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RELATIVISTIC DIMENSIONAL ANALYSIS

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ABSTRACT

The Lorentzian "contraction" for length, time, mass and temperature —taken as functions of a dimensional equation— enable the author to define the mathematical structure of the constants of Nature. In this way the *a priori* determination of all the constants of Nature has been rendered possible.

This new technique for physical investigation shows that the constant of gravitation has a certain variance that, when properly compensated, yields a true universal constant. Upon the introduction of the corrected constant in the Lagrange equations for planetary motion we are led to Einstein's equations and the correct estimate for perihelion advance.

1) Introduction.

Dimensional analysis may be defined as "the study of the invariants for variations of the measuring units", restricting in this way its scope to the study of "adimensional numbers" and the "Principle of Dimensional Homogeneity".

In trying to apply Dimensional Analysis to the study of the relationships existing between quantities referred to two coordinate systems in relative uniform motion we find —under the above quoted assumptions— that the adimensional numbers continue being invariants and, in addition, that all the equations of physics remain invariant due to the presence of the Principle of Dimensional Homogeneity.

Consequently, the extension of Dimensional Analysis to cover relativistic situations must exclude *ab initio* the study of adimensional numbers as well as the acceptance of a Principle of Dimensional Homogeneity.

Under these circumstances, we must not be surprised by the fact that the attempts of applying the Theory of Dimensions to the Relativity Theory have not yet been successful. Disregarding this requirement of dimensional homogeneity —which as is well known originated in a proposition of Maxwell ([1]) of an almost political intention— we will be able to observe, upon the setting up of dimensional equations between inertial frames of reference, that regularities connected with important physical *Invariants* will appear.

2) The Two Types of Relativistic Transformations.

The study of the relativistic methodology shows the existence of two different types of transformations.

One of these types of transformation is permanently referred to both time and space in such a way that we may consider them as real space-time transformations; vgr. the coordinates of a point situated at a moving X-axis transform according to the well known Lorentz formula:

$$x' = \frac{(x - vt)}{\sqrt{1 - v^2/c^2}} \tag{1}$$

The other type of relativistic transformation is referred to "pure" times or "pure" spaces; vgr. the "contraction" of a segment lying along the X axis is determined by the equally well known formula:

$$\Delta x = \Delta x_0 \sqrt{1 - v^2/c^2} \tag{2}$$

similar formulae being obtained for intervals of time, mass and for differences of temperature.

It is important to stress the different structure of (1) and (2): the first one a mixed space-time formula, meanwhile in the second one only spatial quantities appear.

The reduction of the number of variables in the second formula has been rendered possible by the addition of a supplementary requirement. We will turn our attention to the analysis of this extra equation.

3) The Relativistic Convention.

Taking into account the whole group of Lorentz transformations we may derive the two following formulae for the measurement of the length of a segment laid along the direction of relative motion of two inertial coordinate systems:

$$\Delta x' = \frac{\Delta x - v\,\Delta t}{\sqrt{1 - v^2/c^2}} \tag{3}$$

$$\Delta x' = \Delta x \sqrt{1 - v^2/c^2} - v\,\Delta t' \tag{4}$$

In order to derive formula (2) from formula (3) we must introduce the auxiliary condition

$$\Delta t = 0 \tag{5}$$

which actually means that we have measured the length of the segment Δx by taking into account the *simultaneous* value of the coordinates of its ends; i.e. we accept that these two X-coordinates correspond to a same instant t of time. Consequently, this means that the segment coordinates in the X'-axis do not belong to a same instant t' of time.

If, instead, we set up the *simultaneity* of the coordinates in the X'-axis, in other words, if we accept as auxiliary equation

$$\Delta t' = 0\ , \tag{6}$$

we-will obtain, according to eq. (4), a different formula.

The formula determining the relativistic behaviour of a segment will depend on the formulae we choose for the purpose of its determination. Taking into account, successively, the equations (3) (5) and (4) (6) we will obtain the two following results:

$$\Delta x = \Delta x_0 \sqrt{1 - v^2/c^2} \tag{A}$$

$$\Delta x = \Delta x_0 / \sqrt{1 - v^2/c^2}\quad ; \tag{A'}$$

which, translated into current language, means in the first case: the length of a segment *diminishes* with motion; while the second case (A') states that the length of a segment *increases* with motion.

The relativity theory offers, consequently, two alternate possibilities for the behaviour of a segment moving along in its own direction with uniform relative velocity. A choice between the two alternatives stated appears easy at first sight. But a minute attention will prove this task to be a hard one since nothing authorizes us to give preference to one or the other way of stating the simultaneity condition. The selection of the system of equations to which we will ascribe the simultaneity condition is purely a conventional matter. That is equivalent to state that in the relativity theory the behaviour of a moving segment is a purely conventional matter.

As a matter of fact, formula (A) is universally accepted by the physicists, being in consequence, the convention preferred in order to consider the behaviour of a relativistic segment.

4) Relativistic Underterminacy.

The very fact that the behaviour of a segment in the Relativity Theory depends on the conventional attitude we adopt may give rise to serious doubts regarding the physical posibilities of the analysis we are trying to develop. Nevertheless, the problem is not so serious as it appears at first sight since, as we are going to show, the circumstance here appearing as "conventional" implies the existence of an extra degree of freedom in the system of equations constituting the mathematical expression of the laws of physics. The significance of this circumstance is epistemologically concrete since it is referred to the solutions of the systems of determinants we may obtain out of the exponents of the dimensional formulae for the constants of Nature. In such a case the qualification of "conventional" should be better replaced by a statement of "indeterminacy". Leaving this problem for a later chapter we shall pass to analyze the very important problem presented by the behaviour of the quantity *time* in the Relativity Theory.

Along the same lines as for the case of lengths we can obtain, out of the whole group of Lorentz transformations, the following formulae for time transformation:

$$\Delta t' = \frac{\Delta t - \Delta x\, v/c^2}{\sqrt{1 - v^2/c^2}} \tag{7}$$

$$\Delta t' = \Delta t \sqrt{1 - v^2/c^2} - \Delta x'\, v/c^2 \tag{8}$$

which supplemented by the corresponding equations of condition:

$$\triangle x = 0 \tag{9}$$

$$\triangle x' = 0 \ , \tag{10}$$

yields these two formulae for the behaviour of time in the relativy theory:

$$\triangle t = \triangle t_0 \sqrt{1 - v^2/c^2} \tag{B}$$

$$\triangle t = \triangle t_0 / \sqrt{1 - v^2/c^2} \tag{B'}$$

Parallelling what we observed regarding segments we find here that the relativistic behaviour of an interval of time depends on the *conventional* selection of the auxiliary conditions; i.e. the selection of the so called "conditions of *isotopy*" (*) stated by equations (9) and (10). But opposing what we discover in relation to segments, the relativists have accepted universally formula B′ to determine the behaviour of a time interval in the Relativity Theory. It is difficult to explain why in one case (segments) the relativists have accepted one convention and the opposite one in the other case (time). But this is one historical case we can summarize by stating: In the Relativity Theory we perform the measurement of moving rods with units at rest and, viceversa, we appreciate a time interval given by a clock at rest upon comparing it with units of time given by a clock in motion.

5) The Ambivalence of Time Transformation.

The necessity for auxiliary equations —to complement the Lorentz equations in regard to the behaviour of a segment— has not presented difficulties in the history of physics. But in relation to time transformation the difficulties encountered have been very great. Among them I wish to mention the famous case of the "clock paradox" —a problem still unsolved after fifty years of discussions. It would take a long time to develop here, extensively, this problem and the elements of the Theory of Relativity connected with it. As I have made an ample analysis of these

(*) In my paper (5) "The Meaning of the Clock Paradox" I proposed to call the conditions (9) (10) "*Localicity* conditions"; the term "*isotopy conditions*" I adopt here has been suggested by Palacios (6).

questions elsewhere ([2] ([3]) ([4]) ([5]) I limit myself to quote the following statements:

i) The Clock Paradox. As stated above, the equations of Lorentz are characterized by the simultaneous presence of both temporal and spatial parameters. To take them out of their connection by treating them separately with reference to space and time means to leave the scope of the Lorentz transformations: *The equations of Lorentz have physical validity only in regard to space-time intervals.* A fact I have been able to demonstrate in the above quoted paper ([5]) is that the clock paradox has arisen upon the relativistic treatment of non-relativistic entities like "pure" spaces and "pure" times. As shown in the quoted paper, the clock paradox has appeared owing to the implicit (not explicit!) use of an universal t-time instead of the two t and t'-times required by the Relativity Theory.

In concrete words: The clock paradox shows the existence of two types of time transformation in the Relativity Theory (namely, formulae B, B′).

ii) The Lorentz Transformations. As shown above the unavoidable presence of two auxiliary conditions to supplement the Lorentz transformations, in order to determine the formulae for time transformation, leads to the undeterminacy implied by the existence of the two formulae B, B′.

iii) The Wave Mechanics. A very important point regarding Wave Mechanics —which is generally ignored— is the presence, in Wave Mechanics, of *two time transformations.* Professor de Broglie himself has confirmed to me this point about which some literature ([7]) ([8]) ([9]) may be quoted. According to Prof. de Broglie, these two transformations for time correspond to the "time of waves" (formula B′) and to the "time of corpuscles" (formula B).

It is interesting to observe that in Wave Mechanics formula B′ belongs to the "time of waves" and that in Special Relativity this same formula B′ corresponds to the "time of particles" — since the phenomena studied by Special Relativity belong to what in Wave Mechanics is associated to "group velocities".

We arrive at the conclusion that, also in Wave Mechanics, we have to deal with two kinds of time transformation (formulae B, B′).

iv) The Experiment of Michelson-Morley. The correct form for writing the Lorentz transformations should be:

$$
\begin{aligned}
x' &= k \frac{x - vt}{\sqrt{1 - v^2/c^2}} \\
y' &= k\,y \\
z' &= k\,z \\
t' &= k \frac{t - x\,v/c^2}{\sqrt{1 - v^2/c^2}}
\end{aligned}
\tag{11}
$$

in which the constant k is really indeterminated. Lorentz himself made the hypothesis $k = 1$ (accepted since that time in relativity) but, as Ives (10) (11) and Palacios (6) have been able to show, nothing warrants the correctness of this *convention.* On the other hand, from the standpoint of Relativity Theory, the condition $k = 1$ is equivalent to the requirement of equivalence of both inertial systems. But the requirement of the equivalence of the reading of clocks in two inertial systems has led to Palacios (6) (10) to a different statement of the transformations of Lorentz.

Summing up we may say that in the own structure of the experiment of Michelson-Morley is this indeterminacy present which leads to formulae B, B′.

v) The Experimental Determinations. The admitted purpose of the experiments of Ives-Stillwell-Otting was to overcome the undeterminacy introduced by the presence of the unknown k in the equations of Lorentz. According to the statement of Ives (11) an auxiliary experiment —additional to Michelson-Morley experiment— was required, and to this end the first experiment suggested was based on the so called "lateral relativistic Doppler--effect". But, as remarked by Jones (12), this effect cannot furnish any information regarding the relativistic "expansion" of time since it would lead to the two formulae B, B′ according to whether we consider either the emitter or the receiver in motion. This is to say that the lateral relativistic Doppler effect is also endowed with the ambivalence of time transformation.

This was the reason why Ives-Stillwell-Otting decided to employ the so called "longitudinal relativistic Doppler-effect". Unfortunately, as

I have been able to show elsewhere (2), the experiments of Ives-Stillwell-Otting lead to the two following formulae of transformation (*):

$$\nu_{av} = \frac{\nu_b + \nu_r}{2} = \frac{\nu_0}{\sqrt{1 - v^2/c^2}} \tag{12}$$

$$\lambda_{av} = \frac{\lambda_b + \lambda_r}{2} = \frac{\lambda_0}{\sqrt{1 - v^2/c^2}} \tag{13}$$

These two formulae were obtained by Ives in his theoretical papers (10) (11) (13) (14) previous to the experiment. Their physical interpretation means that the relativistic effect increases (at the same time !) the frequency of light and its wave length. I want to express my surprise about the fact that this clear self-contradiction, implied by the formulae (12) and (13), has not been observed before. The decision of Ives to take into account formula (12) was determined by the fact that he had his spectrometer calibrated for frequencies. If he had had his spectrometer calibrated for wave lengths, we could be sure that he would have arrived at opposite conclusions regarding the behaviour of fast moving clocks.

We may conclude that the experiments of Ives-Stillwell-Otting stresses the ambivalence of the formulae for time transformation in the Relati-

(*) The deduction of eq. (12) and (13) proceeds as follows: Taking into account the relativistic formulae for the Doppler effect,

$$\nu = \nu_0 \sqrt{\frac{1 \pm v/c}{1 \mp v/c}} \quad ;$$

$$\lambda = \frac{c}{\nu} = \lambda_0 \sqrt{\frac{1 \mp v/c}{1 \pm v/c}}$$

we obtain:

$$\nu_{av} = \frac{\nu_b + \nu_r}{2} = \frac{\nu_0}{\sqrt{1 - v^2/c^2}}$$

and

$$\lambda_{av} = \frac{\lambda_b + \lambda_r}{2} = \frac{\lambda_0}{\sqrt{1 - v^2/c^2}} \quad ,$$

where the subindices b and r means the blue and red shift, respectivily, produced by the Doppler effect.

vity Theory. Formulae (12) and (13) form a system perfectly equivalent to formulae B and B'.

vi) *The ambivalence of Length Transformation*: The two formulae A, A' lead to the two formulae B, B' through the well known equality (*):

$$\lambda \nu = \lambda_0 \nu_0 = c$$

where λ is the wave length, ν the frequency and c the velocity of light. Taking into account that $\nu = 1/t$ we arrive easily at the conclusion that

(*) This equality means that the fundamental postulate of the Theory of Relativity (namely, the constancy of light velocity) is valid only in the case that the length and time transformations belong, respectively, to formulae A, B or A', B'. Otherwise, it denies the validity of the Relativity Theory. For instance, if we accept for length and time transformations the commonly accepted formulae

$$\overline{} \quad T = T_0 / \sqrt{1 - v^2/c^2}$$

we obtain out of the above equality:

$$\lambda \nu = \lambda_0 \nu_0 (1 - v^2/c^2) = c\,(1 - v^2/c^2)$$

what is against the fundamental postulate of the constancy of light velocity.

If we do not accept, as stated in this paper, formulae A and B for length and time transformation, respectively, we are obliged to deny the validity of the above equality. But in this case we will be obliged to deny many other fundamental concepts of the Relativity Theory like, for instance, the constancy of electric charge under Lorentz transformations which, as is well known, is obtained out of the transformation formulae for charge density and volume

$$\rho = \rho_0 / \sqrt{1 - v^2/c^2} \quad ; \quad dv = dv_0 \sqrt{1 - v^2/c^2}$$

that yield the invariant

$$dq = \rho dv = \rho_0 dv_0 .$$

I want to remark that the above obtainment of Lorentzian invariants has validity only on the base of an accepted "principle of variational homogeneity" which we are going to demonstrate more in advance in this paper.

An additional argument regarding the necessity of a parallelism between the variance of length and time in relativistic transformations may be obtained directly from Lorentz transformations upon observing that for the case of two inertial reference systems (v = const.) we have

$$\frac{\partial x'}{\partial x} = \frac{\partial t'}{\partial t} = (1 - v^2/c^2)^{-1/2} .$$

A more rigorous demonstration may be found through a variational analysis of Minkowsky's interval which shows that the covariant transformation of a space-time tetravector requires a parallel behaviour of the space and time components. But, of course, that treatment is outside the scope of the present paper.

formula A implies formula B and formula A′ is associated to formula B′. The ambivalence of length transformation (determined by the relativity of simultaneity) implies the ambivalence of time transformation (namely, formulae B, B′).

6) Variance of Mass.

Proceeding with our analysis of the bases for a Relativistic Dimensional Analysis we arrive at the moment to determine the behaviour of mass in the Relativity Theory. Fortunately enough, there is complete agreement among the relativists regarding the point so that we can write directly:

$$M = \frac{M_0}{\sqrt{1 - v^2/c^2}} \quad . \tag{C}$$

7) Variance of Temperature.

We will take advantage, for the determination of the relativistic behaviour of temperature intervals, of the Einstein law for energy. Considering that we can ascribe a given mass to any amount of energy we may write for temperature transformations:

$$\theta = \frac{\theta_0}{\sqrt{1 - v^2/c^2}} \quad . \tag{D}$$

8) Relativistic Dimensional Invariants.

Now we proceed to the application of Dimensional Analysis to relativistic situations. If we consider a systems S_0 —possessing an observer that considers himself as at rest— it will be possible to set up a system of quantities $L - M - T - \theta$ that will transform according to the Following equations:

TABLE I

$$L = L_0 \sqrt{1 - v^2/c^2} \tag{A}$$

$$T = T_0 \sqrt{1 - v^2/c^2} \tag{B}$$

$$M = M_0 / \sqrt{1 - v^2/c^2} \tag{C}$$

$$\theta = \theta_0 / \sqrt{1 - v^2/c^2} \tag{D}$$

With the help of the four above transformations we can build up the following *Relativistic Dimensional Invariants* (3) :

TABLE II

$$LM - L/T - L\theta - MT - M/\theta - T\theta$$

These invariants being such owing to mutual compensation of their relativistic variances; namely, $LM = L_0M_0$, $L/T = L_0/T_0$, etc.

It is easy to deduce that different powers of the above invariants will continue to be invariants for any observer regardless his state of relative motion. In consequence, the above invariants will appear as "universal constants" for any observer whatever be his physical condition. This *aprioristic* determination of "universal constants" is confirmed by the experience since, according to TABLE IV, the structure of all the "CONSTANTS OF NATURE" heretofore known obeys the rule of formation we have theoretically developed. Comparing the constants of TABLE IV with those appearing in TABLE III (determined experimentally) we may observe their absolute quantitative and qualitative coincidence.

Summarizing the theoretical and experimental facts deducible from TABLE III and TABLE IV we can say that the Relativistic Dimensional Analysis has made possible the obtainment of an unique formula for all the Constants of Nature:

$$L^{(h+k-j)} M^{k} T^{-h} \theta^{-j} = nr^{-j} \tag{14}$$

$$\begin{aligned}&(k = 0,1\\&(h = 0,1,2,3)\\&(j = 0,1,2,3,4,5)\end{aligned}$$

where n and r are numerical constants.

9) Universal and Empirical Constants.

It is hardly possible for physicists to think that TABLE IV be the result of some simple coincidence. On the contrary, such regular arrangement of all the constants of Nature should be the argument to prove the correctness of our selection of the Lorentz metrical transformations that lead to TABLE I if it were not the case that the opposite selection would have lead to an arrangement equally suitable. It is only on an experimental basis that we may decide between the two possible systems of conventions, and, as we have seen, our only foundation for the system exposed in TABLE I is the experimental evidence regarding the formula for mass transformation: the experimental fact that mass increases with velocity.

We may wish to have some theoretical argument to stress the experimental evidence that proves the correctness of our selection between two equally suitable systems of metrical Lorentz transformations. In trying to find such argument our first attempt is directed toward the proposition of Planck ([15]) about the so called "absolute units". Bus as soon as we try to obtain absolute units out of the arrangement of constants of Nature shown in TABLE IV we meet with the fact that "it is impossible to determine a system of absolute units with the help of all the constants of Nature".

This curious circumstance is so general that we may raise it to the cathegory of a principle determining the structure of the constants of Nature. In trying to analyze it more deeply we must remember the following well known Theorem: "A system of monomes constituted by products and quotients of unknowns has a solution if, and only if, the number of independent monomes is equal to the number of unknowns".

Actually, as it may be observed upon inspection of TABLE IV, in the constitution of the constants of Nature, the number of independent monomes amounts only to three ($LM - L\theta - T\theta$) and the number of unknowns ($L - M - T - \theta$) is four. This is the reason why it is impossible to materialize the project of Planck. Analyzing further the origin of this undeterminacy of the system of constants of Nature we find as an explanation the absence of the invariant (MT) in TABLE IV.

Bridgman ([16]) has given an excelent account of the possibilities of forming systems of absolute units remarking, at the same time, that a necessary requirement is the presence, in the systems of constants selected to this end, of the constant of gravitation. I wish to add that with the help of any empirical constant it is possible to obtain other types of "absolute" units, and in TABLE V I have grouped as "Empirical Constants" all those constants leading to the determination of such units.

I have no idea about the reason why the invariant (MT) does not appear in physical laws but this very fact is responsable for the extra degree of freedom the system of natural laws possess which leads to many special features we will analyze later on in dealing with some epistemological problems of the physical theory.

I consider that at this height of our development we are in a position to enunciate the following:

Principle of Variational Homogeneity: The two members of any physical equation have the same variation in regard to metrical Lorentz transformations.

As stated at the beginning, the requirement of "dimensional homogeneity" gives automatic validity to that requirement but it may be observed that in order to give physical validity to the Principle of Dimensional Homogeneity it has been necessary to introduce artificially in the expression of the physical laws the so called "dimensional constants". But the very important fact is that if we write the laws of physics without giving dimensions to the dimensional constants the "principle of variational homogeneity" is still valid.

This observation leads us to the enunciation of a theorem which I wish to call "*The Second Theorem* π".

"If we write the physical laws in a purely observational form in such a way that we equate the resultant dimensional monome with its pure numerical value, the numbers so obtained are invariant under Lorentz transformations".

10) Tolman's Principle of Similitude.

We can write TABLE I in a purely qualitative way in the following form:

TABLE V

$$L = L_0 x \quad , \; T = T_0 x$$
$$M = M_0 x^{-1} \quad , \; \theta = \theta_0 x^{-1}$$

where x is a simple numerical factor.

The fact that this table condenses just what Tolman [17] [18] [19] called "Principle of Similitude" must attract our attention. In other words, TABLE V shows that the conclusions of the Relativistic Dimen-

TABLE III

CONSTANTS OF NATURE

1)	e^2	$= 2.56650 \times 10^{-40}$	e.m.u.	Electron Charge (square)
2)		$= 2.7989 \times 10^{-39}$	gr cm	Band Spectra Constant
3)		$= 4.0270 \times 10^{-39}$	gr cm^2 deg	Rot. Specific Heat Constant
4)	h	$= 6.62517 \times 10^{-27}$	erg deg^{-1}	Planck's Constant
5)	e^2	$= 2.30674 \times 10^{-20}$	e.s.u.	Electron Charge (square)
6)	k	$= 1.38044 \times 10^{-16}$	erg deg^{-1}	Boltzmann's Constant
7)	e_o	$= 2.07066 \times 10^{-16}$	erg deg^{-1}	Mol. Translational Energy
8)	k'	$= 2.76 \times 10^{-16}$	erg deg^{-1}	Mol. Surface Energy
9)	S_m	$= 7.69352 \times 10^{-16}$	erg deg^{-1}	Mol. Sackur-Tetrode Constant
10)	c_1	$= 1.24795 \times 10^{-15}$	erg cm	First Radiation Constant
11)	a	$= 7.5699 \times 10^{-15}$	erg cm^{-3} deg^{-4}	Energy Dens. of Encl. Radiation
12)	B	$= 4.79931 \times 10^{-11}$	sec deg	Atom. Specif. Heat Constant
13)	c'_1	$= 5.95442 \times 10^{-6}$	erg cm^2 sec^{-1}	First Radiation Constant
14)	c''_1	$= 1.19088 \times 10^{-5}$	erg cm^2 sec^{-1}	First Radiation Constant
15)	σ	$= 5.67283 \times 10^{-5}$	erg cm^{-2} sec^{-1} deg^{-4}	Stefan-Boltzmann's Constant
16)	C_I	$= 1.301 \times 10^{-4}$	erg cm^{-3} sec^{-1} deg^{-5}	Intensity Coefficient
17)	c_2	$= 1.43880$	cm deg	Second Radiation Constant
18)	c_3	$= 0.41784$	cm deg	Third Radiation Constant
19)	A	$= 0.28978$	cm deg	Wien's Displacem. Constant
20)	L	$= 2.55 \times 10^{8}$	$volt^2$ deg^{-2}	Wiedemann-Franz's Law Const.
21)	c	$= 2.99793 \times 10^{10}$	cm sec^{-1}	Light Velocity
22)	N'	$= 7.24405 \times 10^{15}$	cm^{-2} gr^{-1} sec^2 deg	Second Avogadro's Number

ADIMENSIONAL NUMBERS

1)	α	$= 1/137.0373$	Fine Structure Constant
2)	β	$= 4.96511423$	Prop. Constant: c_2/A
3)	γ	$= 3.44339$	Prop. Constant: c_3/A
4)	δ	$= 5.57324$	Prop. Constant: S_o/R_o
5)	N_o	$= 6.02486 \times 10^{23}$	Avogadro's Number

EMPIRICAL CONSTANTS

1)	G	$= 6.66 \times 10^{-8}$	cm^3 gr^{-1} sec^{-2}	Gravitational Constant
2)	A_o	$= 3.60 \times 10^{11}$	e.s.u. cm^{-2} sec^{-1} deg^{-2}	Richardson's Constant
3)	D	$= 0.270$	$cm^{3/2}$	Debye-Hückel's Constant
4)	A'	$= 2.33 \times 10^{-6}$	$cm^{-7/4}$ $gr^{-1/4}$ sec^2	Langmuir's Constant

NOTE: Values c, e, h and k are taken from "Atomic Constants, 1955". Cohen, Du Mond et al. Rev. Mod. Phys. 4, p. 363, 1955. Values a and σ from Birge's "Least Square Adjustment". Rev. Mod. Phys. 13, p. 233, 1941. (No correction made for the new (1954) thermodynamic scale of temperature). Value C_I belongs to I.C.T.

TABLE IV

CONSTANTS OF NATURE

Number	Symbol	DIMENSIONAL INVARIANTS					Adimensional Factor	VALUE (Calculated)
		LM	LT^{-1}	$L\Theta$	$M\Theta^{-1}$	$T\Theta$		
1	e^2	(LM)					$\alpha/2\pi$	2.56659×10^{-40}
2		(LM)					$1/8\pi^2$	2.79889×10^{-39}
3		(LM)		$(L\Theta)$			$1/8\pi^3$	4.02704×10^{-39}
4	h	(LM)	(LT^{-1})				1	6.62517×10^{-27}
5	e^2	(LM)	$(LT^{-1})^2$				$\alpha/2\pi$	2.30674×10^{-19}
6	k		$(LT^{-1})^2$		$(M\Theta^{-1})$		1	1.38044×10^{-16}
7	e_o		$(LT^{-1})^2$		$(M\Theta^{-1})$		3/2	2.07066×10^{-16}
8	k′		$(LT^{-1})^2$		$(M\Theta^{-1})$		2	2.76088×10^{-16}
9	S_m		$(LT^{-1})^2$		$(M\Theta^{-1})$		$-\delta$	7.69352×10^{-16}
10	c_1	(LM)	$(LT^{-1})^2$				2π	1.24795×10^{-15}
11	a			$(L\Theta)^{-3}$	$(M\Theta^{-1})$	$(T\Theta)^{-2}$	$4\gamma^3$	7.56899×10^{-15}
12	B					$(T\Theta)$	1	4.79931×10^{-11}
13	c'_1	(LM)	$(LT^{-1})^3$				1	5.95442×10^{-6}
14	c''_1	(LM)	$(LT^{-1})^3$				2	1.19088×10^{-5}
15	σ				$(M\Theta^{-1})$	$(T\Theta)^{-3}$	γ^3	5.67283×10^{-5}
16	C_1			$(L\Theta)^{-1}$	$(M\Theta^{-1})$	$(T\Theta)^{-3}$	γ^4	1.35764×10^{-4}
17	c_2			$(L\Theta)$			1	1.43880
18	c_3			$(L\Theta)$			$1/\gamma$	0.41784
19	A			$(L\Theta)$			$1/\beta$	0.28978
20	L		$(LT^{-1})^2$		$(M\Theta^{-1})$	$(T\Theta)^{-1}$	$2\pi\gamma/\alpha$	2.55661×10^{8}
21	c		(LT^{-1})				1	2.99793×10^{10}
22	N′		$(LT^{-1})^{-2}$		$(M\Theta^{-1})^{-1}$		1	7.24405×10^{15}

sional Analysis were enunciated in a qualitative form by the great physicist Tolman many years ago. In general lines it can be said that the proposition called "Principle of Similitude" is simply a corollary of the "Principle of Variational Homogeneity". I am quite sure that sooner or later the discussion about the "Principle of Similitude" of Tolman will be reopened (*) and at that time there will be found of importance the following theorem — which I only state since its demonstration is easy taking into account the "Second Theorem π":

THEOREM: All the determinants obtained out of the matrix formed with the dimensional exponents of the constants of Nature vanish.

This theorem, which is a verifiable principle of Nature, is also another form of stating the "Principle of Variational Homogeneity". And it must be observed that the "Principle of Similitude" of Tolman, the "Relativity of Size" of Hoffmann ([20]) ([21]), the Straneo proposition ([22]) trying to impose on the constant of Nature requeriments identifiable with systems of homogeneous linear equations, and, also, Maizlisch's ([23]) "Principle of Projective Covariance" must be considered as corollaries of the principle. Along similar lines we can include the eletromagnetic setting up of the "unified field" theory of Weyl among the corollaries of the P.V.H.

11) The Principle of Equivalence and Newton's Law.

Tolman proved that with his Principle of Similitude the aprioristic qualitative determination of all the laws of Nature was possible. He also pointed out that this was not the case for Newton's law of gravitation; and the main objection against Tolman's Principle of Similitude was that with its help the aprioristic determination of Newton's law of gravitation was impossible. Owing to this circumstance the Principle of Tolman was rejected as a physical principle. It is surprising to observe that nobody, at that time, considered the other possibility: that the "constant" of the gravitational law were not a real constant of Nature.

The equivalence between gravitational mass and inertial mass, implicit in the Newton law of gravitation, has always appeared as demons-

(*) The importance of the Principle of Tolman as a cosmological principle has recently been stressed by the developments of Hoffmann in his "Relativity of Size". In this respect Tolman's Principle must be considered as the limiting Euclidean case of the Conformal Geometry of Hoffman.

trated by the existing isochronism of penduli formed out of different materials. Later on, Eötvöss confirmed the result of the isochronical experiments with the help of his famous balance, demonstrating that the relationship existing between masses of different substances in regard to the gravitational and inertial actions was invariant at the poles and at the equator. Taking into account that at the equator the action of the gravitational force is somewhat opposed by the centrifugal action due to the rotation of the Earth, this experimental determination was considered sufficient proof for the statement of a "Principle of Equivalence" of the gravitational and inertial masses.

All these experiments and analyses have been so amply discussed in the scientific literature that it is surprising that nobody observed that the experiments of Eötvöss —regardless their extreme accuracy— were performed under conditions we could qualify as static ones: i.e. the relationship between the masses (terrestrial mass and the mass in the balance) is referred to *masses at rest*. This is the reason why Eötvöss' experiments say nothing in regard to the relativistic behaviour of masses in relative motion (as, for instance, the masses of a planet and the Sun). In face of such facts —I have analyzed at full length in a previous paper ([24])— it is hardly dubious that a physicist will not qualify the relativistic applying of such experiments as "illegitimate extrapolation of experimental results".

We arrive at the conclusion that nobody has demonstrated that the "constant" of the gravitational law is really a constant of Nature, and that in spite of this lack of experimental evidence the constancy of such an entity is an accepted fact of modern physics. Let us think of the methodological mistake involved in the rejection of the Principle of Similitude on the basis of its inability for determining a "constant" that nobody has yet shown to be a constant of Nature.

12) The Perihelion Advance.

Let us consider the above quoted question from the standpoint of the "Relativistic Dimensional Analysis". In trying to express the gravitational constant in terms of relativistic dimensional invariants we write:

$$[G] = L^3M^{-1}T^{-2} = (LM^{-1})\,(LT^{-1})^2 \qquad (15)$$

what means that the constant of gravitation cannot be split into dimensional invariants because the factor (LM^{-1}) is not a relativistic invariant.

Confronting this result with the splitting of the constants of Nature shown in TABLE IV we are obliged to think of the special character of the constant of gravitation in face of all the other constants of Nature. Notwithstanding this evidence the acceptance of the constant of gravitation is so extended that it will be necessary to give additional demonstrations to the already expressed arguments.

Upon analyzing the "variance" of the constant of gravitation we find:

$$(LT^{-1})^2(LM^{-1}) = (L_0T_0^{-1})^2(L_0M_0^{-1})\ (1 - v^2/c^2),$$

according to TABLE I; i.e. that the actual invariant should be a gravitational constant with the following correction:

$$G_0 = \frac{G}{1 - v^2/c^2} = \frac{6{,}66 \times 10^{-8}}{1 - v^2/c^2}\ \mathrm{cm^3\,gr^{-1}sec^{-2}} \quad ; \qquad (16)$$

and we will call the above expression, really constant, the "corrected constant of gravitation".

In order to demonstrate its efficiency in handling physical situations, let us consider a Coulombian gravitational field to which, in the absence of external forces, we will apply Lagrange's equations. We arrive at the classical integrals for the energy and the angular momentum:

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 - \frac{2GM}{r} = k \qquad (17)$$

$$r^2\frac{d\varphi}{dt} = h\,.$$

Upon replacing for the corrected constant, — $G' = G/(1 - v^2/c^2)$ — and expanding in power series, we arrive, neglecting infinitesimals, at:

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right) - \frac{2GM}{r} = k + \frac{2GMr}{c^2}\left(\frac{d\varphi}{dt}\right)^2 \tag{18}$$

$$r^2\frac{dt}{d\varphi} = h\,,$$

which is the Einsteinian formula for planetary motion.

This shows that the Einsteinian correction consisted exactly in transforming the constant of gravitation into a Lorentzian invariant.

The ulterior transformation of the formula for the calculus of the perihelion advance, shows that the substitution of coordinate time for proper time does not amount to an appreciable value.

Multiplying the first equation by $dt/d\varphi$, and with the help of the second one, we arrive at:

$$\left(\frac{dr}{d\varphi}\right)^2 + r^2 = k\frac{r^4}{h^2} + \frac{2GMr^3}{h^2} + 2GMr \quad ;$$

by means of the classical device of making $r = 1/u$, we obtain,

$$\frac{d^2u}{d\varphi^2} + u = \frac{GM}{r}\left(1 + \frac{3h^2u^2}{c^2}\right). \tag{19}$$

If the term $3h^2u^2/c^2$ were not present, the integration of the equation should be immediate, leading to the classical 2π period. In order to obtain the value of the perturbation we can follow any of the well known methods, which lead to an angle for two successive perihelions of about

$$2\pi + 6\pi\frac{G^2M^2}{c^2h^2}\ .$$

Calculating per century, we arrive, for the case of Mercury, at an angle next to 43″ for the perihelion advance.

This outstanding result is the best proof that the accepted "constant of gravitation" is not a constant of Nature.

13) Epistemological Considerations.

In the above quoted [3] paper I have amply dealt with epistemological questions arising in relation with the Principle of Variational Homogeneity and its physical applications; notwithstanding this circumstance I will make here some remarks on the topic.

It is important to consider at the beginning of these analyses the following corollary of the P.V.H.

Corollary: The determination of the state of relative motion of two frames of reference, with the only help of the constants of Nature, is impossible. This corollary —whose statement is just the inverse proposition to that of the principle— could be enunciated as a principle leading to the "theorem" of variational homogeneity.

Starting from the existence of certain physical entities that appear as invariants under relativistic transformations (the so called "constants of Nature") we arrive at the conclusion that, in the universe, some *absolute* (invariant) entities exist which, in order to be observed, must be decomposed by us in *relative* (variant) elements (our dimensional units).

According to this *realistic* interpretation, the invariants (quantum of action, velocity of light in vacuum, Stephan constant, etc.) do exist by themselves in Nature, but in order to be able to observe them we are obliged to decompose these invariants in various quantities of relative nature; i.e. our four units of measurement: length, time, mass and temperature.

The opposite interpretation is also logically possible: Our quantities of measurement are four and they are altered under relativistic transformations. But there are combinations of these measuring units which, upon mutual compensation of their variance, appear as invariant entities. According to this *idealistic* interpretation, these special combinations are what we interpret as invariants, but this appreciation is only a "hypostasis" of the subjective phenomenon of de relativity of our measuring units — subjective in reference to the observer who does not need to be a human observer.

The circumstance that with the help of all the constants of Nature it is impossible to determine "absolute" units —owing to the vanishing of all the determinants formed out of the dimensional matrix of the constants of Nature— does not permit a decision in regard to the alternative: realism-idealism. In this way, the selection between realism and idealism, in physics, is impossible — due to the presence of the Principle of Variational Homogeneity.

On the other hand, it is interesting to observe that the endless discussion regarding the necessary number of units in physics is overcome by the statement that a measuring unit must possess, as one of its features, a relativistic variance. The statement that we can have no more than four units of measurement follows. And, for instance, some propositions trying to introduce, in electricity, the electric charge as a unit must be considered, from this viewpoint, as senseless.

Another important epistemological consequence of the P.V.H. is that Heisenberg's Principle of Uncertainty is nothing but a corrollary of the Principle of Variational Homogeneity. The appearance of a "Principle of Complementarity" ([3]) follows as the logical consequence of the two possible pictures of our philosophical scheme: realism or idealism.

In reference to the old question about the bidimensionality of electric charge, it is to be observed that both the electrostatic and the electromagnetic charge are invariant under Lorentz transformations. From the viewpoint of the P.V.H. there is no trouble in finding two dimensions for the "electric charge" since they are related by the velocity of light.

I want to conclude this introductory note on P.V.H. pointing out the great generality the Special Relativity may achieve in connection with the Relativistic Dimensional Analysis. Its scope may be enlarged to cover the whole physics as shown by the following statement: "Physics is the study of Lorentz transformation invariants".

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