

Is an Algebraic Cubic Equation the Primitive Instinct beyond Electromagnetic and Nuclear World?

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Abstract Everyone lives his or her life instinctively. Does the instinct originate from the natural world? If the instinct is a rational process, is the natural world rational? Unfortunately, people have not found any rational principle behind the natural world. Because human activities are realized directly through electromagnetic and nuclear forces of entropy-increase, people are difficult to recognize the principle. Compared to the large-scale structure of galaxies, human bodies and their immediate environment are the “microscopic” world. The electromagnetic and nuclear forces which rule the world, however, disappear in the formation of large-scale galaxy structures. Similarly they disappear in the formation of the solar system. My previous papers found many evidences that galaxies are rational. This paper shows that large-scale galaxy structure must originate from an algebraic cubic equation.

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1 Instinct Equation: a Rational Necessary Condition

1.1 Rational structure

By taking spiral galaxy images with long-wavelength electromagnetic waves, we see that a spiral galaxy is essentially a flat (two-dimensional) smooth structure. Therefore, the paper discusses the function of two variables:

$$\rho(x, y) \tag{1}$$

which is the density of material distribution on the Cartesian coordinate plane (x, y) (i. e., the spiral galaxy disk plane). Since ancient times, humans have not known what kind of material distribution is rational. Scientists generally study level curves of any density distribution (i. e., the contours of constant density). *Level curves are everywhere perpendicular to the density gradients.* Since scientists have no basic principle, the level curves and correspondingly the gradients are arbitrary. My study on galaxy structures shows that natural material distribution is not arbitrary. On any spiral galaxy disk plane, there exists at least one net of orthogonal curves. Along any curve of the net, the ratio of material densities on both sides of the curve

is constant along the curve. That is, its left-side density divided by the immediate right-side one is constant along the curve. These curves are, therefore, called proportion curves. At any point on the plane cross two orthogonal curves (see the “cross” in Figures 1 and 2). The “cross” divides the area around the point into four positions. For example, the positions are, from left to right and top to bottom, A, B, C, D respectively. The corresponding densities are $\rho(A), \rho(B), \rho(C)$ and $\rho(D)$. Rational structure means that $\rho(A)/\rho(B)$ equals to $\rho(C)/\rho(D)$. Accordingly, $\rho(A)/\rho(C)$ equals to $\rho(B)/\rho(D)$. Unfortunately, ratio concept applies to discrete density positions only. Galaxies, however, are continuous smooth structures. To generalize the ratio concept to continuous structure, we take the logarithm of the ratio. The result is the difference of two logarithmic densities at the two positions respectively. To be fair, whenever we take a ratio, the distance between the two corresponding density positions is the same. That is, any two positions involving a ratio are separated at the same scale. Now we divide the above difference of two logarithmic densities by the scale. If the scale is small, the result is approximately the variance rate of logarithmic density along the direction joining the two density positions. In mathematical language, the variance rate is the directional derivative along the direction of the two density positions. A proportion curve means that the values of the directional derivatives along the perpendicular directions to the curve are constant along the curve. Of course, the constant values from different proportion curves are generally different.

Since the ratio of the density (1) is approximately proportional to the directional derivative of its logarithmic density, we from now on, study the logarithmic density $f(x, y)$ instead of the density $\rho(x, y)$ itself,

$$f(x, y) = \ln \rho(x, y). \quad (2)$$

1.2 System of rational necessary equations

Now we introduce curvilinear coordinates to describe the above net of orthogonal curves. In fact, galaxy structures depend only on the geometric curves, not the choice of coordinate parameters (see [1]). In [1], I use the following equation

$$\begin{cases} x = x(p, q), \\ y = y(p, q) \end{cases} \quad (3)$$

to describe a net of curves where the letters p, q denote curvilinear coordinates. Originally in [1], I used the letters λ, μ instead of the letters p, q to label the coordinates. The new letters are consistent with the notations in my earlier paper [2]. In fact, the following discussion before the formula (14) is mostly taken from [2]. Given two functions, $x(p, q), y(p, q)$, you have the transformation between the curvilinear coordinates (p, q) and the rectangular Cartesian coordinates (x, y) . It describe a net of curves. Letting the second parameter q be a constant, you have a curve (called a row curve, i.e., the proportion row). That is, the above formula is a curve with its parameter being p . For the different values of the constant q , you have a set of “parallel” rows. Similarly, you have a set of “parallel” columns of parameter q . In this paper, I

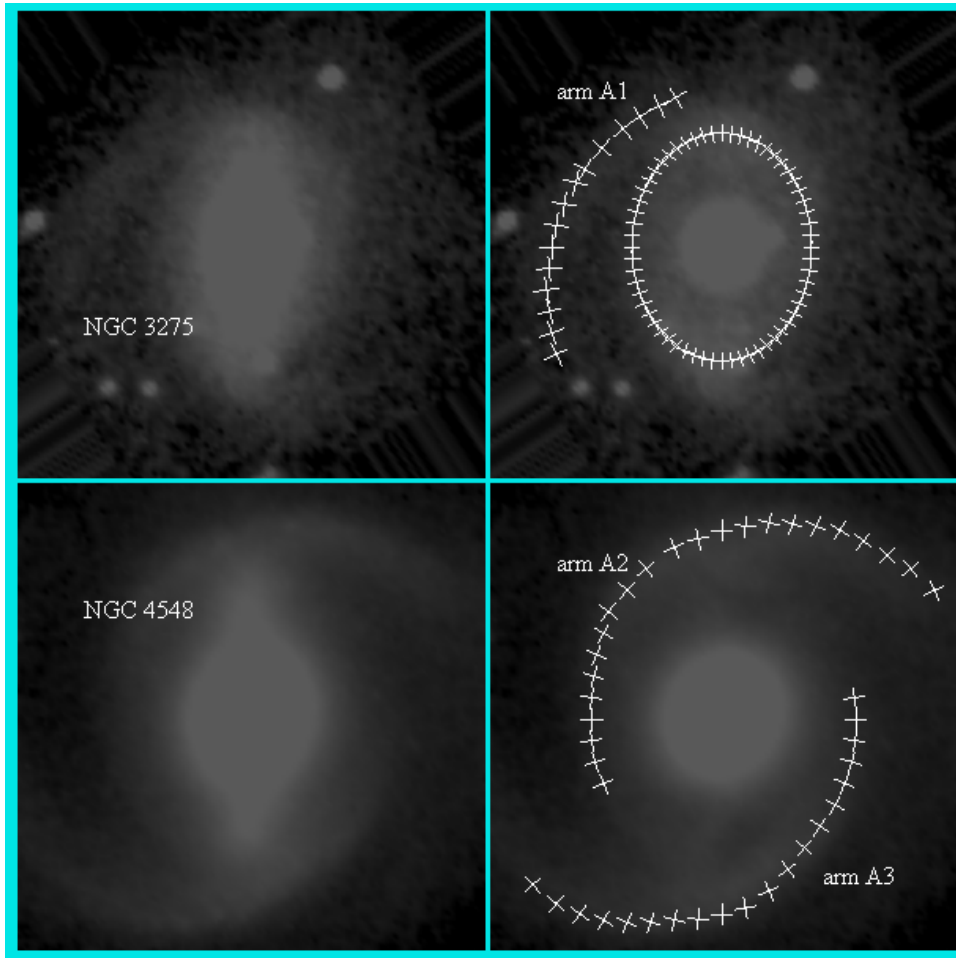


Figure 1: The OSUBGS H-band images NGC 3275, 4548 (left panel, see reference [5]) and the closest solution of instinct equation to the directions of arms and ring (right panel).

am interested in the inverse equation of (3),

$$\begin{cases} p = p(x, y), \\ q = q(x, y). \end{cases} \quad (4)$$

The row curves and the column curves, however, are not necessarily orthogonal to each other. The following equation is the necessary and sufficient condition for the net of curves to be orthogonal:

$$\nabla p \cdot \nabla q = 0. \quad (5)$$

If we imagine p and q were two “density distributions” then the above equation says that the two density gradients are everywhere perpendicular to each other. Accordingly, their level curves are everywhere perpendicular to each other. The angle between the direction of the gradient ∇p and the Cartesian x -axis is denoted by

$$\alpha(x, y). \quad (6)$$

The gradient is along the tangent direction of the row curves. The level curves of the quantity p are exactly the column curves. This is true in reverse for the function $q(x, y)$ with its gradient direction β relative to the x -axis, being along the tangent direction of the column curves. In fact, β is always equal to $\alpha + \pi/2$ because row and column curves are orthogonal to each other:

$$\beta(x, y) = \alpha(x, y) + \pi/2, \quad (7)$$

and

$$\begin{aligned} \cos \alpha &= p'_x / \sqrt{p'^2_x + p'^2_y}, \\ \sin \alpha &= p'_y / \sqrt{p'^2_x + p'^2_y}, \\ \cos \beta &= -\sin \alpha = q'_x / \sqrt{q'^2_x + q'^2_y}, \\ \sin \beta &= \cos \alpha = q'_y / \sqrt{q'^2_x + q'^2_y}. \end{aligned} \quad (8)$$

Now I use $u(p, q)$ (which should be $u(\lambda, \mu)$ in [1]) to denote the directional derivative $\partial f / \partial l_\alpha$, where l_α is the linear length along the direction of row curves whose tangent direction is α , and use $v(p, q)$ (which is $v(\lambda, \mu)$ in [1]) to denote the directional derivative along the direction of column curves whose tangent direction is β . As the previous section 1.1 indicates, the values of the directional derivatives along the perpendicular directions to any row curve are constant along the curve, which means that the level curves of $v(p, q)$ is the row curves. Therefore, v depends only on q . Similarly, the values of the directional derivatives along the perpendicular directions to any column curve are constant along the curve, and u depends only on p . These consist of the rational structure condition (see [1]),

$$\begin{cases} u = u(p), \\ v = v(q). \end{cases} \quad (9)$$

Because we want to find the rational necessary equations of α in coordinates (x, y) , we need the derivatives $\partial f / \partial x$ and $\partial f / \partial y$:

$$\begin{aligned} f'_x &= u(p) \cos \alpha + v(q) \cos \beta = u \cos \alpha - v \sin \alpha, \\ f'_y &= u(p) \sin \alpha + v(q) \sin \beta = u \sin \alpha + v \cos \alpha. \end{aligned} \quad (10)$$

Now the rotation of the logarithmic-density gradient, $f''_{xy} - f''_{yx}$, must be zero. This gives us the first of our two required equations involving α . To begin, we take the y -partial derivative to the first equation in (10). Firstly, we consider the partial derivative to the first factor $u(p)$ in the first term. The corresponding result is $u'(p)p'_y \cos \alpha$. Substitution of the first equation in (8) into the result, we get the common factor $p'_x p'_y / \sqrt{p_x'^2 + p_y'^2}$. Taking the partial derivatives to other terms, we find out that all partial derivatives to the first factors in (10) give the similar common factors, and the contribution of $u(p)$ and $v(q)$ to the rotation is zero. Therefore, we just need to take the partial derivatives to the second factors: $\cos \alpha$, $\cos \beta$, $\sin \alpha$, and $\sin \beta$. That is,

$$\begin{aligned}
& f''_{yx} - f''_{xy} \\
&= (u(p) \cos \alpha \alpha'_x + v(q) \cos \beta \beta'_x) + (u(p) \sin \alpha \alpha'_y + v(q) \sin \beta \beta'_y) \\
&= (u(p) \cos \alpha + v(q) \cos \beta) \alpha'_x + (u(p) \sin \alpha + v(q) \sin \beta) \alpha'_y \\
&= f'_x \alpha'_x + f'_y \alpha'_y = 0.
\end{aligned} \tag{11}$$

This is exactly the first of our two required equations involving α ,

$$\nabla f \cdot \nabla \alpha = 0. \tag{12}$$

Note that we have got rid of the directional derivatives $u(p)$ and $v(q)$. However, the derivatives are related to the proportion curves (3) which define the corresponding rational structure. Instead what we have in (12) is the tangent direction of the proportion rows, $\alpha(x, y)$. However, a scalar angle $\alpha(x, y)$ may not define a vectorial net of orthogonal curves. Therefore, (12) is a necessary equation for rational structure. We can have other necessary equations too. For example, u depends only on p means that the level curves of the ‘‘density’’ $u(p)$ is the column curves, and its gradient (u'_x, u'_y) is perpendicular to the tangent direction of the column curves,

$$-u'_x \sin \alpha + u'_y \cos \alpha = 0. \tag{13}$$

Once again we want to get rid of $u(p)$. To do so, we get the inverse equation of (10),

$$\begin{aligned}
u &= f'_x \cos \alpha + f'_y \sin \alpha, \\
v &= -f'_x \sin \alpha + f'_y \cos \alpha.
\end{aligned} \tag{14}$$

Taking both x - and y -partial derivatives to the first equation of (14) and substituting the results into the equation (13), we get, by using the equation (12),

$$2(f'_y \alpha'_y + f''_{xy}) \cos 2\alpha + (f''_{yy} - f''_{xx} - f_y'^2 \alpha'_x / f_x' - f_x' \alpha'_y) \sin 2\alpha = 0. \tag{15}$$

The reader may wonder what is the resulting equation if we deal with $v(q)$ instead of $u(p)$. The resulting equation is the same one (15). The set of equations (12) and (15) is our system of rational necessary equations. Surprisingly, the equation system has analytic solution which is implicitly given by an algebraic cubic equation called instinct equation, as shown in the following section.

1.3 Instinct equation: a rational necessary condition

Solving α'_x and α'_y from the system of rational necessary equations ((12) and (15)), we get

$$\begin{aligned}\alpha'_x &= f'_y((f''_{yy} - f''_{xx}) \sin 2\alpha + 2f''_{xy} \cos 2\alpha)/(2f'_x f'_y \cos 2\alpha + (f'^2_y - f'^2_x) \sin 2\alpha), \\ \alpha'_y &= -f'_x((f''_{yy} - f''_{xx}) \sin 2\alpha + 2f''_{xy} \cos 2\alpha)/(2f'_x f'_y \cos 2\alpha + (f'^2_y - f'^2_x) \sin 2\alpha).\end{aligned}\quad (16)$$

Differentiating the above expressions to get α''_{xy} , α''_{yx} respectively, and using the above expressions themselves in the final results, we finally get the required solution of rational structure by setting: $\alpha''_{xy} - \alpha''_{yx} = 0$. The corresponding calculation is complicated but straightforward, and the result is the following algebraic cubic equation in $\tan 2\alpha$,

$$a(x, y)\gamma^3 + b(x, y)\gamma^2 + c(x, y)\gamma + d(x, y) = 0 \quad (17)$$

where

$$\gamma = \tan 2\alpha, \quad (18)$$

$$\begin{aligned}a &= (f'^4_y - f'^4_x)(f''_{yy} - f''_{xx})^2 \\ &+ (f'^2_y - f'^2_x)^2(f'_x f'''_{xxx} - f'_x f'''_{xyy} + f'_y f'''_{xxy} - f'_y f'''_{yyy}),\end{aligned}\quad (19)$$

$$\begin{aligned}b &= 2f'_x f'_y (f'^2_x + f'^2_y)(f''_{yy} - f''_{xx})^2 \\ &+ 4f'_x f'_y (f'^2_y - f'^2_x)(f'_x f'''_{xxx} - f'_x f'''_{xyy} + f'_y f'''_{xxy} - f'_y f'''_{yyy}) \\ &- 2(f'^2_y - f'^2_x)^2(f'_x f'''_{xxy} + f'_y f'''_{xyy}) + 4(f'^4_y - f'^4_x) f''_{xy} (f''_{yy} - f''_{xx}),\end{aligned}\quad (20)$$

$$\begin{aligned}c &= 4f'^2_x f'^2_y (f'_x f'''_{xxx} - f'_x f'''_{xyy} + f'_y f'''_{xxy} - f'_y f'''_{yyy}) \\ &+ 8f'_x f'_y (f'^2_x + f'^2_y) f''_{xy} (f''_{yy} - f''_{xx}) \\ &- 8f'_x f'_y (f'^2_y - f'^2_x)(f'_x f'''_{xxy} + f'_y f'''_{xyy}) + 4f'^2_{xy} (f'^4_y - f'^4_x),\end{aligned}\quad (21)$$

and

$$d = -8f'^2_x f'^2_y (f'_x f'''_{xxy} + f'_y f'''_{xyy}) + 8f'_x f'_y f'^2_{xy} (f'^2_x + f'^2_y). \quad (22)$$

The equation (17) is called the primitive instinct equation beyond electromagnetic and nuclear world, or simply called *instinct equation*. A primitive form of the equation was obtained in the year of 2002. Its study should have been resumed after I successfully modeled galaxy structures in 2005 [3]. However, I have stopped my galaxy study since 2005 for the apparent reason. Now I give some explanations to the equation.

Firstly, we discuss the angle α of the tangent direction of the row curves. Since either directions of any proportion curve can be chosen as the positive direction, the variance domain of α can be chosen to be $[-\pi/2, \pi/2]$ for simplicity. Since the labeling of row or column curves is arbitrary, we can not differentiate between α and $\alpha + \pi/2$. This is why our instinct equation involves the angle 2α , not α . Both equations in (16) are the same if α is replaced by $\alpha + \pi/2$. Therefore, the variance domain of α can be further reduced to the following for simplicity

$$\text{Domain of Angle } \alpha : [-\pi/4, \pi/4]. \quad (23)$$

But the real solution to rational structure is four angles: $\alpha, \alpha + \pi/2, \alpha + 2\pi/2, \alpha + 3\pi/2$. Geometrically speaking, instead of giving one angle, any root of the algebraic equation (17) gives four angles which make a “cross” at each point on the (x, y) plane. That is, at any point on the plane cross two orthogonal curves (if the global net of orthogonal curves does exist).

Secondly, note that all of my idea about galaxies has no other theoretical assumption except the one of rational structure which is based on the simple concept of constant ratios or constant values of directional derivatives. Therefore, our result is independent of any choice of coordinates. It depends only on the shape of proportion curves. That is why the coefficients of equation (17) are all homogeneous of the partial derivatives.

Thirdly, the algebraic equation (17) always has at least one root if the logarithmic density is smooth enough and the coefficient a is not zero. That is, we generally have a solution of angle $\alpha(x, y)$ for any smooth density $\rho(x, y)$. However, a scalar angle may not define a vectorial net of orthogonal curves. It resembles the situation that, given two functions, we may not find the third function whose partial derivatives are the given functions. It is highly urgent that mathematicians find the condition of $f(x, y)$ at which a solution of (17) does correspond to a global net of orthogonal curves.

2 Application to Spiral Galaxy Structure

Galaxies have been observed for over eighty years. Their profiles, e.g., exponential disks, logarithmic arms, $R^{1/4}$ de Vaucouleurs law for elliptical galaxies, are studied perfectly but not explained. It resembles the situation that Kepler laws were not explained until Newton provided universal gravity. There are only two types of galaxies, one being three dimensional ellipticals and the other being flat spirals. Elliptical galaxies have little dust while spiral galaxies have a lot. Independent spiral galaxies, i.e., the ones with little disturbance from other galaxies, have very regular structure. They have bulges which are generally small and located at the galaxy centers. The main structure of spiral galaxies is the exponential disks. Spiral galaxies are called either normal spirals (no bar) or barred spirals (where bars are present). The name “spiral” originates from the fact that spiral galaxies present more or less the linear structure of spiral arms, which are clearly the disturbance to the main body.

My PhD thesis [3] and papers [1,4] suggest that both elliptical and spiral galaxies are rational structures. That is, the material density varies proportionally along some particular net of orthogonal curves (or surfaces in the case of three dimensional elliptical galaxies). The presence of arms is the disturbance to the rational structure, and the disturbance produces cosmic dust. *The disturbing waves try to achieve the minimal disturbance and, as a result, they follow the proportion curves of rational structures.*

2.1 Spiral galaxy disks

Spiral galaxy disks are observed to be exponential,

$$\rho(x, y) = d_0 e^{d_1 r} \quad (24)$$

where $r = \sqrt{x^2 + y^2}$, both d_0 and d_1 are constants. The corresponding logarithmic density is

$$f(x, y) = d_1 r. \quad (25)$$

Its gradient is always in the radial direction, and the value of the gradient is the constant d_1 . Therefore, any curve whose tangent direction always makes a constant angle to the radial direction is a proportion curve. This kind of curve is called logarithmic spirals or equiangular spirals or golden spirals, and all the spirals consist of many nets of orthogonal curves (see the Fig. 1 of [1]). Surprisingly the arms of normal spiral galaxies follow the directions of equiangular spirals. This proves in normal spiral galaxies that arms are the disturbance to rational disk structure. The disturbing waves try to achieve the minimal disturbance and follow the proportion curves.

Additionally, the radial lines and concentric circles at the galaxy center are the proportion curves of any exponential disk. The angle α of the radial lines is: $\tan \alpha = y/x$. Substitution of the angle and the logarithmic density (25) into the instinct equation (17) indicates that the set of f and α are indeed its solution.

2.2 Dual-handle structure.

Papers [1,3] prove that a galaxy bar is a rational structure, and is composed of two or three dual handle structures which are generally aligned with each other (but some galaxies, e.g., NGC 1365, are exceptional). The dual handle structure is determined by the orthogonal curves of confocal ellipses and hyperbolas [1]. The corresponding logarithmic density is

$$f(x, y) = (b_2/3)(g^2(x, y) + b_1^2 x^2 / g^2(x, y))^{3/2}, \quad (26)$$

where

$$g(x, y) = \sqrt{\frac{1}{2} \left(r^2 - b_1^2 + \sqrt{(r^2 - b_1^2)^2 + 4b_1^2 x^2} \right)}, \quad (27)$$

both b_1 and b_2 are constants, and $r^2 = x^2 + y^2$ (see[1]). The angle of the tangent direction of the hyperbolas is

$$\tan \alpha = \frac{y g^2(x, y) / (g^2(x, y) + b_1^2)}{x}. \quad (28)$$

Substitution of the angle and the logarithmic density (26) into the instinct equation (17) indicates that the set of f and α are indeed its solution.

2.3 Is the summation of disk and dual-handle structures still rational?

For example, we take the summation of one disk and two dual-handle structures,

$$\begin{aligned} \rho(x, y) = & d_0 \exp(d_1 r) + \\ & b_0 \exp((b_2/3)(g^2(x, y) + b_1^2 x^2 / g^2(x, y))^{3/2}) + \\ & c_0 \exp((c_2/3)(h^2(x, y) + c_1^2 x^2 / h^2(x, y))^{3/2}) \end{aligned} \quad (29)$$

where

$$h(x, y) = \sqrt{\frac{1}{2} \left(r^2 - c_1^2 + \sqrt{(r^2 - c_1^2)^2 + 4c_1^2 x^2} \right)} \quad (30)$$

corresponds to the second dual-handle structure. To resolve the mystery, we calculate the corresponding logarithmic density $f(x, y) = \ln \rho(x, y)$, i.e., the formula (2), and substitute it into the instinct equation to get the angles of proportion curves.

I have nine barred galaxy images from Ohio State University (see [5,6]). We know that galaxy centers correspond to the coordinates $(x, y) = (0, 0)$. But what is the coordinates for other points on galaxy images? In fact, rational structures depend only on the shapes of proportion curves not on coordinates. I have chosen the special formulas to describe proportion curves (see [1,6]), and the images accordingly have coordinates. The coordinates are called optimized ones as explained in the following. I use the formulas (9) and (15) in my paper [1] to describe the proportion curves of galaxy disks and bars respectively. I increase or decrease the unit of the coordinates. When the corresponding theoretical density best fits galaxy bar image, the coordinates are simultaneously determined. The unit of the coordinates is called [Cn]. The fitted galaxy image lengths, disk and bar parameters are given in [6] and in the following Table.

The fitted side lengths which correspond to the square galaxy images are given in the second column of the Table. We can not directly measure galaxy actual sizes but their angular diameters as seen from Earth can easily be measured with telescopes. The third column of the Table is the measured angular side lengths which correspond to the square galaxy images. The fifth and sixth columns are the fitted values for galaxy disk parameters. All other columns are the fitted values for galaxy bar parameters.

The above procedures determine galaxy structures, i.e., the logarithmic density $f(x, y)$. After having the density, we can substitute it into the instinct equation to get the angles $\alpha(x, y)$ of proportion curves. I compare the angles with the real angles made by the tangent directions of galaxy arms or rings. In my sample of galaxy images, only NGC 3275, 4548, 5850, and 5921 present significant arms or rings. I choose the angle $\alpha(x, y)$ from the three possible roots of the cubic equation, which best fits the tangent direction of arms or rings. The result is shown in Figures 1 and 2.

I find out that galaxy rings follow proportion curves exactly. The directions of arms, however, always make an angle to the direction of proportion curves. That means arms do not follow the orthogonal proportion curves. The curves followed by arms should still be proportion curves but they can not compose a net of orthogonal curves. Therefore, the spiral curves of arms are not the solution of instinct equation. Note that the series of ‘‘crosses’’ on the arms (see Figures 1 and 2)

Table 1: The Fitted Disk and Bar Parameter Values

NGC	L_0 [Cn]	Θ [arcsec]	$l_0 (= L_0/\Theta)$ [Cn/arcsec]	Disk		
				d_0	d_1 [Cn ⁻¹]	
3275	22.0	110	0.20	4000	-1.3	
4548	6.1	152.5	0.04	3800	-3.8	
4643	9.2	115	0.08	4400	-1.5	
4665	11.9	148.7	0.08	4700	-1.6	
4930	19.5	108.3	0.18	2600	-1.5	
5701	16.8	105	0.16	4300	-1.3	
5850	27.6	153.3	0.18	3100	-2.5	
5921	7.4	185	0.04	7200	-2.7	
6782	30.4	117	0.26	3600	-1.3	
NGC	b_0	1st Pair		2nd Pair	Handles	
		b_1	b_2		3rd Pair	b_1
		[Cn]	[Cn ⁻³]	c_0	c_1	c_2
					[Cn]	[Cn ⁻³]
3275	134	1.76	-0.2	72	3.25	-0.095
4548	45	1.14	-1.83	49	1.57	-1.627
4643	127	1.23	-1.95	111	1.81	-1.165
				94	2.60	-0.62
4665	317	1.52	-0.84	172	2.76	-0.154
4930	55	3.0	-0.08	56	5.73	-0.023
5701	164	3.25	-0.07	64	5.13	-0.014
5850	41	5.08	-0.014	15	7.16	-0.027
				34	7.23	-0.01
5921	144	0.31	-8.0	121	0.88	-6.0
6782	75	4.33	-0.02	32	6.15	-0.018

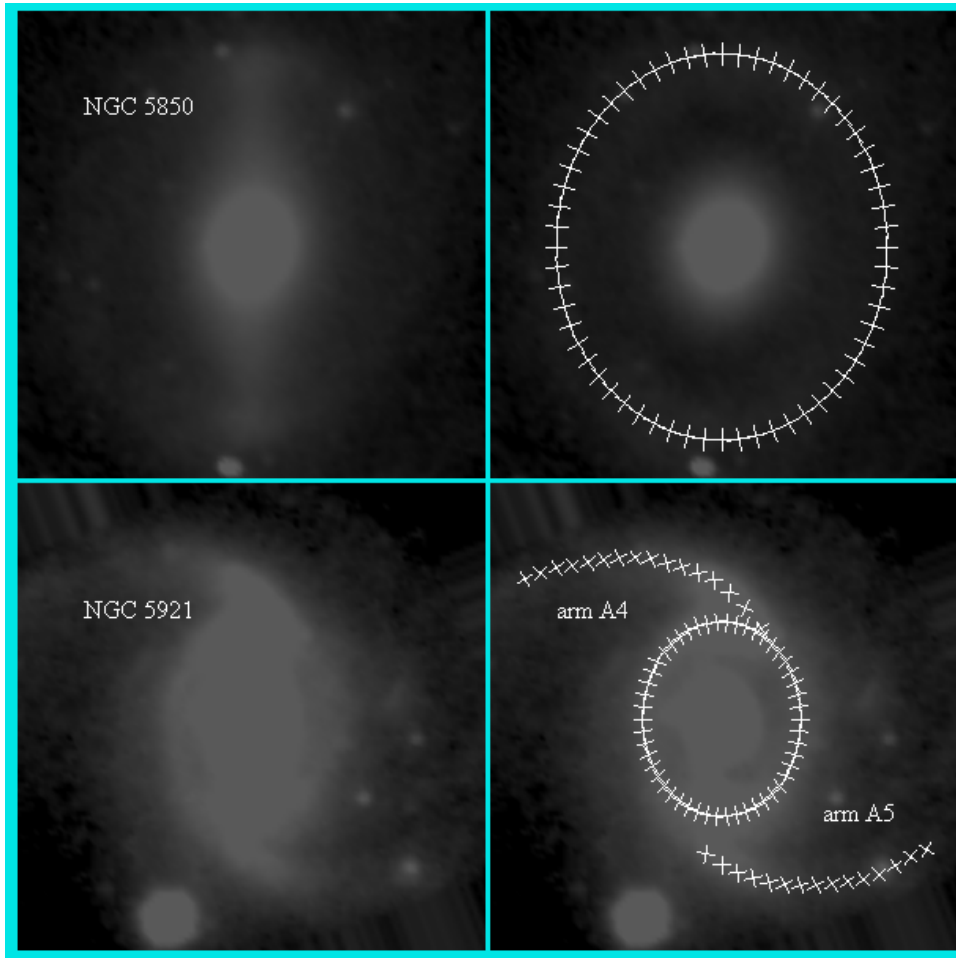


Figure 2: The OSUBGS H-band images NGC 5850, 5921 (left panel, see reference [5]) and the closest solution of instinct equation to the directions of arms and ring (right panel).

look like a falling row of dominoes. This means that arms try to achieve the minimal disturbance to rational structures and, as a result, they follow the (non-orthogonal) proportion curves of the rational structures. For galaxy NGC 4548, the “dominoes” make very small angles to the arm direction when approaching the center parts of bars and make larger angles to the arm direction when approaching the end parts (handles) because the radial components of the gradients of the logarithmic density take larger values when approaching the end parts (see Figures 3 and 4). Fig. 5 is the demonstration of the directional derivatives along the perpendicular directions to the arms for the fitting logarithmic density $f(x, y)$. As indicated by Figures 1 and 2, I did not design a program to choose the curves which best represent the spiral arms. In fact, I visually choose some points which follow the arms. At each point, I use the averaging value of its two neighboring slopes to be the tangent direction of arms at the point. The preliminary calculation as shown in Fig. 5 does suggest that the curves followed by arms are proportion ones, i. e., the directional derivatives of $f(x, y)$ along the perpendicular directions to arms are approximately constant. I find out that the data standard deviation to the mean horizontal lines in Fig. 5 is mainly due to my un-smooth choice of discrete points while the data mean-lines tilting away from the horizontal direction is mainly due to the bad fitting parameters for the logarithmic density $f(x, y)$ (generally due to the bad fitting for galaxy disks).

Note that the directional derivative of $f(x, y)$ is very sensitive to the direction along which the derivative is taken. The maximum absolute value of the derivatives at a point is the absolute value of the gradient at the point. The minimum value is zero. Fig. 5 shows that the standard deviation to the mean value of the directional derivatives along the perpendicular directions to arms is less than ten percent of the corresponding mean value for each arm (A1, A2, A3, A4, or A5). This preliminary result plus all my previous ones in paper [1] proves that large-scale galaxy structure must originate from the instinct equation (17).

In summary, galaxy structures are rational and, generally, bilaterally symmetric. Accordingly the instinct equation admits bilaterally symmetric solutions for orthogonal proportion curves. There exist at most three nets of orthogonal curves. Galaxy rings follow the orthogonal proportion curves exactly but arms never follow the direction of the curves at any point because arm patterns are not bilaterally symmetric.

3 Do-It-Yourself Notes for Layman

Rational structure is a simple concept and it admits a complete solution taking the form of algebraic equation, i.e., the instinct equation (17). Every layman can try a galaxy image to convince himself or herself that rational structure is true.

On the internet are many images of galaxies. Do not be fooled with color images. Color is essentially the different frequencies or wavelengths of light. In fact, the shape of an object or its image is the distribution of light arriving at your eyes from the surface of the object. That is, it is the distribution of light frequency and density varying with the surface of the object. Light of longer wavelength that appears reddish has strong penetrating ability. In other words,

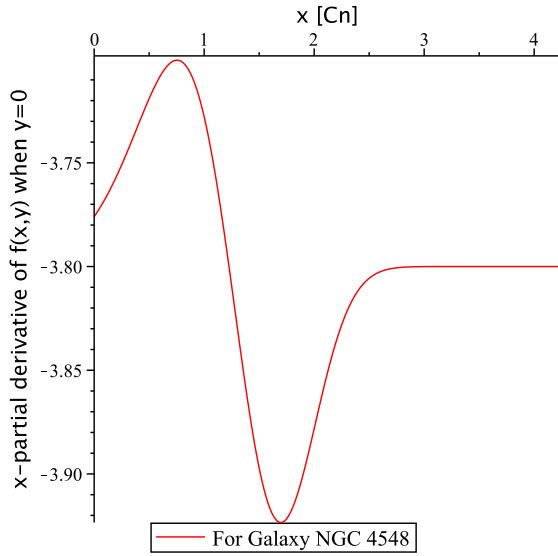


Figure 3: The x -partial derivative, $\partial f/\partial x$, of the fitted logarithmic density on the perpendicular line to the bar, i.e., on the x -axis for NGC 4548, which clearly takes the maximum value at $x \approx 1.6$ [Cn] with the value ≈ 3.92 [Cn $^{-1}$].

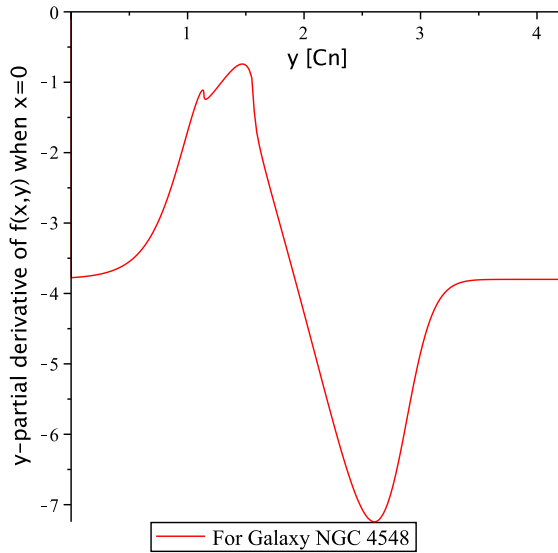


Figure 4: The y -partial derivative, $\partial f/\partial y$, of the fitted logarithmic density on the bar central line, i.e., on the y -axis for NGC 4548, which clearly takes the maximum value at $y \approx 2.6$ [Cn] with the value ≈ 7.2 [Cn $^{-1}$].

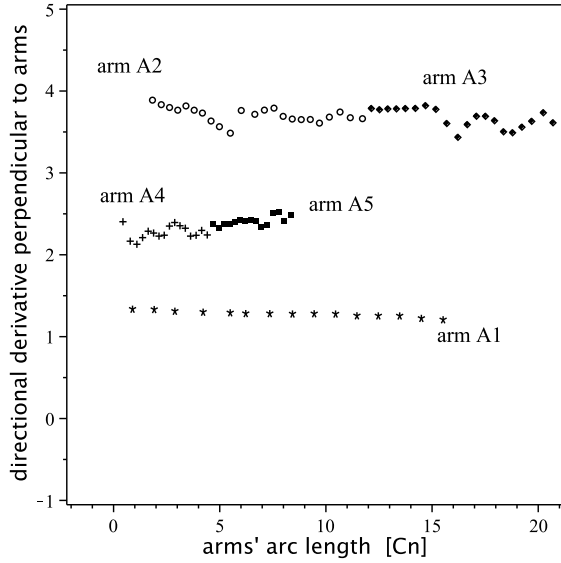


Figure 5: The directional derivatives along the perpendicular directions to arms are approximately constant along the arms. The data sets of the derivatives, A1, A2, A3, A4, and A5, correspond to the arms in Figures 1 and 2, respectively. Because the perpendicular directions are chosen to be inwards to the central bars, the values of the derivatives are all positive.

reddish light refuses to be absorbed by dust or gas. Elliptical galaxies are very clean, with little observation of gas and dust. Therefore, it does not matter to catch which color for you to take the images of elliptical galaxies. Images of the same elliptical galaxy of different colors are very similar and smooth. They are the good demonstration of star distribution in the galaxy. But elliptical galaxies are three-dimensional while their images are two-dimensional. The image of an elliptical galaxy is the cumulative density of stars in the observing directions.

Spiral galaxies are just the opposite. They have a large amount of gas and dust. Although their structures are two-dimensional, they have a certain degree of thickness. Therefore, if we take images of spiral galaxies at the shorter wavelength (i.e., bluish light) then the light from the stars that are behind gas and dust are basically absorbed by the gas and dust. As a result, the image is more or less the distribution of gas and dust. Because the distribution of gas and dust is not smooth, the image looks ugly. Internet images of spiral galaxies are usually short-wavelength ones, therefore, people are daunted by the mysterious look of gas and dust. Therefore, to get an image of spiral galaxy which is mainly stellar density distribution, we take light of longer wavelength from the galaxy, e.g., infrared image. The resulting image is reddish. Although gas and dust have charming and bright colors, they have negligible mass.

Now you can ask for longer-wavelength images of barred galaxies from astronomers. Do not be shy when asking for images because astronomers and physicists are supported by taxpayers. The digital images should represent linearly the light density of galaxies. Most galaxies are inclined to the sky plane, therefore, you ask for de-projected galaxy images so that the galaxies look like face-on. What you ask for is, in fact, an array of real positive numbers which is

proportional to the stellar density distribution, i.e., $\rho(x, y)$ (see the formula (1)).

It is best to model galaxy bars visually. Therefore, you may need some graphic softwares like Maple, Matlab, Mathematica, etc. Otherwise you need to know some computer language with graphic tools. My image analysis is made with C++ language and OpenGL tools. When you try to solve the instinct equation (17) numerically, it would be troublesome if you naively follow the formulas of cubic equation roots. The best way is to calculate one approximate root with some technique like Newton-Raphson Method. Then the cubic equation reduces to a quadratic equation and finally you use the accurate formula of quadratic roots. After you get double-examined result, publish it for discussion with other people.

4 Conclusion

Both the electromagnetic-plus-nuclear world and the world beyond are living ones. But the heaven world is much much simpler. Its instinct can be described by an algebraic cubic equation. Its body texture (rational galaxy structure) admits at most three nets of orthogonal curves. Its blood vessels (galaxy arms or rings) either follow the lines of the texture or cut through the lines proportionally.

The readers may wonder what is the sacred power (the real force) behind the living worlds! The answer is very possibly the so-called universal gravity. Let me repeat my argument from my previous paper [1]:

“1. The well-known fact can not be ignored that gravity is very very weak. For example, it is 10^{-40} times weaker than the electricity between protons. Therefore, humans in the foreseeing future can not design a physical precision experiment which can resolve the 10^{-40} strength of the earth’s gravitational field. That means we have not had a full understanding of gravity. But scientists assume they had it and applied the preliminary results of Newton and Einstein to the whole universe. This resembles the situation of cycles and epicycles in the old geocentric model.

“2. It is a fact that the results of Newton and Einstein deal with the motion of two-bodies. When applied to the free motion of many-bodies, the theories give chaotic results. However, the universe has orderly motion. Whenever a problem involves free many bodies, Newton and Einstein theories have no power. For example, the Bode law of planetary distribution in the solar system has not been explained.

“3. Newton and Einstein theories have no power for the explanation of natural structures. Galaxy structure is the simplest one in the observational world. Every one with common sense must suggest that there exists a law on galaxy structure. Newton and Einstein theories can not provide such law because they are the theories of two-bodies. The law is very possibly the rationality explained in my article.

“4. The mainstream model of the universe (the Big Bang theory) which is based on Newton and Einstein theories, is being declined. A new article [7] describes: Nearly every month new observations arise that pose further challenges to the Λ CDM paradigm: Correlations in galaxy structures [8]; absence of baryon acoustic oscillations in galaxy-galaxy correlations [9]; galaxies

formed already when the universe was 4 to 5 billion years old [10]; dwarf satellites that swarm our own galaxy just like its stars [11]. Observational data [12, 13] strongly suggest a paradigm shift for cosmology.”

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