A TREATY OF SYMMETRIC FUNCTION

Using Sum of Power for Arbitrary Arithmetic Progression for studies of Prime Numbers that coexist within the equation formed through a new conjecture of symmetric function rule of division

Paper Part V

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In remembrance of my beloved father who passed away on the 23rd of June 2009 and my special thanks to my brother Mohd Yunus Abd Shukor[2] for introducing me Fermat's Last Theorem when I was a teenager. Although I didn't get the proof for this theorem, it enhanced my understanding towards developing the generalized equations for Symmetric Function for Sums of Powers, Fermat Last Theorem, Riemann Zeta Function and Sum of power for non-integer R-th power. Lastly, to my sister Nazirah Abd Shukor, thanks for all the supports and patience.

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Abstract. A new approach in deriving Sum of Power series using reverse look up method, a method where a mathematical formulation is constructed from set of data. Faulhaber [1] derived a general equation for Power sums and calculated the terms up to p=17 (i.e. $\sum_{i=1}^{n} x_i^p$). However, these formulae only work for integers from $x_1 = 1$ to $x_n = n$. A depth study on Power series revealed a systematic general equation which applicable for all numbers with a condition that the series should be in an arithmetic progression without the power p (i.e. $\sum_{i=1}^{n} x_i$). The general formulation is given as follows

$$\sum_{i=1}^{n} x_i^p = \sum_{j=0}^{m} \left[\phi_k s^{2m} \frac{\left[\sum_{i=1}^{n} x_i \right]^{p-2j}}{n^{p-(2j+1)}} \right] \qquad [1]$$

Where, $s = x_{i+1} - x_i$, ϕ_m is a coefficient and $\phi_0 = 1$.

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1 Introduction.

The sum of power equation can be expressed into Diophantine equation if *n* is set to 2. When *n*=2, Equation [1] reduces into the form given as follows:

$$x_1^p + x_2^p = \sum_{i=1}^{2} x_i^p = \sum_{j=0}^{m}\left[\phi_k s^{2m}\frac{\left[\sum_{i=1}^{2} x_i\right]^{p-2j}}{2^{p-(2j+1)}}\right] = x_3^p \qquad [2]$$

Where: $p-(2j+1)\geq -1$ if *p* is even, $p-(2j+1)\geq 0$ if *p* is odd, $s = x_{i+1} - x_i$, ϕ_k is a coefficient and $\phi_0 = 1$ and $k = \begin{cases} \frac{p-1}{2}\ for\_odd\_p \\ \frac{p}{2}\ for\_even\_p \end{cases}$

Expanding [2] for odd and even *p* yields,

$$\sum_{i=1}^{n} x_i^p = \frac{\left[\sum_{i=1}^{n} x_i\right]^p}{n^{p-1}} + \phi_1 s^2 \frac{\left[\sum_{i=1}^{n} x_i\right]^{p-2}}{n^{p-3}} + \phi_1 s^4 \frac{\left[\sum_{i=1}^{n} x_i\right]^{p-4}}{n^{p-5}} + \cdots + \phi_{\frac{p-1}{2}} s^{p-1}\left[\sum_{i=1}^{n} x_i\right] \text{ for odd } p. \qquad [3]$$

$$\sum_{i=1}^{n} x_i^p = \frac{\left[\sum_{i=1}^{n} x_i\right]^p}{n^{p-1}} + \phi_1 s^2 \frac{\left[\sum_{i=1}^{n} x_i\right]^{p-2}}{n^{p-3}} + \phi_1 s^4 \frac{\left[\sum_{i=1}^{n} x_i\right]^{p-4}}{n^{p-5}} + \cdots + \phi_{\frac{p}{2}} n s^p \text{ for even } p. \qquad [4]$$

Let $\left[\sum_{i=1}^{n} x_i\right] = w$ and factorizing both equations yields:

$$\sum_{i=1}^{n} x_i^p = \frac{w}{n^{p-1}}\left[w^{p-1} + \gamma_1 w^{p-3} + \gamma_2 w^{p-5} + \cdots + \gamma_{\frac{p-1}{2}} s^{p-1}\right] \quad \text{for odd } p. \qquad [5]$$

$$\sum_{i=1}^{n} x_i^p = \frac{1}{n^{p-1}}\left[w^p + \gamma_1 w^{p-2} + \gamma_2 w^{p-4} + \cdots + s^p\right] \quad \text{for even } p. \qquad [6]$$

Setting *n*=2 for both equations yields

$$\sum_{i=1}^{n} x_i^p = x_1^p + x_2^p = \frac{w}{2^{p-1}}\left[w^{p-1} + \gamma_1 w^{p-3} + \gamma_2 w^{p-5} + \cdots + \gamma_{\frac{p-1}{2}} s^{p-1}\right] = x_3^p \quad \text{for odd } p. \qquad [7]$$

$$\sum_{i=1}^{n} x_i^p = x_1^p + x_2^p = \frac{1}{2^{p-1}}\left[w^p + \gamma_1 w^{p-2} + \gamma_2 w^{p-4} + \cdots + s^p\right] = x_3^p \quad \text{for even } p. \qquad [8]$$

Rearranging these equations yields:

$$w\left[w^{p-1}+\gamma_1 w^{p-3}+\gamma_2 w^{p-5}+\cdots+\gamma_{\frac{p-1}{2}} s^{p-1}\right]-2^{p-1}x_3^p=0 \quad \text{for odd } p. \qquad [9]$$

Or

$$w^p+\gamma_1 w^{p-2}+\gamma_2 w^{p-4}+\cdots+\gamma_{\frac{p-1}{2}} s^{p-1}w-2^{p-1}x_3^p=0 \quad \text{for odd } p. \qquad [10]$$

$$w^p+\gamma_1 w^{p-2}+\gamma_2 w^{p-4}+\cdots+s^p-2^{p-1}x_3^p=0 \quad \text{for even } p. \qquad [11]$$

Further study on Symmetric Function Equation for even p reveals that equation [11] can be factorized by $\left(x_1^2+x_2^2\right)$ with a condition that $p=2\alpha$ and α is an odd number or in the modulo form, $\sum_{i=1}^{2} x_i^p \equiv 0\left(\bmod\left(x_1^2+x_2^2\right)\right)$ with $p=2\alpha$ and α is an odd number.

Therefore, equation [11] can be also expressed as follows:

$$\sum_{i=1}^{n} x_i^p = x_1^p + x_2^p = \frac{\left(w^2+s^2\right)}{2^{p-1}}\left[w^{p-2}+\gamma_1 w^{p-2}+\gamma_2 w^{p-4}+\cdots+s^{p-2}\right]=x_3^p \qquad [12]$$

Since,

$$\sum_{i=1}^{n} x_i^2 = \frac{\left[\sum_{i=1}^{n} x_i\right]^2}{n}+\frac{n(n^2-1)s^2}{12} \qquad [13]$$

Setting n=2, yields

$$\sum_{i=1}^{2} x_i^2 = \frac{w^2}{2}+\frac{s^2}{2}=\frac{w^2+s^2}{2} \qquad [14]$$

Thus,

$$\sum_{i=1}^{n} x_i^p = x_1^p + x_2^p = \frac{\left(x_1^2+x_2^2\right)}{2^{p}}\left[w^{p-2}+\gamma_1 w^{p-2}+\gamma_2 w^{p-4}+\cdots+s^{p-2}\right]=x_3^p \qquad [15]$$

Therefore,

$$\left(x_1^2+x_2^2\right)\left[w^{p-2}+\gamma_1 w^{p-2}+\gamma_2 w^{p-4}+\cdots+s^{p-2}\right]-2^p x_3^p=0 \quad \text{for even } p. \qquad [16]$$

If $p=\alpha 2^t$ then, $\sum_{i=1}^{2} x_i^p$ can always be divided by $\sum_{i=1}^{2} x_i^{2^t}$. Therefore,

$$\sum_{i=1}^{n} x_i^p = x_1^p + x_2^p = \frac{\left(x_1^{2^t}+x_2^{2^t}\right)}{2^{p-1}}\left[w^{p-2}+\gamma_1 w^{p-2}+\gamma_2 w^{p-4}+\cdots+s^{p-2}\right]=x_3^p \text{ for } p=\alpha 2^t \qquad [17]$$

By setting $t = 2$, yields

$$\sum_{i=1}^{n} x_i^{4\alpha} = x_1^{4\alpha} + x_2^{4\alpha} = \frac{\left(x_1^4 + x_2^4\right)}{2^{4\alpha-4}}\left[w^{4\alpha-4} + \gamma_1 w^{4\alpha-6} + \gamma_2 w^{4\alpha-10} + \cdots + s^{4\alpha-4}\right] = x_3^{4\alpha}, \ \alpha \text{ is an odd number.} \quad [18]$$

Or in the modulo form,

$$\sum_{i=1}^{2} x_i^{p} \equiv 0\left(\mathrm{mod}\left(x_1^{2^t} + x_2^{2^t}\right)\right) \text{ with } p = \alpha 2^t, \ (t \in N) \text{ and } \alpha \text{ is an odd number.} \quad [19]$$

It is further conjectured that, if $p = \alpha r^t$ then,

$$\sum_{i=1}^{2} x_i^{p} \equiv 0\left(\mathrm{mod}\left(x_1^{r^t} + x_2^{r^t}\right)\right) \text{ with } p = \alpha r^t, \ (t \in N) \text{ and } \alpha \text{ is an odd number.} \quad [20]$$

2 Rule of Division.

The symmetric function above can be simplified as follows:

$$\frac{x^{t\gamma} + y^{t\gamma}}{x^t + y^t} = p_s \quad [21]$$

Where, t is the multiplier and γ can be divided into two folds. If γ is an odd number p_s is always an integers and if γ is an even number, p_s will fall into special cases.

Special Cases.

Let $q = 2$, there are two cases.

Case 1:

$$q = tp = 2x1$$

$$\frac{(m)^{2x1} + (m+s)^{2x1}}{(m)^2 + (m+s)^2} = p_s = 1 \quad [22]$$

Case2:

$$q = tp = 1x2$$

$$\frac{(m)^{1x2} + (m+s)^{1x2}}{(m)^1 + (m+s)^1} = p_s \quad [23]$$

Not to lose generalization, let $m = x$ and $m + s = y$ .Thus

$$\frac{(m)^{1x2} + (m+s)^{1x2}}{(m)^1 + (m+s)^1} = \frac{(x)^{1x2} + (y)^{1x2}}{(x)^1 + (y)^1} = p_s \quad [24]$$

From Sum of Power Formula,

$$x^2 + y^2 = \frac{\left(w^2 + s^2\right)}{2} \qquad [25]$$

Where,

$x + y = w$ and $y - x = s$

Therefore,

$$\frac{(x)^{1x2} + (y)^{1x2}}{(x)^1 + (y)^1} = \frac{\left(w^2 + s^2\right)}{2w} = p_s \qquad [26]$$

Expanding equation [26], yields

$$w^2 + s^2 = 2wp_s \Rightarrow w^2 - 2wp_s + s^2 = 0 \qquad [27]$$

Solving equation [27], yields,

$$w = p_s \pm \sqrt{p_s^2 - s^2} \qquad [28]$$

Since $x + y = w$ and $y - x = s$,

Thus,

$$x = \frac{p_s - s \pm \sqrt{p_s^2 - s^2}}{2} \text{ and } y = \frac{p_s + s \pm \sqrt{p_s^2 - s^2}}{2} \qquad [29]$$

For composite or prime numbers p_s, s and p_s must be part of Pythagorean triple. Otherwise, p_s would be non-integer division (i.e. fraction).

Example:

Let the Pythagorean triple be

$3^2 + 4^2 = 5^2 \Rightarrow 3^2 = 5^2 - 4^2$

In this case, $p_s = 5$ and $s = 4$

Thus,

$$x = \frac{5 - 4 \pm \sqrt{3^2}}{2} = \frac{1 \pm 3}{2} = 2, -1 \text{ and } y = \frac{5 + 4 \pm \sqrt{3^2}}{2} = \frac{9 \pm 3}{2} = 6, 3 \qquad [30]$$

Therefore,

$$p_s = \frac{(x)^2 + (y)^2}{(x) + (y)} = \frac{(2)^2 + (6)^2}{(2) + (6)} = 5 \text{ and } p_s = \frac{(x)^2 + (y)^2}{(x) + (y)} = \frac{(-1)^2 + (3)^2}{(-1) + (3)} = 5 \qquad [31]$$

Case 2.

It is also found that Fermat's Prime can be formed using this generalized equation with a special case of multiplier equal to 1 (i.e. *t*=1) and $p = 2^n$.

Fermat's Prime is given as follows:

$$2^{2^n} + 1 = P_s \quad [32]$$

$$2^{2^n} + 1 = \frac{x^{1\times 2^n} + y^{1\times 2^n}}{x + y} = p_s \quad [33]$$

Setting $x = 2$ and $x = -1$ thus reducing equation [33] into,

$$2^{2^n} + 1 = \frac{2^{1\times 2^n} + (-1)^{1\times 2^n}}{2 - 1} = p_s$$

$2^{2^n} + 1 = 2^{2^n} + (-1)^{2^n} = p_s$, Since $(-1)^{2^n} = 1$, then

$$2^{2^n} + 1 = 2^{2^n} + 1 = p_s \quad [34]$$

Another interesting relationship found throughout this research is that, if power *tp* of an odd number plus a power *tp* of an even number divides by power *t* of an odd number plus a power of *t* even number it will only give prime number if *p* is a prime number. This relationship can be seen as follows:

$$\frac{x^{tp} + y^{tp}}{x^t + y^t} = p_s \quad [35]$$

Where t is the multiplier, p is the primitive prime and p_s is the symmetric prime formed. Another fascinating character of this symmetric function is that it governs the factorization of Fermat's Last theorem. The example is given as follows:

Let $tp = 120$, which is $tp = 2\times 3\times 4\times 5$, therefore, t can be chosen to be 8 and 120. While p must be only an odd number which are 1 and 15.

Examples:

Case 1.
$t = 8$ and $p = 15$, thus

$$\frac{x^{8\times 15} + y^{8\times 15}}{x^8 + y^8} = p_s$$

p_s will always be an integer whatever the value of *x* and y we chose. Let $x = 1$ and $x = 2$, thus

$$\frac{1^{8\times 15} + 2^{8\times 15}}{1^8 + 2^8} = 5172093368812902229197692841557761 \quad [36]$$

This characteristic also works on the odd number power. Lets consider the power to be $tp = 3 \times 5 \times 7$, then

Case 1.

$$\frac{x^{3\times 35} + y^{3\times 35}}{x^3 + y^3} = p_s$$

p_s will always be an integer whatever the value of x and y we chose. Let $x = 1$ and $x = 2$, thus

$$\frac{1^{3\times 35} + 2^{3\times 35}}{1^3 + 2^3} = 4507202134144815649766055841337 \quad [37]$$

Case 2.

$$\frac{x^{5\times 21} + y^{5\times 21}}{x^5 + y^5} = p_s$$

p_s will always be an integer whatever the value of x and y we chose. Let $x = 1$ and $x = 2$, thus

$$\frac{1^{5\times 21} + 2^{5\times 21}}{1^5 + 2^5} = 1229236945675858813572560684001 \quad [38]$$

Case 3.

$$\frac{x^{7\times 15} + y^{7\times 15}}{x^7 + y^7} = p_s$$

p_s will always be an integer whatever the value of x and y we chose. Let $x = 1$ and $x = 2$, thus

$$\frac{1^{7\times 15} + 2^{7\times 15}}{1^7 + 2^7} = 314455962847312719751120174977 \quad [39]$$

3 Prime Number Formation through the rule of Division Method.

We conjectured that all prime numbers can be expressed into generalized symmetric prime equation given as follows:

$$\frac{x^{tp} + y^{tp}}{x^t + y^t} = p_s \quad [40]$$

Where t is the multiplier and p is an odd or prime number and not of the form 2^n, otherwise [40] would be a non-integer division or fall into special cases as discussed before. The prime number p_s existed with the condition that p also must be a prime number. In this work, we use the SIQS (Self Initialization Quadratic Sieve) algorithm to verify the primality of the number generated by equation [40] for integers less than 90 digits. It is demonstrated by Contini [3] that SIQS algorithm to be faster method in testing the primality of a number compared to other methods such as qs (quadratic sieve) and nfs (number field sieve) for this range. For bigger integers, the primality is done through EMC

(Elliptic Curve Method), developed by Lenstra in 1987 [4]. The calculation of the numbers generated in this paper was done and verified to be prime by an algorithm written by Alpern [5] using SIQS and EMC (Using Rabin probabilistic primality algorithm). Throughout this research discovered how Mersenne and Wagstaff numbers are formed. Therefore, equation [40] can be used as a generalized equation for Mersenne and Wagstaff numbers. This equation explains how derivation of both numbers can be carried out, they are as follows:

Consider Mersenne's Number:

Let, $x = 2$, $y = -1$ and $t = 1$, thus equation [40] reduces to:

$$\frac{2^p + (-1)^p}{2-1} = P_s$$

Since it is conjectured P_s would only be a prime number if *p* is also a prime number. Therefore,

$$2^p + (-1)^p = 2^p - 1 = P_s \qquad [41]$$

Consider Wagstaff's Numbers:

Let, $x = 2$, $y = 1$ and $t = 1$, thus equation [40] reduces to:

$$\frac{2^p + (1)^p}{2+1} = P_s$$

Since it is conjectured P_s would only be a prime number if *p* is also a prime number. Therefore,

$$\frac{2^p + 1^p}{3} = \frac{2^p + 1}{3} = P_s \qquad [42]$$

A thorough studies of prime numbers generated using the generalized equation [40] can be seen as follows:

List of Primes p_s for $p < 200$ is given as follows:

p	$\frac{(m)^{2p} + (m+s)^{2p}}{(m)^2 + (m+s)^2} = p_s\,(m=1, s=1)$	$\frac{(m)^{2p} + (m+s)^{2p}}{(m)^2 + (m+s)^2} = p_s\,(m=2, s=1)$
3	13	61
5	-	4621
7	-	369181
11	-	2414250301
17	-	1282861452271981
19	-	103911691734684541
41	-	10232918959454754965754056541 3396038701
53	-	28900785585664327723593061693 364968422740414514061
109	-	79157154965793818030763743420 89862963295414837600820914397 69502729616807465277868108109 2369443226449741
167	-	17561262072278303301240596936 88537186295145437781718150270

		0500823992087865451604299036663991461393987574618047014097448920831235045345304693324729403113481398 1
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\,(m=3, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\,(m=4, s=1)$
3	193	-
5	-	263761
25	1980707608031856300839037 13	-
71	-	43747854495995047158162182396981155108030352729376743081913957014909088166742863228572199132670401
113	-	22617110674923301132513417306897492084009246012163654422896496276758665997891521419053583486997567147605290740295475412859807654870248388656115611856 73 904881
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\,(m=5, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\,(m=6, s=1)$
3	1021	1933
5	-	-
7	1384716061	-
17	470645699290500173623566 1	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\,(m=7, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\,(m=8, s=1)$
3	3361	-
5	12 001921	-
17	45351498734078493076599 756481	-
19	18494798447421622596671820763 3441	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\,(m=9, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\,(m=10, s=1)$
3	8461	12541
5	-	162612781
7		2170813726621
53	5524939866388874134675852028334013268928542405886912471063168632095161096714064684493609236905439029246 1	-

199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=11, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=12, s=1)$
17	19538535628569028655049557316 533617	-
31	-	37322485120150432013139919634 78174506792744440976994034317 605922401
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=13, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=14, s=1)$
5	-	2056781581
41	-	65571460586120320880453648985 58642156848825362517026665512 93838965979170707707779429762 6434701
67	10444437432441864266174817087 07911062668406267919769633936 27594661273320978942020390684 35226612585954278544568204512 17291512728296170037236900917 2926717	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=15, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=16, s=1)$
17	20128832288030146264909828436 7994932481	
31	-	36428835489931260845060260628 46301822468412411282942332526 2868358547442881
37		20974317935115101939914571968 40093848789283880015774538405 50401340595303930753893215189 77
61		23913233113665604487330325391 66866420195166877075467856091 63907911878925426918306710519 91287676802707209299040945822 23401061093017899821171499141 761
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=17, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=18, s=1)$

7		1713605917321117
11	8654145857477125265076301	-
41	14088617362008386606891377147 47773601370434145709381185995 98209630812293824361922550264 30 449610322701	-
59	-	11427992590464176645909734938 61783483297738029824407948162 82973009018827690231456098472 52910481577441977721207761796 34568741861697721788729305945 7469
157	23340283932133970783690279953 28434167906933678727216208648 09868399152428781963462488334 07438711527623368027462909183 37895249730327959010055119827 06810481872587513980350194822 13887233416605392207006825066 16810083048209783607074741500 92105236343472152134827769938 41354942500910191906923091443 28855256570433880269628733788 97648429900831970702955676520 45707882234638455042615021847 624453416119181	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=19, s=1\right)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=20, s=1\right)$
5	21512570641	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=21, s=1\right)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=22, s=1\right)$
3	-	258061
5	-	67113952621
7	-	17586039089799181
13	11226296683698359593412329352 3053	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=23, s=1\right)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=24, s=1\right)$
3	306913	-
7	-	48533408083087201
31	36283434709630407242293021480 19734123734585986817363309492 7937696223495980842642081	-
41	14028438767717718266173845557	-

	94438483005377552085891747347 82331573098998982482055949738 4160984709964496319734441	
89	43098424925501256706394122891 69604736234249722050169957585 97687361644447632697061544929 18356179673945288563295288424 84251581348271158997952881002 69015438207875775708309397828 02162709885444957738816957917 10875795980136965805348628669 8264345409	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=25, s=1\right)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=26, s=1\right)$
3	425101	495613
7	-	123792856821928477
31	44735751480900486511514550665 73250129840062789952109050505 7572621246678345366805915O1	43333245731510916944699473263 38195349740507878246616176363 105405237478937510421O175581
41	85234816423434277783366263863 50819481760215721720909275365 11182328592918063649344670940 7241724040817059097317950 1	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=27, s=1\right)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=28, s=1\right)$
7	-	295149140003533537
17	-	42239199681237610980312515978 384692682858336497
71	20848632522832149090486839893 33773281583975342902336809810 87265950738989587076571740663 47524274264822991917592628314 96733361716308711479068192162 76198786800899508662043735466 60519 29718127066343819356960 1	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=29, s=1\right)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=30, s=1\right)$
13	-	45696031704501456654968121115 6262221
19	98986197364403461116588685841 419141312794946135153021	-
37	-	13421085079201284716584418387 71062228272457159130090784081 90361949343722236296525644094

		29853690233227485166 1
61	94380524801729596470195542277 88713135638457203126973398862 47734795574253725885907176814 86650949840177702480353549480 58877500132960285435174849098 24426470991305776171200960807 501	-
79	14008882893286713664575258234 66627593633218375796815631229 22977945106432158533221693564 53270150562962900555813598853 05484865415318541081571622477 56838571278415431070616173835 82984321348573465735128299873 7476261220623857786549443421	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=31,s=1\right)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=32,s=1\right)$
3	988033	-
5	-	1257680497921
103		30884349892149219932502641963 02041958596623597721568919313 19407573297847077020095226747 61849023089208028015704219615 34452952952781172863542045603 45065394692224303975673337463 25820379607186149964910415810 90526969059575175768053209146 12153204939570277911222924273 99944276386995891840100730005 39976938833978367361
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=33,s=1\right)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=34,s=1\right)$
3	1263373	-
7	-	2897205977622263581
17	-	18167514982204958977035797695 5585383401760432383 81
19	-	26451419085700882060126774432 3071170634580368928351901 41
29	35103145756599100428795431325 77378801597638308044726303786 5848841576499590833724577869	-
59	23764633562030008893304868912 91152609075277748601734546906 99867092907078497480153499904 72126396022911487266323610494	-

	27916161757737517082514837809 16461080859313430697825059626 2349	
83	-	87509142354111578122067781749 23151761579277927324037549327 27981490441855847504745267750 61653052098761735481745435950 38740278607191044077964860418 30708561912896122277850392540 81683760665175803773964815924 06012335129515870447395435821 934212689367158331261
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ \left(m=35, s=1\right)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ \left(m=36, s=1\right)$
7	4077940236745021921	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ \left(m=37, s=1\right)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ \left(m=38, s=1\right)$
3	-	2202253
5	3941271319021	-
47	12148590845316717071511827383 48755524420141460588370042394 72451620720997165739761438801 56757504268932754703461028435 40134819265920354442791705238 1	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ \left(m=39, s=1\right)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ \left(m=40, s=1\right)$
7	14635019158645223521	-
47	-	13376543239088841507318024812 63957434832773729802081535931 85979304066402432264260392993 14489812676037275939254972072 50089854494138104370639751901 9681
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ \left(m=41, s=1\right)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ \left(m=42, s=1\right)$
3	-	3268861
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ \left(m=43, s=1\right)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ \left(m=44, s=1\right)$

5		15462759367441
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=45, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=46, s=1)$
5	18466886355661	-
19	5298798825318304403180434817 2889194411721864767103668490 0221	-
31	3739898783908478661314012640 3424772720922079172579965732 3252479386811739621612385948 3565212853630301	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=47, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=48, s=1)$
101	91706204305905989941540782751 52052665777818294416003522339 47908696948942284784621827378 43762959092013443323698735929 49596206453188640353455719055 37756329456494741488770647634 50068582082473925802790263004 97943152725720999491668942820 08449770540298793313910155756 67904852274525771346820360907 64223478901017557763350545876 89806947305328001	-
199		-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=49, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=50, s=1)$
23	-	97133359110274989225012407377 26785457708086871373729724369 84828850346364701
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=51, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=52, s=1)$
3	7043713	7606561
59	-	58449346083074509721877457991 91330617529403251599679383752 20659856961024599082394452502 85866671625893618162542090982 22365391608032013748466216632 43987198367446028795395263876 22122595318041461234950241
181		27984416358436478653947616961

		7310669858566262499909431691381662670793796212526809379675794064458100165681997264383712538991295088845859676117938769328092907021217894184592203542244099569304216499782350081414245331043271696542730427670989873971840736639269363494582578300891744676083823852617622431353723426824749720642484144850075274021603465405693084134054679659290383174854294294026651132011036765702448900804549982121639693002740053753693759337875945914586786231585083751948843522479586425158992093728126299048217144484707247485122953757964965138495308222384343346018501667784389116506469543812437469571071250538920401
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=53, s=1\right)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=54, s=1\right)$
23	-	274251119234901428012747052539386500184450964330913740317256923161939844615 81
29	-	1976488801293125523250238888499626949743291090501582055373673683882708432856716140151805645 2231021
73	158189516852569578333134563559620119842101800565015314661382397944794032243863980655892501008818349538809965144018079993118932153164882910505456427762036414622976911635948003177220556558685211304211208662336708841744658714958180600299044398575684481 3	-
181	-	1729816185490061364064692709183677005952805931310078648437265770894737608922701614377511886438696697753558175826345186989999282121733481858684367421999919342400914720248306439635851577780297561293937764599427998425912674777308938100652947146608971645151718486570996284241977104472991626373 5

		51957648540977045023871259709 74025080107224576415716923160 92273971820635638239935304757 42799270803247561572076023487 22674997275728470382271829292 35972690724052067803449932062 82976018829773486120218825116 81233357009979386981681428503 95576529095357451679106882084 72934096523934841691970727804 03182049085761664035721020869 735542340779864301
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=55, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=56, s=1)$
3	-	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=57, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=58, s=1)$
29	39215481263146260885746286342 40838759883308862434092330280 48238761697464542342439311990 757719309741	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=59, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=60, s=1)$
5	157573579056721	180030688657681
109	62786914078115778598686552318 83087727035019478700517322824 80354390051371962828897015931 99159251467407598583633124598 05244933891231859551596088038 81262065585621028172037847816 24188461693332978239622295120 98565740841160196734350688004 08738774811458412000135653497 28066395314831167675997071952 91990694072307652950648968943 86907473790114515742729792909 26930785668115684245465641288 7239441	
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=61, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=62, s=1)$
3	-	15272461

197	2109759566602708789506076237907057941384341161563943829388885169958505946565482714606084881201843241102382160249293050338242436135993326724198234112001037130649314786689597298026659445854058286785024325242349277210009466419319656109755562190902334410321094767257722449785113495164121098515028043067306424290329591349429210019037403623516225151615832334803630910119351203723191803040325909889908834193091574078401759980118848259600294864226399513271101798059079618188706595665712363670139097536477890604765649617570766658749231745479420882903143624314779557404549747629439873256740231478607538833227601364935921451286044508835638473751598305498591132647565086722484989732159626179534119989801070128509197	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \; (m=63, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \; (m=64, s=1)$
3	-	17322241
7	-	5212672924583067390721
79	-	3598269800835201928204571124494718115124334483771963550440044202856663930564957206827500464597076792963745025105193064525769118654621993944654485290595368991313812000751065190410483194152865358081321166711511552422032884435239999502927093348355922977109946138052191327574384382293761
107	42716147697395236591810099483229518480536732887836887001769177338465358932905023797839359336486105320897648768386130218695513308750124483789021736308246338311208395607747030356247115960412147488225105025436621390677479537014335761525886139395158670243005866713041412168092799176553235953916	-

	234895250555836989334476194764352226524959000334826005242391812541340591196858953009315289857	
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=65, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=66, s=1)$
5	339658689590221	-
7	-	7517354826143887520797
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=67, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=68, s=1)$
3	-	22033633
7	-	10724214748546028673217
31	-	1526610826078842024587312565460233846566669139185443729437701689884030987462526379710805905924916810316512716 81
149	-	102367431873588273368080400112896643218898977047890205876635755912112445549976023236526466149351436921954078979489949559904078770701660573408263560661260136536188943912982229549802141588142285951406456983959585849856635718948278200405553218221959941517650686579132328015710128453654602670350173629250453025147778018624703331632640015198287707387363765918392347282323525557447696472078465662147898155022212780252273154599771449642534694111770161406003970804745811031646065956854849931288892185255422605648889796888642195199378395411195253499458 09
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=69, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=70, s=1)$
3	-	24720781
5	-	611608091842861
19	-	35477776360827481371591706248503891553091115422691352946518908 92621
61	-	8451533266098653110982902061216010203891491924389646864582

		20316060214327726184756602100154451616453207474669072604178600384819475962089521432543670544105941077014695803420243561716072841692897996095053864525612442554115501
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=71, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=72, s=1)$
3	26152993	-
11	12330638741404176981645514740996512161	-
181	2225559806578298934781516481946180896254427968497281799749578155207103942700488935409092832897077385219616156960620167075425195956592931282195368538381527704466729732434873630436135951970130293172157469397946411046095977430802146164671734647213903854044136529542882123457993254533037650194495308239866783701873103775038867487123147715859320960256341577880524445290537725018425649734303615102903260447519872596517232368888491682891861585160450718297470485451604212136462345258426529740905438517254796096298772074007209084157273706987661197237662412952395483651739591522117402167832848223735720300546353131296461034345830453257316993767812180288091808976319151750288362881	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=73, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=74, s=1)$
3	-	30824701
5	-	950846038041901
83	20045073621229684439176504814613138640438354415716303206339782062035926856313061455400335625891787718299353979343513203589830461231596618011492582298167847712623552940181374620067304419683322907304444478	-

	37690562012049864340122163958 42908412514943224792755550112 62479303590130800638887993342 58153041575833293	
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=75, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=76, s=1)$
47	-	23617717334867021464052305058 29667557035263721221337988693 13738411568283025721787906756 05717412076049211919401030431 83472142155647933414025673720 90655944876106177715547931617
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=77, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=78, s=1)$
5	1303792250380621	-
31	-	53043874302920942244185239304 25032755163362022961791780169 76648738512044295953374841116 0278474119098250355457399981
83		92212285998686963622106389145 64723139589770809111547673645 73551824761736907160616255267 49908200589619366896066897652 42860004554395265672903179236 56420399952343778072838222344 81652275248195897715887436280 37437109872630793890712209630 14372003988176548561774968964 55737080199567696992751943232 3688834857706541609973
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=79, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=80, s=1)$
3	-	42016321
13	41148256431708011389416465731 98114637865694081	-
107	-	21506176410320989328114010787 47667323547120703942696658193 19505041832442605921162300844 96253562504062974106372977751 77537942656903523948335758031 20008525031694145099377898366 32257671783995202814542520857 50331237098524771626600797446 54523388266082975995396194822 20481414658389390621647092041

		5100840792480892163655012706255137607595635652607565879777201112214512870324180763948749766525463881955789706032961
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=81, s=1\right)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ \left(m=82, s=1\right)$
3	44142733	-
47	-	239863158881380818296866218366209399682534689049895896057324012478089504068690900236633105348214249270543550776775630355529938900148808741269496179153868951274421466619669564381
83	-	30726153595775293417062080849216396642002684291178769411126660986574098782311450164608065182979083558958803491282863411175614349329409909023603821465811262446010284834277982855557146796682766502310067481731421198728571521259843739172483433293936593388717388780446919663665208794858146790049093094059653343132914726 1
199	-	45983424397734164142489036999218198797909400102312095668468107925296672930918661652536011673446548432345514592548277457613687790370235012601900202200968330200384356494193662671466046990989933268423476523743870693506347613026126213338339737232203459440931532981476115698430319084878880413230919949935636013145319357084855082017239859688525649939710153031672260832390018124344268040817197601432476657828708432900215143431985277972479446475786926264878024774781179926746811725909304670479303744210787464566680674172522538630485952125817458773672084034152697933414774984740290751665432126870543746770725220456162535676101029927191228751867510734908289262177130985884704910679859657352343392775343233 8

		60099026742267358666069471382 69541039982528549868364924276 285341
p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=83, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=84, s=1)$
7	-	132937456874360617140481
13	-	17761010903132077458723990996 511968784184850961
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=85, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=86, s=1)$
23	-	17520063855145863886566308567 30072875192602056767444337416 9599737672277301188220565773
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=87, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=88, s=1)$
3	-	61371553
5	3441026514731521	-
11	69729053655109430079475735652 3259400801	-
31	35215520891795426578233074182 98800551892395792696011782118 32634845128886605155398322318 35445806494974806731743012240 1	69545497107926187794226975058 48445169055838396821997351827 27009439635804056922546166303 46296895096838963568528926128 1
61	13741765026043860027182742430 40775558590948664234227893802 31920373943933938905848677428 51885122023648851955931396681 75917599599756115317441339590 91432700072804681114780008625 41394129027824164890867866646 34559350136203914018923710048 01	-
173	41133111812646718494955820901 88991737320582923255032277406 84255764055411467982362974780 98361503349734953987584458781 40248672950166957999587812919 29120851316734254087891543729 42622142754228640798863445841 09209881418251361395368012081 98282604513035368767021228102 41041297487992894089347118234 21674819152616469146510196434 10203755747002621904661083769 07284025451169791095655027142	-

	5100044239908357330228063618 7877815994291007452917981060 7547527710721188697720645564 3655799691620324036878453740 2339819909549956644292801763 1202826712368981632530952220 9729272114350061424119337978 7457175947094165644726280956 6821787941199375448716082131 4795440259262107410362896686 023588754561	
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=89, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=90, s=1)$
5	4122686719927981	-
23	-	127672245596689835053275165883685066397077113044507715108251540134303028508514634626461
41	1546443746010068965283635937136030646805473244779812556646862548452170243218951953559027202462201703873673130272040666579877878906193335705653010594955982701	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=91, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=92, s=1)$
11	-	211712286351993553929794699437374722880 1
19	-	61651947021789841461930119325882931220436909211068285900617387003221441
41	-	2148524640913737873941538223142463750305769787081560661841583361033388025485758655891800089152656838817958299211127358781260908335636866544043144972559494520 1
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=93, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=94, s=1)$
3	-	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=95, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s\ (m=96, s=1)$

3	-	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=97, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=98, s=1)$
3	-	94167613
7	-	83606686639725857255247 7
17	45134699548628080661475915150 42632834048957029065235562561 851981	-
173	49829423467516586355319178881 80409722362181620809497540197 50572379463343610465554801039 40194676420901729981872902152 51709959535984780186640173021 17821184268389078799897485808 81631633611991163699553897509 33937489177028911748790945636 09517292780006905752015042400 09815086432568409746273323677 79090228946941526087807994842 19895094030271533515182069647 35421885408950507365638424901 85320786151682168920037708772 15390573514488853107327066636 69986624425227138852675875323 68689492614162872780914574910 11894660844616531806566260349 62172905999193369835557120488 95020847891374508293831986709 34864904794689606455882643255 26813134069611611314647800796 41319001499315060038885625809 480301571181859661	-
199	-	-

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=99, s=1)$	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=100, s=1)$
5	9617605550269201	-
47	70137059647201006641985716479 09118348582767810465324950254 33770742634063783562316668644 29272109139917403472473098678 78674311789541214475780316911 13969840156728830295048967452 9651840801	-
199	-	-

The tables show that there are plenty of Prime Numbers between symmetric function from 3 to 201 with s=1 and Primitive Prime Numbers from 3 to 199. The largest Prime number has 760 Digits, which occurs at Primitive Prime of 199. Further calculation at higher symmetric function still yielding Prime Numbers. One of the calculation yields 849 Digits prime number at symmetric function of 20005 and Primitive Prime 107 and 2241 digits at primitive prime 281. The Prime Numbers are given as in the table below:

p	$\frac{(m)^{2p}+(m+s)^{2p}}{(m)^2+(m+s)^2}=p_s \ (m=10002, s=1)$
107	10544827069737984769198984969 37050042671850268120760514304 15070267074554624446738277466 31582078995873173003112695271 85824088790356017253839082621 15944070434442477426548043424 85077416816112524945143649115 96585718808520057697527964876 66379758575000045107743758043 13195034177441611723414946422 67236967234130843755524026851 32037589662389280452251136827 40569494018446416742684813121 18251229451680135564373026882 69358358671874793074213856111 28410718735699855404158443890 00454584680028316609503523456 40340300470892612259181535435 41869654015188758306221990218 10129499378957906969817469845 99656909070565602565823521087 97466936597269626914493984440 82597851659596236671930771960 04893674725964461963085915384 31342765308758164304995440064 90153978508623358463531132800 71855784027052767655234278255 09323389382104417436349374061 71435409269295157675460783575 51397181
281	11507067905299776611167663020 41590820051565995672304288311 24854137822480173849330291461 66087367460386669118580647249 30052681875635965617124940310 62326938026349750640865076202 03736318769140528216656033596

	4472248989844786480215390787 2730051146247104009420732127 09055484455984370017901438709 41871109778315248621308833938 57116023750001806964832346954 56845652425601634765804544331 95326205234894535710212918016 61343445362974483779347602458 29467480419735310315398787465 45994364215120136433100775189 29642512402974739144543687552 92850187177261730204602831993 00124337818568808973637552983 57909389258174938281646954112 43542511122033357254015321929 04899795595459197136924762841 59368154553004411610771265684 13432660065806302962675143496 22412353995332590599458604953 89429379542217443921217065586 26162991096290828120669925440 64089486578768469484153493781 35118314274439123021096831283 29712585215930794153512537673 83169103687276687933785930769 68924997164413143479303166003 71368694278043756655437978487 63952301005768258253411506620 22747158624514504613813240860 52972534392344029213382962787 88619269398023138567198725805 61971908424318685623225367830 15832006650231904597854219430 06816367335566888388910482913 68063669216011679104233061204 76759085554361635850266098272 50249977608686085274364306107 54287241023345779323291864929 29619447339982526121537470693 50910704125170615604489774190 05457778718257541251444047159 11745951530896649584258077469 92544865364741165367021792654 33822843667922899463024630654 33038234914351985396816642396 23492573048471675305732180515 44419156458037336165304266504 87641525814851046437401206042 28264701870131020132480675367 92812788617068306857089054408 05633677127195347447403232918 36924697960109739110984501185 6

	956421274804406099125946274853642650645769353270759561232717594273764876973834356898572326312477885282296781279287322146939419019991674256026998483523768730518439732639512064505538178929302257403518174826936797337723228857942019828087658206805864897478365851624509155666800662510712104433891893432782771920257271732546208235698011349885492159938160152224692859955775616498107937050165578493075237308965049978224714356131112735263185117996694216369569734440633805272012279802086924361338564505985692174307382376534038923542 29745901

The conjecture can be further extended to 4th multiplier. Thus

$$\frac{(m)^{4p}+(m+s)^{4p}}{(m)^4+(m+s)^4}=p_s$$

p	$\frac{(m)^{4p}+(m+s)^{4p}}{(m)^4+(m+s)^4}=p_s\left(m=1,s=1\right)$	$\frac{(m)^{4p}+(m+s)^{4p}}{(m)^4+(m+s)^4}=p_s\left(m=2,s=1\right)$
3	241	5521
5	61681	-
7	15790321	-
23	291280009243618888211558641	
37	20988936657440586486151264256610222593863921	-
89	8634347730786151573123090429372562600645891723927646583482687395339003768803707512734187494061649643499761	-
103	-	386531783152876144020182731449343959135022693300714472951917997253128371321604965069594191474418137656374872794086565964814149945507878803494802243387466028177562979672052187265614879780667981521
109	-	109167992351041214117622808752158715710041864962337012541938411764122877108757011265651113366900468059943056313774488057212489097517356343569010154910455355335929669927521681785254699387354746279894587420081

149	152555719444154152886485583223939748198986993832442533088075637120979882772533387735283347387761168907654334066591940778764527613066609386319370398313013370847837566301526085 42961	-
173	120867093326019611839177157235297074116505969733988369933324469280071970457193952714008881807848999338855307188790859782008718571925179316707900155575640644530221132149091098304036391495503738907888172144 6641	-
199	-	-

4 Discussion and Conclusion.

We had demonstrated a new finding in the prime number field through symmetric function. The most important part is the finding of an alternative prime number generation other than Mersenne's number. It was found that the generalized equation can produce more prime numbers than Mersenne's equation at a specific range (i.e. primitive prime range $3 \le p \le 199$). On the other hand, Mersenne's prime of 687 digits occurs at primitive prime 2281 yet from the generalized equation, prime number of 849 digits occurs at primitive prime of 107 and many prime numbers of more than 200 digits at primitive prime less than 199. Therefore, we conclude that the generalized equation can be more efficient in generating prime numbers than Mersenne's prime even though Mersenne's prime can be categorized as part of generalized equation's prime of symmetric function of value 1(i.e. $x + y = 2 + (-1) = 1$).

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