotating point-mass

The Kerr solution, in Boyer-Lindquist coordinates and relativistic units is,

$$\begin{aligned} ds^2 &= \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\varphi\right)^2 - \\ &- \frac{\sin^2\theta}{\rho^2}\left[\left(r^2 + a^2\right)d\varphi - a dt\right]^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2, \end{aligned} \tag{31}$$

$$a = \frac{L}{m}, \quad \rho^2 = r^2 + a^2\cos^2\theta\,,$$

$$\Delta = r^2 - r\alpha + a^2, \quad 0 < r < \infty\,,$$

where L is the angular momentum.

If $a = 0$, equation (31) reduces to Hilbert's [7] solution for the simple point-mass,

$$\begin{aligned} ds^2 &= \left(1 - \frac{\alpha}{r}\right)dt^2 - \left(1 - \frac{\alpha}{r}\right)^{-1}dr^2 - \\ &- r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,, \end{aligned} \tag{32}$$

$$0 < r < \infty\,.$$

However, according to the general formula (2) the correct range for r in (32) is,

$$\sqrt{C(r_0)} < r < \infty\,,$$

where $\sqrt{C(r_0)} = \alpha$. Therefore (32) should be,

$$\begin{aligned} ds^2 &= \left(1 - \frac{\alpha}{r}\right)dt^2 - \left(1 - \frac{\alpha}{r}\right)^{-1}dr^2 - \\ &- r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,, \end{aligned} \tag{33}$$

$$\alpha < r < \infty\,.$$

Equation (33) is the Droste/Weyl solution.

Since the r that appears in (32) is the same r appearing in (31) and (33), taking (4) into account, the correct general form of (31) is,

$$\begin{aligned} ds^2 &= \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\varphi\right)^2 - \\ &- \frac{\sin^2\theta}{\rho^2}\left[\left(C + a^2\right)d\varphi - a dt\right]^2 - \\ &- \frac{\rho^2}{\Delta}\frac{C'^2}{4C}dr^2 - \rho^2 d\theta^2\,, \end{aligned} \tag{34}$$

$$a = \frac{L}{m}\,, \quad \rho^2 = C + a^2\cos^2\theta\,,$$

$$\Delta = C - \alpha\sqrt{C} + a^2\,, \quad r_0 < r < \infty\,.$$

When $a = 0$, (34) must reduce to (2).

If $C' = 0$ the structure of (34) is destroyed, since then $g_{11} = 0\ \forall\ r > r_0 \Rightarrow B(r) = 0$ in violation of (1). Therefore $C' \neq 0$. Equation (34) must have a global arrow for time, whereupon $g_{00}(r_0 = 0$, so

$$\Delta(r_0) = C(r_0) - \alpha\sqrt{C(r_0)} + a^2 = a^2\sin^2\theta\,. \tag{35}$$

Solving (35) for $\sqrt{C(r_0)}$ gives,

$$\beta = \sqrt{C(r_0)} = m \pm \sqrt{m^2 - a^2\cos^2\theta}\,, \qquad (36)$$

having used $\alpha = 2m$. When $a = 0$ (36) must reduce to the value for Schwarzschild's [8] original solution, i. e. $\sqrt{C(r_0)} = = \alpha = 2m$, therefore the plus sign must be taken in (36). Since the angular momentum increases the gravitational mass, and since there can be no angular momentum without mass, $a^2 < m^2$. Thus, there exists no spacetime for $a^2 \geqslant m^2$. To reduce to (2) equation (36) becomes,

$$\beta = \sqrt{C(r_0)} = m + \sqrt{m^2 - a^2\cos^2}\,, \qquad (37)$$

$$a^2 < m^2\,.$$

Equation (34) must be asymptotically flat, so

$$r \to \infty \Rightarrow \frac{C(r)}{\left(r - r_0\right)^2} \to 1\,. \qquad (38)$$

Therefore,

$$\lim_{r\to\infty} \frac{C(r)}{\left(r - r_0\right)^2} = 1\,. \qquad (39)$$

Since $C(r)$ behaves like $\left(r - r_0\right)^2$, must make (34) singular only at $r = r_0$, and $C'(r) > 0\ \forall\ r > r_0$, $C(r)$ is strictly monotonically increasing, so

$$C'(r) > 0,\ r > r_0\,. \qquad (40)$$

Consequently, the conditions that $C(r)$ must satisfy to render a solution to (34) are:

1. $C'(r) > 0,\ r > r_0$;
2. $\lim\limits_{r\to\infty} \dfrac{C(r)}{\left(r - r_0\right)^2} = 1$;
3. $\sqrt{C(r_0)} = \beta = m + \sqrt{m^2 - a^2\cos^2\theta},\ a^2 < m^2$.

I call the foregoing the Metric Conditions of Abrams (MCA) for the rotating point-mass.

The only form admissible for $C(r)$ in (34) that satisfies MCA and is reducible to (2) is,

$$C_n(r) = \left[\left(r - r_0\right)^n + \beta^n\right]^{\frac{2}{n}}\,, \qquad (41)$$

$$\beta = m + \sqrt{m^2 - a^2\cos^2\theta}\,, \quad a^2 < m^2\,,$$

$$r_0 \in (\Re - \Re^-) \quad n \in \Re^+\,.$$

Associated with (31) are the so-called "horizons" and "static limits" given respectively by,

$$r_h = m \pm \sqrt{m^2 - a^2}\,, \quad r_b = m \pm \sqrt{m^2 - a^2\cos^2\theta}\,, \ (42)$$

where r_h is obtained from (31) by setting its $\Delta = 0$, and r_b by setting its $g_{00} = 0$. Conventionally equations (42) are rather arbitrarily restricted to,

$$r_h = m + \sqrt{m^2 - a^2}\,, \quad r_b = m + \sqrt{m^2 - a^2\cos^2\theta}\,, \ (43)$$

$$a^2 < m^2\,.$$

For (34), $\Delta \geq 0$ and so there is no static limit, since by (41),

$$C_n(r_0) = \beta^2 \Rightarrow\ , \qquad (44)$$

$$\Rightarrow \Delta(r_0) = \beta^2 - \alpha\beta + a^2\,.$$

Solving (41) i .e .

$$\sqrt{C_n(r)} = \left[\left(r - r_0\right)^n + \beta^n\right]^{\frac{1}{n}}\,, \qquad (45)$$

gives the r-parameter location of a spacetime event,

$$r = \left[C_n(r)^{\frac{1}{2n}} - \beta^n\right]^{\frac{1}{n}} + r_0\,. \qquad (46)$$

When $a = 0$, equation (46) reduces to $r_0 = \alpha$, as expected for the non-rotating point-mass.

From (46) it is concluded that there exists no spacetime drag effect for the rotating point-mass and no ergosphere.

The generalisation of equation (34) is then,

$$\begin{aligned} ds^2 &= \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\varphi\right)^2 - \\ &- \frac{\sin^2\theta}{\rho^2}\left[\left(C + a^2\right)d\varphi - a dt\right]^2 - \\ &- \frac{\rho^2}{\Delta}\frac{C'^2}{4C}dr^2 - \rho^2 d\theta^2\,, \end{aligned} \qquad (47)$$

$$C_n(r) = \left[\left(r - r_0\right)^n + \beta^n\right]^{\frac{2}{n}}, \quad n \in \Re^+,$$

$$r_0 \in (\Re - \Re^-)\,, \quad \beta = m + \sqrt{m^2 - a^2\cos^2\theta}\,, \quad a^2 < m^2\,,$$

$$a = \frac{L}{m}\,, \quad \rho^2 = C_n + a^2\cos^2\theta\,,$$

$$\Delta = C_n - \alpha\sqrt{C_n} + a^2\,,$$

$$r_0 < r < \infty\,.$$

Equation (47) is regular $\forall\ r > r_0$, and $g_{00} = 0$ only when $r = r_0$. There is no event horizon and therefore no Kerr black hole.

By (47) the correct expression for the Kerr solution (31) is,

$$\begin{aligned} ds^2 &= \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\varphi\right)^2 - \\ &- \frac{\sin^2\theta}{\rho^2}\left[\left(r^2 + a^2\right)d\varphi - a dt\right]^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2\,, \end{aligned} \qquad (48)$$

$$\Delta = r^2 - r\alpha + a^2, \quad a = \frac{L}{m}, \quad \rho^2 = r^2 + a^2\cos^2\theta,$$
$$a^2 < m^2, \quad m + \sqrt{m^2 - a^2\cos^2\theta} < r < \infty.$$

When $a = 0$ in (48) the Droste/Weyl solution (33) is recovered.

4 The vacuum field of the rotating point-charge

The Kerr-Newman solution is, in relativistic units,

$$\begin{gathered} ds^2 = \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\varphi\right)^2 - \\ - \frac{\sin^2\theta}{\rho^2}\left[\left(r^2 + a^2\right)d\varphi - adt\right]^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2, \end{gathered} \tag{49}$$
$$a = \frac{L}{m}, \quad \rho^2 = r^2 + a^2\cos^2\theta, \quad \Delta = r^2 - r\alpha + a^2 + q^2,$$
$$0 < r < \infty.$$

By applying the analytic technique of section 3, the general solution for the rotating point-charge is found to be,

$$\begin{gathered} ds^2 = \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\varphi\right)^2 - \\ - \frac{\sin^2\theta}{\rho^2}\left[\left(C_n + a^2\right)d\varphi - adt\right]^2 - \frac{\rho^2}{\Delta}\frac{C_n'^{\,2}}{4C_n}dr^2 - \rho^2 d\theta^2, \\ C_n(r) = \left[\left(r - r_0\right)^n + \beta^n\right]^{\frac{2}{n}}, \quad n \in \Re^+, \\ r_0 \in (\Re - \Re^-), \quad \beta = m + \sqrt{m^2 - (q^2 + a^2\cos^2\theta)}, \\ a^2 + q^2 < m^2, \quad a = \frac{L}{m}, \quad \rho^2 = C_n + a^2\cos^2\theta, \\ \Delta = C_n - \alpha\sqrt{C_n} + q^2 + a^2, \\ r_0 < r < \infty. \end{gathered} \tag{50}$$

Equations (50) give the overall general solution to Einstein's vacuum field when $\Lambda = 0$. The associated Metric Conditions of Abrams (MCA) for the rotating point-charge are,

1. $C_n'(r) > 0, \ r > r_0$;
2. $\lim\limits_{r\to\infty} \dfrac{C_n(r)}{\left(r - r_0\right)^2} = 1$;
3. $\sqrt{C_n(r_0)} = \beta = m + \sqrt{m^2 - (q^2 + a^2\cos^2\theta)}$, $a^2 + q^2 < m^2$.

From (50) it is concluded that there exists no spacetime drag effect for the rotating point-charge, and no ergosphere.

Equation (50) is regular $\forall\, r > r_0$, and $g_{00} = 0$ only when $r = r_0$; $r_h \equiv r_0$. When $a = 0$ in (50) the general solution for the point-charge (15) is recovered. If both $a = 0$ and $q = 0$ in (50) the general solution (2) for the simple Schwarzschild point-mass is recovered. There is no event horizon and therefore no Kerr-Newman black hole.

By (50) the correct expression for the Kerr-Newman solution (49) is,

$$\begin{gathered} ds^2 = \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\varphi\right)^2 - \\ - \frac{\sin^2\theta}{\rho^2}\left[\left(r^2 + a^2\right)d\varphi - adt\right]^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2, \\ a = \frac{L}{m}, \quad \rho^2 = r^2 + a^2\cos^2\theta, \quad \Delta = r^2 - r\alpha + a^2 + q^2, \\ q^2 + a^2 < m^2, \quad m + \sqrt{m^2 - (q^2 + a^2\cos^2\theta)} < r < \infty. \end{gathered} \tag{51}$$

If $a = 0$ in (51) the correct expression for the Reissner-Nordstrom solution (16) is recovered. If $q = 0$ in (51) the correct expression for the Kerr solution (48) is recovered. If both $a = 0$ and $q = 0$ in (51) the correct expression for Hilbert's (i. e. the Droste/Weyl) solution (33) is recovered.

5 The Einstein-Rosen Bridge

The Einstein-Rosen Bridge [9] is obtained by substituting into the Droste/Weyl solution (33) the transformation,

$$u^2 + \alpha = r, \tag{52}$$

which carries (33) into,

$$\begin{gathered} ds^2 = \left[\frac{u^2}{(u^2 + \alpha)}\right]dt^2 - \\ - 4\left(u^2 + \alpha\right)du^2 - \left(u^2 + \alpha\right)^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \\ -\infty < u < \infty. \end{gathered} \tag{53}$$

Metric (53) is singular nowhere, and as u runs $-\infty$ to 0 and 0 to $+\infty$, r runs $+\infty$ to α then α to $+\infty$, thereby allegedly removing the singularity at $r = \alpha$. However, (53) is inadmissible by (2): (52) is not a valid form for $C_n(r)$ for the simple point-mass. This manifests in a violation of MCA. Indeed,

$$\lim_{u\to\infty}\frac{C(u)}{u^2} = \lim_{u\to\infty}\frac{\left(u^2 + \alpha\right)^2}{u^2} \to \infty, \tag{54}$$

so the far field is not flat. The Einstein-Rosen Bridge is therefore invalid.

6 Interacting black holes and the Michell-Laplace dark body

It is quite commonplace for black holes to be posited as members of binary systems, either as a hole and a star, or as two holes. Even colliding black holes are frequently alleged (see e. g. [10]). Such ideas are inadmissible, even if the existence of black holes were allowed. All solutions to the Einstein field equations involve a single gravitating body and a test particle. No solutions are known that address two bodies of comparable mass. It is not even known if solutions to such configurations exist. One simply cannot talk of black hole binaries or colliding black holes unless it can be shown, as pointed out by McVittie [11], that Einstein's field equations admit of solutions for such configurations. Without such an existence theorem these ideas are without any theoretical basis. McVittie's existence theorem however, does not exist, because the black hole does not exist in the formalism of General Relativity. It is also commonly claimed that the Michell-Laplace dark body is a kind of black hole or an anticipation of the black hole [10, 12]. This claim is utterly false as there always exists a class of observers who can see a Michell-Laplace dark body [11]: ipso facto, it is not a black hole. Consequently, there is no theoretical basis whatsoever for the existence of black holes. If such an object is ever detected then both Newton and Einstein would be invalidated.

Dedication

I dedicate this paper to the memory of Dr. Leonard S. Abrams: (27 Nov. 1924 — 28 Dec. 2001).

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