Reasons for relativistic mass and its influence on Duff's claims that dimensionful quantities are physically nonexistent

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Abstract. The main argument against the relativistic mass is that it does not tell us anything more than the total energy tells us, although it is not incorrect. It is shown that this is not true, because new aspects of special relativity (SR) can be noticed. One reason for this definition is to show a relation between time dilation and relativistic mass. This relation can be further used to present a connection between space-time and matter more clearly, and to show that space-time does not exist without matter. This means a simpler presentation than is shown with Einstein's general covariance. Therefore, this opposes that SR is only a theory of space-time geometry. Phenomenon of increasing of relativistic mass with speed can be used for a gradual transition from Newtonian mechanics to SR. The postulates, which are used for the definition of SR, are therefore still clearer and the total derivation of the Lorentz transformation is clearer. Such derivation also gives a more realistic example for the debate regarding Duff's claims. It gives also some counter-arguments for some details of debate about physical nonexistence of dimensionful units and quantities. These details are why three elementary units exist and why a direct physical measurement is not the only possibility for physical existence. Such derivation thus shows that relativistic mass is differently presented to us as relativistic energy.

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1. Introduction

An understanding of the elementary physical theories and of their various aspects is important both for students and for researchers of still undiscovered theories, such as quantum gravity. Relativistic mass (m_r) is used here for a different interpretation of the theory of special relativity (SR). The shortest way to define m_r is:

$$m_\mathrm{r} = W/c^2 \,, \tag{1}$$

where W is the total energy and c is the speed of light. One argument against using m_r is that it confuses students. The main argument against using m_r is that there is no sense to do so [1, 2, 3, 4], as it does not tell us anything more than W tells us; although it is not incorrect. But, here the reasons will be shown that the definition of m_r is useful:

- A new relation between time dilation and m_r is presented.
- This relation can be further used to present a connection between space-time and matter more clearly, and to show that space-time does not exist without matter. This means a simpler presentation than it is shown with Einstein's *general covariance* [5, page 847].
- The next sense is to show, how the increasing of m_r with speed can be used for a pedagogical gradual transition from Newtonian mechanics to SR.
- The modified postulates of SR additionally clarify SR and the derivation of the Lorentz transformation.
- Such derivation gives a more realistic example for the debate on Duff's claims [6, 7] of physical nonexistence of the physical units and the dimensionful constants (PND). Thus, the debate about Duff's claims becomes clearer.
- The derivation with m_r also gives arguments that three elementary units are necessary, why not more or less.
- Even laymen are very familiar with the equation $E = mc^2$, but (1) is only generalization of this equation and so derivation is more familiar to laymen.
- Other reasons for m_r can be found in [2, 8] and in references therein.

Thus m_r gives us a different view as relativistic energy. In section 2 it is shown somewhat a different derivation of SR, where m_r is used. It is evident also, how insight in SR is clearer. In section 3 is alternatively shown how m_r is increased at acceleration. This is shown in semi-SR approach and crosses to full-SR approach. This additionally visualize SR. In section 4 it is shown how space-time and matter are connected. It is shown with the common interpretation of SR and with this new interpretation with m_r that space-time does not exist without matter. This connection is also shown from other aspects. In section 5 it is shown influence of m_r on some details of Duff's claims for PND. Duff's claims are widely acknowledged in science community, but some arguments are not clear. One of Duff's provocative questions is also, why there are precisely three elementary units (kg, m and s), why not less or more. Some clarifications and completion of its claims will be given in this paper.

2. Derivation with use of m_{r}

Let us imagine a trolley inside of a moving rocket that moves perpendicularly to the direction of the rocket. When the rocket increases velocity, the trolley moves a little bit slower than before (according to a rest observer). The common explanation is that a cause is time dilation. But, an alternative explanation can be that the transversal momentum is constant with velocity, hence increasing of m_{r} means smaller velocity of such a trolley. Such explanation shows a relation between m_{r} and time dilation. Such a relation also confirms that space-time does not exist without matter.

An essential difference of such interpretation of SR with the common interpretation of SR is that here the following transformations are used:

$$m = m_{\mathrm{r}}/\gamma\,, \tag{2}$$

$$t' = t''/\gamma\,, \tag{3}$$

where m is mass of the trolley inside of the rocket, t' is transformed time inside of the rocket which is obtained with the common Lorentz transformation,‡ and γ is defined as

$$\gamma = \left(1 - \left(\frac{v}{c}\right)^2\right)^{-1/2} \tag{4}$$

with v the velocity of the rocket. Of course, it is understandable that physics is the same if these transformations are used. This transformation already gives that the trolley inside of the rocket moves slower because of conservation of the momentum in the transversal direction.

But, let us show a detailed derivation in order how things will be clearer. We will also see how input postulates can be simplified. Einstein's postulates of the SR are

(i) The laws of physics are the same for all observers in an inertial state of motion.

(ii) All inertial observers always measure the speed of light as being the same.

Let us add to these two postulates still two known, acceptable postulates

(iii) Space is isotropic for all observers.

(iv) A maximal speed inside of every inertial system is a speed of light c'' (which is not necessarily equal to c).

But, let us omit postulate (ii). The reason for the omission of this postulate is that postulate (ii) will be derived in the following paragraphs. Otherwise it seems to the author that this is a slightly more empirical postulate than the other three ones, and it is less self-evident.

Let us synchronize clocks in another inertial system so that we see them to move with the same rate as our clocks. Of course, this does not mean synchronization in the opposite direction. Therefore, a speed of light in the transversal direction regarding an observer in a rocket (with respect of the our transformations) can be simply calculated. An observer from the stationary system sees the speed of light equal to c, of course. He

‡ Time for a rest observer is commonly assigned as t.

can use Pythagoras' theorem, and therefore he calculates that the observer in the rocket sees a transversal speed of light c''_{trans} equal to

$$c''_{\mathrm{trans}} = c/\gamma \,. \tag{5}$$

Postulate (iii) also gives that the longitudinal speed of light equals to the transversal speed of light, this means

$$c''_{\mathrm{long}} = c/\gamma \,. \tag{6}$$

Because of equal status of both inertial systems, an observer in a moving system must calculate γ in (4) with the same value, but this gives that he sees smaller velocity of the inertial system (v'') than it is seen by the first observer:

$$v'' = v/\gamma \,, \tag{7}$$

therefore, from his point of view all velocities are proportionally smaller.§ It is said for another observes that he sees the same γ, but not the same c. This is one step closer to calculation with dimensionless numbers [6, 7], which are physically more fundamental.

The derivation of the *Lorentz transformation* can be started in the similar way as in the common derivation of SR; this means that we begin with the two initial equations of the Lorentz transformation:

$$x'' = \eta(x - vt) \,, \tag{8}$$

$$x = \eta(x'' + v''t'') \,, \tag{9}$$

where x and t are space and time coordinates of a stationary system and x'' and t'' are space and time coordinates of a moving system (for instance, a rocket), which moves with velocity v in the direction x.‖ η means a factor of Lorentz contraction and it still needs to be calculated by applying (6) and (7).¶ Hence, a calculation gives that η equals γ. These equations also give how time is transformed:

$$t'' = \gamma^2(t - vx/c^2) \;, \tag{10}$$

$$t = t'' + v''x''/c''^2 \;. \tag{11}$$

Now let us respect that everything in the moving system is moving slower, hence also processes in brains and also its clocks. Therefore, we can use the transformation (3). (3) together with (6) to (11) gives back the common equations of the Lorentz transformation and hence gives the equal speed of light in all inertial systems. So, this is a transition from the above three postulates (i), (iii) and (iv) to the common two postulates (i) and (ii). But, further analyses will be done with t'' as it is calculated in (3).

§ If this was not true, the velocity of the first system v'' would exceed c'', what is against postulate (iv).

‖ If we are more precise, then the minus sign can stand before $v''t''$ in (9) because, in truth, v'' means opposite velocity. In this way those four equations (8) to (11) become still more symmetric.

¶ The procedure with η (γ) is also used in the common calculation of the Lorentz transformation.

The smaller speed of light in (5) and (6) (and proportionally smaller all velocities, for instance (7)), can be compensated by larger mass, so the momentum in the transversal direction is preserved.

(10) and (11) are less symmetric, because factors before (8) to (11) are γ, γ, γ^2, and 1, but in the common derivation they are all equal to γ. However, we can also notice some simplifications:

- simplifications of the postulates (i) and (ii),
- simpler and clearer calculation of γ,
- dimensionless numbers are more frequently used,
- presentation with momentum is clearer, (It will be further seen),
- we can use the minus sign for velocity v''.

3. Relativistic mass at acceleration in the longitudinal direction

Now it is seen, how it is with conservation of the momentum in the transversal direction. Let us see still more clearly, how it is with movement in the longitudinal direction. Let us look at, how it is with increasing of W with velocity. This can also be additionally clarified with use of m_r. Therefore this is a further visualization of these equations.

For the beginning, let us naively assume that space is Euclidean, and that acceleration increases W and hence also m_r. Then the equation for increasing of energy of an accelerating body is:

$$\mathrm{d}W = c^2\mathrm{d}m_\mathrm{r} = m_\mathrm{r}a\mathrm{d}x = m_\mathrm{r}v\mathrm{d}v\,, \tag{12}$$

where a means acceleration, x means distance and v means velocity. A solution of (12) is

$$2\ln(m_\mathrm{r}/m_{\mathrm{r}0}) = (v/c)^2\,, \tag{13}$$

where $m_{\mathrm{r}0}$ was mass at velocity zero, and ln is the logarithm with base e. The result is incorrect, because the real relation is

$$m_\mathrm{r} = \gamma m_{\mathrm{r}0}\,. \tag{14}$$

But, additional supposition should be that longitudinal distances x_1 in the rocket seen from the rocket are larger than the same distances x seen from the rest system. (This is *Lorentz contraction.*) This can also be comprehended from (8) to (11):

$$\mathrm{d}x_1 = \gamma\mathrm{d}x\,. \tag{15}$$

If this is corrected in (12), the new equation is:

$$c^2\mathrm{d}m_\mathrm{r} = m_\mathrm{r}\gamma a(\gamma\mathrm{d}x) = m_\mathrm{r}(\gamma v)\gamma\mathrm{d}v\,. \tag{16}$$

and the result is (14), what is correct. The equation, similar to (16) is known from the common calculations of SR:

$$\mathrm{d}W/\mathrm{d}t = \gamma^3 mva\,. \tag{17}$$

It is interpreted that longitudinal relativistic mass (m_{rt}) equals

$$m_{\mathrm{rt}} = \gamma^3 m_{\mathrm{r}0}\,. \tag{18}$$

But, in the present example the part γ^2 is attributed to length-contraction and not to m_{rt}.

Therefore it is obtained with the use of acceleration, how W expressed with m_{r} increases with increasing of velocity.

- Thus, properties of the mass (m_{r}), such as inertia or rest, are also important, not only properties of its energy counterpart (W).
- At the same time, this is also a pedagogical gradual transformation from Newtonian mechanics to SR.

4. A connection between matter and space-time

A remark is possible that we see a larger mass inside the rocket, but from the rocket they see us that we have a larger mass. Therefore, it seems that larger mass is not realistic. However, this is exactly the same problem as in the common Lorentz equations, where relations for time show the same paradox. Yet, SR is a correct theory, and both the common interpretation of SR and the interpretation with m_{r} are correct. In short, it is not incorrect that mass-time relationship is only one-way. It is important that the introduction of m_{r} explains dilation of time and the connection between matter and time. It is not necessarily to look at the same time from two inertial systems; it is enough to look at once from one inertial system and at another time from another inertial system.

This slower speed of time with increasing of m_{r} can also be generalized out of SR to big and small elementary particles. If a human body was made from the same particles, but 1000 times lighter ones, the speed of time would seem to us much smaller than now. Hence one second would seem very long. (This example is not relative, because it gives the same result from both observers.)

This can be generalized still further. A fly feels a longer second than an elephant, because of smaller mass of the fly brain the brain processes are faster. Although particles are not smaller, this also can be an analogy for reduction of the "mass". Another example from biology is a cold lizard or a warm one. For the first one a second seems shorter. Therefore various examples of different time flows have a very similar key foundation, this is the momentum or movement.

We know from the common interpretation of SR that rest matter cannot be accelerated to $v = c$. It can only be approached to this speed. But, anywhere close to c this matter is moving, always we can find an inertial system, where this matter is at rest. The speed of a photon equals c. We cannot find an inertial system where it is at rest. Time flows where rest matter exists, but time does not flow for a photon. Therefore rest matter defines that time flows; hence time is dependent of rest matter. Thus, this can be found also from the common interpretation of SR.

Interpretation of SR with m_r tells us still more. It tells us that speed of time depends on largeness of mass. Therefore this is another clarification that space-time does not exist without rest matter. Hence, this is a simplified explanation of what is shown with general covariance [5, page 847].

Hence formally, one time is really attributed to every point of space, but truly time flows only if rest matter is present, or differently saying, that matter is a reference for this space-time. Therefore space-time without rest matter does not exist.

But one detail should still be clarified. Seemingly, time flows for a photon

- because it has some frequency,
- if it is calculated for rest matter that time does not flow at $v = c$, this does not mean automatically that time does not flow for a photon. Precisely said, it means only that time does not flow for rest matter if it is accelerated to $v = c$. And, of course, it never reaches this speed.

Indeed, frequency of a photon is dependent from rest matter, or from inertial system, where this matter is at rest. But, privileged inertial system does not exist. Therefore rest matter cannot be ignored where the existence of photons is mentioned. Thus, photons exist because of rest matter. It is similarly in general relativity, where it is claimed that gravitational waves exist independently of matter. But indirectly they are connected with matter.

Connection between elementary particles and space-time was indirectly found also by Cramer's Transactional interpretation of quantum mechanics [9]. Namely, a photon does not fly into empty space, but checks with "hand-shaking" if there is any other particle, and then flies.

Let us think in the approximation, where space-time is continuous, therefore it is not grained. In the continuous area, points can never build up a straight line, because always is valid $0 \times x = 0$, despite even when $x = \infty$.[+] Therefore three-dimensional space can be partitioned to smaller pieces of three-dimensional space, but it cannot be partitioned to two-dimensional objects. Hence space-time is not built up from rest pictures of space, but it also includes time transitions between these pictures. Therefore we always have "five dimensional volume", hence matter is another "dimension". But it seems that space-time is not continuous, but is grained. Despite of this, the smallest cell also implicitly includes matter.

Oas [4] commented that acceleration gives rise to energy but not to a larger number of particles. (So, by him, m_r does not tell us anything new according to energy.) But, this also tells us that m_r is increased inside of elementary particles, therefore elementary

[+] This is different as limits, for instance, $\lim_{x\to 0} \sin(x)/x = 1$, because $x = 0$ is never reached.

particles are these essential things.* So, dimensionless numbers μ_i are generators of all space-time. $\mu_i^2 = m_i^2 G/(\hbar c)$, where m_i are masses of various elementary particles, G is the Newton gravitational constant, and $\hbar$ is the Planck constant. Fotini Markopoulou claims similarly [10]. Hence again, elementary particles are necessity for the existence of space-time.

Hence energy shows a property of matter, this is inertia, that is expressed with the momentum. Energy also shows another property of matter that can be at rest. Of course, energy of photons is not at rest, but regarding all above, space-time does not exist without rest matter, therefore energy of photons also does not exist without rest matter.

5. Influence of m_{r} on Duff's claims

Some Duff's suppositions [6, 7] are:

(i) The existence of the elementary dimensionful constants G, $\hbar$ and c is necessarily related with the existence of elementary units such as kg, m and s.

(ii) G, $\hbar$ and c are only conversion factors between the elementary units.

(iii) The elementary dimensionful constants are always redundant according to elementary dimensionless constants such as μ_i and α.

(iv) If a dimensionful constant, for instance c, is variable, in this case c/c_0 is not an elementary dimensionless constant, such as μ_i and α. c_0 means initial value of the c.

(v) Elementary units do not exist in final results of measurements.

(vi) Points (iii) and (v) means for PND.

(vii) The number of necessary units is zero, because the number 3 is not more privileged than, for instance, the number 7.

The author agrees with points (i) to (v). But he does not agree with points (vi) and (vii).

According to point (vii), a provocative Duff's questions is also why precisely three elementary units (m, s and kg) exist, why not, for instance, two or seven ones. But, in the example above it is evident that those three units form almost a complete set, because time does not exist without matter and it is a part of space-time. We can also see that this is connected with conservation of the momentum. At the same time we can conclude, that the "Cube theory" [6] is not an appropriate answer for Duff question and that its connection with Duff's question is only an accident.

Duff's main objection is the that dimensionless quantities occur in physical calculations, and that the dimensionful quantities are only mathematical tools, but they are not physically realistic. This can be answered with a comparison with the

* Rest matter is built up from elementary particles, but it can be built up also from black holes. Maybe those two things are the same. The answer is hidden in a quantum gravity theory.

quantum mechanics. Let us look at, for instance, Feynman's derivation in [11]. Complex amplitudes here are summed up, and probability for one event is proportional to the square of this sum. Differently said, their linear impact is important. We can ask ourselves, if those imaginary numbers are physically real. In the above example they are indirectly physically real, because they are used in mathematical procedure. Use of imaginary numbers means also that the partial result of calculation is not physically real. If any derivation without imaginary numbers exists, still ever it will be a conclusion that a partial result is not physically real. One example of a view from a different aspect is Zeilinger-Brunkner's one [12].

The physical units behave similarly as amplitudes. For instance:

$$ln(2\text{kg}) - ln(1\text{kg}) = ln(2) - ln(1) + ln(\text{kg}) - ln(\text{kg}) = ln(2) - ln(1)\,. \tag{19}$$

$ln(\text{kg})$ is not defined (is not directly physically real), but it is canceled in the calculation, similarly as imaginary numbers cancel with squaring. Duff has also not shown any example where the physical units are not necessary to use in a derivation.

Duff also asserts for PND. c means one relation between time and length. Therefore analogically, we can say that the radian is approximately a relation between rectangular lengths, the analogous question is if the radian is physically non-existent. But a legal answer is that it exists physically.

For point (iii) Duff gives examples with variation of dimensionless constants. He gives examples with Planck's, Schrödinger's, and Stoney's units. For instance, a formula for a speed of light in Schrödinger's units (c''') is

$$c''' = c/\alpha \tag{20}$$

But, the derivation with m_r gives also an example for variation of c according to an inertial system, therefore dimensionless constants do not vary. So this is a new example and it is more realistic. This m_r example gives also more importance on constancy of dimensionless units than on constancy of c. Other comparisons can also be made: If all particles become 1000 times lighter, c would seem much smaller than it is. Feeling is also a physical phenomenon; the common interpretation of SR is based also on the same feeling in another inertial system. Similarly is with the above mentioned examples with the animals.♯ Existence of four dimensional space-time is important. Thus, momentum is not only a dimensionless number, but it contains units m, s and also kg. So such an indirect importance as at imaginary numbers in quantum mechanics.

Duff's ideas mean also lowering of necessary elementary physical parameters, what was also done by SR (all inertial systems are equivalent) and by general relativity (gravity is (curved) space-time). If we respect the Zeilinger-Brukner [12] interpretation, this reduction was also done in it, because it reduces all to information.

Of course, the above examples do not finish the debate in the trialogue. μ_i are masses written in more natural units, not only any dimensionless numbers. Above

♯ Either interpretation with constant μ_is or common interpretation of SR with constant c shows on importance of mass as the main building block of the space-time. So space-time is not an elementary unit where mass can be ignored.

debate means also questions of quantum gravity and it is not yet developed. The author thinks that Duff's ideas used in quantum gravity can also show a nontrivial nature.

6. Conclusion

It is not easy to visualize SR with only $c = constant$. But the additional presentation with $m_{\rm r}$ helps us to better visualize SR. The presentation with $m_{\rm r}$ together with a more precise common interpretation of SR also opposes that SR is only the theory of space-time geometry. It is not enough to show the symmetry of four-momentum to space-time, it should also be shown why this symmetry exists. The connection of space-time and mass shows that kg, m and s form one triplet. This is used to oppose Duff's claims for PND.

For a *theory of everything* we need to go to foundations. The postulates are also foundations of physics, not only formulae. A property of the equations in SR is that they are hyperbolic. But this is a consequence of the postulates, it is not a fundamental property. Some dimensionless constants are also foundations of physics. It is beneficial that they appear already in pre-theories of theory of everything, such as in SR.

It is naturally to upgrade the above visualization of SR with $m_{\rm r}$ to general relativity. A good ways to it are [13, 14], for instance. Such many sided interpretations of formulae should also be written in other fundamental physical theories, for instance in quantum mechanics. One example, where this is done, is the Zeilinger-Brukner interpretation of quantum mechanics [12]. A generalization of a new aspect of a known theory can give new cognition.

This presentation has helped the author to better visualize SR. With this reason he thinks that also others will better understand SR. It is a trend in teaching of fundamental physics, that it should be as abstract as possible; but visualization is not so important. For instance, it is known that our closed universe exists without any space out of universe. But, although external space does not exist, it is easier to visualize it. Similarly, it is better to visualize $m_{\rm r}$.

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